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Demand drives growth all the way: Goodwin, Kaldor, Pasinetti and the Steady State

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A demand-driven alternative to the conventional Solow–Swan growth model is analysed. Its medium run is built around Marx–Goodwin cycles of demand and distribution. Long-run income and wealth distributions follow rules of accumulation stated by Pasinetti in combination with a technical progress function for labour productivity growth incorporating a Kaldor effect and induced innovation. An explicit steady state solution is presented along with analysis of dynamics. When wage income of capitalist households is introduced, the Samuelson–Modigliani steady state 'dual' to Pasinetti's cannot be stable. Numerical simulation loosely based on US data suggests that the long-run growth rate is around 2% per year and that the capitalist share of wealth may rise from about 40 to 70% due to positive medium-term feedback of higher wealth inequality into its own growth.

Key words: Wealth distribution, Income distribution, Heterodox economic growth, Cambridge theory
JEL classifications: D31, D33, D58, D50

1. Introduction

What sets the long-run growth path of the economy? Following Solow (1956) and Swan (1956), the conventional view is that growth is determined by factors contributing to aggregate supply—capital deepening, population increase and long-run growth of labour productivity. 'Potential output' increases accordingly.[1]

The obvious alternative is to analyse growth from the side of demand. How do effective demand, endogenous productivity growth, and shifting income and wealth distributions influence and constrain the economy in the present and over time?

In addressing this question, the model presented here has eight salient characteristics.

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[1] Similar comments apply to Ramsey-type models which basically add a fancier saving function to Solow–Swan.

First, as in almost all growth theory, profit income is assumed to flow directly to households (ignoring interest, dividends, capital gains and all the other channels via which households receive payments from business). *Unlike* mainstream models, our specification initially maintains a household class distinction between 'capitalists' who receive profits and 'workers' who get both labour and capital income. Capitalists save at a higher rate than workers. Wages received by capitalists are discussed towards the end of the paper, where it is shown that a 'dual' solution to the model in which capitalists vanish cannot be stable if they receive some labour income.

Second, growth models distinguish between 'fast' and 'slow' (or 'state') variables. The former vary in a 'short' to 'medium' run. Fast-moving *level* variables include output (X in what follows), employment (L) and gross capital formation (I). Slowly moving level variables include the capital stock (K), capital owned by capitalists (K_c), population (N) and labour productivity ($\xi = X / L$). They are all fixed in the short run. Their rates of change, however, vary in the same time frame as the fast variables.

Third, in analysing growth it is convenient to work with *ratio* variables including the output/capital ratio or 'capital utilisation' (u), the profit rate (r), the investment/capital ratio (g), employment relative to population (λ), etc. Time derivatives of slowly moving ratios such as productivity ξ, the capital/population ratio ($\kappa = K / N$) and the share of wealth held by capitalists ($Z = K_c / K$) are also determined in the short run. Contemporary mainstream models typically do not address dynamics of ξ and Z.

Fourth, in all time frames, mainstream models presuppose full employment of labour and capital and the existence of an aggregate production function with associated marginal productivity conditions which determine income distribution.[2] In contrast, we assume that u, r and other fast variables are determined by interaction between functions $u(r\ldots)$ for effective demand and $r(u\ldots)$ for distribution. Both relationships have parameters included and also depend on κ, ξ and Z.

Fifth, as opposed to the Solow–Swan assumptions, the specifications of $u(r\ldots)$ and $r(u\ldots)$ are based on observed business cycle behaviour in rich economies. To reduce dimensionality, we suppress cyclicality in growth analysis and assume that *levels* of r and u are set by the joint solution of $u(r\ldots)$ and $r(u\ldots)$. In so doing, we omit explicit discussion of a Goodwin–Marx growth cycle but draw upon its empirical underpinning. Indeed, this is a central feature of the argument.

Sixth, dynamics of aggregate capital K (measured at cost) are driven by real net investment. At prevailing output levels, capital is *not* a scarce factor of production subject to decreasing returns. Rather, its *level* sets the scale of the macro system. Its *growth* stimulates technical change.

Seventh, even though we do not assume full employment or decreasing returns to capital, dynamics of κ drive the state variables towards a steady state at which their growth rates would be equal at a level largely determined by population and productivity growth. We maintain the standard assumption that the growth rate of population (n) is exogenous.

Eighth, away from the steady state, levels of fast variables are determined by the demand and distribution functions with their associated parameters. If the system were *at* a steady state (which will not be attained in finite time), equalised growth rates would override the effects of some demand-side parameters on levels of κ, ξ and Z. But demand does lead growth 'all the way' towards the steady state.

Finally, we are dealing here with a fairly complicated system. Its behaviour will to a large extent be described in terms of diagrams and signs of partial derivatives. More

[2] We include a 'production function' $L = \xi X$, which holds at a point in time. A neoclassical cost function and marginal productivity conditions are replaced by the distributive relationship (equation 4) below.

detailed analysis in terms of equation specifications and parameters will be provided in footnotes as we proceed.

Our specification draws freely on the works of several Keynesian economists, notably from the University of Cambridge. In the short run, aggregate demand and distribution interact according to Richard Goodwin's (and ultimately Karl Marx's) model of cyclical growth in which a tighter labour market leads to a higher wage share and lower profit rate. Distribution influences demand via differential saving rates across classes and profitability figures in the determination of planned investment.

Over time, Nicholas Kaldor's technological progress function along with induced innovation describes how productivity growth responds to the installation of new capital and shifts in the income distribution. Luigi Pasinetti pioneered the theory of wealth inequality. We adopt his approach by working with two distinct classes and tracing their wealth holdings over time. There can be sustained growth with the capitalists' share of wealth settling between zero and one. We present illustrative numerical simulations of how economic activity and wealth concentration may change over time.

2. Kaldor's and other stylised facts

Almost all growth models are set up to converge towards steady states at which their ratio variables are constant, reflecting the fact that shares of level variables in income and wealth cannot trend up or down indefinitely. Sixty years ago, Kaldor (1957) set out six characteristics of long-run economic growth that have become canonical in the literature as 'stylised facts'. He posited that 'over long periods':

i. labour productivity, ξ, grows at a steady exponential rate $\hat{\xi} = (d\xi / dt) / \xi$;
ii. the ratio of capital to the population, κ, grows at a steady rate $\hat{\kappa}$;
iii. the profit share π is stable;
iv. the profit rate r is stable (with $r = \pi X / K = \pi u$);
v. the ratio of output to capital u is stable; and
vi. the real wage, ω, grows at the same rate as labour productivity.[3]

We can add

vii. the employment ratio, $\lambda = L / N$, is stable in the long run;
viii. in standard national accounts including household and business sectors, undistributed corporate profits and taxes are major sources of saving (both at a rate of 100%); distributed profits as well as capital gains on equity flow predominantly to high-income households who have substantially higher saving rates than those further down in the size distribution whose incomes mostly come from wages (and fiscal transfers);
ix. in the (u, π) plane for rich economies, there is an observed clockwise business cycle around a stationary point with π leading u as the economy emerges from a trough or swings down from a peak.

[3] In advanced economies, for at least three decades, productivity has increased more rapidly than the real wage so that the profit rate r and share π have increased with a stable or even falling real wage ω. One could see this weakening of labour's position as a rejection of Kaldor's stylised facts of stable wage/profit shares. We, however, interpret this socio-political development as outside our economic framework and discuss how to model it in terms of shifts of parameters (and schedules in Figure 1) below.

3. Model design

Stylised fact (ix) is the basis for a model of the medium run. The wage share $\psi = 1 - \pi = \omega / \xi$ falls as the economy emerges from a slump—the real wage stagnates while productivity grows. If investment demand responds to higher profits, capital utilisation u and employment $L = X / \xi$ rise. With π and u both increasing, r goes up as well. A tighter labour market ultimately bids ω and ψ up. Profits are squeezed and firms implement labour-saving technical change. A downswing or 'crisis' ensues.[4]

This cycle narrative appears in Marx's *Capital* and *Theories of Surplus Value*, and was formalised by Goodwin (1967). For present purposes, we adopt Goodwin's relationships between income distribution and effective demand. Specifically, r responds negatively to λ (high-employment profit-squeeze) and u responds positively to r (profit-led demand).

Observations (i), (ii), (v) and (vii) all apply to real variables at a steady state. Three level variables evolve over time—the total capital stock K (or alternatively the capital/population ratio κ), the quantity of capital controlled by capitalists K_c and labour productivity ξ. A steady state can be characterised by constant values of *two* ratio variables. One is $Z = K_c / K$ or the capitalists' share of wealth. The other is $\zeta = \kappa / \xi = \lambda / u$ or the ratio of capital 'depth' to productivity which also equals the ratio of the employment rate to capital utilisation. Constant Z and ζ, respectively, imply that the pairs K_c and K, and κ and ξ change at the same exponential rate. Away from the steady state, Z and ζ have their own proper dynamics stated in the form of differential equations. Their levels at a point in time determine u, r and λ.

Growth of capital depth $\hat{\kappa}$ is driven by the investment/capital ratio $g = I / K$ with g responding positively to r and u. (In standard notation for any variable x, $\dot{x} = dx / dt$ and $\hat{x} = \dot{x} / x$.)

Productivity growth $\hat{\xi}$ can be modelled following Kaldor's demand-side explanations. Over the years, he introduced two versions of a 'technical progress function'. In the first (Kaldor, 1957), $\hat{\xi}$ is driven by $\hat{\kappa}$, with investment serving as a vehicle for more productive technology. The second (Kaldor, 1966) ties productivity growth to the output growth rate $\widehat{X}$ via economies of scale. To avoid too many logarithmic derivatives, we follow the earlier variant. We also assume on Marxian lines that increasing tightness in the labour market will bid down the profit rate and induce innovation to speed productivity growth.

On these assumptions, we show below that the ratio ζ converges to a steady state with $\dot{\zeta} = 0$. The long-run investment rate g is affected by income distribution and is not equal to an exogenously determined 'natural' level as in supply-driven models. The employment rate and income distribution adjust to support the steady state so that observations (iii), (iv), (vi) and (vii) apply. The stylised facts mentioned in (viii) are typically modelled in one of two ways. Following the traditional 'Cambridge equation', one approach is simply to assume that the saving rate from profit income exceeds the rate from wages. This version is relevant to determination of macro equilibrium and growth, but says nothing about accumulation of wealth.

[4] The idea that the wage/profit distribution can influence effective demand traces back to the *General Theory* (Keynes, 1936; Steindl, 1952). Beginning with papers by Rowthorn (1982) and Dutt (1984), the distribution versus demand linkage has been under active discussion. Bhaduri and Marglin (1990) is an influential summary. Following Keynes's (1939) repudiation of a counter-cyclical real wage, the mainstream version of the dependence of distribution on the level of activity eventually emerged as a real wage Phillips curve. Econometric evidence about Marx–Goodwin cycles appears in Barbosa-Filho and Taylor (2006), Flaschel (2009) and Kiefer and Rada (2015).

Pasinetti's (1962, 1974) distinction between two classes of households shifts the focus to wealth. In an initial specification, capitalists receive only profit income rK_c on their capital K_c; workers get the rest of income $(X - rK_c)$. The classes' saving rates from income are s_c and s_w, respectively, with $s_c > s_w$.

These assumptions underlie dynamics of capital concentration Z, with saving and investment setting the growth rates of K_c and K. Under appropriate assumptions discussed below, $\dot{Z}$ will converge to zero with $Z > 0$, setting up a joint steady state with ζ. There is also a possibility that it will diverge towards a maximum possible level discussed below, in an 'anti-dual' solution noted by Darity (1981).

In the next section, we specify the short- to medium-run equilibrium of the economy in terms of the level of aggregate demand and the functional distribution of income. These expressions and their dependence on Z and ζ then allow us to spell out the details of the two-dimensional (Z, ζ) long-run dynamical system.[5]

4. Short and medium term

The distributive side of temporary equilibrium can be set up in terms of either the profit share (π) or profit rate ($r = \pi u$). The latter gives more tractable short-run and steady state specifications so we opt for that.
A convenient formulation for gross investment is

$$g = I / K = g_0 + \alpha r + \beta u. \tag{1}$$

Household saving per unit of capital is

$$\sigma = s_c rZ + s_w \left[(1-\pi)u + r(1-Z)\right] = (s_c - s_w)rZ + s_w u. \tag{2}$$

Setting up macroeconomic balance just in terms of private investment and saving is traditional, but does not fit the data. Besides investment, exports and government purchases of goods and services are demand injections; imports and taxes are significant leakages.[6] Let ι be a coefficient relating these injections to capital, with ν scaling leakages to output. The macro balance condition becomes

$$(g + \iota) - (\sigma + \nu u) = 0.$$

To simplify algebra until we get to simulations below, we hold $\iota = \nu = 0$. On this assumption, an expression for u becomes

$$u = \left[1 / (s_w - \beta)\right]\left\{g_0 + [\alpha - (s_c - s_w)Z]r\right\}. \tag{3}$$

[5] Dutt (1990) and Palley (2012) point out that variation in Z must play a role in long-run macroeconomic adjustment. This fact is not widely recognised, but is highly relevant to contemporary debate. As far as we know, the significance of ζ and its dependence on dynamics of κ and ξ have not been noted previously.

[6] For details, see Taylor (2017).

An increase in r raises g by a factor α and σ by a factor $(s_c - s_w)Z$ so that demand will be profit-led if $\alpha > (s_c - s_w)Z$ and the 'Keynesian' stability condition for u, $s_w > \beta$, applies.[7]

On the distributive side, we formulate the Marx–Goodwin profit-squeeze rule as a relationship between the profit rate and the employment ratio

$$r = \mu_0 - \mu_1 \lambda = \mu_0 - \mu_1 \zeta u \tag{4}$$

with $\lambda = \zeta u$, or employment is proportional to output as the expression after the second equals sign shows. If μ_0, $\mu_1 > 0$ a higher level of u or ζ increases λ, causing the rate of profit to fall: $dr / du < 0$ and $dr / d\zeta < 0$.

Equations (3) and (4) specify equilibrium relationships between demand and distribution. We will see as we proceed that the inequality relationships

$$\alpha > s_c > s_w > \beta$$

will help assure dynamic stability of the system. In a Marx–Goodwin cycle model, equations (3) and (4) would be 'nullclines' (loci along which $\dot{u} = 0$ and $\dot{r} = 0$) of a medium-run (business cycle frequency) dynamical system in the (u, r) plane. It would generate clockwise cycles around a stationary point. As discussed above, we suppress this cyclicality to concentrate on growth in the three-dimensional (κ, ξ, Z) system with the joint solutions to equations (3) and (4) setting levels of u and r.[8]

To explore comparative statics, we can totally differentiate equations (3) and (4) to find partial derivatives (denoted by subscripts) of u and r with respect to Z and ζ:

$$u_Z = -(s_c - s_w) r / \Delta < 0 \tag{5-u}$$

$$u_\zeta = -[\alpha - (s_c - s_w) Z] \mu_1 u / \Delta < 0$$

$$r_Z = (s_c - s_w) r \mu_1 \zeta / \Delta > 0 \tag{5-r}$$

$$r_\zeta = -(s_w - \beta) \mu_1 u / \Delta < 0$$

with

$$\Delta = (s_w - \beta) + \mu_1 \zeta [\alpha - (s_c - s_w) Z] > 0.$$

Figure 1 presents a graphical representation of how u and r respond in the short to medium run to shifts in Z and ζ. Econometric results suggest that in high-income economies demand is weakly profit-led so the $u(r)$ schedule is relatively steep in the

[7] If s is the overall saving rate, the standard Keynesian stability condition is $s > \beta$. Data suggest that $s_w > \beta$ also applies.

[8] Formal stability analysis of Marx–Goodwin cycles is readily available in the literature, for example: Taylor (2004) and Flaschel (2009).

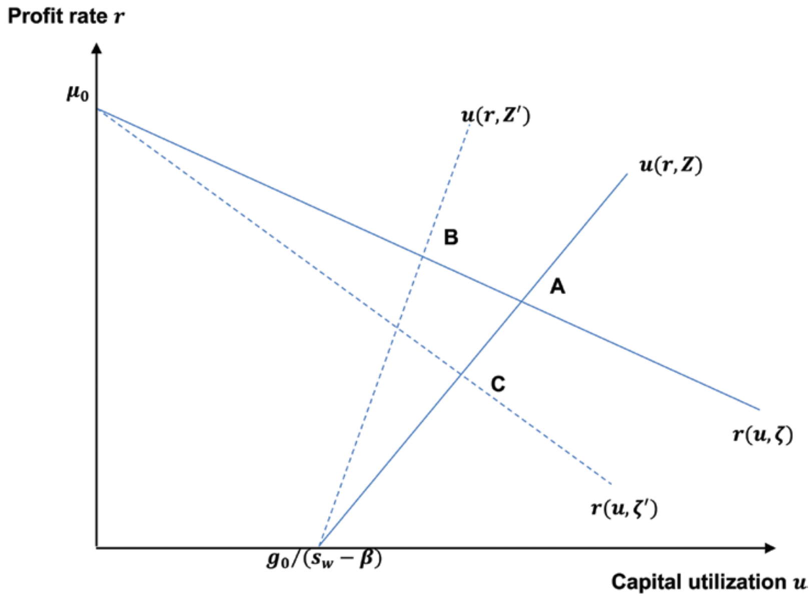

Fig. 1. *Short- and medium-run equilibrium as a function of Z and ζ. An increase in* Z *lowers* u*, raises* r *and shifts the equilibrium to point B. Higher ζ lowers* r *and* u *and shifts the equilibrium to point C.*

(u, r) plane. The $r(u)$ curve shows more responsiveness. The intercepts on the horizontal and vertical axes follow from equations (3) and (4) with $r = 0$ and $u = 0$, respectively.

The point of intersection of the schedules, A, is the short- to medium-term equilibrium of the economy. From equation (2), a higher value of Z shifts profit income from low-saving worker to high-saving capitalist households, lowering demand u for any given level of r: the $u(r)$ schedule becomes steeper. The new equilibrium point is B. The outcome is $u_Z < 0$. Because of a weaker profit-squeeze, the profit rate responds positively to Z, $r_Z > 0$. With $\pi = r / u$, we have $\pi_Z > 0$. The magnitude of π_Z is important in the analysis below of long-run stability of Z.

An increase in ζ strengthens the profit-squeeze for any given level of u, causing the $r(u)$ schedule to have a steeper negative slope so that r falls. Due to the profit-led demand regime, u also falls: $u_\zeta < 0$ and $r_\zeta < 0$. With a stable value of u, we get $\pi_\zeta < 0$. The new equilibrium point is C.[9]

As discussed in footnote 3, we analyse the model with a stable configuration of the schedules in Figure 1. Over recent decades, the $r(u)$ schedule may have shifted upward for socio-political reasons while $u(r)$ shifted to the left because of non-expansionary macroeconomic management. Observed increases in π and stable or decreasing u have been the outcomes. A reduction in the ability of labour to bid for higher wages when the labour market is tight (lower μ_1) has a similar effect on the profit share.

[9] An alternative medium-run model can be based on wage-led demand and a high-employment wage-squeeze (decreasing returns to labour in a neoclassical specification or 'forced saving' by workers in antique terminology). In a diagram (e.g. Figure 1), the slopes of $u(r, Z)$ and $r(u, \zeta)$ would be negative and positive, respectively. An increase in Z would reduce both u and r, making $\pi_Z > 0$ if the slope of $r(u, \zeta)$ is relatively shallow (the elasticity of substitution is high in a neoclassical version). Wage-led/wage-squeeze appears to fit the cyclical data less well than profit-led/profit-squeeze. If $\pi_Z < 0$, long-run dynamics of Z will be stabilised (see discussion of equation (20) below).

5. Dynamics of productivity and capital stock

Immediate interest lies with the growth rate of the capital stock. From equation (1), Z affects $\hat{k}$ ambiguously. Higher Z lowers u but raises r which decrease and increase investment, respectively. If output is relatively insensitive to distribution, then $g_Z > 0$.[10] The shifts in r and u just noted imply that $g_\zeta < 0$.

Dynamics of the capital-population ratio $\kappa = K/N$ are the heart of all growth models. In growth rate form κ evolves over time according to

$$\hat{\kappa} = g - \delta - n \tag{6}$$

with δ as the rate of depreciation and n as the exogenous population growth rate.

As discussed above, following Kaldor and Marx labour productivity growth can be assumed to respond to capital formation and distribution,[11]

$$\hat{\xi} = \gamma_0 + \gamma_1 \hat{\kappa} - \gamma_2 r. \tag{7}$$

Putting equations (6) and (7) together gives the growth rate equation for ζ: $\hat{\zeta} = \hat{\kappa} - \hat{\xi}$ or

$$\dot{\zeta} = \zeta\left[(1-\gamma_1)(g-\delta-n) - \gamma_0 + \gamma_2 r\right]. \tag{8}$$

The equation shows a trade-off between g which boosts accumulation and r which retards productivity growth—increases in both variables raise $\dot{\zeta}$. At a steady state with $\dot{\zeta} = 0$, if one rises, the other must fall. With $\zeta > 0$ and using boldface to signal variables at steady state we have

$$\boldsymbol{g} = \left[\delta + n + \gamma_0/(1-\gamma_1)\right] - \left[\gamma_2/(1-\gamma_1)\right]\boldsymbol{r} = \bar{g} - \left[\gamma_2/(1-\gamma_1)\right]\boldsymbol{r} \tag{9}$$

or simply the balancing condition

$$\bar{g} = \boldsymbol{g} + \left[\gamma_2/(1-\gamma_1)\right]\boldsymbol{r}$$

in which $\bar{g} = \delta + n + \gamma_0/(1-\gamma_1)$ is the traditional long-run investment/capital ratio, equal to the sum of rates of depreciation, population growth and Kaldorian productivity growth $\gamma_0/(1-\gamma_1)$.[12] Along a trajectory towards the $\dot{\zeta} = 0$ point, a higher profit rate boosts $\hat{\zeta}$ by cutting $\hat{\xi}$. At the steady state itself, $\boldsymbol{r}$ and $\boldsymbol{g}$ must adjust to the 'natural rate' $\bar{g}$. With $\hat{\kappa} = \hat{\xi}$, both variables can grow indefinitely at the rate $\bar{g} - \left[\gamma_2/(1-\gamma_1)\right]\boldsymbol{r} < \bar{g}$. Simple closed-form expressions for $\boldsymbol{g}$ and $\boldsymbol{r}$ are provided in equations (22) and (23) below.

Because $g_\zeta < 0$ and $r_\zeta < 0$, equation (8) is a stable differential equation with $d\dot{\zeta}/d\zeta < 0$ at the steady state. Figure 2 plots $\hat{\zeta} = \hat{\kappa} - \hat{\xi}$. The slopes of the schedules show that an increase in ζ cuts into investment but spurs productivity growth. A higher

[10] If the model is set up with π instead of r responding to ζ then $g_Z > 0$ unambiguously.

[11] The mainstream 'induced innovation' literature beginning with Hicks (1932) also points in the direction of a negative response of $\hat{\xi}$ to r, consistent with microeconomic analysis of firm behaviour. Rezai (2012) discusses the Kaldor and Marx effects in a model of growth and distribution in more detail.

[12] We ignore the potential equilibrium at $\zeta = 0$ which corresponds to the pre-capitalist state of zero employment and/or zero capital stock.

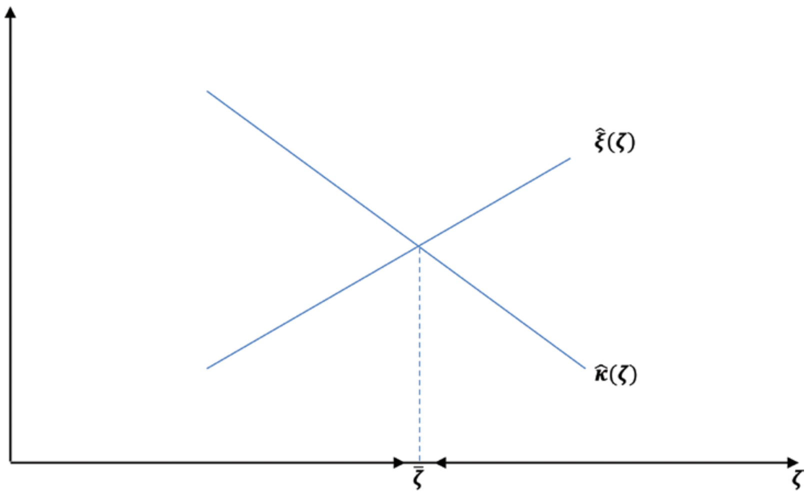

Fig. 2. *Dynamics of* ζ *. There is a steady state at* $\bar{\zeta}$.

base rate γ_0 of productivity growth shifts the $\hat{\xi}(\zeta)$ locus upward, leading to a lower level of steady state $\bar{\zeta}$ while the steady state growth rate of the underlying variables increases.

6. Wealth dynamics

Capitalist households receive income on their wealth holdings (ignoring any wage income at this stage), so their capital stock evolves according to

$$\widehat{K}_c = s_c r - \delta \tag{10}$$

or

$$\dot{K}_c = (s_c r - \delta) K_c.$$

Because $r_Z > 0$, dynamics of K_c are unstable. As will be seen, the instability can be offset by the evolution of Z and ζ. Meanwhile, along with equations (6) and (8), equation (10) describes our three-dimensional dynamical system.

Total capital stock grows at the rate of aggregate saving per unit of capital (equation 2) minus depreciation,

$$\widehat{K} = \sigma - \delta = \left(s_c - s_w\right) rZ + s_w u - \delta. \tag{11-σ}$$

Alternatively,

$$\widehat{K} = g - \delta = g_0 + \alpha r + \beta u - \delta. \tag{11-g}$$

Stability of Z can be analysed using either equation (11-g) or equation (11-σ) for $\widehat{K}$. Begin with the latter.

With $\widehat{Z} = \widehat{K}_c - \widehat{K}$, we have the differential equation for Z,

$$\dot{Z} = Z\{[s_c(1-Z) + s_w Z]r - s_w u\}$$

It is easier to conduct stability analysis in terms of $\pi = r / u$ instead of r and u separately, so we rewrite this equation as

$$\dot{Z} = Z\{[s_c(1-Z) + s_w Z]\pi - s_w\}u. \tag{12}$$

From Figure 1, an increase in Z pushes up π. From equation (12), $\dot{Z}$ can go up, destabilising dynamics around a Pasinetti steady state with $0 < Z < 1$ if the difference between s_c and s_w is small (see discussion below). After quick contemplation of the characteristics of a Pasinetti solution, we return to the details of convergence.

6.1. Pasinetti steady state wealth

For stock variables in a steady state, the ratios of their changes over time to their levels must all be equal. Combining the ratios for workers' and capitalists' capital gives the relationship

$$1 - \boldsymbol{Z} = \left[s_w / (s_c - s_w)\right]\left[(1-\boldsymbol{\pi}) / \boldsymbol{\pi}\right] \tag{13}$$

If $\boldsymbol{\pi} < 1$ and $s_w > 0$, there will be some saving from wages. The workers' share of capital, $1 - \boldsymbol{Z}$, has to be positive at steady state, setting an upper bound on $\boldsymbol{Z}$.

For capitalists' gross saving, from equation (10), the change-to-level ratio is $s_c r K_c / K_c = s_c r$. It must be equal to the economy-wide ratio of gross investment to capital, setting up Pasinetti's famous equation

$$s_c \boldsymbol{r} = \boldsymbol{g}. \tag{14}$$

This formula implies that Piketty's (2014) $r > g$ condition is a corollary of steady state accounting. It is not some new law of capitalism.

Finally, equating workers' change-to-level capital ratio to overall capital stock growth gives an expression alternative to equation (13),

$$1 - \boldsymbol{Z} = s_w (1 - \boldsymbol{\pi}) \boldsymbol{u} / (\boldsymbol{g} - s_w \boldsymbol{r}).$$

If $\boldsymbol{u}$ and $\boldsymbol{r}$ are relatively stable, then this expression shows that the investment/capital ratio and concentration of wealth are positively related from the side of saving. Steady states in mainstream and the demand-driven model at hand bear a strong family resemblance.

6.2. Stability of the Pasinetti steady state

Returning to dynamics of Z and holding ζ constant, define

$$f(Z) = \left[s_c(1-Z) + s_w Z\right]\pi - s_w = \left[s_c\pi - s_w\right] - (s_c - s_w)\pi Z. \tag{15}$$

The derivative of f is

$$df / dZ = f_Z = -\left(s_c - s_w\right)\pi + \left[s_c(1-Z) + s_w Z\right]\pi_Z \tag{16}$$

in which $\pi_Z > 0$. From equation (12), we have

$$\dot{Z} = Zfu \tag{17}$$

and

$$d\dot{Z} / dZ = Z\left[f_Z u + f u_Z\right] + fu. \tag{18}$$

For a Pasinetti steady state, we need $f(\boldsymbol{Z}) = 0$ in equation (17), or

$$s_c \boldsymbol{r} = s_w \left[\boldsymbol{r} + (1-\boldsymbol{\pi})\boldsymbol{u} / (1-\boldsymbol{Z})\right], \tag{19}$$

which states that the growth rates of the capital stocks of both household classes have to be equal. Capitalist households receive only capital income, while worker households have additional income from wages (the second term in brackets). Rearranging equation (19) gives the explicit solution for $\boldsymbol{Z}$ appearing in equation (13).

To check on stability of the Pasinetti solution, substitute equation (19) into equation (16) to get

$$f_Z = -\left(s_c - s_w\right)\pi + s_w(\pi_Z / \pi). \tag{20}$$

The Pasinetti steady state will be locally stable if the right-hand side is negative, requiring s_w to be well below s_c and π_Z small (or negative if the medium run is wage-led/wage-squeeze). If these conditions are not satisfied, Z will diverge towards zero or the maximum level permitted by workers' saving (i.e. the Samuelson–Modigliani dual analysed below or the Darity anti-dual solution). The potential divergence arises from positive feedback. An increase in Z raises π which from equation (16) can push up $\dot{Z}$—this is the destabilising linkage via workers' saving noted above. On the other hand, higher π strengthens the stabilising term $-\left(s_c - s_w\right)\pi$, which can hold Z below one.

If we use equation (11-g) instead of equation (11-σ) to set $\widehat{K}$, working through similar analysis gives a stability condition as

$$\left(\alpha - s_c\right) r_Z > -\beta u_Z.$$

From equations (5-u) and (5-r), we have $-u_Z = r_Z / \mu_1 \zeta$, so the inequality becomes

$$\left(\alpha - s_c\right)\mu_1 \zeta > \beta. \tag{21}$$

The implication is that we need a small β (weak accelerator), strongly profit-led demand $(\alpha > s_c)$, a strong profit-squeeze $(\mu_1 > 0)$ or some combination to assure dynamic stability.

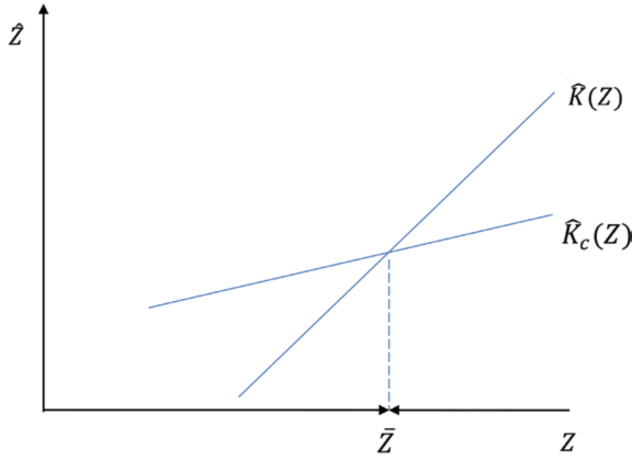

Fig. 3. *Dynamics of* $\boldsymbol{Z}$ *around a Pasinetti steady state at* $\overline{\boldsymbol{Z}}$.

For either version, Figure 3 is a visualisation of dynamics of Z. For stability, $\widehat{K}$ must respond more strongly than $\widehat{K}_c$ to an increase in Z.

7. Explicit steady state solution

With productivity growth responding to r in equation (7), Pasinetti's formula (equation 14) is a bridge between steady state solutions of ζ and K_c. Substituting equation (14) into equation (9), letting
$A = \gamma_2 / (1 - \gamma_1)$, and solving gives values for r and g,

$$\boldsymbol{r} = \bar{g} / (s_c + A) \tag{22}$$

and

$$\boldsymbol{g} = s_c \bar{g} / (s_c + A). \tag{23}$$

In practice, γ_2 and A will be small, but they create space for a long-run investment rate $\boldsymbol{g}$ differing from $\bar{g}$. For the reasons discussed in connection with equation (9), $\boldsymbol{g} < \bar{g}$.

Because $\bar{g} = \delta + n + \gamma_0 / \left(1 - \gamma_1\right)$, a higher value of γ_0, the base rate of technical progress, leads to a higher long-term investment/capital ratio. The same is true of the Kaldor technical progress coefficient γ_1 if γ_2 is relatively small. A higher capitalist saving rate s_c reduces the profit rate but stimulates capital formation. Animal spirits (and workers' saving, etc.), on the other hand, do not affect r and g at a steady state. Imposing a given investment/capital ratio $\boldsymbol{g}$ on equation (3) means that u and r would have to adjust if g_0 were to increase. Such a response is a 'theorem of accounting', valid if the system is really *at* a steady state but is not relevant in other circumstances.

Finally, one can plug equations (22) and (23) into the investment function (equation 1) and solve for steady state $\boldsymbol{u}$. The result turns out to be

$$\boldsymbol{u} = \left[1/\beta(s_c + A)\right]\left[(\alpha - s_c)\bar{g} - g_0(s_c + A)\right]$$

The condition $\alpha > s_c$ (with a strong inequality) discussed in connection with equation (21) is needed here to assure $\boldsymbol{u} > 0$.

8. Digression on capitalist wage income

Before turning to long-run nullclines, it makes empirical sense to take a quick look at a specification in which capitalists receive wage income. The richest 1% of US households receive around 7% of total labour compensation (largely through bonuses and stock options). How does this fact influence growth dynamics?

Equations (12) and (18) permit two well-known steady state solutions with $\dot{Z} = 0$ to exist, one with $\boldsymbol{Z} = 0$ and the other with $f(\boldsymbol{Z}) = 0$. The former is the 'dual' steady state proposed by Samuelson and Modigliani (1966). A simple example arises when $s_c = s_w = s$. If saving rates are equal, workers are identical to capitalists, *except* for the fact that they also receive wages. Using this extra source of income, workers can out-save capitalists so that in the long run, Z goes to zero. In more detail, from equation (15), $f(0) = s(\pi - 1) < 0$. At $Z = 0$, equation (18) becomes

$$d\dot{Z}/dZ = s(\pi - 1)u < 0,$$

so the dual equilibrium is stable. For uniform saving rates, Pasinetti apparently reduces to Solow–Swan.

But in fact it is easy to show that even if saving rates are equal Solow–Swan breaks down if capitalists get wage income. Suppose that the capitalist class receives a share $1-\theta$ of the wage bill $(1-\pi)X$, then their saving is $S_c = s_c\left[rK_c + (1-\theta)(1-\pi)X\right]$ and workers' saving $S_w = s_w\left[r(K - K_c) + \theta(1-\pi)X\right]$. Using these expressions, an extended version of equation (12) is

$$\dot{Z} = Z\left[[s_c(1-Z) + s_w Z]r - s_w u - [s_c(1-\theta) + s_w\theta](1-\pi)u\right] + s_c(1-\theta)(1-\pi)u.$$

If $Z = 0$ then $\dot{Z} = s_c(1-\theta)(1-\pi)u > 0$ so the Samuelson–Modigliani steady state is unstable when $\theta < 1$. So long as capitalists receive some wage income, they can accumulate wealth at $Z = 0$. As noted in connection with equation (13), saving from workers' wages means that Z cannot reach a value of one. Similarly, saving from capitalists' wages can support a positive value of Z even if saving rates are equal.

9. Accounting background for simulations

In annual data for the US economy, imports typically exceed exports so the rest of the world is a macroeconomic net lender. The sum of government current spending on goods and services, transfers to households and net interest minus taxes is positive, making the sector a net borrower. The combined government and foreign sector is a net lender, meaning that it is accumulating wealth. Here is an explicit formulation.

In the notation introduced in connection with equations (1) and (2), in current data, we have $\nu u - \iota > 0$. Let K_ϕ be capital controlled by the foreign/government (FG) consolidated sector, and $\Phi = K_\phi / K$. Wealth accumulation is

$$\dot{K}_\phi = \nu X - \iota K$$

or

$$\widehat{K}_\phi = (\nu u - \iota) / \Phi.$$

There is no feedback from Φ to Z and ζ, so accounting consistency ensures that Φ will converge to a steady state with $\widehat{K}_\phi = g$ if the other two state variables do so. Its steady state level will be

$$\Phi = (\nu \boldsymbol{u} - \iota) / \boldsymbol{g},$$

or the ratio of FG net lending to overall gross investment. Levels of wealth held by the two classes and the FG sector must sum to K.[13] It is possible for the FG sector to be a net debtor in steady state, that is, $\Phi < 0$. In that case, household wealth holdings would sum to $K - \Phi$, or capital plus consolidated foreign and government debt, as in standard national financial accounting.

10. Nullclines

The next step is to assume that a Pasinetti steady state exists. We can examine slopes of its nullclines by using the derivatives of $\dot{\zeta}$ in equation (8) and $\dot{Z}$ in equation (12) with respect to ζ and Z. Pasinetti's formula (equation 14) is valid only at steady state, so we cannot employ it directly.

At a Pasinetti equilibrium, we get $\partial \dot{Z} / \partial \zeta < 0$ from equation (12) because $\pi_\zeta < 0$. Equation (20) already shows when $\partial \dot{Z} / \partial Z < 0$ near a Pasinetti steady state. The nullcline for ζ is a bit trickier. The discussion of equation (8) above suggests that $\partial \dot{\zeta} / \partial \zeta < 0$. In equation (1), if u is relatively insensitive to Z while $r_Z > 0$, then $\partial \dot{\zeta} / \partial Z > 0$ via $g_Z > 0$ along with $r_Z > 0$. We end up with a Jacobian with the sign pattern

	Z	ζ
$\dot{Z}$	−	−
$\dot{\zeta}$	+	−

The signs say that in the vicinity of a steady state, the $\dot{Z} = 0$ nullcline will have a negative slope, with the $\dot{\zeta} = 0$ locus sloping upward. The Routh–Hurwitz conditions for local stability (trace < 0, determinant > 0) are satisfied.

11. Numbers

Table 4 is a social accounting matrix (or SAM), very loosely based on US data, for an economy with a capital stock of 80 (trillion dollars). Output, defined as value-added

[13] In equation (13), the workers' share will now be $1 - \boldsymbol{Z} - \Phi$.

		Current spending		Demand injections		
		Worker uses of income	Capitalist uses of income	FG outlay	Investment	Row totals
		13.588	0.912	2	3.5	20
Worker wages	12					15.6
Worker profits	3.6					
Capitalist wages	0					2.4
Capitalist profits	2.4					
FG income	2					2
Macro balance		2.012	1.488	0	−3.5	0
Column totals	20	15.6	2.4	2	0	

Fig. 4. *SAM for simulations (initial capital stock = 80).*

plus FG leakages, is 20. To avoid a lot more algebra in the model, corporate sector accounts, capital wage income and flows of fiscal and financial transfers (exceeding 10% of GDP in the American economy) have been suppressed. The numbers in the matrix are *not* realistic in this sense. Even so, they suggest three observations.

The profit rate is 7.5%, with capitalists receiving an income of 2.4 corresponding to a share of 40% in total wealth or capital. Their income of 13% of GDP (total demand minus FG income) approximates the share of the top 1% of households in the USA. Workers would outnumber capitalists by a factor of almost one hundred, so the discrepancy in incomes per household is vast.

Using their profit income, capitalists provide 42.5% of total saving. Implied saving rates are $s_c = 0.62$ and $s_w = 0.117$.

Initially, net saving of the FG sector is set to zero (fiscal and foreign deficits are equal), but this condition does not have to hold over time in the simulations. Values of other parameters are reported in the appendix.

On the basis of the SAM, Figure 5 shows nullclines for the model. There is a unique Pasinetti equilibrium, with $\boldsymbol{Z} = 0.69$ and $\zeta = 2.42$.

12. Simulation results

Based on these numbers for a stylised US economy, we simulate our model to gauge whether current trends of increasing wealth and income inequality may persist and to demonstrate that long-term growth projections can be derived from models of the demand side. Figure 6 presents simulation results from the model. Figure 6a and 6b show relatively slow convergence of GDP and the capital stock to approximately 2% growth, reflecting the intrinsic dynamics of the growth equation (6). The level of income per capita grows exponentially in Figure 6l.

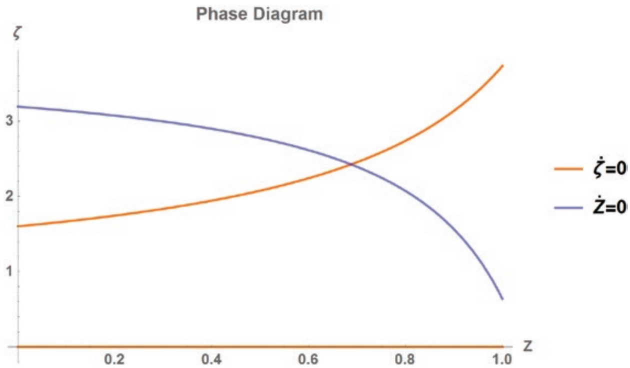

Fig. 5. *Phase diagram based on Figure 4 data for* Z *and* ζ.

Figure 6d shows a sustained increase in Z. Figure 6e and 6g illustrate how the dynamics of Z incorporate positive feedback. The utilisation rate u declines and the profit rate r goes up over time, in line with the description in Figure 1 of the effects of a higher level of Z. Because $\pi = r / u$, the profit share in Figure 6h rises steadily. Together with high capitalist saving, the shift in the income distribution towards profits sets up increasing Z from equation (13). The shifts in saving in Figure 6i and 6j mirror these trends. The higher profit rate spurs investment in Figure 6k.

In Figure 6c, the auxiliary variable ζ is fairly stable in the range between 2.0 and 2.5 (with a steady state value of 2.42). With $\zeta = \lambda / u$, the decreasing utilisation rate forces employment to drop off in Figure 6f. Finally, the evolution of FG net lending and wealth share in Figure 6m and 6n reflects dynamics of u and g. The FG sector switches from being a net lender to a borrower, but plays a secondary role in the overall dynamics of the model.

Figure 7 shows the effects of a recession caused by an adverse demand shock in the year 2067 (five decades after the base year) to autonomous investment to produce an immediate 6% reduction in GDP, followed by gradual recovery. After the shock, variables revert towards the steady state with somewhat higher profits and investment (Figure 7g, 7h and 7k) than in the unshocked simulation. With lower employment in the recession (Figure 7f), r and g jump above the unperturbed model's trajectories towards the steady state and then slowly decline. Capital utilisation in Figure 7e is lower and wealth concentration in Figure 7d rises as saving by capitalists goes up in Figure 7i and workers' saving in Figure 7j drops off.

In Figure 7c, $\zeta = \kappa / \xi$ rises, in part due to faster growth of capital but also driven by a slower increase in productivity induced by the higher profit rate over time. Towards the end of the simulation, both productivity and income per capital fall by around 1% in comparison to the recession-free simulation. Deviations in trajectories towards the steady state are restrained, but visible. A *favourable* short-run demand shock would be beneficial for a long time. Demand does indeed drive growth all the way.

13. Final thoughts

Drawing heavily on multiple strands within Post-Keynesian economics, we construct a heterodox model of economic growth which ties short-run variables describing

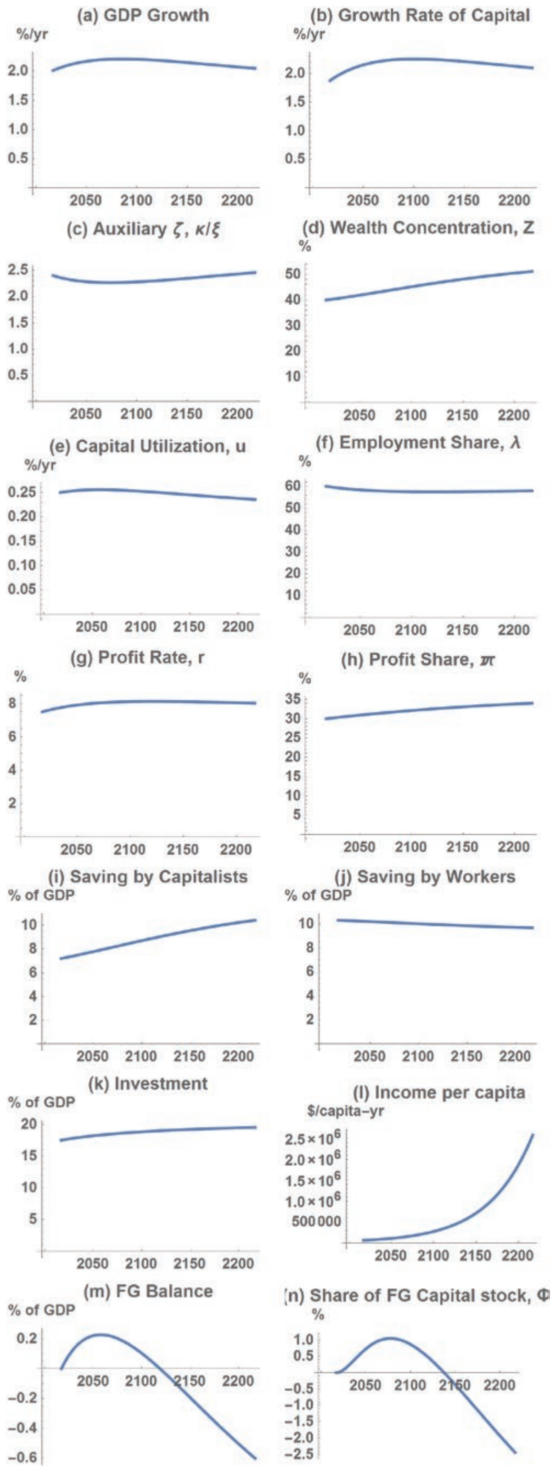

Fig. 6. *Time plots for simulation.*

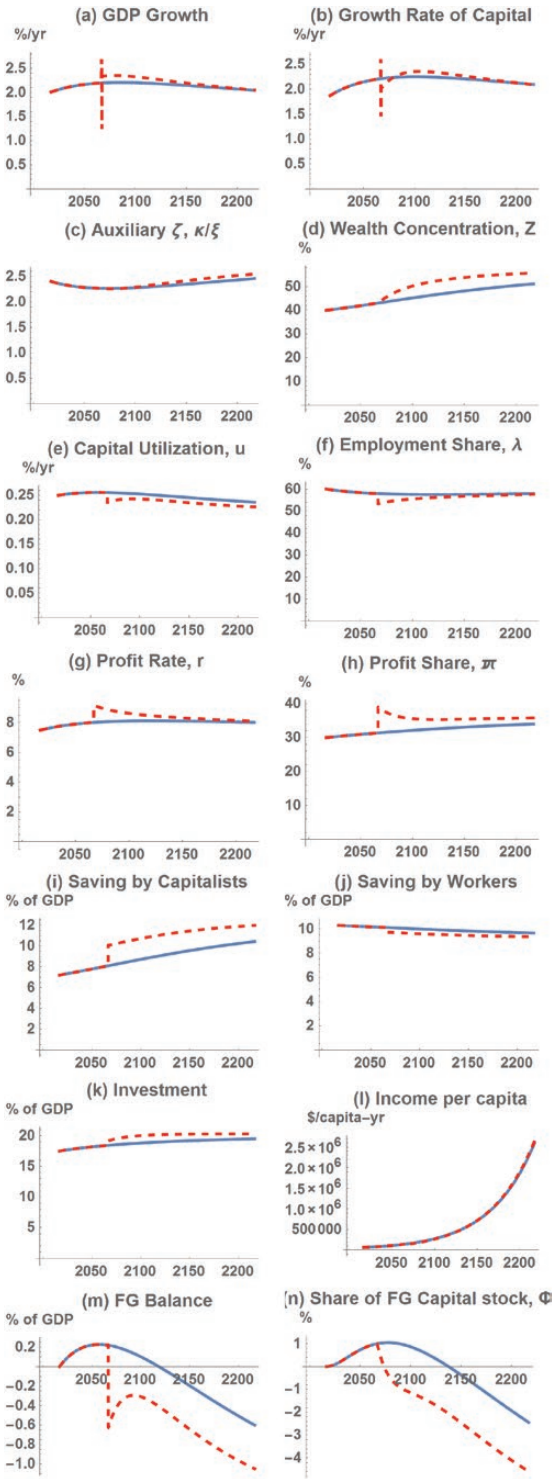

Fig. 7. *Time plots for baseline (solid) and shocked (dashed) simulations.*

aggregate demand and the distribution of income to long-run variables including the stock of capital, the distribution of wealth, and the productivities of labour and capital. Our alternative to the Solow–Swan model allows us to point to important similarities in steady state relationships as well as differences in the dynamics towards a steady state. In contrast to its neoclassical cousins, our demand-driven economy is prone to divergent instabilities. Even if the conditions for stability are satisfied and variables such as the equilibrium rate of growth of capital are set by supply-side parameters, demand and income distribution adjust and determine the equilibrium distribution of wealth.

Applying our model to a stylised data for the US economy, we find a rising concentration of wealth associated with a falling employment ratio and a more concentrated distribution of income. The reason is that via the paradox of thrift, a higher *level* of Z cuts into effective demand. The resulting downward pressure on employment pushes up the profit rate and faster *growth* of wealth and income inequality. This narrative has a degree of verisimilitude in wealthy economies over recent decades. To retain analytic tractability, we assume no active policy in counteracting such developments and leave this important question for future research.

These results show that one need not rely solely on supply-side explanations for economic growth. Interactions between income distribution and effective demand, endogenous productivity change, and dynamics of wealth have their own roles to play even in a simple model such as the one presented here. Important features of advanced capitalist societies, such an elaborate financial sector with multiple assets and independent dynamics of their prices, active fiscal and monetary policy, and open economy complications (as outlined in Foley and Taylor, 2006, and more recently Taylor *et al.*, 2015), allow for more realistic interactions and deserve further exploration.

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Appendix

Parameters are scaled to match the US economy as represented in the SAM of Figure 5. In particular, parameter values are the following:
Saving, investment and FG parameters:

$$s_c = 0.62,\ \ s_w = 0.117,\ \ g_0 = -0.015,\ \ \alpha = 0.6,\ \ \beta = 0.059,\ \ \nu = 0.1,\ \ \iota = 0.025\,.$$

Distribution parameters:

$$\mu_0 = 0.225,\ \ \mu_1 = 0.25\,.$$

Parameters for capital, labour productivity and population (assumed to follow logarithmic growth) dynamics:

$$\delta = 0.025,\ \ \gamma_0 = 0.01,\ \ \gamma_1 = 0.5,\ \ \gamma_2 = 0.01,\ \ n_{2017} = 0.005,\ \ L_\infty = 500.$$