Journal of Economic Literature, Vol. 8, No. 1, March 1970

The Distribution of Labor Incomes: A Survey

With Special Reference to the Human Capital Approach

By JACOB MINCER

Professor of Economics, Columbia University, and Senior Research Staff, National Bureau of Economic Research. *This paper is based, in large part, upon ongoing research in human capital at the National Bureau. This research is financed by grants from the Carnegie Corporation and from the Economic Development Administration of the United States Department of Commerce. I am grateful to Gary Becker, Barry Chiswick, and Victor Fuchs for helpful comments, and to Masanori Hashimoto and Sara Paroush for most competent assistance.*

EMPIRICAL KNOWLEDGE about income distribution has grown during the past two decades at a rate much greater than that of any other comparable period. The stock of information on individual incomes increased enormously with the appearance of the 1950 and 1960 Population Censuses and BLS Surveys of Consumer Expenditures, the periodic Current Population Surveys of the Census Bureau, and the Michigan Surveys of Consumer Finances, to mention only the most important sources. At the same time, high speed electronic computers made possible the processing and statistical analyses of these massive collections of individual observations.

While traditional and mainly speculative preoccupations with factor shares, ability vectors, and stochastic processes continued in evidence, the dominant form of research was empirical. A still modest but significant change in emphasis is emerging in recent empirical work: beyond the usual and necessary efforts to summarize, measure, and occasionally interpret the bewildering varieties of statistical frequency distributions of income, some of the new studies attempt multivariate statistical analysis of "causal" factors associated with size of individual income.

Parallel conceptual developments are more general and more striking. Interest has shifted from the consumption function and its role in short-run fluctuations in income and employment to the production function as a key concept in the study of economic growth. The focus on income as a determinant of consumption behavior is giving way to an interest in income as a dependent variable. Thus the analysis of causes of income variation in the aggregate and among individuals is returning to the mainstream of theoretical and empirical research. One impressive outgrowth of this shift is the rapidly developing literature on human capital. The human capital approach is intimately related to the study of income distribution: costs and returns to investments in human capital are measured in the first instance by earnings differentials. Consequently, there is a growing recognition of the importance of investment in people as an underlying principle in theoretical and empirical analysis of income distributions.

This review concentrates largely on the emerging human capital approach to the analysis of personal income size distribution. This approach is no reflection on the actual or potential value of alternative approaches to the subject. Fortunately, the field of income size distribution is already blessed by a number of excellent surveys containing detailed summaries of past and recent work. The surveys usually appear as necessary background to the author's own contribution and represent valiant efforts at an orderly exposition of a field which lacks conceptual unity.[1] Since they contain rather full accounts of a wide spectrum of analyses and findings, however disparate and fragmentary, the present rather specialized exposition should be viewed as a complementary rather than competing effort. Nevertheless, the traditional interest in the role of chance, ability, and opportunity will not be neglected.

In the distribution of income, as in all statistical distributions, the most dramatic cases are those at the tails. Some readers will be disappointed that no mention is made here of "the super-rich who own America" or of the many poor who are not even in the labor force. The present survey is limited to earnings of labor. Even within this narrow compass of the survey, a variety of topics are omitted. To the extent that the omissions cannot be blamed on the limitations of the literature, separate surveys are called for.

The traditional approaches centering on differences in opportunity, ability, and chance, are briefly reviewed in the next few pages. This review serves both as contrasting and complementary background to the human capital approach. Following the introductory review, the survey is devoted to an exposition of the development and application of the human capital model as an instrument of analysis of income distribution. The order of the exposition is both chronological and methodological. Starting with the simplest "schooling model" (section 1) which relates earnings to schooling we move to the more general formulation of the relation between earnings and human capital investments (section 2) of which schooling is only a part. Continuing investments which follow the completion of schooling and their effects on working life "earnings profiles" of individuals are the subject of special attention in section 3. The implications of individual differences in self-investments and in rates of return on them are individual differences in levels and shapes of earnings profiles. These implications, it is shown, can yield an understanding of the characteristic shapes and of comparative magnitudes of parameters of observed statistical distributions of earnings in the aggregate and in component schooling and age groups. Following the review of such "qualitative" analysis, section 4 describes econometric attempts to answer the question: how much of the observed inequality in earnings is attributable to individual differences in the sizes of human capital investments? The correlation between investments and earnings is interpreted in section 5 in terms of the interplay of factors which influence human capital investment decisions. In the last few sections there is a discussion of distinctions between long- and short-run income inequality, of transformations of personal into family distributions of earnings, and of some research agenda.

Traditional Approaches

In the literature which dates back to Ricardo, the major *economic* approach to

[1] The most thorough treatments are in Reder [51, 1969], Bjerke [9, 1969], Lydall [35, 1968, Chapter 2], Schultz [56, 1965], and Kravis [32, 1962, Chapter 6]. The monographs of Kravis, Bjerke, and Lydall are products of original empirical research. They contain useful data and analyses of income distributions in the U.S. (Kravis), Denmark (Bjerke), and a set of international comparisons (Lydall).

income distribution is the functional or factor-share approach. In the past, this approach was motivated by an identification of the trinity of factors of production with corresponding and distinct social classes. The approach continues to flourish in the literature despite the blurring of such social class identifications and despite the recognition that under modern conditions the variance in labor incomes is the dominant component of total income inequality. Remote links exist between the functional and the size distribution of total incomes, but this approach does not address itself to the distribution of labor incomes.[2]

To be sure, the heterogeneity of rewards to individual workers did not escape the attention of classical economists. Their comments are summed up in two famous principles: first, Smith's compensatory principle is conditional on the strength of competitive forces in the labor market. Labor mobility produces earnings differentials which tend to equalize "net advantages and disadvantages" of work. Second, Mill's and Cairnes' doctrine of "noncompeting groups" proclaims in effect the absence of labor mobility resulting in real income differences, produced and perpetuated by socially, legally, and culturally imposed and inherited stratifications.

A great deal of research on labor markets is directed toward the assessment of the relative validity of the two principles. Though this research is abundant, as is the passion surrounding it, it is often vague. In a recent summary, Reder [49, 1962] concludes that evidence favors the competitive hypothesis, at least insofar as long-run differences in average occupational and industrial wages are concerned. Wage differentials which compensate for differences in risk and in the cost of living have also been noted as illustrations of compensatory differentials. Indeed, Friedman [20, 1953] has shown that risk-taking behavior can produce an over-all pattern of income distribution which, like the observed distribution, is positively skewed and humped. Mincer [40, 1957 and 41, 1958] has shown that a similar shape is likely to arise in consequence of compensatory differentials due to costs of occupational training. Some of the predictions of the compensatory principle are often rejected *prima facie:* on the whole, occupations in which work is more unpleasant and unstable command lower, not higher wages. The costs of occupational training, however, can reconcile these apparent contradictions. The reconciliation requires sufficiently large training costs and a negative correlation between occupational skill and both unpleasantness and instability. Becker's theory of specific training [3, 1964] provides a possible explanation of the latter. In occupations requiring similar training costs, compensatory differentials are more clearly observable.

The doctrine of noncompeting groups is rarely accepted in the extreme form enunciated by Mill. It usually denotes a recognition of the importance of "institutionally" determined inequalities of opportunity. In its moderate versions this doctrine has had many adherents, but it has not produced any cumulative theoretical developments in the analysis of income distribution. It did contribute, however, by its emphasis on a variety of environmental factors, to a pragmatic statistical approach. In this spirit,

[2] Half a century ago Dalton [17, 1920] complained that the emphasis of economists on the functional distribution of income in effect largely relegates the personal distribution of incomes, "a problem of more direct and obvious interest", to the status of a residual to be studied if at all, "by plodding statistical investigations, which professors of economic theory were content to leave to lesser men".

Garvy [24, 1952], who quoted Dalton several decades later, saw little progress in the meantime: he points out that the American Economic Association's volume on *Readings in the Theory of Income Distribution* (1946) contains only one article on income size distribution [11, Bowman, 1945].

Garvy's remark can now be updated with reference to the International Economic Association's recent volume on *The Distribution of National Income* (1968). It contains only one paper exclusively devoted to the analysis of income size distribution [50, Reder, 1968].

such "institutional" or "demographic" factors as sex, age, occupation, education, location, and parental wealth have become objects of multivariate analyses of statistical associations with individual income differences [1, Adams, 1958; 30, Hill, 1959; Morgan *et al.*, 47, 1962]. It is true, of course, that without the guide of a theoretical framework, the statistical formulations and the interpretations attached to them are often insecure and ambiguous. Nevertheless, they are a welcome departure from the largely speculative, single-factor theories that have traditionally competed for attention in this field.

According to Stigler [58, 1966], "the major modern noncompetitive force on wages is the labor union." Specialized research on union wage effects is voluminous[3] but rarely related to the overall pattern of income distribution. On this question Rees [52, 1962] concludes that "unions have probably raised many higher income workers from above the middle to a position closer to the top of the income distribution." While this effect is not clearly described as an increase or decrease in the equality of the income distribution, "it seems closer to the latter than to the former."

One version of "noncompeting groups" which has been elevated to a theory of income distribution is the view that worker differences in productivity, and hence in earnings, are due to differential abilities. Starting with Galton [23, 1869] this idea has fascinated geneticists, psychologists, statisticians, and economists. Referring to the alleged normal distribution of intelligence, Pigou (1920) dramatized the issue by posing the question: if capacities are normally distributed, why is this not true of earnings?

One answer, stated by Reder [50, 1968], is that the distribution of capacities is not the same as the distribution of marginal productivities. The transformation of the former into the latter depends on cooperating factors and on the form of the production function of services which these capacities help to produce. Another response is to question any arbitrary definition of relevant abilities, to the point of denying the possibility of measuring earning capacity independently of earnings.

Indeed, recent research is now attempting to infer the distribution of abilities from the distribution of earnings. And because variables such as education, age, and location also affect earnings, the distribution of abilities are studied in earnings distributions of homogeneous groups of workers [54, Roy, 1950; 9, Bjerke, 1969]. Thus the relevance of ability theories is reduced to residual variations in income distribution, after the systematic factors have been netted out. This orientation is, however, too narrow because it ignores the interaction between ability and the systematic variables. A possible link is restored on theoretical grounds in Becker's recent formulation of the human capital model [4, 1967]. But in this formulation, to be discussed later, the role of abilities is neither exclusive nor dominant.

Mathematical Models of Chance and Ability

It may seem paradoxical that one of the most popular approaches to the analysis of income distribution singles out residual variation, or "chance," as the principal source of income inequality. Actually the "chance" factor is not conceived as a residual, but as a substitute for, or net effect of all kinds of causes which are too numerous and obscure to be treated explicitly. This is the standard rationale of applied probability models. As we are increasing our insights into the subject, the need for such models is decreasing together with the size of the residual variation which is left over.

The basic stochastic model dates back to Gibrat [25, 1931]: starting with some initial distribution, individuals are subjected to the

[3] For a basic reference and analysis see H. G. Lewis [34, 1963].

play of "chance." If they experience increases or decreases in income unrelated to income levels, the central limit theorem guarantees that, in time, the distribution of income will approach normality, regardless of the form of the initial distribution. If this specification of "random shock" applies to percentage rather than absolute changes, the process converges to a log-normal distribution. The latter has been observed to fit aggregate distributions of income better than the normal distribution, hence the name "law of proportionate effect."

The model recently received a number of interesting elaborations: Champernowne [14, 1953] has shown that when certain assumptions are introduced about the distribution of the proportionate random shock, the income distribution converges to a Pareto distribution.

Aitchison and Brown [2, 1957] have shown that even if the law of proportionate effect produced a log-normal distribution only within homogeneous subgroups ("trades"), the aggregate distribution would remain log-normal so long as variances in the component distributions are of the same size and the means of the components are log-normally distributed.

Since income at any time is portrayed as a sum of uncorrelated random shocks in all previous periods, the implication of these models is that the variance of income must grow over time.[4] Yet empirical observations of aggregate income inequality remain relatively stable. In his model, Rutherford [55, 1955] turns this apparent defect into a virtue. He applies the random shock to age cohorts. The income variance must increase with age for each age cohort, but given a relatively stable age distribution, the aggregate variance does not change much. Rutherford also observes that the observed distributions do not fit the log-normal distribution very well: a graph of the cumulative distribution on log-normal probability paper is more akin to an S-curve (the distribution is "leptokurtic") than to a straight line. This, Rutherford shows, will result from aggregation when population frequencies decline with age (exponentially in his formulation). If, in addition, income grows with age, the implied positive correlation between means and variances by age groups will impart (additional) positive skewness to the aggregate.[5] Among the several stochastic models, Rutherford's is the richest in empirical predictions, providing a standard which a substantive economic model should match.

Mandelbrot [36, 1960] substitutes an additive process for the multiplicative (proportionate) random shock. He notes that the characteristic shape of the aggregate income distribution remains similar when the empirical definition of income is varied (wages and salaries, self-employment, taxable income, etc.). If the differences in the definitions of income are due to alternative inclusions or exclusions of additive components, this is precisely the condition under which a Pareto-Levy distribution arises. This distribution is rather realistic: two-tailed, positively skewed, with a Pareto upper tail. Mathematically, the result is generated by the repeated application of a weighted sum of independent random elements.

In a subsequent paper, Mandelbrot (1962), provides another interpretation of such a process. Abilities can be factor-analytically represented as weighted sums of uncorrelated components. The distribution of weights determines the specific mathematical function within the general family of distribution functions. The effective distributions are in

[4] If Y is income and E, the "random shock,"

$$Y_t = Y_0 + \sum_{i=1}^{t} E_i$$

and $\sigma^2(Y_t) = \sigma^2(Y_0) + t\sigma^2(E)$, assuming $\sigma^2(E_i)$ same for all i.

[5] Such a correlation in subsets of a distribution is easily visualized as stretching the right tail of the distribution.

turn produced by a selection process, in which component abilities required by particular jobs are matched with abilities that different individuals will supply in combinations reflecting their comparative advantage.[6]

The alternative application of the same mathematical argument to stochastic and to ability models is not new. It was, in fact, the way ability theorists tried to find an answer to Pigou's question of whether a skewed distribution of earnings can be associated with an allegedly normal distribution of abilities. The argument, originating with Boissevain [10, 1939], runs as follows: earnings are proportional to ability. But ability is a multi-dimensional concept. If component abilities combine multiplicatively, as "random shock" does in stochastic models, the resulting implications are: symmetric distributions of component abilities produce positively skewed aggregates, hence earnings. If the component abilities are positively intercorrelated, skewness is augmented. And if the variances of the components are unequal, humpedness (leptokurtosis) will result [54, Roy, 1950].

Roy studied distributions of physical output (piecework) in samples of homogeneous workers. Results were mixed, though positive skewness was encountered more often than symmetry or negative skewness. Similar results were found for wages of homogeneous groups of workers by Bjerke [9, 1969].

Lydall [35, 1968] argues that if the random shock models are accepted, ability models must be applied only to new entrants into the labor force. Lydall accepts the view of ability as a multiplicative combination of components, but rejects the "random shock" hypotheses. He replaces the latter by psychological findings to the effect that abilities and their variances grow with age—at least up to middle age. Thereafter, levels decline, though variances do not. The empirical implications are clearly similar to Rutherford's and quite realistic. But Lydall's restoration of ability from the more modest residual role assigned to it by Roy and Bjerke to a dominant place in the overall distribution verges on the tautological: stating facts about distributions of productivities comes dangerously close to stating facts about distributions of earnings. Both sets of facts require explanation.

The attention paid to the mathematical random shock and ability models is perhaps undeserved. The models seem rather superficial in focusing on an unexplained category and in the single-minded objective of theoretically reproducing a presumed mathematical form of the aggregative distribution. Still, an acquaintance with these models may be useful. Residual distributions may be best treated in a probabilistic fashion. More instructive is the explicit treatment and the demonstrated flexibility of the mathematical structure of the arguments. As we shall see, similar structures reappear in very different substantive models, thereby assuring at least as much explanatory power as the just described models are capable of producing.

The Human Capital Approach

1. The Schooling Model. Common to the mathematical models described in the previous section is the view that the distribution of earnings is unaffected by individual choice. The exogenous variables have economic effects, but do not pertain to economic behavior.

In contrast, human capital models single out individual investment behavior as a basic factor in the heterogeneity of labor incomes. Indeed, the first model of this kind [40 and 41, Mincer, 1957 and 1958] starts by assuming a complete absence of environ-

[6] This rough sketch does not do justice to Mandelbrot's sophisticated analysis. His approach, incidentally, represents a solution to a scheme of income distribution proposed by Tinbergen [60, 1956]. For a more detailed discussion see Bjerke [9, 1969].

mental inequalities. These assumptions are not inherent in the human capital approach and have been relaxed. They were initially imposed in order to reveal the effects of individual choice unhindered by the non-competitive forces which are so prominent in the traditional literature.

The model takes the length of training as the basic source of heterogeneity of labor incomes. Training raises productivity, but the time spent in training necessitates postponement of earnings to a later age. Individuals undertake various amounts of training in the expectation that their occupational incomes in the future will be sufficiently large to compensate for the cost of training. For simplicity, the cost of training is restricted to foregone earnings, the dominant component of private training costs. In a competitive equilibrium, the distribution of earnings is such that the present values of future earnings, discounted at the market rate of interest are equalized at the time training begins.

The model is formulated in terms of training periods which are completed before earnings begin. It, therefore, applies strictly to schooling rather than to all occupational training. Assume that no further investments in human capital are undertaken by individuals after completion of their schooling, and that the flow of their earnings is constant throughout their working lives. Then, the equalization of present values of earnings for individuals with s_1 and s_2 years of schooling, and with n_1 and n_2 years of working life, results in the following ratio of annual earnings:[7]

$$(1)\qquad k_{2,1} = \frac{E_{s_2}}{E_{s_1}} = \frac{e^{-rs_1}(1 - e^{-rn_1})}{e^{-rs_2}(1 - e^{-rn_2})}$$

Here r stands for the market discount rate or for the internal rate of return on the differential investment which it must equal, E is annual earnings, and e is the base of natural logarithms. If n_1 and n_2 are large, k approaches the value $e^{r(s_2-s_1)}$. More compactly, putting $s_2=s$, $s_1=0$, $k_s \to e^{rs}$. Alternatively, when $n_1=n_2=n$, regardless of the length of working life, $k_s=e^{rs}$, exactly. These formulations, incidentally, highlight the principle that it is not the shorter pay-off period of the more educated, but the postponement of earnings that is the basic cause of differentials in earnings. Empirically, both formulations are roughly correct,[8] so that (1) can be written:

$$(1a)\qquad \lg E_s = \lg E_0 + rs$$

If the competitive assumptions are relaxed, internal rates of return cannot be equated with the market rate of interest and generally differ among individuals.

(1a) can remain serviceable, however, with r interpreted as a group average internal rate of return on schooling, while individual differences in r and in $\lg E_0$ are impounded in a statistical residual.

Relation (1) makes percentage differentials in earnings a linear function of time spent at school. This transformation is one source of the tendency toward positive skewness in the distribution of earnings: for a symmetric distribution of years of schooling would imply a positively skewed distribution of earnings. Indeed, unless the distribution of schooling is highly negatively skewed, a positive skew will be imparted to the distribution of earnings.[9]

Relation (1) also implies that relative dis-

[7] Using continuous discounting, the present value for s and n is:

$$V_s = E_s \int_s^{n+s} e^{-rt}dt = \frac{1}{r} E_s e^{-rs}(1 - e^{-rn})$$

$k_{2,1}$ is obtained by equating $V_{s_2} = V_{s_1}$

[8] Recent data suggest that the differences in lengths of working lives are rather small.

[9] Positive skewness in earnings obtains so long as

$$1 - \frac{d_u}{d_l} < rd_l,$$

where d is the interval (in years) between the median and a symmetric percentile (u—upper, l—lower) [45, Mincer, 1969].

persion and skewness in the distribution of earnings are greater the larger the absolute dispersion in the distribution of schooling. Earnings inequality and skewness are also greater the higher the rate of return.[10] If for example, barriers to investment in schooling produce a higher rate of return to schooling, greater inequality will result.

The model does not predict the shape of the distribution of schooling. Empirical schooling distributions are more likely to be positively skewed when the average level of schooling is low. In the United States the distribution has even become negatively skewed in the most recent cohorts, though not in the aggregate. Even in the recent cohorts, the negative skewness in schooling is not sufficient to create a negatively skewed distribution of earnings.

Chiswick [15, 1967] applied model (1a) to a comparative analysis of income inequality among different regions in the U. S. and a few other countries. Regressions of (log) earnings of males over 25 years of age on their schooling in each of the regions do, indeed, show that inequality (variance of logs) and skewness (measured by the third moment in the distribution of log-earnings) are larger the greater the variance in the distribution of schooling and the higher the rate of return as estimated by the size of the regression slope in (1a). These factors jointly explain over a third of the differences in inequality among regions, and the rate of return is apparently the more important factor. Lydall [35, 1968], who did not employ the rate of return as an explicit variable, also found that the dispersion in the distribution of schooling is a significant factor in explaining differences in the inequality of employment incomes within a wider set of countries.

Within a region, however, the explanatory power of the schooling model is quite low, R^2s running a little over 10% in the various states in 1959, and less than 10% in the regression based on individual observations of the 1960 Census 1:1,000 sample.[11] Moreover, the regression slope which serves as an estimate of the rate of return to investment in schooling is from a half to two thirds the size of the rates of return calculated directly [*cf.* 29, Hansen, 1963; 3, Becker, 1964; 28, Hanoch, 1965]. Evidently, the residuals in that regression are very large and related to the schooling variable. That is to say, the schooling model, which says something about differences in earnings among schooling *groups*, is a rather blunt instrument when applied to the whole distribution of individual earnings.

Equation (1a) was applied by Lydall [35, 1968] and Mincer [45, 1969] to grouped data (mean earnings of males classified by schooling). Logs of earnings yield a better fit than dollar earnings.[12] The slope remains low as in Chiswick's regressions. However, when average earnings of all individuals in a schooling group are replaced by earnings of individuals who have the same amount of labor force experience, measured by years elapsed since completion of schooling,[13] not only does the statistical fit improve, but the slope rises to a level within the range of the rates of return directly estimated.

2. The General Earnings Function. The need to incorporate post-school investments, such as "experience" into the earnings model is evident. Developments in that direction were facilitated by Becker's [3, 1964] formulation of a general earnings function within the context of a comprehensive theory of investment in human capital. Earn-

[10] This is apparent by taking variances across individuals in eq. (1a) and by varying r in the expression in note 9.

[11] The state R^2s are higher partly because they are derived from data grouped by income brackets.

[12] An error crept into Lydall's test. While the equation (p. 90) properly relates log-earnings to years of schooling, the statistical test (p. 95) relates log-earnings to logs of years of schooling. The proper form is, therefore, mistakenly rejected.

[13] Standardization by age improves the regressions only slightly [15, Chiswick, 1967].

ings of individual i in period j is expressed as a sum of the returns on all his previous net investments and the earnings from his "original" endowment [3, equation 33]. Earnings are "gross" if current investments are included in the concept, "net" if not included. The gross earnings function is:

$$E_{ji} = X_{ji} + \sum_{t=0}^{j-1} r_{ti}C_{ti} \tag{2a}$$

and the net earnings function:

$$Y_{ji} = X_{ji} + \sum_{t=0}^{j-1} r_{ti}C_{ti} - C_{ji} \tag{2b}$$

Here C_{ji} are net investment costs of person i in period j and X_{ji} is the "raw" earnings stream that would obtain without investments. An additional subscript may be added if it is desired to distinguish between different forms of investments (*e.g.*, training, migration, recreation). The average rate of return may differ among persons, periods, and investments.

The schooling model (1a) can be shown to be a special case of (2): if investments are restricted to time costs of schooling: $(C_t = E_t)$, if the rate of return is the same in all periods and for all individuals, $(r_{ti}=r)$, and if $X_{ji}=E_0$ is fixed and the same for all. Substituting in (2b):

$$E_{j>s} = E_0 + r\sum_{t=1}^{s} E_{t-1} = E_0(1+r)^s \tag{1b}$$

This is the discrete form of equation (1a).

The general implication of equations (2) is that the distribution of earnings depends not only on the distribution of investment costs C_i, but also on the distribution of the rates of return r_i, and the correlation between these parameters. The distribution of original capacities X_i is also relevant, but we take these as given. For fixed X and r, the distribution of earnings would be of the exact same form as the distribution of investment costs, while for fixed X and C, earnings would vary as does r_i, which can in that case be interpreted as an index of "ability." Several specific conclusions are:

1. Even if rates of return r_i and investment costs C_i were symmetric and uncorrelated, the distribution of earnings, at least insofar as the component of returns $r\sum C$ is concerned, would be positively skewed—another application of a multiplicative mathematical structure, familiar from the models of ability and chance. The skewness would be strengthened, if a positive correlation between r_i and C_i can be assumed.

2. Consequently, the larger the accumulated human capital component $r\sum C$ in earnings, the more skewed is the distribution of earnings likely to be. Thus skilled workers are expected to have a more skewed earnings distribution than unskilled workers.

3. During the investment period, defined by positive net investments, the subtraction of current investment costs C_j in (2b) weakens the importance of $r\sum C$, more so at younger than at older ages, since $r\sum C$ cumulates with age. Positive skewness, therefore, increases with age.

Though empirically confirmed, these implications are theoretically conditional on a zero or positive correlation[14] between r_i and C_i. It is tempting to postulate a positive correlation between the two, based on the greater incentive to invest of more able people. Becker's later work [4, 1967], however, suggests that the correlation across individuals is not necessarily positive, though it would be under conditions of equality of opportunity, as he defines it.

4. Compared to the special case (1a), the implications about earnings inequality are now broadened to include the effects of average levels of investment and dispersion in rates of return among individuals. Chiswick [16, 1968] illustrates these effects when he introduces dispersion in r in the schooling model, while assuming no correla-

[14] Strictly speaking, a small negative correlation need not be ruled out. Also, the term X_{ji} and its correlation with the other terms of (2b) are disregarded.

tion between rates of return r_i and years of schooling s_i: From $\lg E_i = \lg E_o + r_i s_i$, it follows[15] that:

$$(3) \qquad \sigma^2(\lg E_i) = \bar{r}^2\sigma^2(s) + \bar{s}^2\sigma^2(r) + \sigma^2(s)\sigma^2(r)$$

In other words, inequality in earnings is large not only when r and $\sigma(s)$ are large, as we have already learned, but also when $\sigma(r)$ and the average level of schooling $\bar{s}$ are large.

Several recent studies have found a negative correlation between average income and income inequality by states in the U. S., and between average levels of schooling $\bar{s}$ and income inequality. Chiswick [16, 1968] shows that this is no contradiction of (3): the *partial* correlation is positive, once $\bar{r}$ and $\sigma(s)$ are held constant.[16]

Note that in (3),

$$\sigma^2(\lg E) = \bar{s}^2\sigma^2(r), \quad \text{for fixed } s,$$

and

$$\sigma^2(\lg E) = \bar{r}^2\sigma^2(s), \quad \text{for fixed } r.$$

If r_i can be taken as an index of individual ability, the interpretation is: (a) individual differences in ability create greater relative (and absolute) differences in earnings at higher levels of schooling, and (b) differences in schooling create greater differences in earnings at higher levels of ability.

3. The Life Cycle of Earnings: Optimization, and Levels of Earnings by Schooling and Age. Variation of earnings over the life cycle is an important source of income inequality. This variation was disregarded in the schooling model, but it can be analyzed by means of earnings function (2): A specification of changes in C_{ti} over the life cycle traces out the "age profile" of earnings of individual i.

[15] By a theorem of L. Goodman (1960).

[16] Of course, this finding does not explain the negative correlation between level and inequality of income by region. Chiswick [16, 1968] provides some interesting conjectures on this matter.

Generally speaking, the fact that age-earnings profiles slope upward over a large part of the life cycle is a consequence of the tendency to invest in human capital at young ages. According to Becker [3 and 4, 1964 and 1967] this tendency is due to the following incentives: 1) with finite lifetimes, later investments produce returns over a shorter period, so total benefits are smaller; 2) to the extent that investments in human capital are profitable, their postponement reduces the present value of net gains; and 3) a person's time is an important input in his investment, but the consequence of human capital accumulation is an increase in the value of his time. Investments at later periods are more costly, because foregone earnings (per hour) increase. The incentive to shift from learning to earning activities follows, except in the special or temporary cases when productivity in learning grows as fast or faster than productivity in earning.

Should we then not expect an early and quick accumulation of all the desired human capital even before individuals begin their working life? The answer of human capital theory to this question is twofold. Investments are spread out over time, because the marginal cost curve of producing them is upward sloping within each period. They decline over time both because marginal benefits decline and because the marginal cost curve shifts upward.

Specifically, the argument [6, Ben-Porath, 1967; 4, Becker, 1967] visualizes the individuals as firms which produce additions (Q) to their own human capital stock (H), by combining their human capital with their own time (T) and with other market resources (R) in a production function:

$$(3) \qquad Q = f(H, T, R)$$

Attempts to increase Q within a given period run into diminishing returns: Costs rise with the speed of production. Thus the marginal cost curve in Fig. 1 is upward sloping:

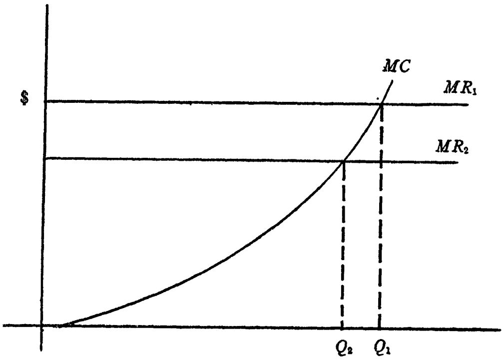

Figure 1

The marginal revenue of adding a unit of Q to the capital stock is the discounted flow of future increases in earning power. For reasons indicated, the benefits of later investments decline. The MR curve slides downward with increasing age, tracing out a declining pattern of investment over the life cycle.

The decline is reinforced if MC shifts to the left with increasing age. As already mentioned, this is not a logical necessity, but will happen if the increasing cost of time—greater earning power in the market—is not matched by increased efficiency of the larger stock in producing additional human capital. A recent attempt by Ben-Porath [7, 1968] to test this "neutrality" empirically, suggests that investments decline over the life-cycle faster than would be predicted by the mere downward slide of MR on a fixed MC in Fig. 1.

Investments, however, need not decline throughout the life-cycle. Ben-Porath [6, 1967] has shown that the optimization process may lead to an increase in investment during the early stages, because of "corner solutions": The initial stock (H) may be so small that even an input of all the available time, other resources not being highly substitutable, produces less than the optimal amount of output. As the stock increases, investment output will increase for a while until optimum is reached with an input of less than the total available time. At this point investments and time devoted to them begin to decline. The initial period of complete specialization in the production of human capital may be identified with the period of schooling, though more rigorously by the absence of earnings.

The optimization process described above applies explicitly [6, Ben-Porath, 1967] to gross investments in human capital. Note, however, that the predicted decline in gross investment applies *a fortiori* to net investment, if depreciation increases with age. This would be true, even if the depreciation rate were constant.

The implications of this analysis for age variations in earnings can now be spelled out in terms of earnings model (2):

$$\text{(4a)} \qquad \frac{dE_j}{d_j} = r_j C_j > 0, \quad \text{when } C_j > 0,$$

and

$$\text{(5a)} \qquad \frac{d^2E_j}{d_j{}^2} = r_j \frac{dC_j}{d_j} < 0, \quad \text{since } \frac{dC_j}{d_j} < 0.$$

The simplifying assumptions here are:

$$\frac{dX_j}{d_j} = 0, \quad \text{and} \quad \frac{dr_j}{d_j} = 0.$$

Their relaxation is not likely to change the conclusions.[17] Gross earnings slope upward in a concave fashion during the post-school net investment period after labor force activities begin to dominate. The profile of net earnings, which is better approximated by observed earnings, has a steeper slope, and peaks and declines somewhat later than gross earnings.[18] Its shape is also concave—at least eventually.

Fig. 2 indicates the shapes of gross earnings E_j and of net earnings Y_j during the

[17] *Cf.* note 24 for empirical statements about the constancy of X_j throughout most of the working life.

[18] From (2b):

$$\frac{dY_j}{dj} = rC_j - \frac{dC_j}{dj} > \frac{dE_j}{dj}$$

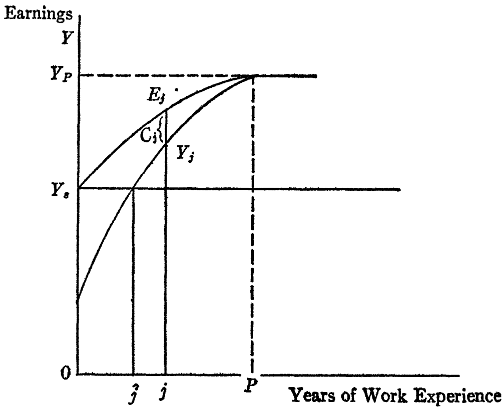

Figure 2. Earnings Profiles

post-school investment period OP. $\hat{j}$ is the year of "overtaking" when $Y_j = Y_s$, earnings obtainable without post-school investment.

With the same initial earnings capacity, individuals who invest more have steeper slopes, at time j, than those who invest less, provided their average rate of return r is not excessively smaller. This is the basic explanation for the fanning out of earnings profiles with age and for increases of variances of earnings with age, so long as individuals with different investments in human capital comprise the earnings distribution.[19] This conclusion holds strictly for gross earnings. Net earnings would initially be lower for those who invest more, later surpassing those of the smaller investors. The variance of observed earnings might, therefore, initially decline with age. However, the time ($\hat{j}$) it would take for a trained person to overtake an untrained is rather short:

Let $Y_{j1} = X_j$, and

$$Y_{j2} = X_j + r_j \sum_{i=0}^{j-1} C_i - C_j$$

Then $Y_{j1} = Y_{j2}$, when

$$r_j \sum C - C_j = 0, \quad \text{or} \frac{C_j}{\sum_{0}^{j-1} C_i} = r_j$$

If C_j were all the same, the time of overtaking $\hat{j} = 1/r_j$. Since $C_{j-1} < C_j$, $\hat{j} < 1/r$, less than a decade, if $r = 10\%$.

Although reversals of dollar variances have not been observed, they have been observed in relative variances [47, Morgan, 1962; 45, Mincer 1969]. The analysis of relative variances suggests that attention be paid to logarithmic rather than to dollar profiles of earnings. A logarithmic version of the earnings function (2) was used in the special case of the schooling model (1a). This model can be expanded to cover post-school investments by means of the following device [5, Becker and Chiswick, 1966]: Let k_j be the ratio of investment costs C_j to gross earnings E_j in period j. This ratio can be viewed as the fraction of time (or a "time equivalent," if investment costs include direct outlays as well as time costs) the worker devotes to the improvement of his earning power. His net earnings in year j are, therefore, smaller by this fraction than they would be if he did not invest during that year.

$$C_j = k_j E_j, \quad \text{and} \quad E_j = E_{j-1} + rC_{j-1} = E_{j-1}(1 + rk_{j-1})$$

By recursion, therefore:

$$E_j = E_0 \prod_{i=0}^{j-1} (1 + r_i k_i)$$

and, assuming $k \leq 1$, r relatively small, this can be expressed:

$$\text{(6)} \qquad \lg E_j = \lg E_0 + \sum_{i=0}^{j-1} r_i k_i$$

Since net earnings $Y_j = E_j(1-k_j)$:

$$\text{(6b)} \quad \lg Y_j = \lg E_0 + \sum_{i=0}^{j-1} r_i k_i + \lg (1 - k_j)$$

Assuming, as in the schooling model that

[19] Assuming that the ranking of investments across individuals is roughly the same in the different periods.

$k_i = 1$ during the school years:

$$\text{(6c)} \quad \lg E_j = \lg E_0 + rs + \sum_{i=s+1}^{j-1} r_i k_i$$

Let

$$P_j = \sum_{i=s+1}^{j-1} k_i,$$

the cumulative "time" expended on the post-school investments and assume r_j the same for all j. Then:

$$\text{(6d)} \quad \lg E_j = \lg E_0 + r_s s + r_j P_j$$

If $r_s = r_j$, this last expression is basically the same as the schooling model (1a). The total "time" spent investing $(s+P_j)$ replaces the time spent in schooling (s), and (P_j) is the number of "years" devoted to net post-school investments.[20]

The shape of the logarithmic earnings profile is upward sloping, as long as $k_j > 0$ and concave, *a fortiori*, if the dollar profile is upward sloping and concave. If individuals who accumulate more human capital spend more time accumulating it, logarithmic age profiles also fan out and produce the observed increase of log-variance with age. If years of schooling are viewed as indicators of total "time" an individual intends to invest over his lifetime, age-profiles of upper schooling groups will typically grow faster, percentagewise, than those of lower schooling groups, in given age intervals.

Summing up the implications: 1) at given ages, earnings profiles grow faster (in dollars and percentages) at higher levels of human capital investments; 2) dollar and log-variances of earnings increase with age. These implications have been repeatedly verified in empirical studies [32, Kravis, 1962; 35, Lydall, 1968; 40 Mincer, 1957; 47, Morgan, 1962].

[20] If on-the-job training is the bulk of it, this formulation provides a method of measuring on-the-job training in the same time units as schooling. In [44, Mincer, 1962] all net post-school investments were interpreted as on-the-job training.

The special assumptions and some additional implications of the human capital model which underlie the preceding discussion can be clarified by the following diagram:

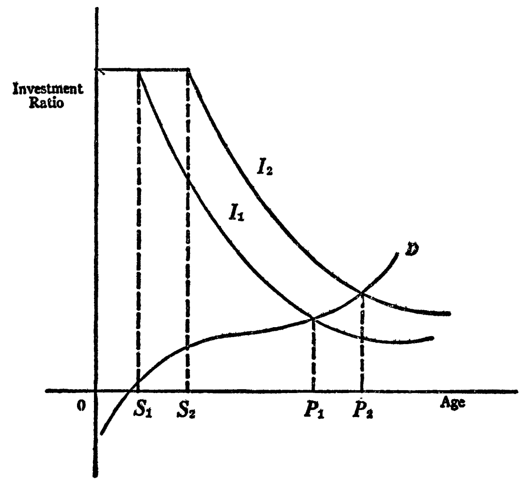

Figure 3

In Fig. 3 I is gross investment in human capital measured as a fraction of obtainable earnings, and D is the fraction by which such earnings are diminished as a result of depreciation. The net investment fraction is $k = I - D$, at each age. If retirement were compulsory and investment had no effect on non-market productivities, gross investment would terminate at retirement age. Otherwise, as is assumed in the diagram, gross investment remains positive throughout the expected life span. Retirement here can be viewed as endogenous, its timing being related to the decline in earning power, that is to the time at which depreciation outstrips gross investment.[21]

Depreciation is portrayed as a function of age, initially negative (appreciation), rising

[21] For this to be valid, the substitution effect must dominate in the relevant supply of labor function. *Cf.* Lewis [33, 1957]. At a deeper level, the lifespan itself is endogenous, since it can be lengthened by investment. An illuminating analysis of this problem is contained in Grossman [27, 1969].

slowly and accelerating at later ages. The diagram shows age profiles of investment of two individuals: Assuming the same life span, it is plausible that $I_2 > I_1$ at each age. Consequently, net investment $k_2 > k_1$ at each age. The empirical implication, it will be recalled, is that earnings of the larger investor grow faster, relatively and absolutely, at given ages. An additional implication shown by the diagram is that earnings of the larger investor decline later in life: the more educated retire at a somewhat older age, though they do not necessarily have a longer working life, since it begins after a longer schooling period.

In the diagram the schooling period $s_2 > s_1$, and this is an indication that total "time" invested of individual (2) is larger. It does not follow, however, that individuals who have more schooling also spend more "time" in post-school investment. In the special case of parallel investment profiles illustrated in the diagram, the larger investor spends no more "time" in post-school investment than the smaller investor. If the investment ratio of the larger investor declines faster, the smaller investor may experience faster growing earnings in the age interval S_1P_1 than the larger investor does in the corresponding age interval S_2P_2. But so long as the age-investment profile I_2 is above I_1, dollar investments are larger at each year of experience,[22] hence the dollar *experience* profile of earnings of the large investor must be steeper.

Empirical evidence does show that earnings of the more educated peak later, grow faster in dollar terms at given years of age as well as at given years of labor force experience, grow also relatively faster (in logs) at given ages, but no faster[23] at given years of experience. Across schooling groups, these findings reflect a positive correlation in the dollar volumes of schooling and post-school investment, and a negligible correlation between time equivalents of these investments.

4. Dispersion and Skewness of Earnings. School is a convenient empirical indicator of investment in human capital. However, differences in investment behavior among individuals with the same number of years of schooling are ignored (averaged) in the observable typical earning profile of such a group.

This average conceals individual differences in the quality of schooling and in post-school investments. These considerations suggest that even if investments ceased after completion of schooling, a dispersion in earnings would be observed reflecting schooling quality, as well as individual ability and other residual factors. The fact of post-school investments, however, means that even within a schooling group, the variance of earnings changes with age. The nature of these changes is predictable, given the correlations between schooling and post-school investments. If these correlations are positive in dollar terms within schooling groups, as they were observed across schooling groups, dollar variance tends to increase with age monotonically. If they are negligible in time equivalent terms, logarithmic variances tend to decline during the first decade of experience and increase thereafter. Empirical observation of variation of earnings within schooling groups shows dollar variances increasing strongly with age, but somewhat less clear U-shaped patterns of relative variances [45, Mincer, 1969].

It will be recalled that Lydall [35, 1968] explains the age patterns of earnings and of earnings inequality as a reflection of growth of abilities with age and experience, as observed by developmental psychologists. This, however, need not be viewed as an alternative interpretation, unless the observed patterns of age changes in productivity are

[22] The experience scale is not given by age, but by years spent in the labor force, assumed to start at completion of schooling.

[23] In most data the logarithmic *age profiles* of annual earnings of various school groups are clearly divergent (fan out). In U.S. data, the logarithmic *experience profiles* of annual earnings are convergent. Interestingly, the convergence vanishes in estimated hourly earnings data (based on Fuchs [22, 1967]).

somehow intrinsic to the individuals. Psychologists do note that it is difficult to isolate intrinsic age patterns in productivity from age-changes affected by the individuals' adaptation, such as training or health care, that is, investments in human capital. To the extent that adaptation is important, the analysis of human capital investment behavior contributes to the understanding of the observed "learning curves".[24] Of course, the converse may also be true.

The psychological data to which Lydall refers may simply reflect effects of differential training on productivity, a major focus of the human capital model. The data may also show differential growth in productivity even without differential training (investments) among the individuals. But this possibility is also implicit in the human capital model, so long as the existence of investment is not denied. For simplicity, look at gross earnings in equation (2):

$$E_{ji} = X_i + r_i \sum_j C_{ji}$$

Assume that intrinsic dispersion $\sigma^2(X_i)$ does not change with age. For given levels of investment C_i, r_i the average rate of return can be viewed as an index of ability. Assume $\sigma^2(r_i) > 0$. The variance of earnings will grow with age only when $C_{ji} > 0$, even if $\sigma^2(C_{ji}) = 0$. This is most simply shown by neglecting the X_i term.

$$\sigma^2(E_{ji}) = \left(\sum_j C_j \right)^2 \sigma^2(r_i)$$

Since the first term on the right hand side increases with j, the variance rises with age.

[24] In this connection note the following statement in a recent survey of developmental psychology [8, Birren, 1968]: "Except for individuals with cumulative injuries or problems of health, worker performance up to age 60 should be little influenced by physiological changes in aging," and, "at the present time there is little evidence to suggest that there is an intrinsic age difference in learning capacity over the employed years, *i.e.*, up to age 60."

A similar monotonic growth of relative variances can be derived from the logarithmic formulation (eq. 6).

A general approach is to assume both $\sigma^2(C_{ij}) > 0$ and $\sigma^2(r_i) > 0$. The empirical implications are basically the same as before, except for the conclusion (previously shown in eq. (3)) that at given years of experience, earnings inequality should increase with level of schooling.

Let us once again summarize the important, though not unconditional, predictions of the human capital analyses:

1. *Inequality:* (a) Relative and dollar dispersions are positively related to rates of return and to the dispersion in the distribution of schooling.

(b) Dollar dispersion increases with schooling and experience. Relative dispersion increases with experience, though initial reversals are plausible.

(c) The relation between level of schooling and relative dispersion is complex: it is likely to be positive at given experience intervals but it need not be in the aggregate.[25]

2. *Skewness:* There are several distinct, but not mutually exclusive explanations: (a) the distribution of schooling is roughly symmetric. This creates a tendency toward positive skewness of earnings. More specifically, if symmetry is assumed in investments measured in time equivalents, as well as in "raw earnings" X_i, two implications follow: Dollar skewness tends to be positive, but logarithmic skewness negative. Moreover, when subgroups are distinguished by average level of investment (such as by schooling and experience), positive dollar skewness tends to be larger and negative log-skewness smaller the higher the schooling or experience level. (b) Interaction of ability and investment: even if the distribution of dollar in-

[25] Empirical findings are mixed, often showing a U-shaped relation. This may be due in part to less rapidly growing experience profiles in the higher schooling groups, and to differential age distributions, a consequence of strong educational trends.

vestments C_i and of average rates of return r_i were symmetric, but r_i and C_i not excessively negatively correlated, earnings would be positively skewed. (c) Dollar variances increase with age and with schooling. The resulting positive correlation between means and variances of component distribution is sufficient to create aggregate positive skewness. It also creates humpedness (leptokurtosis) in the shape of the aggregate distribution, as both Rutherford's [55, 1955] and Hill's [30, 1959] mathematics prove in a different context.[26]

Note, in conclusion, that the predicted phenomena are relatively well established features of empirical earnings distributions.

Even in terms of the qualitative implications reported thus far, the predictive range of the human capital model clearly exceeds that of the random shock models. The stochastic models say nothing about age and experience profiles of earnings. They do little more than predict a monotonic growth of variances with age. Distinctions between aggregates and components, or between absolute and relative parameters, are outside the reach of these models.

5. Quantitative Accounting of Earnings Inequality: The Correlation between Earnings and Investment in Human Capital. The human capital model provides interpretations for a large variety of qualitative features of observed distributions of earnings. Major interest, however, attaches to the quantitative dimension of earnings inequality. Can the model help in measuring the extent to which observed inequality in earnings can be attributed to individual differences in investment?

By definition of the earnings function (2), if investment costs and average rates of return were available for each worker, all of the inequality, except a random component, would be accounted for. Comprehensive information on individual investments is almost entirely restricted to schooling, measured in years. Information on post-school investments, except perhaps for medical care, is available only in a highly fragmentary fashion.

As already mentioned, the heroic statistical specification of the schooling model (1a) yields very low explanatory power and biased estimates of rates of return. This may be due to the exclusion of post-school investments. In principle, their inclusion can be accomplished by the expanded model in equation (6).

$$(6)\quad \lg Y_{ij} = \lg Y_0 + r_s s_i + \sum_{t=1}^{j-1} r_t k_{it} + \lg(1 - k_{ij}) + u_{ij}$$

Even if information on post-school investments k_{ij} were available, individual rates of return could not be directly observed. The residual u_{ij} would therefore contain differentials in school quality not measured by years, as well as individual differences in average rates of return. Note that the rates of return to schooling r_s and to post-schooling investment r_j are allowed to differ. The partial coefficient r_s measures the rate of return to schooling alone, a concept not measured in the usual calculations.[27] Its estimate in (6) is unbiased, as it applies to schooling, net of post-school investments.

Of course, neither k_{ij} nor even typical k_{js} within groups are observable. However, the effects of the latter on earnings could be estimated by expressing k_j as a declining function of j, as optimization theory suggests. A statistical function would be:

$$(7)\quad \lg Y_{ij} = \lg Y_0 + r_s s_i + f(r_j, k_j) + v_{ij}$$

[26] It is interesting to recall at this point that in puzzling over the paradox of presumably symmetric abilities but skewed earnings, Pigou proposed the possibility that aggregate skewness might result from a merger of homogeneous, symmetric components. H. P. Miller [38, 1955] offered some rather weak evidence in support of this hunch, but no explanation why the merger should produce positive skewness.

[27] They measure rates of return to the mix of incremental schooling and of post-school investment.

Compared to residual u in (6) residual v in (7) is augmented by individual variation in post-school investments as well as by errors in using a functional form $f(r_j, k_j)$. To that extent (7) understates the explanatory power of the investment variables s_i and k_{ij}.

The functional form of the investment profile in (7) must be fitted by experiment, since there is no theory to specify it. It can be shown that a linearly declining net investment profile yields a parabolic earnings profile in logs [45, Mincer, 1969; 31, Johnson, 1969].

If $k_j = k_o - (k_o/T)j$, where k_o = investment ratio at the start of work experience, T = span of investment period. Then:

$$\text{(7a)} \quad \lg Y_j = \lg Y_0 + r_s s + rk_0 j - \frac{rk_0}{2T} j^2 + \lg\left(1 - k_0 + \frac{k_0}{T} j\right) + v$$

If investment k_j declines exponentially:

$$k_j = k_0 e^{-\beta j}$$

Then:

$$\text{(7b)} \quad \lg Y_j = \lg Y_0 + r_s s + \frac{rk_0}{\beta}(1 - e^{-\beta j}) + \lg(1 - k_0 e^{-\beta j}) + v$$

$\lg E_j$ is the same, except that it excludes the term before the residual v.

In this case, the earnings function E_j is a familiar growth curve, known as the Gompertz curve.[28]

Some experiments with *individual observations* of the 1:1,000 U. S. Census Sample of 1960 indicate that the Gompertz curve fits somewhat better than the parabola, when the distribution is restricted to about 40 years of experience in each schooling group.[29] The results of applying this equation to the individual annual earnings of white, urban males are: (a) A coefficient of determination of 35 percent. (The coefficient rises to 55% when weeks worked (in logs) are included in the regression). (b) An estimate of the rate of return to schooling above 10%, and not clearly different from the rate of return to post-school investment.[30]

Even with the use of only two variables, years of schooling and of experience, the explanatory power of specification (7) compares quite favorably with results of statistical studies of comparable microeconomic data which employ a large number of explanatory variables on a more or less *ad hoc* basis.

Adams [1, 1958] found a coefficient of determination of 43 percent in an analysis of earnings of white males in 1949 reported in the Michigan Survey. In addition to education and a parabolic age variable, his explanatory factors included geographic region, city size, employment status, and occupation. Hill [30, 1959] achieved a correlation of about the same magnitude in an analysis of a 1953 sample of male full-time wages and salaries in Britain. His variables included age, occupation, region, and industry. Morgan et al. [47, 1962] explain 34 percent of the variance of wage rates of family heads in the

[28] An interesting implication of the Gompertz experience profile is that human capital accumulates at a rate proportional to the percentage gap between the current and eventual peak level. It can also be shown that [*cf.* 19, Eisner and Strotz, 1963] this pattern of capital accumulation arises as an optimal life-time distribution when the cost function of producing human capital is quadratic (in logs). In an analysis of optimal time-distribution of investments specific to firms, Rosen [53, 1969] derives the same Gompertz formula for "age-progress curves" of firms.

[29] Johnson [31, 1969] applied the parabolic fit (7a) to grouped earnings profiles estimated by Hanoch [28, 1965]. Johnson interprets the k coefficient as gross, not net investment. This enables him to estimate an assumed fixed depreciation rate. Its estimate was about 6% per year.

A similar procedure applied to the micro-data yields a very small difference in explanatory power, and a very small estimated depreciation rate, whether the parabola or Gompertz are used. However, in contrast to Johnson's work, these experiments [45, Mincer, 1969] excluded individuals past age 65.

[30] If correct, this suggests that there is little error in calling the directly calculated rates by the name "rates of return to schooling."

1959 Michigan Survey by regression on education, age, occupation, city size, geographic mobility, employment status, as well as indexes of ability and motivation. Hanoch [28, 1965] gets a similar coefficient of determination for 1959 annual earnings of males in the Census 1:1,000 sample, using schooling, age, race, location, marital status, weeks worked, family size, and industry.

To repeat, in the statistical regression based on (7) a third of earnings inequality is attributable to schooling, measured in years, and to *average* (within schooling groups) variation of post-school investment with age, The remainder contains individual differences in post-school investment, in quality of schooling, in ability, and in other "transitory" income variations. Because the first two are components of human capital investment, regressions based on (7) understate the explanatory power of human capital investments.

One indirect procedure [45, Mincer, 1969] makes it possible to include the additional contribution of individual variation in post-school investment that regression (7) does not capture. Recall that at a relatively early stage of work experience ($\hat{j}$ in fig. 2) each investor reaches the "overtaking point": This is the level of earnings (Y_s in fig. 2) a worker would receive if he ceased investing after completion of schooling. Net earnings of individuals at that stage of their careers are not affected by their post-school investments. Consider the distribution of earnings of individuals observed only in the "overtaking" period: Denote $\sigma^2(u')$ the residual variance from the regression of log-earnings at overtaking on years of schooling. On the assumption of homoscedasticity, $\sigma^2(u')$ is an estimate of the residual variance in the expanded, but not directly observable, regression model (6). Therefore, the ratio of $\sigma^2(u')$ to aggregate earnings inequality $\sigma^2(\lg Y_{ijs})$, subtracted from unity, is an estimate of R^2, the proportion of total earnings inequality attributable to measured variation in schooling (in years only) and in post-school investments. Several variants of this procedure applied to earnings of white, urban males in 1959 yield estimated R^2 ranging from a half to two thirds, with lower coefficients for all annual earnings and higher for full-period earnings.[31]

6. Ability, Opportunity, and Investment. Do we necessarily expect a strong correlation between investments in human capital and earnings? The answer to this question is neither obvious nor invariant. Not surprisingly, a better understanding of the relation between investment and earnings requires an understanding of the factors determining investment.

Becker's recent work [4, 1967] treats this problem in a conventional and highly illuminating demand-supply framework. The analysis yields insights into the meaning of gross and partial correlations between investment and earnings, suggests approaches to the question whether the inequality in earnings is due mainly to inequality in opportunities or in abilities, and clarifies some of the normative issues that arise in the context of income distribution.

In effect, the investment-earnings relation is viewed as a "reduced form" resulting from two simultaneous structural relations: demand functions (D_i) relate individual investments to marginal rates of return on them, and supply functions (S_i) relate the obtainable volume of funds for such investment purposes to their marginal "interest" costs.

The amounts individuals invest, their marginal and average returns, and therefore their earnings are simultaneously and optimally

[31] The estimated R^2 is an underestimate, because variation in the quality of schooling is still left in the residual. The variation is not negligible. Judging by figures quoted by Becker [3, 1964], the coefficient of variation in expenditures on a college education in New York state alone was no less than the coefficient of variation in the national aggregate distribution of years of schooling.

determined by the intersections of the D_i and S_i curves, as shown in Fig. 4. Here DC is total investment (in dollars). Earnings are given by the area under the demand curve ODPC, that is by $\bar{r}C$, where $\bar{r}$ is the average height of the relevant segment of the demand curve.

The downward slope in the demand curves represents diminishing returns to self-investments. Presumably, limitations of human capacity, the "fixed human body," produce this effect. Differences in levels of demand curves represent individual differences in productivities, or abilities. On the supply side, curves are upward sloping to represent the increasing difficulty of financing investments of increasing size. There are differences among individuals in the costs of the same volume of investment: for example, students differ in family wealth and in costs and availabilities of loans and of scholarships. These differences are represented by the dispersion in levels of supply curves.

It is important to note that the dispersions in D_i and in S_i can be viewed as broadly corresponding to inequalities of ability and of opportunity, respectively, and that some of the factors involved are not independent: Tastes and motivation can be reflected in greater productivity (higher level of D) as well as in greater willingness to reduce consumption in order to finance investment (lower S). Discrimination can be reflected both in lower D and in higher S. More important, some of the dispersion in demand curves represents differences in opportunity disguised as differences in ability: the importance of the home environment and of social contacts is unlikely to have been isolated and "costed" as a component of investment. It is nonetheless true, that social policy can more easily affect the dispersion in supply curves than that in the demand curves. With fixed demand curves, an evolutionary or political change in the distribution of opportunities will alter the distribution of average rates of return, thereby affecting the distribution of earnings as well as the correlation between investment and earnings.

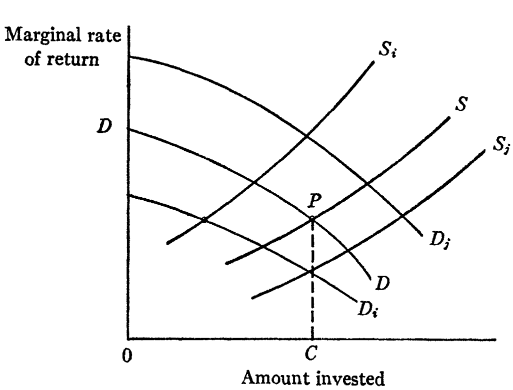

Figure 4

Among the possible configurations of the distributional equilibrium in Fig. 3, three special cases are of particular interest—1) Equality of opportunity: this is defined by a common supply curve for all individuals. Here investments, marginal (and average) rates, and earnings are all positively related. 2) Equality of ability: This is defined by a common demand curve. Here investment and earnings are positively related, but investment and rates of return are negatively correlated. 3) A perfect positive correlation between ability and opportunity:[32] Individuals with greater productivity have lower costs in financing their investments. Again investment and earnings are positively related, but investment and rates of return can relate positively or negatively.

Each of these and only these conditions yield a perfect correlation between investments and earnings. If both inequalities of ability and of opportunity are sizeable, a substantial correlation between investment and earnings must reflect primarily a strong correlation between ability and opportunity.[33]

[32] A negative correlation is conceivable, but not plausible.

[33] An interesting and important research question is, to what extent does this correlation reflect motivation

Otherwise, there is a range of intersection points at the same levels of investment, creating residual dispersion in the regression of earnings on investments. If, in addition, the scatter of equilibrium points in Fig. 4 has a downward slope, as is suggested by findings that rates of return are, on the average, higher at lower levels of schooling [29, Hansen 1963; 3, Becker, 1964; 28, Hanoch, 1965], the inference is that inequalities in opportunity are larger than inequalities in ability.

Statistical studies of factors associated with earnings often include indicators of investment such as schooling and age, as well as of ability, such as IQ scores, and/or of opportunity such as family background or wealth. The structural model proposed by Becker can be helpful in interpreting some of the findings resulting from such studies. For example, the less accurate the measurement of investment, the stronger the positive correlation between ability and opportunity at measured levels of investment. The more precise the measurement of investment, the more negative this correlation becomes. Hence appropriate additional ability variables should remain significant in the earnings regressions, even net of investment. However, opportunity indicators may lose their significance or become "perverse," as the measurement of investment is improved.

One interesting insight is relevant to empirical studies which infer distributions of ability from distributions of earnings. The distribution of ability is not observable: it is given by the heights of the demand curves at fixed levels of investment. This distribution cannot be equated with the conditional distributions of earnings at fixed levels of investment. Only a subset of the ability distribution is reflected in a conditional earnings distribution, and the ability subsets observable at different levels of investment are not comparable. What is more important, given the distribution of abilities (a fixed set of D curves), the conditional distributions of earnings change with a change in opportunities. For example, if inequality of opportunity (dispersion in S) diminishes, the conditional variance in earnings diminishes as well. But this is not to be interpreted as an equalization of abilities. Such a change, incidentally, gives rise to an increased correlation between investment and earnings. These are two sides of the same story, and not competing hypotheses in explaining the change.

There are some interesting insights into normative questions, such as: under what conditions does more equal opportunity or greater productive efficiency increase the equality of earnings? An efficient distribution of earnings is one in which marginal rates of return on human capital investments are the same for all individuals. Hence given a dispersion of ability, equality of opportunity does not suffice to bring about maximum product, unless the common supply curve is horizontal—no segmentation in the capital market. Note, however, that maximum efficiency can also be achieved with highly unequal opportunity, when a perfect correlation of ability and opportunity yields a horizontal locus of intersections of S and D curves in Fig. 4. An increased equality of opportunity (without change in the slopes of the S-curves) may, therefore, mean decreased efficiency. It will, to be sure, reduce the inequality of earnings. On the other hand, increased efficiency resulting from flattened S-curves, without a change in their dispersion, increases the inequality of earnings.

Perfect equality of opportunity and maximum efficiency would characterize a meritocratic society. Such a society, which many consider attractive, is quite consistent with large earnings inequalities, all of which depend on differences in ability. If abilities are

and encouragement of abler individuals which is independent of family background and wealth, or links between parental wealth, environment, and ability? Social mobility is furthered by the former and inhibited by the latter.

believed to be invariant and inherited, this vision becomes rather less attractive. However, technical and cultural changes and associated uncertainties would continue to reshuffle abilities and earnings, providing safeguards against a new type of caste system.

Even if some objectives are ultimately inconsistent with one another, this need not be true about improvements in conditions which are not generally optimal. For example, if inequality of opportunity dominates, and if the correlation between ability and opportunity is far from perfect, a reduction in inequality of opportunity is likely to lead to greater efficiency and greater equality in earnings. If it is also true that the rate of return on investments in human capital is, on the average, higher than on other forms of capital, a greater aggregate investment in human capital will also produce greater efficiency and greater equality.

7. Long and Short Run Inequality. Once ability and opportunity are introduced as determinants of investment, earnings differentials can no longer be considered as wholly compensatory. Rents or "profits" from investment in human capital arise as the differences between returns ($\bar{r}\,C$), where $\bar{r}$ is the average rate of return, and repayment costs ($\bar{i}\,C$), where $\bar{i}$ is the average interest cost. They appear in Fig. 4 as differences between the areas under the individual demand and supply curves. Barring some perverse configurations of the curves, it is easily seen that the size of profits is positively correlated with earnings as well as with investments. The present value of profits is $PV_i = 1/r_i\ (\bar{r}_i - \bar{i}_i)C_i$, where r_i is the discount factor. Present values also tend to correlate positively both with earnings and with investment. It is interesting to observe that (relative) inequality in profits may or may not be smaller than the inequality in current earnings. In the special case when the curves have a constant and common elasticity, Becker shows that factors determining this "ultimate" inequality are exactly the same as those determining the inequality in current earnings.

If the distinction between the long and short run is defined in terms of length of the accounting period, observed earnings inequality tends to diminish as the accounting period is lengthened. The major components of individual income variation which average out in this process are: the longer-run age profile of earnings, and the short run cyclical and random fluctuations. If age variation in earnings over the working life is attributable to the life-cycle allocation of net post-school investments, the inequality of earnings among individuals, in which this variation is eliminated, would be given by the variance of earnings at the "overtaking" stage of experience. A comparison of the variance in earnings of individuals with about a decade of labor force experience with the aggregate variance, shows the former to be about 25 percent smaller than the latter.[34]

Estimates of the contribution of short-run temporal fluctuations in earnings to the overall variance in earnings are comparable in size to the age effects [32, Kravis, 1962; 28, Hanoch, 1965; 45, Mincer, 1969]. In a procedure[35] which smoothes out both short-run and life-cycle variation, Summers [59, 1956] found earnings inequality reduced by over a half.

Short-run variation in earnings is the exclusive focus of random shock theories of income distribution. It will be recalled that these models generally predict increasing variances of earnings with age as well as certain specific mathematical forms of the earnings distribution. The greater predictive

[34] Note that this method of eliminating age variation in earnings is not quite the same as an extension of the accounting period to cover all of the working life.

[35] Summers estimated lifetime average annual distributions from panels of 500 spending units, separated by one year. The estimates are based on parameters of first-order difference equations, separate for each of ten year age intervals. The estimates for a 40-year period were not very sensitive to assumed initial distributions.

power of the human capital model was pointed out before. Where the two models yield similar predictions, they interpret the empirical phenomena differently. Thus, the stochastic models interpret temporal variation in income as a matter of chance. In contrast, human capital models view much of the temporal variation as a systematic and persistent consequence of cumulative investment behavior. Discrimination between the two views can be sought in so-called panel correlations of earnings of the same cohort in two different time periods. The random shock models predict a fixed rate of decay in the coefficients of determination, as the interval between the two time periods is lengthened. It also predicts a fixed correlation for a fixed size interval, regardless of the life-cycle stage. Accumulating empirical evidence, [21, Friedman 1957; 30, Hill, 1959; 32, Kravis 1962; 45, Mincer, 1969] indicates, however, that the decay in panel correlations is weak and decelerating with span. It also shows weak correlations in intervals bracketing the "overtaking" age, and strong correlations afterward [44, 45 Mincer, 1962, 1969]. Both phenomena are consistent with human capital analysis, particularly when it is remembered that observed earnings are net rather than gross of investment.

The finding that the systematic (investment) component may account for a large part of the temporal variation in earnings does not preclude a residual random component; panel correlations are certainly less than unity. But even the residual component need not have the stochastic properties specified in the random shock models. Instead of being independent of the previous level of income, thereby creating an explosive variance, the random "transitory" component may be unrelated to a latent, "permanent" level of income, so that the variance does not change much over time, if at all. Under this formulation, introduced by Friedman [21, 1957], the contribution of the short-run fluctuation to total income inequality was estimated from income and consumption data to be as much as 30 percent. This figure is roughly comparable to the contribution of the dispersion in weeks worked during the year to annual earnings inequality.[36]

The short-run, within-year, and cyclical variation in earnings reflects in large part labor turnover, job and labor force mobility, unemployment and illness. In many ways, not as yet fully worked out, economic theory [3, Becker, 1964; 51, Reder, 1969] predicts that the incidence of the employment phenomena is inversely related to age (experience) and to the level of human capital. The greater average departure from full period work the larger the added dispersion in earnings introduced by it. For these reasons variances of earnings are augmented at low levels of skill and early stages of experience. The short-run component is, therefore, a force in the direction of negative skewness [57, Staehle, 1943; 46, Morgan, 1962; 32, Kravis, 1962].

These amendments work in the opposite direction to some of the predictions of human capital models in which the short-run component of earnings was ignored. Since earnings are a product of wage rates and of "hours" of work, the degree to which these amendments tend to obscure effects previously implied depends on the size of the inequality (log variance) in "hours" relative to the inequality in wage rates.

8. Female and Family Earnings Distribution. The relative contribution of employment ("hours") dispersion to earnings inequality is not small in population groups with full and permanent labor force attachment, but can be quite substantial in groups whose attachment is, on the average, weak. Men and women (particularly married men

[36] However, the two components of income variation are not coextensive. Some of the employment variation is "permanent," and some of the age variation is discounted by consumers.

and married women) exemplify these differences in labor force behavior. The distribution of earnings of all adult males is, therefore, roughly similar to the distribution of earnings of full-period male workers. For women, however, the all earnings distribution is quite different from the full-period earnings distribution. Thus the inequality in annual earnings of all women workers is larger than the inequality in the comparable male distribution, while the opposite is true of full time earnings [61, Woytinsky, 1953].

Some of the differences between earnings distributions of male and females are explainable by the effects of labor supply behavior on human capital investment decisions. Individuals who expect to spend only a part of their adult lives in the labor force have weaker incentives to invest in forms of human capital which primarily enhance market productivities than persons whose expected labor force attachment is permanent. Women are likely to invest less than men in vocational aspects of education and particularly in on-the-job training. This is reflected in the comparative (to males) structure of their full-time earnings by flatter age-earnings profiles, smaller variances within school and age classes, and a lesser aggregate inequality of earnings [40, Mincer, 1957].

One important aspect of income distribution, not yet discussed, is the definition of the recipient unit. Analyses of consumption behavior and notions of economic welfare are more closely linked to family rather than to personal distributions of income. From this point of view *sums* of male and female earnings are of greater interest than the differences.

As a matter of arithmetic, dollar dispersion in family earnings is a positive function of the variances in earnings of family members and of the correlation between these earnings. For relative dispersion the relevant correlation is between levels and shares.[37] The sign and size of these intra-family correlations depend, in part, on labor supply functions. In standard economic theory the amount of time a person allocates to money earning activity is positively related to his money earning power (wage rate), and negatively to his total real (family) income. The positive correlation between earning powers of husbands and wives is therefore a factor which increases the variance of family earnings relative to the variance of husband's earnings. However, the income effect is a force which tends to reduce the variance of family earnings. The empirical results are: a larger dollar variance and a somewhat smaller relative variance in family earnings than in husbands' earnings. [42, 45, Mincer, 1960, 1969]. The results are entirely consistent with parameters of market labor supply functions that have been estimated for married women [42, 43, Mincer, 1960, 1962; 13, Cain, 1966].

It should be noted that these findings have been observed in data which include property income in family income. Because of the well known underreporting of non-employment incomes, it cannot be concluded that total family income, correctly measured, would behave similarly. Nor can we conclude that real family income patterns are similar to the observed money income distributions. Indeed, the partial labor force attachment of wives is largely the obverse of the importance of their non-money contributions to real family income.

In Lieu of a Conclusion

The relative success of the human capital model in explaining a variety of features in the observed distribution of earnings is actually something of a surprise. This is be-

[37] Let $Y_F = Y_H + Y_W = Y_H(1 + R_W)$, where $R_W = Y_W/Y_H$

Then

(a) $\delta^2(Y_F) = \delta^2(Y_H) + \delta^2(Y_W) + 2\,\text{Cov}\,(Y_H, Y_W)$

And

(b) $$\delta^2(\lg Y_F) = \delta^2(\lg Y_H) + \delta^2 \lg (1 + R_W) + 2\,\text{Cov}\,[\lg Y_H, \lg (1 + R_W)]$$

cause the model does not directly apply to cross-sections. The theory deals with lifetime behavior of individuals, not with differences among individuals of different ages. There are special cases where the distinction between longitudinal (cohort) analysis and contemporaneous (cross-section) analysis would not matter. These are the cases of a stationary economy, or of an economy in which changes are "neutral" with respect to categories entering the human capital model. In the more general case, however, modifications introduced by secular change should be taken into account when the models are applied to cross-section.

There are a few recent empirical studies of moving cross-sections of income [39, Miller, 1960; 12, Brady, 1965; 18, David, 1969; 45, Mincer, 1969]. Abstracting from cyclical and other fluctuations in the economy, the findings confirm some of the qualitative features of cross-sections: concave age profiles of income, differential age profiles by occupation or education, and variances (in dollars or logs) increasing with age. There are severe data limitations in this area, but it deserves a great deal of research: Cohort analyses should help not only in a better understanding of the cross-section, but also in providing insights into secular changes in income distribution.

Another very important issue to which occasional allusions were made in the survey is the distinction between effects of human capital investments on productivity and on the allocation of time to earning activity. Over a period of time, earnings are a product of hourly wage rates and hours worked during the period. The latter are affected by human capital investments as well as the former, in a number of ways. The theory of the effects of worker and employer human capital investments on labor mobility and employment distributions [3, Becker, 1964] can be elaborated and applied to sort out the effects in observed earnings distribution. For sex, age, occupation, and industry comparisons of earnings, the employment effects are clearly a basic research need.

At a deeper level, there is no shortage of positive and normative questions about investment in human capital as a dependent variable. But this takes us beyond the scope of an already overlong survey.

References

1. ADAMS, F. G. "The Size of Individual Incomes: Socio-Economic Variables and Chance Variation," *Rev. Econ. Statist.*, Nov. 1958.
2. AITCHISON, J., and BROWN, J. A. C. *The Lognormal Distribution*, Cambridge University Press, 1957.
3. BECKER, G. S. *Human Capital*, Columbia University Press for NBER, 1964.
4. ———. "Human Capital and the Personal Distribution of Income," *W. S. Woytinsky Lecture No. 1*, University of Michigan, 1967.
5. BECKER, G. S., and CHISWICK, B. R. "Education and the Distribution of Earnings," *American Economic Review*, May 1966.
6. BEN-PORATH, Y. "The Production of Human Capital and the Life-Cycle of Earnings," *J. Polit. Econ.*, August, 1967.
7. ———. "The Production of Human Capital Over Time," Paper presented at NBER *Conference on Research in Income and Wealth*, Nov. 1968.
8. BIRREN, J. E. "Psychological Aspects of Aging," in *International Encyclopedia of the Social Sciences*, Vol. 1, 1968.
9. BJERKE, K. *Income and Wage Distributions*, Copenhagen, May 1969.
10. BOISSEVAIN, C. H. "Distribution of Abilities Depending Upon Two or More Independent Factors," *Metron*, 1939.
11. BOWMAN, M. J. "A Graphical Analysis of Personal Income Distribution in the United States," *American Economic Review*, Sept. 1945.
12. BRADY, D. S. *Age and Income Distribu-*

tion, Social Security Administration, Research No. 8, Washington, 1965.
13. CAIN, G. G. *Married Women in the Labor Force*, The University of Chicago Press, 1966.
14. CHAMPERNOWNE, D. G. "A Model of Income Distribution," *Econ. J.*, 1953.
15. CHISWICK, B. R. "Human Capital and Personal Income Distribution by Region," unpublished Ph.D. dissertation, Columbia University, 1967.
16. ———. "The Average Level of Schooling and the Intra-Regional Inequality of Income: A Clarification," *Amer. Econ. Rev.*, June, 1968.
17. DALTON, H. *Some Aspects of the Inequality of Incomes in Modern Communities*, G. Routledge & Sons, 1920.
18. DAVID, M. "Time-Series Versus Cross-Section Lifetime Earnings Patterns in Different Occupational Groups," Paper No. 6941, *Social Systems Research Institute*, University of Wisconsin, August 1969.
19. EISNER, R., and STROTZ, R. H. "Determinants of Business Investment," in *Impacts of Monetary Policy*, by the Commission on Money and Credit, Prentice Hall, 1963.
20. FRIEDMAN, M. "Choice, Chance and the Personal Distribution of Income," *J. Polit. Econ.*, August, 1953.
21. ———· *A Theory of the Consumption Function*, Princeton University Press, 1957.
22. FUCHS, V. R. "Differentials in Hourly Earnings by Region and City Size," Occasional Paper 101, NBER, 1967.
23. GALTON, F. *Hereditary Genius*, Macmillan 1869.
24. GARVY, G. "Inequality of Income: Causes and Measurement," *Stud. Income and Wealth*, Vol. 15, NBER, New York, 1952.
25. GIBRAT, R. *Les Inégalités Economiques*. Paris, Sirey, 1931.
26. GOODMAN, L. "On the Exact Variance of Products," *J. Amer. Statist. Assoc.*, December, 1960.
27. GROSSMAN, M. "The Demand for Health," unpublished Ph.D. dissertation, Columbia University, 1969.
28. HANOCH, G. "Personal Earnings and Investment in Schooling," unpublished Ph.D. dissertation, University of Chicago, 1965.
29. HANSEN, W. L. "Total and Private Rates of Return to Investment in Schooling," *J. Polit. Econ.*, April, 1963.
30. HILL, T. P. "An Analysis of the Distribution of Wages and Salaries in Great Britain, *Econometrica*, 1959.
31. JOHNSON, T. "Returns from Investment in Schooling and On-the-Job Training," unpublished Ph.D. dissertation, North Carolina State University at Raleigh, 1969.
32. KRAVIS, I. *The Structure of Income*, University of Pennsylvania, 1962.
33. LEWIS, H. G. "Hours of Work and Hours of Leisure," Proceedings of the Industrial and Labor Relations Association, 1957.
34. ———. *Unionism and Relative Wages in the United States*, University of Chicago Press, 1963.
35. LYDALL, H. *The Structure of Earnings*, Oxford Press, 1968.
36. MANDELBROT, B. "The Pareto-Levy Law and the Distribution of Income," *Int. Econ. Rev.*, May 1960.
37. ———, "Paretian Distributions and Income Maximization," *Quart. J. Econ.*, Feb., 1962.
38. MILLER, H. P. "Elements of Symmetry in the Skewed Income Curve," *J. Amer. Statist. Assoc.*, March 1955.
39. ———, "Annual and Lifetime Income in Relation to Education 1939–59," *Amer. Econ. Rev.*, Dec., 1960.
40. MINCER, J. "A Study of Personal Income Distribution," unpublished Ph.D. dissertation, Columbia University, 1957.
41. ———, "Investment in Human Capital

and Personal Income Distribution," *J. Polit. Econ.*, August, 1958.
42. ———, "Labor Supply, Family Income, and Consumption," *Amer. Econ. Rev.*, May, 1960.
43. ———, "Labor Force Participation of Married Women," in *Aspects of Labor Economics*, Princeton Press for NBER, 1962.
44. ———, "On the Job Training; Costs, Returns, and Some Implications," *J. Polit. Econ.*, Supplement, October 1962.
45. ———, "Schooling, Age, and Earnings," in *Human Capital and Personal Income Distribution*, NBER in progress, 1969.
46. MORGAN, J. N., ET AL, *Income and Welfare in the United States*, McGraw-Hill, 1962.
47. ———, "The Anatomy of Income Distribution," *Rev. Econ. Statist.*, August, 1962.
48. PIGOU, A. C. *The Economics of Welfare*, Macmillan, 1920.
49. REDER, M. W. "Wage Structure Theory and Measurement," in *Aspects of Labor Economics*, Princeton Press for NBER, 1962.
50. ———, "The Size Distribution of Earnings," in *The Distribution of the National Income*, International Economic Association, St. Martin's Press, 1968.
51. ———, "A Partial Survey of the Theory of Income Size Distribution," in *Six Papers on the Size Distribution of Income and Wealth*, Lee Soltow ed., Columbia Press for NBER, 1969.
52. REES, A. *The Economics of Trade Unions*, University of Chicago Press, 1962.
53. ROSEN, S. "The Supply of Human Capital," mimeograph, NBER, 1969.
54. ROY, A. D. "The Distribution of Earnings and of Individual Output," *Econ. J.*, Sept. 1950.
55. RUTHERFORD, R. S. G. "Income Distributions: A New Model," *Econometrics*, July, 1955.
56. SCHULTZ, T. P. *Personal Income Distribution Statistics of The United States*, prepared for the Joint Economic Committee of Congress, 1965.
57. STAEHLE, H. "Ability, Wages, and Income," *Rev. Econ. Statist.*, Feb., 1943.
58. STIGLER, G. J. *The Theory of Price*, 3rd Edition, Macmillan, 1966.
59. SUMMERS, R. *An Econometric Investigation of the Size Distribution of Lifetime Average Annual Income*, Technical Report No. 31, Stanford University, 1956.
60. TINBERGEN, J. "On the Theory of Income Distribution," *Weltwirtschaftliches Archiv*, 1956.
61. WOYTINSKY, W. S. *Employment and Wages in the United States*, Twentieth Century Fund, 1953.