General Equilibrium

Problems and Prospects

Edited by

Frank Hahn and Fabio Petri

Routledge Siena Studies in Political Economy

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2002

Routledge
Taylor & Francis Group
LONDON AND NEW YORK

General Equilibrium

In recent years highly renowned economists have called the usefulness of general equilibrium theory into question. This superb new book brings together leading economic theorists with important new contributions to the ongoing debate.

General equilibrium theorists including Frank Hahn, Alan Kirman, Franklin Fisher, Michael Magill, Mordecai Kurz and Michio Morishima debate strengths, weaknesses, recent advances and possible futures with leading 'dissenting' thinkers including Duncan Foley, Pierangelo Garegnani, Herbert Gintis and Bertram Schefold who seek to explain their rejection of the general equilibrium approach and the alternative directions they consider more promising. The book contains critical surveys of the state of the arts and new results in many central sub-fields of general equilibrium theory and of value theory more generally. Also, the confrontation between advocates of different research programmes produces important clarifications on the basic disagreements and open issues, eliminating misunderstandings and creating the opportunity for more fruitful debate among economists of different persuasions. The topics discussed include existence of equilibrium in infinite-horizon economies; incomplete markets; indeterminacy; stability; the relevance of the 'Cambridge controversies' for general equilibrium theory; expectation formation and learning; the microfoundations of macroeconomics; the long-period method; game-theoretic and evolutionary approaches to behaviour modelling; statistical equilibria; the differences between the classical and the general equilibrium approaches to value and distribution, and the roles of general equilibrium theory.

General Equilibrium: Problems and Prospects will be of essential interest to serious economic theorists from all schools of thought.

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Introduction by Fabio Petri

1 AIM AND CONTENTS OF THIS BOOK

1.1 The present book is based on the lectures delivered at the XII Workshop (4–11 July 1999) of the International School of Economic Research of Siena University.

The purpose of these yearly "Summer Schools" is to give a number of internationally selected advanced graduate students and junior faculty members exposure to the major contending research programmes, each time in a different important field of economic research, and at the same time, the opportunity of direct contact with the leading figures in the field, profiting from the fact that the lecturers are in residence for the entire week of the Workshop – a fact which also favours interaction among the lecturers themselves.

The topic of the 1999 Summer School was "General Equilibrium: Problems, Prospects, Alternatives". The Economics Department of Siena University, which selects the Workshops' themes, chose to dedicate a Workshop to this topic in view of the fact that, while research on general equilibrium (GE) theory continues and is exploring new fields, in recent years some well-known GE specialists have expressed doubts as to its usefulness as a positive theory, and some other ones have shifted to exploring very different approaches. It was therefore decided to have a Summer School on three main themes: (1) the GE specialists' assessment of the state of health of the GE research programme, together with some assessment by outside critics, including a historical perspective on the development of GE theory; (2) the main research strategies now being pursued by GE specialists; (3) the possible alternatives.

The attempt was accordingly made to have lecturers reflecting the present diversity of opinions on these issues. Unfortunately not all invited scholars were able to come, and as a result some important viewpoints were not represented at the School. Nonetheless the diversity of positions was, to put it mildly, considerable, and debate was often lively (its flavour is given by Chapter 18, which records the interventions in the Final Discussion session which concluded the School). In order to decrease the risk of a presentation too biased in one direction, this volume has two Editorial Introductions, written by economists of different persuasions.

One session of the School was dedicated to the memory of Prof. Bruno Miconi, who had been among the proponents and organisers but sadly disappeared shortly before the School was held. Bruno's was a lively, stimulating presence whose premature departure is greatly regretted by the Economics Department of Siena University, which dedicates this volume to his memory.

1.2 I start by illustrating the contents of the book. As indicated, one main aim of the School was to ascertain the opinion of the practitioners of GE theory on the latter's state of health, and to survey the main research programmes currently carried on by GE specialists. In this area, the volume includes (in alphabetical order by author) ten contributions:

1 A chapter by Elvio Accinelli of the results on existence in infinite-horizon GE models;
2 A chapter by Aloisio Araujo[1] and Wilfredo Maldonado on the prospects, offered by the known, and by some new, results on learning processes, for the issue of equilibrium selection when there are multiple equilibria in macroeconomic models; it emerges that the learning dynamics can cause the economy to converge to steady states, to cycles, or to stochastic equilibria (sunspot equilibria);
3 A chapter by Franklin Fisher of the history of studies of the stability of GE, with an explanation in non-mathematical terms of his results on stability when one abandons the hypothesis that the auctioneer prevents disequilibrium decisions from being implemented;
4 A chapter by Frank Hahn on why macroeconomic analysis should detach itself from the Arrow–Debreu model and become more realistic, and on one way to achieve this aim: by admitting that macro variables may enter the individual agents' decision processes;
5 A chapter by Mordecai Kurz of his work on a theory of expectations alternative to rational expectations, with illustrations of the explanatory capacity of this theory when applied to stock markets volatility, to exchange rate fluctuations, and to the risk spread;
6 A chapter by Michael Magill and Martine Quinzii on incentives and stock markets, demonstrating that the new financial securities (call and put options etc.) are essentially "able to collectively mimic the ideal system of Arrow securities", i.e. to overcome the moral hazard problem caused by variable entrepreneurial effort which makes that "ideal system" not implementable;
7 Another chapter by Magill and Quinzii, on the difference which it makes, to the overlapping generations model with production, to assume that not only intra-generational but also intergenerational transfers are possible: it is shown that when intergenerational transfers on the bond market are permitted by infinitely lived intermediaries, then many of the important policy recommendations which are based on the Diamond model can change in significant ways, and some inefficiencies of the Diamond model disappear; it is further shown that, if capital once installed in a firm cannot be transferred to other firms without incurring significant "adaptation costs", then the market for the transfer of ownership of firms, i.e. the stock market, provides a natural mechanism for the intergenerational transfer of funds;
8 A chapter by Michael Mandler on the possibility that, when technology is of the activity–analysis type (i.e. when in each industry there is only a finite number of fixed-coefficients alternative production technologies), then, in intertemporal

equilibria interpreted as sequential equilibria with perfect foresight, the equilibrium behaviour of agents may endogenously generate indeterminacy of subsequent periods' equilibria, what points to a further difficulty of the interpretation of Arrow–Debreu intertemporal equilibria as perfect-foresight equilibria;

9 A multifaceted chapter by Michio Morishima on Schumpeter, which starts from Walras; presents a different view of the stability problem in GE theory from Fisher's; summarises Morishima's reasons (centering around Say's Law) for dissatisfaction with Walras's own model; proceeds to argue that Schumpeter is fundamentally Walrasian although with Böhm-Bawerkian elements, and that Schumpeter's innovator–entrepreneur shifts the path of temporary equilibria, which nonetheless still converges to a long-run equilibrium, like the sequence of temporary equilibria of the Walrasian model when the latter is corrected through the integration of elements from Hicks and La Volpe (but Hicks too is criticised for an assumption in *Value and Capital* which, according to Morishima, excludes the possibility of capital accumulation); continues with a discussion of Schumpeter's views on the connections between economics and other social sciences, views which are contrasted with those of Morishima's teacher Takata (who argued for a broad economics, a view which Morishima approves of); and ends with an indication of Paretian influences on *Capitalism, Socialism and Democracy*;

10 A chapter by Hamid Sabourian which, starting from the difficulty of proving that the behaviour of perfectly competitive economies with a continuum of agents is the limit of the behaviour of economies with a large but finite number of agents, surveys the recent research – up to some still unpublished results – which has tried to supply game-theoretic foundations to competitive behaviour, in the area of repeated games of the Cournot-Bertrand oligopoly type, and in the area of bargaining and matching models.

1.3 The other main aim of the School was to survey some of the reasons advanced to explain the rejection of the GE approach and the pursuit of alternative research directions. In this area the volume includes (again in alphabetical order) six contributions:

1 Duncan Foley explains, with examples, his statistical equilibrium approach to the determination of market prices;

2 Pierangelo Garegnani argues that, contrary to a widespread opinion, the modern versions of GE theory are not left unscathed by reswitching and reverse capital deepening: to this end he resumes an older approach to stability (consisting in studying the stability of general equilibria by concentrating on disequilibrium in a single market, whose price is varied parametrically, while all other markets are assumed to be in equilibrium) in order to bring to the fore the possible instability of savings–investment market(s) in intertemporal equilibria, owing to the same reasons which may cause reverse capital deepening in long-period analyses;

3 Herb Gintis, after explaining his reasons (based largely on endogenous contract enforcement) for rejecting the Walrasian GE theory, advocates a rejection of the notion of *homo oeconomicus* in favour of a notion of *homo reciprocans*;

4 Heinz D. Kurz and Neri Salvadori present an overview of the development of the theory of value, with special attention to the differences between the classical and the neoclassical approach and to the changing attitude toward the long-period method; they also argue, and provide two analytical examples, that the long-period method may be a way to achieve more quickly, in some cases, results which would be very tiresome to reach through intertemporal equilibrium analysis;
5 My chapter illustrates the distinction between the traditional (long-period) and the recent (neo-Walrasian) versions of general equilibrium[2] and argues that the distinction is the key to a correct appreciation of the "Sraffian" critique of the neoclassical approach; in particular, it shows that long-period general equilibria, in spite of complete disaggregation, need the conception of capital as a single factor given in "quantity" but of endogenously determined "form", and that neoclassical macroeconomics needs that same conception in order to derive the decreasing labour demand curve and the decreasing investment schedule; it then presents the classical-Keynesian alternative, briefly illustrating its implications for the theory of growth and for the theory of distribution;
6 Bertram Schefold starts his chapter by summarising earlier work of his, which shows the possibility of very counterintuitive (and in all likelihood unstable in spite of being unique) intertemporal equilibria, owing to a technology associated with reswitching; he thus points, with a different method, to the same problem highlighted by Garegnani, the possibility of instabilities in intertemporal equilibria arising from the production side, and not from income effects. He then discusses the potential applications of the classical approach, stressing its greater compatibility with historical specificity, and supplying examples of its applications to the study of the composition of output, to technical choices in energy supply, to joint production, to income distribution, and to inflation caused by rises of the rate of interest.

Of these six chapters, four are by economists who have, each one in his own way, drawn inspiration from the work of Piero Sraffa, advocated the resumption of the method of long-period positions and of the classical approach to value and distribution, and taken part in the Cambridge controversies in capital theory on the critical side (The School was indeed the first occasion, after many years, in which several important representatives of opposite sides in the Cambridge controversies in capital theory met and discussed again.). Much of the debate at the School centred on the theses advanced by these economists, and many interventions in the Final Discussion came back on the problems raised in their chapters – and understandably so, because the alternative advocated (although not without differences, as a careful reading of their chapters and interventions in the Final Discussion will show) by these economists appears to be at the present time the most serious challenge to the neoclassical approach. This is because it is at present the most consistent and developed one (not least because of the advantage of a long and rich tradition behind it), and because its advocacy entails disagreements with the neoclassical approach which go deeper than other criticisms, questioning both the abandonment in modern GE theory of the method of long-period positions, and the continuing faith in factor substitution mechanisms on which, more or less explicitly, mainstream economics continues ultimately to base its analyses. I will come back on these points later in the present Introduction.

Then there is a last chapter, by Alan Kirman, who kindly accepted the task of opening the Final Discussion session with a "summing-up" lecture which was to be an attempt at a first evaluation of the positions presented at the School.

A record of the Final Discussion concludes the volume (and makes, in my opinion, very interesting reading, because that session offered several lecturers the opportunity to express disagreements or further to clarify their position). The Final Discussion also includes interventions by two lecturers at the School, Graciela Chichilnisky and John Geanakoplos, whose very interesting lectures regrettably could not be included in the present volume.[3]

This volume contains several surveys of frontier research, and some new results, on topics connected with general competitive equilibrium analysis; it contains illustrations of research directions alternative to GE theory; and it is a good place to start for those who want an update on the evolution of the "Cambridge debates".

Apart from Kirman's chapter whose natural place was just before the Final Discussion, the other chapters are arranged in alphabetical order by (first) author.

The remainder of this Introduction comments on some important aspects of what came out of the School. Section 2 notices that not even the practitioners of GE theory defend the latter as a *positive* theory of value (i.e. as a theory indicating with sufficient approximation the results of the working of market economies), and tries to suggest a possible explanation of why so much research nonetheless adopts the GE framework. Section 3 discusses some misunderstandings, which emerged at the School, of the "Cambridge" capital-theoretic critique; in particular it argues that the belief that the criticism was only aimed at aggregate production functions has prevented many economists from perceiving the true consequences of the Cambridge critique; even intertemporal equilibria are touched by these consequences, as shown by the arguments of Schefold and Garegnani. Section 4, in order to answer a question raised at the end of the Final Discussion, surveys the evolution of the Cambridge controversy, and advances some critical comments on Joan Robinson's contribution to that controversy.

2 ON THE PREVAILING ATTITUDE TOWARD GENERAL EQUILIBRIUM THEORY AMONG ITS PRACTITIONERS

2.1 A striking aspect of the papers collected here is that in none of them is GE theory explicitly defended as a good guide to the actual behaviour of market economies. That GE theory should be criticised by those who argue for alternative approaches, is only to be expected; it is on the contrary surprising – at least, I was surprised – that none of the lecturers who work within its framework and try to extend it to new problems spent some words openly in defence of its usefulness, not even in the Final Discussion session. Kirman confirms that the School was not unrepresentative in this respect, by writing that "apart from those who use computational GE models, this (i.e. the belief that GE 'does actually have something to tell us about real economic phenomena') is not a widely used justification for the use of GE theory". (At the School there was no specialist of computable GE models; but applications must be justified by theory, and the situation described by Kirman, and confirmed by the present volume, implies that GE theoreticians do *not* think that GE theory justifies the use which is made of computable GE models.)

But then why do so many energies still go into developing GE theory, e.g. into extending it to infinite horizons, or to incomplete markets? I do not think that a clear answer to this question emerged at the School. Still, some hints can perhaps be found in the contributions to this volume, and this section is dedicated to discussing them. One part of the answer appears to lie in an insufficient acquaintance with the existing alternatives. If, owing to little contact with alternative approaches, in some academic environments the acceptance of GE theory as the sole existing theory of value is total, the economists moulded by those environments will find it only natural that the GE framework should be the one within which one explores new problems, since they can see no alternative. Another part of the answer no doubt lies in the usefulness often attributed to GE models as an ideal benchmark; but then the next question to be asked is, in what sense is a GE a "benchmark", and why is this "benchmark" considered so important?

I try now to expand a little on these issues.

2.2 A difficulty with conceiving of alternatives to the supply-and-demand approach to value and distribution surfaces, in my opinion, in some of the contributions in this volume.

This seems to be for example the case with the useful survey by Accinelli, which admits that intertemporal equilibria over an infinite horizon present several problems additional to those of the finite-horizon case, but – in spite of its title – stops short of trying to assess how grave these problems are for the validity of the entire approach and whether one should try and explore different approaches.

A second and particularly interesting case is the chapter by Franklin Fisher, which, after a beautiful summary of the history of the research into the stability of general equilibria from Samuelson onwards, admits what in my own contribution is called the "impermanence problem" of the modern versions of GE, i.e. the impossibility to assume that the equilibrium as defined in these versions does not change during the disequilibrium processes which should bring it about, with a consequent inability of the equilibrium based on the initial data to give an indication of the situation the economy might be tending to. Fisher's conclusion is that "the present state of GE theory must therefore be regarded as unsatisfactory or incomplete when it comes to the provision of a positive theory of value"; but the sole indication towards an alternative is that we need a "serious modelling of disequilibrium". Thus Fisher appears to share a view which is not infrequent nowadays among economists of neoclassical formation who are becoming dissatisfied with GE theory as a positive theory of value: namely, that if one drops the GE approach, then the theory of value and distribution must restart more or less from scratch. Kirman's chapter is clearer in rejecting GE theory, but on the constructive side it expresses very much the same view.

I can only attribute such a view to an imperfect acquaintance with the voluminous non-neoclassical literature which in recent decades has developed alternative approaches to value, distribution, employment and growth. Some of this literature is remembered in the chapters by Kurz and Salvadori, by Schefold and by myself. A lack of familiarity with these non-neoclassical alternatives emerges, or is honestly admitted, in some of the interventions in the Final Discussion.

A lack of familiarity with non-neoclassical approaches can also be suspected behind the fact that some of the research presented at the School was couched or interpreted in general equilibrium terms while in fact it needed not be, because concerned with *partial* problems whose study only needs that other prices and quantities be given, and not necessarily given by a neoclassical equilibrium; or because also interpretable in terms of other approaches.

One example is the chapter by Mordecai Kurz, whose basic insights are applied to the stock exchange, to fluctuations of the exchange rate, to the risk premium, i.e. to particular markets, and would appear to be still applicable to these markets even if distribution and growth (which he takes as given) were determined in non-neoclassical ways.

Another example is the first of the two chapters by Magill and Quinzii. This chapter argues that, in a model where the output of firms depends not only on outside influences but also on entrepreneurial effort (which is assumed to be the sole input besides capital), the entrepreneur's choice of financial instruments (the choice of the combination of debt, equity and options), when combined with knowledge of his wealth, his ability and his attitude to risk (informations which, the authors argue, are fairly well known owing to the market's active collection of information on the past record of managers and on their personal wealth), is fully revealing of his effort decision and therefore of the future output of the firm (apart from the uncertainty deriving from outside shocks). It is therefore possible to determine an equilibrium in which every agent correctly anticipates the effort each entrepreneur will invest in his firm, and in this way the moral hazard problem deriving from the effort decision of entrepreneurs is surmounted. Now, there seems to be no reason why this insight should not be applicable also to a classical-Keynesian world where wage labour is another input, real wages are given, prices are determined by normal costs of production, and aggregate demand is determined by exogenous expenditure and the multiplier, so that the way entrepreneurial effort can influence profits is by influencing the market share of the firm and its cost of production.

An example of analysis which is not partial-equilibrium but which is unnecessarily interpreted as referring to neoclassical models is the contribution by Araujo and Maldonado. Their analysis considers an intertemporal one-step forward-looking model where, if agents have expectations for the next period state variables (prices and quantities, in general) given by the probability distribution μ_{t+1}, then x_t is the current value of the state variables that equilibrates the markets, where the vector x_t is given by the equation $Z(x_t, \mu_{t+1}) = 0$ where the function Z incorporates preferences, endowments, technology, governmental policies etc. The question the chapter asks is: under various assumptions as to how μ_{t+1} is determined by the past history of the economy, will the sequence of temporary equilibria converge to something definite, e.g. a steady state, or a k-cycle? It seems to me that there is nothing in this very general structure which obliges one to interpret the equation $Z(x_t, \mu_{t+1}) = 0$ as determining a neoclassical temporary equilibrium, or indeed an equilibrium in any possible sense of the word. The equation might just as well be interpreted as determining a reaction function: the model might e.g. refer to Harrod's world where the state variables are levels of output of the various goods the economy produces, levels of productive capacity in the several industries, desired levels of capacity utilisation and of inventories, and the expectations are about the evolution of demand. And perhaps the interpretation of the results would then be more convincing, because the interpretation in terms of temporary equilibria runs against at least two serious difficulties: (1) general equilibrium temporary equilibria are not in general unique (macroeconomic aggregate formulations are able to avoid this problem only through arbitrary assumptions), so the equation $Z(x_t, \mu_{t+1}) = 0$ cannot be assumed to have a unique solution, and then the dynamics of the learning rule are not well defined; (2) the assumption that in each elementary period the economy reaches a temporary equilibrium means that one is assuming instantaneous equilibration, a fairy tale which – as implicit in Fisher's lecture, and argued explicitly in mine – has no clear implications about the behaviour of economies not continuously in equilibrium. There is therefore room, and perhaps greater legitimacy, for researches applying the very general insights obtained by the studies surveyed by Araujo and Maldonado to problems arising in non-neoclassical analyses.

2.3 I come now to the "benchmark" role of intertemporal general equilibrium. Hicks (1933/1982: 32) minimised the usefulness of that notion of equilibrium for the understanding of actual economies, by describing it as only "a standard of comparison", the "model of a *perfectly working* economic system" characterised by perfect foresight, useful for little more than making it clear that the existence of fluctuations is due to the absence of perfect foresight (cf. Petri, 1991: 276): but in this way the intertemporal equilibrium was viewed as describing the ideal situation which is not realised only owing to "imperfections" (e.g. the incompleteness of markets which creates room for a disturbing influence of erroneous expectations). In the present volume, this role emerges very clearly in the opening pages of Magill and Quinzii's first chapter, and again when Fisher and Kirman mention the importance generally attributed to the connection, established by the Fundamental Theorems of Welfare Economics, between general competitive equilibria and Pareto-efficiency. Fisher writes that these theorems "underlie all the looser statements about the desirability of a free-market system"; Kirman concurs with Fisher that these theorems "are the cornerstones for the arguments in favour of economic liberalism". Clearly such a use of these theorems presupposes a belief that GE theory describes with sufficient approximation the result of the unfettered working of competitive markets.[4] But even when assigning a more important role to "imperfections", as long as one continues to believe that the basic forces at work in market economies are the neoclassical supply-and-demand ones, one will be understandably interested in studying the situation those forces would generate if unimpeded by "imperfections".[5] This is, I suspect, the main reason for the continuing research efforts in general equilibrium theory: its assumptions are admitted to be unrealistic, but the vision of the basic forces acting in a market economy as supply-and-demand forces pushing toward some equilibrium is not abandoned. Such a suspicion is certainly not allayed by the instances in which economists, who express strong doubts as to the correspondence between general equilibria and actual economic outcomes, then reveal a continuing acceptance of neoclassical ways of thinking. Frank Hahn for instance, who has often repeated that GE theory is not a good guide to the actual working of market economies, in his chapter in this volume shows a great reluctance to depart from basic neoclassical tenets (e.g. the tendency to complete market clearing if prices move sufficiently fast).[6]

This may contribute to explaining why the "Cambridge criticism" has caused so much heat. That criticism claims that the forces based on factor substitution mechanisms, which are the foundation of the neoclassical approach to distribution and to employment, are in fact non-existent because the logical basis on which they were argued to exist is faulty. The view of general equilibria as describing the outcomes of "perfectly working" economic systems undisturbed by expectational complications is thereby radically rejected; the claim is, that unemployment, crises etc. are not the result of mistaken expectations and other "imperfections" (such e.g. as those discussed by the New Keynesians), but instead, more simply and more radically, of the non-existence of the forces which, in the absence of those "imperfections", should bring about a general equilibrium according to the neoclassical approach.

The inability to abandon the neoclassical "vision" also explains why it is so difficult for some neoclassically-trained economists to understand how the classical approach can claim the importance of long-period analysis, can claim that product prices tend to long-period levels, and yet can deny that a competitive economy tends toward a long-period *equilibrium* as characterised by neoclassical theory, e.g. characterised by the full employment of labour. For economists accustomed to taking the marginalist factor substitution mechanisms for granted, and relying on expectations and other "imperfections" in order to explain short-period deviations from complete equilibrium, the long period is the situation in which expectations have had time to be corrected and the economy must therefore have reached a full-employment equilibrium[7]: the differences from, say, a Friedman or a Lucas are therefore ultimately of minor importance, turning only, essentially, on the speed of the error–correction mechanisms. When one rejects the neoclassical forces based on factor substitution, the tendency of mistaken expectations to be corrected[8] no longer implies that the economy will tend to the full employment of resources nor to a specific income distribution, it only means that product prices will tend to their long-period levels determined by income distribution, and that in each industry productive capacity will tend to become the one required by the evolution of demand; but income distribution, and aggregate output and employment, will remain to be determined (and there seems to be little alternative here to accepting the classical insights for the explanation of distribution,[9] and the principle of effective demand for the determination of aggregate output). So the existence, noticed by Kirman, of some similarity between the modern classicals and Lucas at the level of method – the belief in the existence and importance of long-period tendencies where an autonomous role of expectations tends to disappear – only means the acceptance of a method central to economic theory since Adam Smith, but does not at all mean a similarity on the forces determining distribution and employment.

In the present volume this important issue is discussed in the chapter of Heinz Kurz and Neri Salvadori, who remind the reader that, against the Marshallian interpretative tradition nowadays taken up e.g. by Samuel Hollander, there is both historical and analytical evidence that the classical approach is radically different from the marginalist/neoclassical one.[10] One important implication, briefly remembered in my paper, is that the recent loss of familiarity with the long-period method in no way testifies to deficiencies of that method, because its abandonment was due to the difficulties *the neoclassical approach* finds when determining a long-period position, and not to deficiencies of the thesis that there is a tendency of relative product prices toward the prices yielding a uniform rate of return on supply price.

3 MISUNDERSTANDINGS AND CLARIFICATIONS

3.1 It cannot then come as a surprise that another important aspect which emerges from this volume is, that the sides in the Cambridge controversies in capital theory have yet to reach a stage of sufficient mutual understanding. One of the aims of the School was to re-establish communication between the two sides in that debate. Pursuing such an aim was indeed opportune, because, as it came out, significant difficulties of communication persist; but the School showed that these difficulties originate now mostly on the neoclassical side; as Prof. Foley remarks in the Final Discussion, the chapters in this volume show that the "modern classicals" understand, and can produce new results in, modern general equilibrium theory; the converse is unfortunately not always true, as shown by the misunderstandings of the critics' arguments which emerge in some contributions in this volume.

3.2 Some important misunderstandings of Sraffa's work emerge for example in the otherwise excellent chapter by Mandler. Mandler identifies long-period analysis with steady-state analysis; this is a frequent misconception, which, among other things, makes it very difficult to understand why traditional marginalist analyses attempting the determination of disaggregated general equilibria needed all the same a *given* endowment of "capital" conceived as a single factor; it has been one of the main causes of the difficulties of communication in the Cambridge controversies. I attempt to dispel this particular misunderstanding in endnotes 11 and 13 of my chapter in this volume (and at greater length in Petri (1999: 27 ff.)). Here I limit myself to remembering that steady states belong to what Marshall would have called *secular*, rather than long-period, equilibria, and that the main point to remember in order to have the distinction clear is that the speed with which the relative amounts in existence of the several capital goods can vary is of a greater order of magnitude than the speed with which accumulation can change the overall "quantity of capital" (if one believes in that notion) or more generally the average productive capacity of the economy, so that it is legitimate to treat the latter as fixed while allowing the composition of capital to adjust.[11] A second misunderstanding emerges when Mandler writes that "Sraffa fixes the aggregate quantities produced and finds multiple equilibrium prices for those quantities" and interprets this as meaning that in Sraffa there is an "indeterminacy argument"[12] (so Mandler believes that his sequential indeterminacy results can give some support to Sraffa's claim). But the prices that Sraffa determines are not "equilibrium prices" in the sense of being associated with equilibrium on factor markets, they are the *long-period* relative product prices[13] necessarily associated with a parametric income distribution both in a classical and in a neoclassical approach, and the "indeterminacy" is simply the fact that, until either the real wage or the rate of profits are given, the equations determining these prices have one degree of freedom, which is precisely what makes it possible to vary income distribution parametrically.[14] So the existence of this degree of freedom is no "indeterminacy argument" in the sense of a critique advanced against neoclassical theory, because exactly the same degree of freedom can be found in the subset, of the equations of (long-period) general equilibrium of Wicksell or of Walras, which determines relative product prices (cf. endnote 10 of my chapter). What Sraffa does, among other things, is to show that his equations, when interpreted as a subset of the equations of *long-period* general equilibrium systems, can be analysed in isolation and exhibit important properties which go against the notion of "capital" as a factor of production. Mandler perhaps has correctly sensed that Sraffa believed that competition cannot determine distribution in the same way that it determines relative prices, but the reason is not the above "indeterminacy", it is rather that the only approach arguing for a fundamental analogy between the competitive determination of the prices of factor services and of the prices of any other good, i.e. the marginalist/neoclassical approach, is judged by Sraffa to be untenable, because faulty in the way it "closes" that degree of freedom (through the condition of equality between demand for and supply of "capital" conceived as a single factor given in "quantity" but not in "form").[15]

Another important misunderstanding emerges in Fisher's intervention in the Final Discussion, where he repeats the frequent argument that "Neoclassical general equilibrium theory does not require the existence of aggregate capital as a factor in an aggregate production function". As explained already in 1970 by Garegnani, the versions of marginalist theory relying on aggregate production functions are of secondary importance only; the important theoretical issue is the legitimacy of the conception of capital as a single factor of variable "form", a conception needed not only – as I explain in my chapter (Section 3 and endnote 15) – for the determinability of the general equilibria endowed with sufficient persistence (i.e. long-period equilibria) which all the founders of marginalism aimed at determining, but also for the plausibility of the neoclassical substitution mechanisms, in neo-Walrasian general equilibria (as argued by Schefold and Garegnani) as much as in traditional marginalist analyses. The untenability of that conception of capital means not only that a long-period equilibrium is indeterminable, but also that there is no foundation for the traditional view of investment as a decreasing function of the rate of interest, what means that investment cannot be presumed to have a tendency to adapt to savings: concentration on the issue of the legitimacy of aggregate production functions has prevented many economists from perceiving the true, extremely important consequences of the Cambridge critique.

3.3 A noteworthy aspect of the present volume is indeed the discussion of these consequences for intertemporal GE models, in the contributions by Schefold and by Garegnani.

In the first part of his chapter Schefold presents in more concise form an argument he had advanced in 1997. He intends to show that reswitching causes problems of instability in intertemporal equilibria. He finds a way – by deriving the necessary form of utility functions *ex post* – to construct examples of intertemporal equilibria characterised by the paths of prices and quantities which he wants to obtain; he then considers intertemporal equilibria (without contingent commodities, and over a finite number of periods), which are stationary for a certain number of periods, then undergo an increase in labour supply (e.g. due to immigration) and start a transition to a new stationary state with a higher labour employment. By assuming a single consumer, he proves that the equilibrium is unique and Pareto-efficient. He then proves that, with a given real wage, relative product prices and the technique adopted by an economy in intertemporal equilibrium tend to become the dominant ones in a long-period sense, the ones on the outer envelope of the long-period wage curves. Therefore, the intertemporal equilibrium he builds exhibits Sraffian, or long-period, prices in the initial and in the final periods; but because of the change mid-way in labour employment these two long-period situations differ in the real wage and in the technique chosen. Schefold then assumes that the economy has at its disposal such a finite array of alternative techniques – characterised by reswitching and reverse capital deepening – and starts from such a real wage that the only way to *increase* employment in the latter periods (and thus maintain full employment in all the equilibrium's periods) without net savings is to change to a technique associated with a *higher* real wage. The transition to a higher level of employment is therefore associated with an increase, not a decrease of the real wage.[16]

The point of the exercise is to show that, if the long-period wage curve of an economy exhibits reswitching and reverse capital deepening, then the only way to absorb an increased labour supply without increasing savings may be to pass to a higher, rather than a lower, real wage even in an intertemporal equilibrium framework. Thus, Schefold argues, there may be cases in which an economy has a unique intertemporal equilibrium, but a paradoxical one which cannot be reached by supply-and-demand adjustments: although he does not attempt a formal proof of the instability of the equilibrium, he convincingly notices that in his example, if in the first period after the increase in labour supply there were excess labour supply because the wage is still the one appropriate to the earlier periods, the auctioneer would lower, not raise, the real wage,[17] but this would cause a decrease, not an increase, in labour demand in those periods.[18] Schefold concludes

> that if the technology is such that employment can be increased only through techniques which imply reswitching, the more plausible outcome is unemployment, and that the full employment equilibrium is only of formal relevance.
>
> (Schefold, 1997: 489)

The exercise just described focuses on the difficulty of reaching equilibrium on the labour market, assuming equilibrium on the savings–investment markets. The same instability would show up on the savings–investment markets, Schefold continues, if one assumed that labour is continuously fully employed at a constant level, but that the average propensity to save decreases from a certain period onwards. Then, if techniques are such as to generate reverse capital deepening, the only way to have investment decrease would be to decrease the rate of interest; while the auctioneer, facing an excess demand for savings, would increase the rate of interest.

3.4 Garegnani attempts to explore essentially the same problem via a different route: he constructs supply and demand curves for savings in an intermediate period of an intertemporal equilibrium, and argues that the demand curve for savings can be as "perversely" shaped in an intertemporal equilibrium, as the demand curve for "capital" the value factor in long-period analyses, giving rise to possible instabilities and multiplicities of equilibrium, or to the existence only of equilibria with implausible levels of the distributive variables ($w = 0$, or $r = -1$, in certain periods at least).

In order to arrive at the analysis of the savings–investment market in an intermediate period of an intertemporal equilibrium, Garegnani resumes an approach to the study of stability on single markets, which was the implicit one in traditional analyses, e.g. of the demand for labour, but which has afterwards been neglected in favour of processes with disequilibrium simultaneously on many markets. The method consists in assuming equilibrium on all markets but the one of interest, in dropping the equilibrium condition regarding the market under study, and in replacing it with two equations, one determining the quantity demanded, and the other the quantity supplied of the good or factor in question: These quantities being derived from the firms' and consumers' decisions, together with an appropriate budget condition guaranteeing that there will be equilibrium on all other markets (this budget condition will be discussed shortly).[19] The extra degree of freedom (the number of equations has increased by one, that of variables by two) is "closed" by treating the price of that good or factor as a parameter.

For the labour market, the construction has an immediate economic interpretation and is the one which makes it possible to grasp the effects which the marginalist approach attributes to real wages kept above their equilibrium level: the effect is unemployment, but with no disequilibrium on any other market because the unemployed labourers, obtaining no income, cannot purchase goods. The budget condition guaranteeing that there is equilibrium on all other markets is in fact here the condition that the income which can be employed to purchase goods is only the income from factor supplies which find purchasers.

When one applies the same method to the market of a product, then one must assume equilibrium on all other factor and product markets, and one must assume that there is an external agency (e.g. the state, or foreign markets), which buys the excess supply of the product, or supplies the excess demand for it, at the same time taxing or subsidising consumers for the value of the excess supply or demand.[20] Thus e.g. if labour and land produce two products, corn and meat, with variable coefficients, one can assume equilibrium on the markets for labour, land and corn, and treat the relative price of meat (in terms e.g. of the price of corn) as a parameter. If, starting from the equilibrium prices, one raises the price of meat, this will require – in order to maintain prices equal to costs of production – a change in income distribution in favour of the factor relatively more utilised in the production of meat; the assumption of full employment of resources, together with a supply of corn equal to the new demand for corn, will then imply a new supply of meat unequal to the new demand for meat coming from the income of fully-employed factors. Let us then, e.g., assume an excess supply of meat: in order to have disequilibrium only on the meat market, one must assume that the excess supply of meat is bought by an external agent, e.g. the state, but then to maintain the equality between the value of aggregate product supply and the value of aggregate product demand on all other markets, one must also assume that there is a tax on factor incomes equal in value to the value of the excess supply of meat. This is not generally realistic, but the sign of the disequilibrium thus determined may still be argued to give an indication of the tendency of the price in the real economy in a neighbourhood of equilibrium, because agents will usually have the possibility to spend temporarily more than their income by running down their cash reserves, or to spend less than their income by accumulating cash. And the method allows a clearer examination of the influences acting on the chosen market, thus permitting to locate more precisely the roots of the phenomena which are thus discovered. Clearly, when there is equilibrium also on the chosen market, then the budget constraint becomes the usual one.

In an earlier and as yet unpublished (but widely circulated) paper based on his 1993 Sraffa Lecture, Garegnani (1994) applies this method to the savings–investment market i.e. to the ensemble of markets for the new capital goods produced and sold at an intermediate date, say date t, of an intertemporal equilibrium. The price treated as a parameter is the own rate of return of a representative consumption good[21] between date t and date $t+1$. The same consumption good, of date t, is taken as numéraire. Assuming equilibrium on all other markets means assuming the full employment of capital goods at all *other* dates, and equilibrium on the labour and on the consumption goods' markets at *all* dates. (In order to have equilibrium on the markets at subsequent dates, the special budget assumption here is that one must assume that the income spent on those markets corresponds to the employment of the capital goods *demanded*, and not of the capital goods supplied, at date t.) So the resources, available for the production of capital goods to be made available at date t, are well defined and, since these resources are fully employed, all that remains to be determined is the *composition* of the production of capital goods of date t.

Garegnani assumes that production takes one period, and that all capital goods are circulating capital goods; so the *supply* of capital goods of date t comes from the utilisation of that part, of the labour and capital goods available at date $t-1$, which is not employed for the production of consumption goods of date t. The *demand* for the capital goods of date t is derived from the demands for consumption goods and capital goods[22] at date $t+1$, which, via cost minimisation (and the assumption of full employment of labour at date t), determine the desired capital goods of date t. The composition of the *supply* of capital goods at date t can then be determined by assuming it to be the same as the composition of the demand for capital goods at date t: one then has disequilibrium of the same type on all markets for capital goods offered at date t, so, in effect, only one disequilibrium.[23] The prices are all determined because there are as many equations as unknowns,[24] so one can calculate the value of the intended supply of capital goods at date t and call it *gross savings of date t*, and one can determine the value of the intended demand for capital goods at date t, and call it *gross* (ex ante) *investment of date t*. They will depend on the value of the own rate of return, i.e. of the rate of interest, of the numéraire between dates t and $t+1$, which is taken as parametrically given.

A necessary condition for equilibrium with non-zero prices is that date-t investment so defined be equal to date-t savings, i.e. to the value of the supply of date-t capital goods: this shows that in intertemporal equilibrium models too there are (gross) investment and savings, in each period where there is production of capital goods, and that they must be brought into equality in order to obtain plausible equilibria.

It may be assumed that the inequalities between investment and savings tend to influence the rate of interest in the usual way. This is because, since the prices at which capital goods are valued are the same in the determination of investment and of savings, and given the assumption of equal composition of supply and demand for date-t capital goods, the inequality between savings and investment will mean an inequality of the same sign between the supply of, and the demand for, each capital good at that date. Thus if, e.g., date-t investment is lower than date-t savings, this must be assumed to cause a tendency for date-t capital goods to decrease in (discounted) prices relative to date-$(t+1)$ goods, what in terms of undiscounted prices would mean a lower interest rate between dates t and $t+1$.[25] The question is, whether such a variation of the rate of interest will eliminate the discrepancy between investment and savings. Traditionally, it would be argued that the lower interest rate will induce firms to employ more capital-intensive production methods, i.e. (given the assumption of full employment of labour) to increase their investment, and that therefore the equilibrium on the savings–investment markets is stable. But this traditional thesis rests on neglecting the changes in relative prices accompanying changes in the interest rate; this variation of relative prices is what permits "capital reversal" in long-period models, and an analogous dependence of relative prices on the rate(s) of interest also exists in intertemporal models; so, Garegnani (1994) argues by utilising some of the numerical examples already used in his 1970 article to highlight reverse capital deepening, analogous "perversities" can happen in intertemporal equilibria as well, undermining the stability of the savings–investment market, or preventing the existence of equilibria with equality of savings and investment, so that the only equilibria exhibit zero wages or a rate of interest equal to -1.[26]

3.5 In his chapter in the present volume, Garegnani shows that the same analysis can be applied to the simpler model (essentially the one in Hahn, 1982) of a two-period, two-products intertemporal equilibrium, and that even in this very simple model the dependence of relative prices on distribution (in this case, on the own rate of return of the good chosen as numéraire) can make the equilibration of the savings–investment market problematical.[27]

Garegnani concludes that it is wrong to believe that if only futures markets for consumption goods existed so that savings decisions today entailed increases in the demand for future consumption goods, then there would be no "Keynesian" problem: the rate of interest may still be incapable of bringing about the equality of today's investment and today's full-employment savings.

He adds (cf. also his interventions in the Final Discussion) that these results show that neoclassical theoreticians have been wrong in believing that the reasons for the possible multiplicities and instabilities of Arrow–Debreu general equilibria are to be found in income effects only: even without any income effects causing problems on consumption goods' demand side or on factors' supply side, instabilities may originate in the production side: technical choices may work in a direction contrary to that necessary for the plausibility of neoclassical theory; this possibility, long hidden by the unjustified faith that capital could be treated just like labour or land, puts the last nail in the coffin of the neoclassical belief in substitution mechanisms pushing toward a supply-and-demand equilibrium.

This, as Mandler notices in his intervention in the Final Discussion, raises a question: as is well known, it is a theorem that the weak axiom of revealed preferences for aggregate consumer excess demand is a sufficient condition for the uniqueness also of equilibria with production,[28] what Mandler interprets as meaning that heterogeneity of agents is necessary for the multiplicity of equilibria; Schefold appears to agree; Garegnani on the contrary appears to claim that this is only true because the production in the theorem is *atemporal* production, i.e. excludes production of capital goods; so there appears to be here a clear analytical point of disagreement, which further inquiry should be able to settle. If Garegnani is right, it should be possible to produce examples of intertemporal equilibria where the weak axiom is valid for the aggregate consumer excess demand, and yet equilibrium is not unique. If it will turn out that this is not possible, then the problem will remain of fully understanding the effects of the situations that Schefold and Garegnani have shown to be possible.

So clearly there are open issues and room for more analytical work. But I, for one, would be surprised if Schefold's and Garegnani's results were found not to imply the instabilities that these authors see as entailed by them; relative prices do tend to long-period prices even in intertemporal equilibria, so the results based on long-period prices such as the possibility of reverse capital deepening must also reappear, one way or another, in intertemporal equilibria.

3.6 It may now be clearer why the idea, that the Cambridge critique was only aimed at disputing the legitimacy of aggregate production functions, has been deeply misleading.

This idea was strictly associated with the mistaken idea that there was only one notion of disaggregated general equilibrium, the neo-Walrasian one, where the aggregability of capital was not necessary for the specification of the equilibrium; and this has obscured the indispensable supply-side role of the conception of capital as a single quantity for the determination of sufficiently persistent (i.e. long-period) disaggregated equilibria.

The same idea has also allowed attention to be limited to the supply-side problem (misconceived as it was) of the representability of the capital endowment in terms of a scalar, thus directing attention away from the demand-side role of the conception of capital as the substance demanded and supplied on the savings–investment market. The consciousness that the marginalist approach had not been born on the basis of aggregate production functions (and that therefore it did not rely on their existence for the arguments in support of its plausibility) has made it easy for neoclassical economists to concede the non-existence of the conditions for aggregability, while the lack of attention to the demand-side role of the traditional conception of capital has made it possible not to realise the relevance of reverse capital deepening for the issue of the forces pushing toward a supply-and-demand equilibrium. It should now be possible to enter a more fruitful stage of debate, where the issues at stake are clearer.

4 TOWARD A HISTORICAL PERSPECTIVE ON THE CAMBRIDGE CONTROVERSIES

4.1 How successful was the School at re-establishing communication on capital theory will only become visible with time. Progress on such issues cannot but be slow; the important thing is that the seeds of reflection be sown, and I think they have been.

I would like now to help the sprouting of these seeds by attempting to answer a question, posed toward the end of the Final Discussion, and which received only very brief answers then, because time was running out. The question (posed by Sabourian) was why, in the critics' opinion, their criticisms appear to have been so far unable to persuade the majority of the profession that the neoclassical approach is radically vitiated: is it ever possible that the neoclassicals after so many years of debating have not understood yet the argument? (The question of course was meant to imply that it is more likely that the criticisms *were* understood, but were not found convincing.) Anticipating the possibility of such a question, I had included some elements of an answer in Section 1 of my chapter; but evidently it was not enough, so I attempt now to supply some more elements of what I think is the answer.

I argue in my chapter that the answer to this question is that, indeed, in spite of so many years of controversy, the neoclassical side has not well grasped the critical argument yet. Confirmation that this is the case emerges from what I have observed earlier in this Introduction, especially in Sections 3.2 and 3.6. What I want to do now is advance some (tentative) explanation of this persistence of misunderstandings. I will suggest three main reasons: (1) the confused state of capital theory at the time the Cambridge controversy started; (2) the deficiencies, in the first bouts of debate, of the critical arguments themselves, in particular of those of Joan Robinson; (3) the apparent ignorance by the neoclassical side of the more recent critical contributions on capital theory.

4.2 When in 1953 the controversy was started by Joan Robinson, there was no generally accepted treatment of capital among marginalist economists. In the 1930s, there had been heated debates on capital theory among marginalist economists, with Hayek, for example, openly rejecting the conception of capital as a single factor;[29] but no clear conclusion had emerged, except for the unease with the measurement of capital as an amount of value, reflected in Lindahl's and Hicks's shift to the temporary equilibrium method, but not otherwise widely spread. The uncertainty about how best to introduce capital into general equilibrium is clear, for example, in the treatise on general equilibrium by Robert E. Kuenne (1963), which discusses Walras only to conclude that the nature of capital and of the origin of a positive rate of interest are best discussed in terms of stationary economies, but then is unable to choose between the different views of J.B. Clark, Böhm-Bawerk, Fisher, Knight, Wicksell, Metzler etc. The proposal to reinterpret the "atemporal" general equilibrium of Cassel and Wald as an intertemporal equilibrium with dated commodities, and to deal with capital in this way, a proposal advanced by Koopmans in 1957, by Debreu in 1959, and most explicitly by Malinvaud (1961), evidently had not, by the time of Kuenne's book, won the field; indeed, as late as 1970 Bent Hansen was still not following that proposal, and was including in his *Survey of General Equilibrium Systems* a Wicksellian model of disaggregated long-period general equilibrium where the endowment of capital is measured as a given amount of value (cf. Hansen, 1970: Chapter 17). The uncertainty and oscillations are well exemplified by Hicks, who after advocating in 1939 the temporary equilibrium method as a way to avoid the conception of capital as a single factor (a "fund"), later had second thoughts, admitted grave problems in that method, and reaffirmed in 1963 his faith in the possibility of conceiving capital as a single quantity in some physical sense (cf. Petri, 1991, and Section 6 of my chapter in this volume), although never daring to go back to treating capital as a single factor K in his formal analyses.

Very importantly, there are clear signs in the 1940s and 1950s of a loss of contact with the notion of long-period equilibrium: Hicks in *Value and Capital* confuses it with a secular stationary state (Garegnani, 1976); Kuenne (1963) also tends to make the same mistake; and an interesting confirmation comes from the so-called Patinkin controversy (or Classical Dichotomy controversy) in monetary theory which rage from 1943 to the early 1960s: Patinkin, like Oskar Lange before him and most of the interventions which follow, is unable to understand that the equilibrium, of the marginalist theorists who in order to determine the price level had appended a Fisherine equation $MV = PT$ to a system of equations only determining relative prices, is not a neo-Walrasian but instead a long-period equilibrium, which therefore implicitly assumes that the relative average cash balances of the individual agents are endogenously determined, much like the relative endowments of capital goods (Petri, 1982). Also striking is that Arrow (1959) admits that disequilibrium adjustments take time and that production is going on while prices adjust, so that the equilibrium is a long-run notion,[30] without realising that then the formalisation of the equilibrium cannot be a neo-Walrasian one, including among its data given amounts of the several capital goods.

More research is no doubt necessary to enrich the picture and make it more precise. What seems indubitable is that the uncertainty as to how best to include capital in the theory of value did not induce a suspicion that the supply-and-demand approach to value and distribution might be radically mistaken; the generally accepted position, at the time the Cambridge controversy started, appears to have been that the basic marginalist view of the determinants of distribution, although it was difficult to put it down in completely satisfactory terms in fully disaggregated models, was no doubt correct, so that one might as well enunciate it in the simplest possible terms with the help of one-good models such as Solow's growth model. So, although the general equilibrium theorists were increasingly turning to intertemporal neo-Walrasian formulations, the conception of capital as a single factor and the notion of capital–labour substitution remained the basis of all practical applications (international trade theory is another example). It was not perceived that, as I argue in my chapter, without the belief in the correctness of the conception of capital as ultimately a single factor, the neo-Walrasian formulations lose all right to claim to be even only "ideal benchmarks".

4.3 Joan Robinson was part of this world, and in spite of the conversations she had with Sraffa, she was not, for many years, capable of abandoning the marginalist ways of thinking she had absorbed when a young Marshallian. Given her prominent role in the controversy (because of her reputation, and because she started it), the insufficiencies of her arguments weighed heavily on the controversy. The misunderstanding that the criticism was only aimed at aggregate production functions is largely her fault. A detailed examination of Joan Robinson's views and of their evolution cannot be attempted here, but it must at least be noticed that her opening attack in 1953 was indeed only aimed at aggregate production functions, and that the accusation in that article, that when using a production function $O = f(L, C)$ it is never made clear "in what units *C* (i.e. capital) is measured" (Robinson, 1953–1954: 81), was not aimed at questioning the notion of a capital–labour ratio measurable independently of distribution and positively influenced by the real wage, a notion in fact accepted in the sequel of the article, but rather, essentially, at arguing that capital could be precisely measured only in a state of equilibrium i.e. of realised expectations, and that therefore the production function had been "a powerful instrument of miseducation" because it had helped people forget the Keynesian arguments on the importance of expectations for the determination of the real wage (through their influence on investment and thus on the price level, given the money wage) and of the pace of accumulation.[31]

In subsequent years she was increasingly able to abandon neoclassical factor-substitution notions, but she developed a very defective view of the logic of the theory she had absorbed when young, a view which perpetuated the misleading concentration on aggregate production functions as the target of criticism. In her article "Capital theory up to date" (1970) the object of the criticism, called neo-neoclassical theory, is explicitly identified with the single-good, aggregate-production-function versions, furthermore caricatured as assuming that capital is a physically homogeneous substance and as assuming instantaneous costless adjustments; and the article is replete with misunderstandings of the differences between the various versions of the marginalist approach: the aggregate-production-function approach, instead of being described as a simplified application of the conception of capital as a single factor of variable "form", is seen as "derived from Walras" (p. 311); no recognition is given to the existence of a distinct marginalist approach to distribution, because the sole approaches to distribution which are recognised to have existed are the classical one (given real wages), the Marxian one (class struggle), and thirdly the Marshallian one in which "there is a normal rate of profit and the real wage emerges as a residual" but this normal rate of profit remains unexplained until "an extension of Keynes' *General Theory* into the long period finds a clue to the level of profits in the rate of accumulation and the excess of consumption out of profits over saving out of wages" (p. 315); Walras is liquidated with the apodictic statement that he "does not have a theory of profits at all" (ibid.). These theses are also in Robinson (1969); in Robinson (1974: 209) even Wicksell is accused of having no theory of distribution: "Marshall's normal profits and Wicksell's natural rate of interest were supposed to apply to a capitalist economy but their level was never explained". So, as her comment (1971) to Fisher (1969) also makes clear, to Joan Robinson the aggregate-production-function versions *were* the neoclassical theory of distribution.

Unfortunately, for many years there lacked (in English) contributions capable of clarifying these issues. The true analytical roles of the conception of capital as ultimately a single factor in the marginalist approach – to make it possible to treat its endowment as a datum of the equilibrium while leaving the endowments of the several capital goods to be determined endogenously, and to permit to speak of capital–labour substitution in the same terms as for physically measurable factors, thus supplying a basis for Say's Law by making it possible to consider the interest rate as the price bringing investment into equality with savings – were not made sufficiently clear in the writings of the critics published in English, up to the mid-seventies.[32] (Garegnani had clarified the first role in his 1958 Cambridge Ph.D. thesis, then published in Italian in 1960, but the results of that thesis were only published in English, in a condensed version, in 1990; he had also clarified the second role, but again in Italian, in a 1964–1965 article which was translated into English in 1978. The earlier availability of Garegnani's analyses in Italian helps one to explain the greater impact of the Cambridge criticism in Italy.)

4.4 It may for example be noticed that no one criticised at the time an important misreading of the implications of reverse capital deepening in Samuelson's widely read "Summing-Up" which concluded the 1966 *Quarterly Journal of Economics* Symposium on reswitching. In that article Samuelson conceded that reswitching and reverse capital deepening were perfectly possible, but the only negative consequence he admitted was a danger of "dynamic instability" (i.e. a danger that full-employment paths may not converge to a unique steady state) for some neoclassical growth models: he never admitted that the neoclassical foundations of Say's Law were undermined and that as a consequence the full-employment assumption itself became indefensible. Thus he wrote that, because of the possibility of reverse capital deepening, "after sacrificing present consumption and accumulating capital goods, the new steady-state equilibrium can represent a rise in interest rate!" (Samuelson, 1966: 246), where it is clearly taken for granted that the only way to accumulate capital goods is by "sacrificing present consumption",[33] i.e. the full employment of resources is taken for granted;[34] and where – since there is no mention of state intervention – it is unclear how Samuelson thought that in the presence of reverse capital deepening the accumulation of capital goods corresponding to a decision to sacrifice present consumption would come into being, since in this case an excess of savings over investment would require an *increase* of the interest rate in order to stimulate an increase of investment, while the market's tendency according to supply-and-demand analysis would rather be to *depress* the rate of interest. Unfortunately the connection between the traditional theory of capital, and Say's Law, had by that time become much less clear owing to confusions in the theory of aggregate investment: the traditional foundation of the belief that investment was a decreasing function of the rate of interest, i.e. the decreasing demand curve for capital the value factor, was based on the full employment of labour, and became therefore quite shaky after the admission, with Keynes, of the possibility of persistent unemployment; hence a series of doubtful attempts (starting from Keynes's own) to maintain the traditional investment function while at the same time admitting the possibility of persistent unemployment, attempts (very briefly mentioned toward the end of my chapter) which no doubt contributed to obscuring the logic of the marginalist approach.

4.5 In the absence of interventions explicitly stressing the true reasons for the importance of the notion of capital as a single factor, and in particular re-establishing a correct understanding of the central role in traditional marginalist analyses of the notion of long-period equilibria, and the illegitimacy of basing traditional analyses on neo-Walrasian equilibria, the road was open for Fisher, Hahn, Stiglitz or Bliss to recognise only two types of equilibrium notions, the neo-Walrasian ones, and steady states (mistaken for long-period equilibria);[35] now, both are notions in which (differently from traditional long-period equilibria) a given "quantity of capital" does not appear in the specification of the equilibrium; on this basis, they could argue that "aggregate capital" was only used by neoclassical theorists in the "unrigorous" analyses based on aggregate production functions, and accordingly – and with the support of Joan Robinson's writings – they interpreted the Cambridge criticism as directed solely at the legitimacy of aggregate production functions, and could thus conclude that the criticisms left "rigorous" GE theory unscathed: a thesis accepted also by some of the critics, e.g. Nuti (1976). So Fisher (1971) was able to reply to Joan Robinson that neoclassical equilibrium theory did not need the existence of aggregate production functions.

4.6 Only with Garegnani (1976) and the subsequent contributions mentioned in Section 1 of my lecture, arguments were supplied capable of surmounting these confusions. The clarification is still continuing, as this volume shows. But it cannot be surprising that so many misunderstandings persist, once one notices that there has been up to now no mention of *any* of the papers in this second wave of critical contributions in the writings of Solow or Bliss or Stiglitz or Fisher or Burmeister or Hahn.

So, yes, the criticism was not and still is largely not well understood, because misinterpreted as aimed only at the legitimacy of aggregate production functions, rather than at the indispensable roles of the notion of capital as a factor of production in the different versions of the marginalist approach; and the basic reason for this state of affairs is the insufficient clarity of neoclassical economists as to the logic and development of their own approach, in particular their lack of familiarity with the once-dominant notion of long-period equilibria and more generally with the traditional method of long-period positions.

But no doubt, after this volume, things will change?

NOTES

1 Prof. Araujo had already accepted to speak at the School and sent the preliminary draft of his contribution, when because of an illness he was unable to come. As his chapter had already been included in a pre-print volume distributed to the participants to the School, it seemed only natural that it should be included in the present volume.

2 I call neo-Walrasian the (intertemporal or temporary) general equilibria which include among the data of equilibrium a given vector of initial capital goods' endowments, but which contrary to Walras take into account the changes over time of relative prices, via futures markets or via expectations.

3 Graciela Chichilniski's lecture was based on Chichilnisky (1997, 1999); John Geanakoplos's lecture was based on Dubey *et al.* (2000). (*Warning*: many of the works – not these ones – cited in this Introduction are not listed in its References to avoid repetitions, because listed among the References of my chapter in this volume.)

4 It also presupposes a neglect of the influence of advertising and other selling efforts on tastes, and an enormous underestimation of the pervasive presence of externalities both in consumption (demonstration effects, fashions, envy etc. largely dependent on the social values induced by a market economy) and in production, what would empty the Fundamental Welfare Theorems of relevance even if the competitive equilibrium they refer to were a good representation of the results of the working of market economies.

5 Fisher also mentions the role of general equilibrium as analysing "positions from which there is no incentive to depart"; but the characterisation of these positions offered by general equilibrium theory is not the only possible one and depends strictly on the neoclassical conception of the functioning of market economies. For example, the explanation I propose of wage rigidity in my first intervention in the Final Discussion implies that, the moment the belief is abandoned that

wage decreases will bring about increases of employment, then it is no longer plausible that the positions from which "there is no incentive to depart" must be (when the real wage is not zero) positions of equality between supply and demand for labour.

6 In Hahn's chapter the proposal to admit that macro variables may enter the individual agents' decision processes is illustrated with examples where, Hahn writes, "To simplify I have taken a perfectly competitive economy with markets clearing at every date by prices. If it took time for prices to be changed, say the auctioneer needs time to do so, then of course one needs to consider various rationing schemes". So, Hahn implies, the capacity of prices continuously to clear all markets can only be doubted owing to the slowness with which prices may adjust. Unemployment in his examples can only arise due to search. I am also struck by Gintis's last intervention in the Final Discussion, where he takes it for granted that a decrease of labour costs (due to a decrease of taxes on employment) will increase employment, without feeling the need to specify the mechanism guaranteeing such an outcome (a Keynesian would notice that, if the decrease in taxation causes a decrease in state expenditure, the more likely outcome is a decrease of employment).

7 I am leaving aside here other "imperfections", e.g. price rigidities, which might prevent the reaching of equilibrium even when expectations have become the correct ones; the argument would not be essentially changed, the role of these "imperfections" would still be that of impeding the working of the forces pushing toward the full employment equilibrium, forces whose existence is not denied.

8 The statistical equilibrium approach proposed by Duncan Foley takes the expectations with which agents come to the market as given (and arbitrary); now, clearly the actual working of markets is on the contrary heavily dependent on habitual modes of behaviour, resulting from repetition of markets; the usefulness of the approach will therefore become clearer when it will be used to study the results of the repetition of markets, as the author himself admits. Analogously, in the bargaining-and-matching models surveyed by Sabourian in the second part of his chapter, what is studied is sequential games internal to a single opening of the market, and the perplexing thing is that agents are assumed to come to the market as if it were the first and sole time: so no room is allowed for the learning which agents have acquired in past market openings (a learning which they are going to use to avoid past mistakes), nor for the awareness that the game is a repeated one, which also is no doubt very important in reality. The lack of attention to these aspects of reality (which were on the contrary central to the analyses, aiming at determining long-period prices, with which the supply-and-demand approach was born) seriously limits the interest of the results surveyed by Sabourian; and a similar objection applies to the first half of his chapter, where the analyses surveyed assume a given number of firms, thus neglecting the very important pressure toward no-surplus pricing due to free entry (again, an aspect central to long-period analyses).

9 Gintis's notion of *homo reciprocans* is also not alien to a classical perspective, as a contribution to the explanation of the birth of the social conventions, customs, and coalitions which enter crucially in the classical approach to the determination of income distribution.

10 The very clear and useful historical part of the chapter by Kurz and Salvadori is followed by a less convincing part, on New Growth theory and on examples of the usefulness of the long-period method. The long-period method comes dangerously close to being identified with the study of steady states: the examples given of the usefulness of the long-period method are in fact examples of the advantages of assuming steady growth, and the method of long-period positions is said to have "made a re-appearance in more recent times especially with the so-called 'new' growth theories", which again are concerned with steady states, seen furthermore generally as asymptotes of intertemporal equilibria. This part also contains the surprising statement that Lucas in his theory of endogenous growth "abandoned one of the characteristic features of all neoclassical theories, that is, income distribution determined by demand and supply of factors of production: if we concentrate on the 'balanced path', capital in the initial period *cannot* be taken as given along with other 'initial endowments'". But concentration on "balanced paths" in no way prevents distribution from being determined neoclassically, as shown by Solow's growth model where the steady growth income

distribution is not determined on the basis of a given capital stock but is nonetheless determined in a fully neoclassical way: and indeed in that model, without the neoclassical forces determining distribution (in or outside steady growth), it would be unclear why the steady growth should be a full-employment situation; now, the full-employment character of all New Growth theories should suffice to show that the same holds in their case and that it is therefore mistaken to state, as the chapter goes on to state, that "some of the so-called 'new' growth theories belong within the realm of what we have called 'classical' economics" – as if a formal similarity in some aspects of the steady-growth analytical structure could be taken to mean a similarity as to the forces which are seen as determining distribution.

11 The same argument holds for keeping population constant: in fact it too changes over time, but slowly enough relative to the speed with which the composition of production and of capital adapts to demand, as to make it fully legitimate (except for secular analyses) to treat it as given.

12 Mandler (1999: 696) even more explicitly writes that, from the degree of freedom of his equations, "Sraffa concludes that competition leaves relative prices indeterminate". But, revealingly, no supporting quotation is produced.

13 The long-period nature of Sraffa's prices, and their independence from any assumption as to labour employment, show the absence of any basis for the argument (due to Hahn, 1982, and remembered by Mandler) that Sraffa's prices will only be significant for the fluke cases in which initial endowments are precisely the ones which would generate Arrow–Debreu equilibria with relative prices constant through time (cf. Petri, 1999: 41, 55).

14 So Sraffa writes: "The result of adding the wage as one of the variables is that the number of these now exceeds the number of equations by one and the system can move with one degree of freedom" (Sraffa, 1960: 11). In these equations Sraffa takes as given the quantities produced, but this is because his equations have a double role (cf. Petri, 1990: 174): (1) to show that the classical approach to value and distribution is not logically inconsistent, i.e. to show that relative prices and the residual distributive variable are well determined once one takes as given what the classical authors took as given in what Garegnani has called the "core" of their approach: quantities produced, available production technologies, and the real wage or the rate of profits (For my own attempt to sketch the method associated with such a procedure cf. Petri, 1990: 175–6); and (2) to show that the same equations, when interpreted as a subset of the equations of a marginalist long-period general equilibrium, have implications which destroy the notion of capital as a single factor of production; in this second role, constant returns to scale are implicitly assumed, and one may well forget about the given quantities (and assume instead, for example, a given labour employment) if one restricts the analysis to simple production with no land (i.e. if one assumes the conditions of the non-substitution theorem), what is legitimate if the purpose is only critical and not constructive.

15 The different treatment of capital in Walras or in the neo-Walrasian versions is not discussed in Sraffa's published writings; we will have to wait for a careful examination of his unpublished papers to know his views on these versions.

16 Actually the change in real wage intervenes some periods before the change in labour supply, because the change in real wage takes some periods to alter relative prices so as to make a change of technique convenient.

17 Some more considerations on this issue are offered by Schefold in the original presentation of the argument (1997: 500–501, endnotes 38 and 39). It remains to be ascertained, where the tâtonnement will end up when the unique equilibrium is unstable.

18 Or simply no change in labour demand if at a lower wage rate the technique remains the previous one. I am neglecting here the effects on labour demand deriving from the changes in consumer choices, but these can be made as small as one likes through an opportune choice of the utility function, cf. Schefold (1997: 502, endnote 38).

19 The same method appears to be the one implicit in Hicks's notion, in *Value and Capital*, of an *imperfectly stable* system, by which he means a system of general equilibrium equations where a fall in the price of a commodity relative to its equilibrium level makes the demand for it greater than the supply "when other prices are adjusted so as to preserve equilibrium in the other markets" (Hicks, 1946: 66).

20 Cf. e.g. Hicks (1946: 110–111) where however Hicks takes all other prices as given, what is only made possible by the absurd assumption of a given number of decreasing-returns-to-scale firms.

21 Garegnani assumes that there is only one consumption good.

22 In fact Garegnani assumes that the economy ends at date $t+1$, so at that date there is no demand for capital goods.

23 In order to reach an intuitive grasp of the procedure, it may be helpful to concentrate first on a simpler case. Imagine that there are only three dates, 0, 1 and 2; that there is only one product, corn, which is both a (circulating) capital good, and a consumption good; and that the production of corn requires one period and is governed by a differentiable production function $C = F(K, L)$. The endowments are the initial stock of corn C_0, and the endowments of labour at the first two dates L_0, L_1 (at $t = 2$ the economy ends, so no production is started, and the endowment of labour at that date is irrelevant). Assume further, for simplicity, that the demand for corn for consumption purposes at dates 0 and 1 is fixed, and that the supply of labour at each date is rigid and equal to the endowment. Then the supply of corn as capital at date 0, K_{0S}, is given, and its full employment together with the full employment of L_0 determines C_1; hence the supply of corn-capital at date 1, K_{1S}, is uniquely determined. The demand for corn-capital at date 1, K_{1D}, is determined by the full employment of L_1 and by the optimal capital–labour ratio for that period, which depends on the rate of interest between dates 1 and 2. In this simplified example, of course, the savings–investment market is the same as the market for corn-capital of date 1, and it is stable. But now assume that the given productive resources of date 0 can be used to produce several different capital goods, which may be used in variable proportions to produce, together with L_1, the corn of date 2. Then there arises a need to determine the relative proportions in which these capital goods are supplied at date 1. Since investors are indifferent among the several capital goods as long as the rate of return is the same, an excess demand for a capital good, inducing an increase in its price and thus a decrease in the rate of return on its purchase price, will tend to redistribute investment toward other capital goods, and this may be captured by assuming that, if there is excess demand for capital goods, this excess demand will tend to distribute itself proportionately among all capital goods as long as the rate of return on supply price is uniform.

24 Given his critical purpose, Garegnani can grant the uniqueness of the solution for each parametric value of the own rate of return of the numéraire.

25 Owing to the condition of prices equal to costs of production, a decrease in the prices of capital goods of date t relative to the consumption goods of date $t+1$ will mean a decrease of the rentals of factors of date $t-1$ (paid at date t) relative to the factors of date t (paid at date $t+1$), what will imply a decrease also of the costs of production and prices of consumption goods of date t relative to consumption goods of date $t+1$, and thus a lower own rate of return of the numéraire commodity between dates t and $t+1$.

26 In case no equilibrium exists with equality between savings and investment of date t, the implausible result will be, either (if savings remain greater than investment) equilibria with an infinite price of goods of date $t+1$ relative to goods of date t (i.e. a zero discounted price of the latter goods in terms of the former ones), i.e. a rate of interest equal to -1, or (if investment remains greater than savings) equilibria with unemployment of labour of date t, i.e. a zero wage for that labour.

27 See paragraph 21 of his paper. Paragraph 26 is also very important in that it explains why it would be misleading to view the existence of futures markets as implying a demand for capital goods which is specific to each of them in a way not unlike that of a consumption good. The point is that (since savers are not interested in specific capital goods but only in future income, and therefore consider the several capital goods perfect substitutes the moment the investment of savings in any of them is equally convenient) the process of allocation "of the demand for future income among potential sources of supply . . . is no more relevant for an explanation of the rate of interest . . . than the analogous allocation of the aggregate demand for corn among the several farmers is for the explanation of the price of corn". Garegnani comes back on this issue in his first intervention in the Final Discussion.

28 This is true, strictly speaking, only if the number of equilibrium price vectors is finite (i.e. a continuum of equilibrium price vectors is not excluded – think of indifference curves with kinks): the equilibrium quantities are however in all cases uniquely determined, cf. Mas-Colell *et al.* (1995: 609, endnote 39).
29 Cf. e.g. Hayek (1936).
30 "There has been a position strongly held in recent years that the American economy is basically competitive, in that neither firms nor labour unions have, in fact, much control over prices, despite superficial appearances. The present model suggests that the evidence, to the extent that it is valid, relates only to equilibrium and, therefore, to long-run situations" (Arrow, 1959: 49).
31 Cf. e.g. "When an unexpected event occurs, the three ways of evaluating the stock of goods part company and no amount of juggling with units will bring them together again" (1953–1954: 84); "... the wage bargain does not determine the real wage. Keynes' argument was developed to deal with short-period situations, but it applies with full force to equilibrium positions" (p. 98); "To discuss accumulation we must look through the eyes of the man of deeds, taking decisions about the future, while to account for what has been accumulated we must look back over the accidents of past history. The two points of view meet only in the who's who of goods in existence to-day, which is never in an equilibrium relationship with the situation that obtains to-day" (p. 100). A quantity of capital independent of distribution, and determining a decreasing demand curve for labour and a single real wage compatible with the full employment of labour, appears on pp. 96–98; on p. 99 a fall of the rate of interest "encourage[s] the use of more mechanised techniques" and "accumulation may be conceived to push down the rate of profit, and raise the factor ratio". Thus the production function is criticised, essentially, because by hiding the imperfect adaptation of the composition of capital to changes, it induces people to forget that "the comparison between equilibrium positions with different factor ratios cannot be used to analyse changes in the factor ratios taking place through time, and it is impossible to discuss changes (as opposed to differences) in neo classical terms" (p. 100). The main aim, in other words, appears to be a denial that accumulation is determined by the decisions to save; but on the effects of accumulation there is little in the analysis with which a Pigou might have disagreed: "The production function, it seems, has a very limited relevance to actual problems, and after all these labours we can add little to the platitudes with which we began: in country Gamma, where the road builders use wooden shovels, if more capital had been accumulated in the past, relatively to labour available for employment, the level of real wages would probably have been higher and the technique of production more mechanised, and, given the amount of capital accumulated, the more mechanised the technique of production, the smaller the amount of employment would have been" (ibid.).
32 The distinction between long-period and neo-Walrasian equilibria is still absent in the recent book on capital theory (favourable to the critical side) by Ahmad (1991).
33 I dispute this thesis in the second part of my chapter.
34 This assumption is again implicitly taken for granted in Samuelson (1976, cf. in particular pp. 20–21 and endnote 7) and in Burmeister (1991).
35 The confusion of long-period positions with steady states also frequently appeared in critical writings, e.g. in Robinson (1953–1954: 88), Harris (1973: 100).

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Introduction by Frank Hahn

General Equilibrium theory has many enemies. Some object to its mathematical formulation and so to its abstraction. Some find the fundamentals i.e. rationality and greed etc. objectionable. But the most strident critics are the neo-Ricardians (unlike their guru Sraffa). I have never found it easy to see their objections other than that general equilibrium does not lend itself to Marxian theory. Even there that is only partially true as the extensive work of Roemer (1981, 1982) shows. In other words, the neo-Ricardians are very close to general equilibrium. For instance, 'double switching' can readily be shown as possible in a pure neoclassical context. So can everything Sraffa had to say on the choice of technique and ditto for capital aggregation. The 'classical world' is, so it seems, also driven by maximising agents.

Given that I believe these statements to be truthful, readers may be surprised to find me co-editing and co-chairing a get together which had the purpose of 'reconciling' neo-Ricardian and neo-classical theory. I confess that I did so in the belief that the weight of theoretical arguments might cause a change of mind by the neo-Ricardians. Clearly I was mistaken. I cannot complain since for many years, I argued the matter with the Cambridge neo-Ricardians without success and I should have known better.

A GENERAL REMARK

It is of some importance to understand what the argument is about which sadly is not often the case. No economic theory is true, it is at best an approximation to the truth. Unlike what the neo-Ricardians seem to think, general equilibrium theorists for the most part know this as can be seen by the effort which has been devoted by the best of them to improve the theory e.g. by Radner and Mas-Colell, and by their reluctance to use it empirically. It has been found that in general there are too few markets for the invisible hand to do its job, in particular its intertemporal job. One cannot in general exclude multiple equilibria, in the canonical version there is no room for fiat money, indeed exchange or interaction of any kind is not modelled, and so on and so on.

Although I never believed it when I was young and held scholars in great respect, it does seem to be the case that ideology plays a large role in economics. How else explain Chicago's acceptance of not only general equilibrium but a particularly simplified version of it as 'true' or as a good enough approximation to the truth? Or how to explain the belief that the only correct models are linear and that the von Neuman prices are those to which actual prices converge pretty smartly? This belief unites Chicago and Classicals; both think that the 'long-run' is the appropriate period in which to carry out analysis. There is no empirical or theoretical proof of the correctness of this. But both camps want to make an ideological point. To my mind that is a pity since clearly it reduces the credibility of the subject and of its practitioners. There are many reasons for thinking market economies superior to command economies without relying on the 'Fundamental Theorem of Welfare Economics' and there are often strong arguments for departing from a market economy which have nothing to do with a classical theory of value.

Let us call these various approaches paradigms if they are logically coherent. I once called them rules of grammatical argument. On the general equilibrium side there are simple rules: if an agent believes it can improve its market outcome it will do so. Hence only positions where this is not possible are equilibria. But there is nothing to say that markets and beliefs about markets signal real improvement possibilities. It is an unfortunate characteristic of much of neoclassical literature that it is supposed that rationality entails only believing what is really the case and knowing what is relevant to know. Correct statement of the neoclassical paradigm is not easy and not often encountered. Of course it is also not often explicitly stated. But to avoid mistakes the distinction between what is believed and what is true is very important.

It is also not easy to characterise the neo-Ricardian paradigm. But reading their contributions here suggests its rules are something of the following sort: as far as possible let the analysis proceed without invoking any psychological variables. Like the Lucasians they are only interested in the long run. However, the foundation of value theory is based on technology, and linear constant returns, while distribution is the outcome of class struggle.

In preparing this preface I was forcibly struck by how much the currently most extreme neoclassical school, the Lucasians and the neo-Ricardians overlap. Keynes quipped that in the long run we are all dead. Both the schools engage in only long-run analysis. The neo-Ricardians claim that the economy 'gravitates' to long-run equilibrium, while the Lucasians take this, it seems, for granted. Both are committed to a thorough going determinism: there is a unique long-run equilibrium else the gravitation argument would not deliver. Lucas is quoted as claiming that non-long-run positions are very transitory because the path of actual economies are saddles which follows if actual economies are on optimum accumulation paths. Extremists of all sorts eventually meet.

I now turn to individual contributions. In doing so I distinguish three groups: (1) the Conformists (General Equilibrium Theorists); (2) the Revolutionaries – they want to replace the neoclassical economics by something quite new; and (3) the neo-Ricardians.

GENERAL EQUILIBRIUM

In what follows there are a number of straightforward general equilibrium papers. They exemplify what was said earlier, the continuing attempt to make the theory more widely applicable. First Accinelli considers an infinite dimensional economy and asks how serious the possible cases of non-existence are. These are largely technical and turn on the choice of topology amongst other difficulties. More than fixed points is involved. There are more difficulties than critics have thought of and yet one sensibly makes the judgement that they are not crippling.

General equilibrium theorists have made extraordinary efforts to prove that an equilibrium exists. There are no necessary conditions for that and indeed cases of non-existence are known. General equilibrium theorists have been content to make sure that their version satisfies sufficient conditions to avoid what would literally be nonsense. Yet that is not without economic interest for when these conditions cannot hold in a particular case we learn something about the limits of competitive analysis.

Fisher (*Stability and dynamics in general equilibrium systems*) took on a difficult lacuna in existing general equilibrium theory, the absence of a plausible dynamics. We know that even with tâtonnement which indeed has no claim to realism, convergence to equilibrium is not assured. While I share the economist's preference for equilibrium analysis, it would be nice to have more theoretical support for the view that the economy always seeks an equilibrium of the model one has adopted. The difficulty in the present case is that the postulates of perfect competition in general equilibrium makes it impossible to designate any economic agent as changing price (Arrow, 1959). Fisher has made heroic attempts to extend the Hahn–Negishi stability result to a production economy. He can, as the reader will see, get quite a long way and of course without the use of an auctioneer.

Morishima (*Schumpeter and general equilibrium*) has written a characteristically learned contribution. It is known that Schumpeter admired Walras (and math), but had not much mathematical knowledge and so I have never had a clear idea what he made of general equilibrium. Anyone in the same boat is advised to read Morishima's contribution.

Sabourian (*Game theory and general equilibrium*). These have a long standing relation since von Neumann's article was published in the *Ergebnisse Eines Mathematischen Kolloquiums* in 1937. Arrow and Debreu used Game theory in their existence proof of 1954. Since that time there have been many more manifestations. The basic idea in these latter has been to demonstrate the coincidence of Nash equilibrium with the equilibrium of a competitive economy. Since not all equilibria are Pareto-efficient it is clear that such a result can only be expected in specially constructed games. Sabourian who discusses bargaining theory and the well known assertions of Rubinstein and Wolinsky (1990) reaches sensible conclusions (see also Gale 1986a,b).

Hahn (*General equilibrium and macroeconomics*) notes that the popular demand for micro foundations for macroeconomics has simply led to a bastard microeconomics where a single agent defined as representative acts exactly as an ordinary agent of micro theory would. Macro is transformed to the microeconomics of Robinson Crusoe who is as fictitious as his model. Since many economists seem satisfied with this muddled attempt at micro foundations it is no surprise that rational expectation equilibrium is so extraordinarily popular and largely unjustified. Hahn's suggestion in this chapter is to reverse the search for micro-foundations of macro to one of macro-foundations of micro. He shows that it is perfectly reasonable to suppose that actual agent's decisions depend on both micro and macro (i.e. aggregated) information. It is a beginning, he feels not only of sense but of honesty.

Magill and Quinzii (*Incentive role of the stock market*). These two authors are well known for their contributions to the finance literature in a General Equilibrium context. In this chapter they are concerned with the well known moral hazard of the executive of a firm using effort which is unobservable. Hence investors cannot as such calculate the firm's outcome for each 'state'. There are a finite number of these and both investors and executives can observe each other and make deductions since the firm will offer derivative securities which span the state space.

The outcome is a somewhat baroque model together with some highly implausible assumptions. It is a peculiar result for a Frenchwoman presumably wedded to theoretical elegance and an Englishman with a traditional hostility to 'system building'. I am neither, but I confess that I do not care for this sort of exercise unless it is designed to show how crazy a belief in General Equilibrium solutions for these problems really is.

Magill and Quinzii (*The stock market as intergenerational transfers*). It is with relief that one reads this. It is clearly written and it is interesting. Although it too has some heroic assumptions, somehow they are acceptable as a means of gaining insight into models in which the young can lend to the old by means of bonds or shares (the old will be dead as the debt matures). In addition we have production properly, although simply, modelled. The paper then proceeds via Diamond's famous early construction, and via David Gale to consider the various cases which can arise. Some of these are rather striking and unexpected, but all have some relevance to the design of social security. In spite of its simplifications (of which a serious one seems to me to be the neglect of uncertainty), it is a workmanlike contribution which even 'practical' men and women can take seriously.

Mandler (*Sequential indeterminacy problems*). Debreu proved that his economies have generically (locally) unique equilibria. (The equilibria are isolated.) The neo-Ricardians have denied that this is the case in economies where technology is described by a series of activities (so that isoquants can have corners). Mandler shows that lack of substitutability is not a problem (leading to indeterminacy) if attention is concentrated on long-run equilibrium. In such an equilibrium one starts with a given endowment of factors and all future prices and allocations are related to this. However, if we study a sequence economy and that at each date the endowment is the endogenous result of agents' earlier choices, then determinacy is no longer assured.

Mandler lays out his argument clearly and proves what he claims. But he also notes the irony of indeterminacy being absent in the usual long-run equilibrium which is the one on which neo-Ricardians concentrate.

There is one last remark which I wish to squeeze in, which may be appropriate for a volume such as this. It seems to me that almost every thing in this book is intelligent and some of it is thought-provoking. But nothing is conclusive and ready for practical use. Only the very simplest economic ideas are of that sort and there are very few of those.

I myself have always had a rather modest view of economic theories. In the first place they are Gedanken Experiments and a substitute for real experiments which to put it mildly are hard to perform in economics. In the second place they are a sort of quality control.... they enable one to spot nonsense in policy arguments. These seem to me sufficient rewards and there is no reason to try and save our self respect by applying our theories *tout court*.

THE REVOLUTIONARIES

The brilliant summing up of the conference by Alan Kirman is noted. However, he distributed at the School two further papers (Kirman, 1998, 1999). Both of these are thought-provoking. I shall only have space to notice 'Aggregate Activity and Economic Organisation'.

In this, Kirman proposes that interaction between agents as well as its manner is important for an understanding of the economy in general and macroeconomics in particular. Organisation for interaction reduces the calculations which agents will need to make (this, in a different way, is a point I also make in my contribution). It also carries the implication that the 'law of large numbers' will not help avoid macro-disturbances. Kirman then uses a nice model of Durlauf to make his main points. These are good and almost certainly important points.

There is quite a bit more to come, however it would be inappropriate to give a detailed exposition here. Kirman studies networks and their evolution, as well as contagion of expectations. The latter is made good use of in remarks on the stock exchange. Moreover, the general (stochastic) approach means that long-run equilibrium is consistent with changing stock prices and ownership.

In all of this Kirman has obviously benefited from visits to Santa Fe – it certainly fits with his World-Ausschaung. This is a helpful and promising route out of mechanical theorising and to my mind is much more convincing than most macroeconomic models we now have. But we will need to learn a great deal more about actual distributions of types and of the manner in which they update beliefs before we can make proper use of these tempting avenues.

Gintis (*Endogenous contract enforcement and general equilibrium theory*) has contributed an interesting and well argued criticism of General Equilibrium and some suggestions for improvements. His criticism of Walrasian General Equilibrium theory is that: (1) there is no attention given to the enforcement of contracts; and (2) that it is not based on evidence. Under (1) he says that humans are neither rational (in the appropriate sense) nor greedy and he proposes to replace 'homo economicus' by 'homo reciprocans' (with a propensity to cooperate). I am sure that this is an element of the truth but I for one would be sorry if his suggestion were to be adopted. One of the advantages of neoclassical economics is its deep sense of 'original sin' and one of its triumphs is to show how, in spite of that, an economy can be organised without an organiser (self-organising). Note 'can be'. It is one of the objections to Marxist theory that by showing that some action is in the interest of a 'class', it believes to have explained it. Certainly Gintis' suggestion is reminiscent of the 'class conscious' agent. Subsequently Gintis takes Walrasian theory to task for not explaining why it 'requires private ownership'. He does not mention Hayek's view on information nor does he mention innovations. His treatment here is extremely superficial as was that of the market socialists of 70 years ago. (Incentive theory has made great strides since then.) He approvingly quotes a delphic comment by Holmstrom that the sole merit of competition is an 'optimal extraction of information'. One is not told whether this is potential or actual information and the view strikes me as not very clever without some defining and arguing. (After the war espresso coffee came to England. At first it was expensive and the coffee bars were very crowded. Soon competition made great improvements. What information was revealed by this?)

After this shaky introduction there is a discontinuous improvement. Some of it is fairly technical (in contradistinction to this introduction) and I do not consider it appropriate to summarise it. But it is shown that objects, the quality of which cannot be observed or contracted, leads to their being in excess supply (Labour, efficiency wage, goods prices exceed marginal cost, Stiglitz debt contracts.). In deducing unemployment under this rubric Gintis ignores competition between firms for labour when effort has different utility implications for different workers. Similar cases of quality competition and incentives for managers are analysed game-theoretically. It is shown that incentives may be cheaper when there is competition. In all of these examples one gets deviations from first best. (There are many more to be had.) But the new 'homo' only makes his/her appearance later.

Of course common interest games are the obvious vehicles for a 'propensity' to cooperate. But so are ordinary two person extensive games. Gintis shows what we have come to expect. But there is one extra twist which is nice – he investigates the conditions in which the new homo can 'invade' the old homo's game. There is then an outline of an evolutionary account of the emergence of the new homo. (But only in an example.)

I enjoyed the chapter and believe that the reader will do so also. Not least, Gintis has provided an extensive list of references.

Foley (*Statistical equilibrium and financial arbitrage*) has made a very interesting contribution which is best left unsummarised since he uses some concepts not familiar to many economists which are very well introduced in his own essay. The central observation is that agents transact at stochastically given intervals and efficient transaction arrangements minimise the entropy of the economy. This sounds straightforward, but there are some subtleties to get out of the way. Clearly one has to suppose that transactions are costly and there is a coincidence of wants problem to be tackled. But Foley is the only contributor to escape some of the more mechanistic aspects of the problem.

Mordecai Kurz has become well known by his work on expectations and uncertainty. In particular he proposes to replace 'rational expectation' by 'rational belief'. The latter only contains what can be deduced from a (long) run of data and hence does not include the certainty that the process observed is stationary. I have some sympathy for his views but it is doubtful that 'rational beliefs' are greatly more credible than rational expectations. On the other hand they help to explain certain happenings on the stock exchange.

THE NEO-RICARDIANS

Petri (*A Sraffian critique of general equilibrium theory and the classical-Keynesian alternative.*) is a fully registered neo-Ricardian and the reader will have to make up his own mind. I shall deal with only one of his points: the impossibility of aggregating capital goods and the alleged need of neoclassical theory to do so.

Everyone agrees that aggregability is non-generic and have done so long before neo-Ricardians got going (Nataf, Gorman, Klein, Leontief, Fisher). Nonetheless K was used by many for the same reason that Keynes used 'Investment'. It simplified the analysis: indeed made it possible. It would be nice to be able to claim that neo-Ricardians showed us that dreadful mistakes could result from this short cut. They did not. Instead they spent their energies on double switching and never once to my knowledge argued that that was important. It was easy to use such results to show that there in general was no index of capital which was inversely related to the rate of interest. The trouble with this correct assertion is that it has a sting in the tail: the long-run equilibrium may be unstable and so 'gravitation' is in peril. That is one of the main criticisms of both camps: since they do not provide a theory of convergence the neo-Ricardians don't seem to realise that the main possible relevance of their results is to a dynamics of the economy (There is only one neo-Ricardian attempt at dynamics known to me and that is badly flawed since it ignores changes in the values of capital goods during adjustment.). Indeed Petri's chapter shows that they are determined to ignore all their ill-behaved cases when it comes to 'gravitation'. Lucas also gives up since he considers only one of an infinity of paths to justify his view that the economy is normally in long-run equilibrium.

It is repeated frequently here that neo-classical theory requires aggregated capital. It comes from the idea that in measuring, say, the marginal product of labour we must only vary labour, and it is thought that unless it is to be either zero or positive we must keep some co-operating inputs fixed (Capital (K)). But the dual of a programme shows that we can estimate the shadow prices of all given inputs as the difference made to the minimum production cost by a small change in any one of them. If there are rigidly fixed proportions then neo-classical economics also delivers the sensible answer without aggregation. What more can one want?

I next turn to Kurz and Salvadori (*Is there a 'classical' theory of value and distribution?*). They give an interesting account of some classical economists but then turn to the present classical vs. neoclassical debate. Like all their colleagues they attack the logical foundation of the latter. This is surprising in view of the powerful logicians from Arrow–Debreu to Mas-Colell who have devoted their talents to general equilibrium. Of course that is not an argument, but it is an explanation of surprise and *a priori* scepticism on my part. This is so especially since Sraffa considered himself only to have delivered the introduction to a critique of neo-classical economics.

That Kurz and Salvadori are devoted to the long-period methodology is an explanation of my surprise. For they surely understand that what they designate as long-run prices are what the rest call 'shadow prices' associated with a deterministic steady state. What possible use are they when the actual situation is quite different? Here are some of the more important points:

1 The economy is stochastic so that 'gravity' takes one to a distribution of shadow prices.
2 In any case not all the paths the economy can take are saddle point paths.
3 If the economy starts far enough away from the unique long-run equilibrium there are paths 'gravitating' towards the latter which may take a 100 years to get close. One has to pay attention to this problem if one wants to maintain that shadow prices are 'good enough'. This in turn means that one needs to have a theory of market prices which we are told cannot be had.

4 Like the Lucasians the long-run view has a chance of working if we live in a fully deterministic world. 'There is always a unique account of how the economy was bound to get where it now is'. A piece of 'nonsense on stilts'.

I fear that all of this sounds harsh especially since the contribution I am discussing is amongst the clearest and best by the neo-Ricardians. So I urge the reader to peruse it.

Garegnani (*Saving and investment in a system of general intertemporal equilibrium.*) has contributed a long piece, related, as he tells us, to a much longer book-length one. He finds it difficult to say things briefly. As an instance take the long and rather convoluted discussion of the 'Factor-Price Frontier', why it is negatively inclined and why there is a point (r, 0) and (0, w) on it. But we all have different habits and it is wrong to grumble. More serious for me is that I did not discover what the argument was about. It is conducted in a two good, two period context, so one should be able to follow. But either I am too old or the habits of thought are (for me) so peculiar, that I am baffled, so that the reader can expect no help here. But this does not mean that no reader will be more fortunate than I have been and see Garegnani's contribution in a clear light. Let me remind them, that we are out to discover (a) whether neo-Ricardian theory proves without ambiguity that neo-classical analysis requires an aggregate measure of capital (but not of labour or land?) and (b) that marginal productivity theory is incoherent. Sometimes I find Garegnani attacking these questions directly.

Schefold (*Applications of the classical approach.*) has taken as his task to examine what application can be made of neo-Ricardian theory to practical matters. The chapter is largely based on a 1997 book which he has written. He says that the neo-Ricardian theory takes technology as fixed so that for many applications I regard it as unhelpful (What is the consequence of higher oil prices, lower interest rates etc. on relative prices and labour intensity of production?). Schefold is not what I have called an extremist so he does not insist on the applicability, referring only to long-run equilibrium. As usual, no mention is made of uncertainty.

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1 Existence of GE

Are the cases of non-existence a cause for serious worry?*

Elvio Accinelli[†]

In this work, we attempt to characterize the main theoretical difficulties in the proof of the existence of competitive equilibrium in infinite dimensional models. We shall show cases in which it is not possible to prove the existence of equilibrium and some others in which, although the existence of equilibrium can be proved, the equilibrium prices seem not to have natural economic interpretation. Nevertheless in pure exchange economies, most of these difficulties may be avoided by mild restrictions on the model. In productive economies new specific problems appear, for instance non-convexity of the production sets or non-boundedness of the feasible allocation sets. To prove the existence and the efficiency of the equilibrium in productive economies we need some strong hypotheses about the technological possibilities of each firm.

1.1 INTRODUCTION

The existence of Walrasian equilibrium can be established in considerable generality. In this work we will offer an examination of the existence problems in infinite dimensional economies. Although the title of this work does not suggest any restrictions to some special models, we limit ourselves to consider economies in infinite dimensional spaces; the advantage of this approach is its generality, and the cost is the necessity to follow the methodology of functional analysis. The well known difficulties that arise in finite dimensional cases to prove the existence of equilibrium, like the non-convexity of preferences or other of this kind, survive in infinite dimensional cases, and at the same time new problems appear. A successful formulation of this question not only requires novel technical arguments and a modification of our finite dimensional intuition, but also a new mathematics. Every economic problem requires its own appropriate mathematical treatment. For instance, to prove the existence of Walrasian equilibrium in a finite dimensional model, fixed point arguments were required; to prove the existence of the equilibrium in more realistic models, like infinite economies, i.e. economic models that

* Work for the XII Workshop on "General Equilibrium: Problems, Prospects and Alternatives" 4–11 July 1999, at Certosa di Pontigniano, Siena, Italy. The author is grateful to Paulo K. Monteiro for helpful comments that were very important for this work, and thanks Celina Gutierrez and Fabio Petri for several comments and suggestions.
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consider the consumption space as a subset of an infinite dimensional vectorial space, new and more sophisticated mathematical arguments, specially new topological arguments, are required. So, it may be that to obtain success in providing correct answers to new and old questions yet not answered in economics, new mathematical arguments will be necessary. The economic problem determines the mathematical tool that is applied to obtain the solution and its precise formulation; this does not mean that we have to take a mathematical tool and then look for application.

Looking at commodities as physical goods which may differ by date or by the states of the world in which they become available, and allowing infinite variation in these contingents, the generalization of the classical model of GE to models of economies with infinitely many commodities looks natural.

While the extension of the classical general equilibrium model to an infinite dimensional setting gives new answers to relevant questions of the economic theory, new theoretical challenges appear; for example, new cases where it is still now theoretically impossible to prove the existence of the Walrasian equilibrium or where the mathematical interpretation or economic meaning of this equilibrium is not concrete or natural. Recent researches done by different authors tried to obtain new theoretical tools or more general conditions for existence of the Walrasian equilibrium on these models. For instance, the condition in preferences defined in (Mas-Colell, 1986) and known as properness plays an important role in compensating for the absence of interior points in positive cones in some Banach spaces. We shall analyze this and other conditions that play analogous roles.

For finite dimensional theory the finite dimensional linear space, R^n is a canonical space, the situation changes drastically in more general conditions. By contrast with the finite dimensional cases there is not a canonical infinite dimensional commodity space. Different economic applications require models involving different (non-isomorphic) infinite dimensional linear spaces.

For instance, the usual finance models describe the commodities as being a stochastic process, so this suggests the space of square integrable functions L_2 as the commodity space, while in growth theory the typical space is the space of essential supremum bounded functions L_∞, in which each $x(t) \in L_\infty$ may be interpreted as an inter-temporal allocation.

As Araujo and Monteiro (1989) show for finance models the requirement of infinite marginal utility for consumption at zero, makes that typically (or in the usual *jargon* generically) on initial endowments, the Walrasian equilibrium does not exist. This means that taking into account the endowments as parameters and if infinite marginal utilities at zero hold, the existence of equilibrium is accidental. More formally, if on the consumption space a measure is available, this property implies that the set of endowments such that the equilibrium exists is a zero measure set.

In contrast, in growth theory for every strictly positive endowment, the existence of equilibrium can be proved. As we shall see in what follows, in this case the problem is the concrete interpretation of the equilibrium prices in both senses, mathematical or economical.

In models that allow for many different characteristics, we are led to consider the Borel signed measures on a compact metric space K as the commodity space, where K represents the commodity characteristics and a positive measure x on K, represents a commodity bundle comprising a quantity of some of these characteristics, see Mas-Colell (1975).

It is important to notice that in infinite dimensional models, if the commodity space is not a Hilbert space (like $L_p, p \neq 2, 1 \leq p \leq \infty$), then on this space one cannot find a single C^2 strictly quasi-concave utility function assuming its maximum on the interior of the consumption set and giving rise to a continuously differentiable demand function. Moreover, Araujo proves that if the commodity space is non-semi-reflexive, i.e. the dual space is not equal to the dual of its dual (for instance L_1) and if the consumption space has a non-empty interior, then there does not exist a quasi-concave continuous utility function giving rise to a well defined demand function on it (Araujo, 1987). By means of the *excess utility function* the Negishi approach avoids most of the difficulties related with the non-existence of a differentiable demand function. This approach allows to introduce differential methods in infinite dimensional models (see for instance Accinelli (1996)), and gives a deep intuition of the structure of the equilibrium set. Nevertheless this approach is very dependent on utility functions and also on the properties of the preferences. The Negishi approach also depends on the Pareto-optimality of the equilibrium, and thus on the topological properties of the commodity spaces that guarantee the existence of Pareto optimal allocations. Some examples of well behaved economies which have not Pareto optimal allocations are given in Araujo (1985).

In our work we will consider models in which preferences may be representable by utility functions. In finite dimensional economies, every continuous preference is representable by a utility function. However, on infinite dimensional spaces this result may not be useful, because in general we lack separability,[1] besides continuity, countable boundedness must be added[2] (Monteiro, 1987). Nevertheless in Richard and Zame (1986) it is proved that in a positive cone of a normed vector lattice, uniformly proper, continuous and convex preferences have a continuous utility function representation.

Prices will be elements of a dual space L^* i.e. a space of real continuous linear forms on the topological vector space L, in which the commodity space is included. Mathematical possibilities and economic meaning of some properties of our models will depend only on the pair (L, L^*) selected. In finite dimensional spaces, the dual space and the space are the same thing; it is natural to view prices and commodity bundles as vectors in R^l, we suppose that there are l commodities. The value of a commodity bundle x at prices p is given by $p \cdot x = \Sigma_{i=1}^{l} p_i x_i$. Hence, each price vector defines a linear function on the commodity space L. So, prices are in the dual space $(L)^*$, and for the case $L = R^l$ we see that $L = (L)^*$, on infinite dimensional spaces this equality holds if and only if L is a Hilbert space.

1.2 STRUCTURAL CHARACTERISTICS OF THE INFINITE DIMENSIONAL MODELS

To prove the existence of Walrasian equilibrium in infinite dimensional models is a cause of serious worry because to prove its existence is to prove the logical consistency of the model. There are cases in which it is not possible to prove the existence of Walrasian equilibrium, but in most of them the main difficulties may be avoided by mild restrictions on the hypotheses of the model. As we said in Section 1.1, different (non-isomorphic) infinite dimensional spaces arise in different economic situations. In contrast with the finite dimensional case we point out the following six characteristics of the infinite dimensional spaces, each of them gives rise to a new theoretical challenge to give a positive answer to the question of the existence of equilibrium:

1 *Non-uniqueness of the topology* While in finite dimensional vector spaces there is only one Hausdorff linear topology (Aliprantis and Border, 1994), an arbitrary infinite dimensional vector space admits more than one linear topology.

2 *The possibility of the existence of non-continuous linear functionals* In a finite dimensional space every linear functionals is continuous, remember that the kernel of a linear functional is a finite dimensional subspace and then it is closed, so continuity of linear functionals follows. In infinite dimensional vector spaces there may exist non-continuous linear functionals. To observe this, consider the vector space of every continuous function from $\Re$ in $\Re$ with compact support, $C_c(\Re)$. Let $x(t) = 0\ \forall t \notin [a,b]$ be a continuous function in $[a,b]$, considering the norm given by the supremum in $[a,b]$. Consider the linear functional defined by the integral $\int_{\Re}: C_{[a,b]} \to \Re$. It is easy to see that this is a non-continuous linear functional. To prove the claim consider: the function $x_n(t)$ that takes a value equal to 1 if $t \in [0,n]$, that is equal to 0 out of the interval $[-1, n+1]$, and is linear in $[n, n+1]$, in this case x_n is in $C_{[a,b]}$ but $\int_{\Re} x_n(t) = n+1$ thus, the linear functional is not bounded in the closed unit ball, and then it is non-continuous.[3]

Recall that a positive linear functional should be interpreted as representing the economic concept of prices. In many commodity spaces positive linear functionals are automatically continuous,[4] nevertheless, not every Riesz space[5] admits strictly positive linear functionals. This is the case of $\Re^N$, the vector space of all real sequences on N. This follows, because the topological dual of this space is the space of the sequences in $\Re^N$ whose terms are zero, except for finitely many of them, see Aliprantis and Border (1994). Remember that a functional p is positive on a Riesz space E, if for each $x \in E^+$ (the positive cone of E), $\langle p, x\rangle \geq 0$, and is strictly positive if $x > 0$ implies $\langle p, x\rangle > 0$, for all $p \in (L^*)^+$.

Continuity of prices is in part a mathematical condition, and reflects the choice of the topology, and in several settings is a weak requirement. Nevertheless the choice of topology has a strong economic meaning. It is possible to have the existence of equilibrium allocations supportable only by non-continuous linear functionals (prices).[6]

3 *Multiplicity of dual spaces* A characteristic of the infinite dimensional economic models is that the pair–commodity-price is described by a dual system $\langle L, L^*\rangle$ where L is the commodity space and its dual L^* is the price space. We don't care where L^* comes from, we just need that the dual space itself be another vector space (so an infinite dimensional vector space may have several (non-isomorphic) dual spaces).

A dual system is a pair $\langle L, L^*\rangle$ of vector spaces together with a function $(x, x^*) \to \langle x, x^*\rangle$, from $L \times L^*$ into $\Re$ satisfying:

- The mapping $x^* \to \langle x, x^*\rangle$ is linear for each $x \in L$.
- The mapping $x \to \langle x, x^*\rangle$ is linear for each $x^* \in L^*$.
- If $\langle x, x^*\rangle = 0$ for each $x^* \in L^*$, then $x = 0$.
- If $\langle x, x^*\rangle = 0$ for each $x \in L$, then $x^* = 0$.

Each space of a dual pair can be interpreted as a set of linear functionals on the other. A locally convex topology τ on L is said to be compatible with the dual pair $\langle L, L^* \rangle$ if for each continuous linear functional f in (L, τ) there exists $x^* \in L^*$ (the topological dual of (L, τ)) such that $f(x) = \langle x, x^* \rangle$.

4 *Lack of continuity of the wealth map* The wealth map $(x, p) \rightarrow \langle p, x \rangle$ where $x \in L$ and $p \in L^*$, is jointly continuous in the finite dimensional case, in the infinite dimensional spaces it has sense to ask for the joint continuity, and we will see that the answer depends on the topology of these spaces. Indeed this map is jointly continuous in the norm, but it is not jointly continuous if one of the spaces in the dual pair is given with a weak topology and the other one with a norm topology.[7] (Aliprantis and Border, 1994).

5 *Attainable sets need not be compact* The first problem is that some of the sets which are bounded in finite dimension may not be bounded in an infinite dimensional setting. For instance, if the commodity space is $L = L_\infty([0, 1])$, and the price $p \in L_1([0, 1])^+$ is not 0, then the budget set $B = \{x \in L_\infty([0, 1]): \langle p, x \rangle \leq \langle p, w \rangle\}$ is never bounded, see Mas-Colell and Zame (1991).

The second problem that arises is that the absence of the property of norm-compactness of the unit ball is a characteristic of the infinite dimensional spaces (this is the claim of the Riesz theorem: *A normed linear space is finite dimensional if and only if its closed unit ball is compact*[8]). Moreover, if the space L is not semi-reflexive,[9] then there exists a bounded and closed set with the weak topology $\sigma(L, L^*)$ that is not compact with this topology, see Schaefer (1996).

6 *Non-supportability of convex sets* Two disjoint non-empty convex subsets can be separated by a non zero continuous linear functional, provided one of them has an interior point, so this condition is always guaranteed a in the finite dimensional case, but it is no longer valid for infinite dimensional spaces.

Then taking C to be the set of consumption bundles strictly preferred to x with usual convex preferences, in infinite dimensional models the existence of supporting prices is not guaranteed.

1.3 EXAMPLES OF PURE EXCHANGE ECONOMIES WITH NON-EXISTENCE OF EQUILIBRIUM

From now until Section 1.6, in which we will introduce production, we restrict our attention to pure exchange economies. There are N consumers characterized by their consumption spaces in the positive cone of a locally convex, topological vector space. We will restrict ourselves to models with a finite number of consumers, because models with a double infinity of traders and commodities like overlapping generations, are not provided with the first welfare theorem (Samuelson, 1958) and in most of the approaches to proving the existence of the Walrasian equilibrium in infinite dimensional cases its efficiency is a point of departure. Each commodity space is endowed with an order structure, given by consumers preference relation $\succeq_i$. Preferences are a complete pre-order, monotone and convex[10] on the consumption set. Each consumer has an initial allocation (*endowment*) w_i that belongs to the positive cone in his consumption space. An economy will be represented by the set:

$$\mathcal{E} = \{(L, L^*), \succeq_i, w_i, i = 1, 2, \ldots, N\},$$

where (L, L^*) is a dual system that shows the topological characteristics of the consumption space.

Let us begin this section recalling the definition of the Walrasian equilibrium:

DEFINITION 1 A Walrasian or competitive equilibrium is a pair $(p, x), x \in L, p \in L^*$ such that $x \in \mathcal{B}_i(p)$ and $\bar{x} \succ x$ implies $\langle p, \bar{x} \rangle \geq \langle p, w_i \rangle$, where $\mathcal{B}_i(p) = \{x \in L_+ : \langle p, x \rangle \leq \langle p, w_i \rangle\}$, is the budget set of agent i.

With the success obtained by the Black and Scholes formula the finance models have received a great stream of interest. Theorems on existence of equilibria for models like this, where a trading strategy is an adapted process θ specifying at each state s and time t the number $\theta_t(s)$ of units of the security to hold, and where θ is a square integrable process, were obtained by Araujo and Monteiro (1989). However in Araujo and Monteiro (1988) it is showed that for these kinds of models, if the utility functions satisfy the Inada conditions,[11] the set of the endowments for which the economy does not have an equilibrium is residual. This means that generically the most useful models in finance do not have a Walrasian equilibrium.

More formally, Araujo and Monteiro have proved that for economies with separable utilities $u_i : L_p^+ \to \Re, 1 \leq p < \infty$ (for (S, μ) a measurable space),

$$u_i(x) = \int v_i(x(s), s) d\mu(s), \tag{1.1}$$

where v_i is concave, monotone and differentiable, and such that the derivative at $(0, s), v'(0, s) = \infty$ for each s, the set of endowments that allows us to prove the existence of equilibrium is of first category, this means that the set of endowments where the economy has an equilibrium is a countable union of nowhere dense sets on L_p^+ (Araujo and Monteiro, 1989). Recall that a set A is called nowhere dense if its closure has empty interior.

Nevertheless, if the endowments are positive ($w_i \in L_p^+ - \{0\}$) the condition that establishes that $v'(w(s), s)$ belongs to L_q (the dual space of L_p, where $(1/p) + (1/q)$) is sufficient to prove the existence of equilibrium in the space generated by $[-w, w]$, $w = \Sigma_{i=1}^n w_i$[12] (in finance setting $p = 2$). (This condition is equivalent to the assertion that utilities are proper[13] in all individually rational weak optimum (Mas-Colell and Zame, 1991).) This result also shows that even the supportability of the Pareto optimal allocation is not a typical property.

1.3.1 The possibility of emptiness of the Pareto optimal set

As we said before, some methods to study the existence of equilibria are strongly related with the existence of the Pareto optimal allocation, one of these is the Negishi approach.

Let us begin this section with the definition of ordered vector space:

In a *Riesz space E* (which is a partially ordered vector space that is at the same time a lattice), an ordered interval is any set of the form:

$$[x, y] = \{z \in E : x \leq z \leq y\}. \tag{1.2}$$

If the dual pair $\langle E, E^* \rangle$ is symmetric,[14] where E^* is the normed dual space of E, then the intervals of E are $\sigma(E, E^*)$ compact. (The norm dual L^* of a normed space $(L, \|\cdot\|)$ is the vector space of all norm continuous linear functional on L equipped with the operator norm, also denoted $\|\cdot\|$. Recall that the norm dual of a normed space is a Banach space).

Working with the dual pair of the bounded real sequences as the commodity space, and the space of absolutely summable sequences as the dual space (l_∞, l_1),[15] Araujo (1985) proves that if we relax the assumption of continuity of the preferences with respect to the Mackey topology it is possible to obtain economies without Pareto optimal allocations. This result follows from the fact that the second dual space of the space of bounded sequences with the Mackey topology, $((l_\infty, \tau_{Ma})^*)^*$ is isomorphic to the dual space of absolutely summable sequences $(l_1)^*$, because $(l_1)^* = l_\infty$. Then with this topology, l_∞ is a semi-reflexive space, that is a locally convex space for which $L = (L^*)^*$, in our case with the Mackey topology $((l_\infty)^*)^* = l_\infty$.[16] Araujo proves that if the economy $\mathcal{E} = \{(L, L^*), \succeq_i, w_i, i = 1, 2, \ldots, N\}$, where L is a semi-reflexive space has weakly continuous preferences, and the feasible set is bounded and closed for the weak topology, then $\mathcal{E}$ has Pareto optimal allocations. Moreover the existence of a Pareto optimal allocation is equivalent to the semi-reflexivity of the commodity space.[17] Then if we relax the topology, the space of absolutely summable sequences will not be a semi-reflexive space, and then it is possible to obtain economies on l_∞ without Pareto optimal allocations.

The Mackey topology has the following economically interesting property: she reflects the property of *impatient* or myopic behavior. A preference relation displays an impatient behavior if the present consumption is preferred to future consumption, and the taste for future consumption diminishes as the time of consumption recedes into the future.[18] The intuitive relation between Mackey continuity and myopia is clear. As long as a consumption plan x is equal to a in a finite time and never grows beyond some finite bound in infinite time, the myopic agent is indifferent to the change from a to x, that is $a \succeq x$ and $x \succeq_i a$. However, if $x(t) - a(t) > \epsilon$, at some finite time, (if more is better) then $x \succeq a$ no matter how small $\epsilon > 0$, even if $x(t) = 0$ beyond some $t = \tau_0$ and a is positive for all time. On the other hand, $\succeq$ is a Mackey continuous preference, if and only if, whenever x is infinitely close to a in finite time $a \sim x$, i.e. $a \succeq x$ and $x \succeq a$, no matter how a grows (never beyond some finite bound) for infinite time.

Notice that the assumption of weak continuous preferences or utilities, is a restrictive condition, because weakening a topology on a given space, its continuous functions set generally diminishes. Recall that by definition, a function u of X into Y, is continuous if and only if the inverse image of an open set $V \subset Y$ is an open set on its domain, so the more open sets there exist on X, the more continuous functions there are, and as every open set in a weak topology is open in a finer one, our above notice follows. Symmetrically: the stronger (finer) the topology on a given space L, the more continuous functions there are (Narici and Beckenstein, 1985).

It is important to describe now some natural topologies for infinite dimensional spaces, the most interesting is the weak topology $\sigma(L, L^*)$ and the Mackey topology $\tau(L, L^*)$. The weak topology is the weakest topology for which the map $x \to \langle x, x^* \rangle$ is continuous, for each $x^* \in L^*$. In terms of convergence of nets, x_α converge to x in this topology if $\langle x_\alpha, x^* \rangle \to \langle x, x^* \rangle$, for each $x^* \in L^*$, for this particularity, this topology is called the topology of pointwise convergence. The Mackey topology is the topology for which convergence $x_\alpha \to x$ means $\langle x_\alpha, x^* \rangle \to \langle x, x^* \rangle$, uniformly for $x^* \in \sigma(L^*, L)$ – compact subset of L^*. That is the net $x_\alpha \to x$ if for each $\sigma(L^*, L)$ – compact convex subset A of L^* we have: $\sup\{|\langle x_\alpha - x, x^* \rangle|, x^* \in A\} \to 0$. This topology is called the topology of uniform convergence.

If the topology on L is weak enough, then L^* can be very small, too small to be sensitive. One of the major results on duality theory, the Mackey–Arens theorem, establishes that:

All locally convex topologies τ with the same continuous linear functional L^* lie between the weak topology and the Mackey topology. In other words, the dual of $\sigma(L, \sigma(L, L^*))^*$ is just L^*, and the dual of $\tau(L, \tau(L, L^*))^*$ is L^* too, even though $\tau(L, L^*)$ is generally a finer topology than $\sigma(L, L^*)$. Moreover, $\tau(L, L^*)$ is the finest topology for L which leaves L^* as the dual space of L. Clearly the finest topology is the richest in continuous functions (Narici and Beckenstein, 1985). It follows from the Hahn–Banach theorem that all equivalent topologies have the same closed convex sets, and the same weakly bounded sets too. A set $A \in L$ is weakly bounded if for each $x^* \in L^*$, the set $\{\langle x_\alpha, x^* \rangle x \in A\}$ is bounded in $\Re$.

As a corollary of the above claim it follows that all topologies consistent with a given dual pair have associated the same set of upper semi-continuous quasi-concave functions. The proof is a straightforward conclusion of the fact that u is a quasi-concave function if and only if, the set $\{x: u(x) \leq \alpha\}$ is convex for each α. If these sets are closed in some topology they are closed in all consistent topologies.[19]

So in the above cited work, Araujo proves that continuity with respect to the Mackey topology is the best assumption of this kind that guarantees the existence of a Pareto optimal allocation.

To show an example of "well behaved" economy without Pareto optimal allocations, let us consider the *possibility utility set*:

$$\mathcal{U} = \{(u_1(x_1), \ldots, u_n(x_n)) \in \Re^n : (x_1, \ldots, x_n) \text{ is a feasible allocation}\}, \tag{1.3}$$

an n-tuple $(x_1, \ldots, x_n)$ is called a feasible allocation whenever $x_i \geq 0$ holds for each i and $\Sigma_{i=1}^n x_i \leq w$ where w is the total endowment. An exchange economy satisfies the *Closedness condition* whenever its utility space $\mathcal{U}$ is a closed set.

Note that if each consumer has monotone preferences, then the set U is bounded above by $(u_1(w), \ldots, u_n(w))$. The weak compactness of the interval $[0, w]$ together with quasi-concavity and Mackey upper semi-continuity of each u_i implies that the economy satisfies the closedness condition.[20] But the converse is not true. To prove this claim, consider the following example (Aliprantis *et al.*, 1989).

Example 1 Consider the exchange economy with Riesz dual system $(C[0, 1], ca[0, 1])$, with two consumers with utility functions $u_1(x) = \int_0^1 x(t)dt$ and

$u_2(x) = \int_0^1 \sqrt{x(t)}dt$, and total endowment $w = \mathbf{1}$. (Keep in mind that $ca[0, 1]$ is the norm dual of $C[0, 1]$ equipped with the sup norm.)

The interval $[0, \mathbf{1}]$ is not weakly compact. Nevertheless the utility space of this economy is the set: $\mathcal{U} = \{(u_1, u_2) \in \Re^2_+, u_1 + (u_2)^2 \leq 1\}$, which is a closed set.

As we said above, the weak compactness of the order interval $[0, w]$ is a sufficient condition for the existence of a Pareto optimal allocation, but in Mas-Colell (1975) a weaker condition was given:

For each exchange economy that satisfies closedness condition, the set of Pareto optimal allocations is non-empty. This claim follows as a consequence of the Zorn lemma.[21]

The following example shows that without upper semi-continuity in Mackey topology the utility possibility set may not be closed and then a Pareto optimum may not exist (Araujo, 1985):

Example 2 Consider an exchange economy, with dual pair (l_∞, l_1) and utility functions:

$$u_1(x) = \sum_{n=1} \frac{x_n}{2^n} \quad \text{and} \quad u_2(x) = \lim_{n \to \infty} inf\, x_n$$

and endowments $w_1 = \mathbf{1}$; $w_2 = \mathbf{1}$.

It is easy to see that both utility functions, u_1 and u_2 are concave, monotone and norm continuous functions. In order to establish that u_2 is not Mackey upper semi-continuous, let us consider $x_n = (0, 0, \ldots 0, 1, 1, \ldots)$, where there are zeros in the first n positions, note that $x_n \to 0$ in the Mackey topology, while $\lim_{n \to \infty} u_2(x_n) > u_2(0)$.

The utility possibility is: $U = \{(a_1, a_2) \in \Re^2 : a_1 < 2, a_2 \leq 2, \text{ or } a_1 = 2, a_2 = 0\}$, it is not a closed set.

Then assuming continuity of preferences with respect to a stronger topology than the Mackey topology, it is possible to obtain a large class of economies without Pareto optimal allocations and then without equilibrium.

In Aliprantis *et al.* (1989) it is proved that if the consumers exhibit impatient behavior[22] then the closedness condition is satisfied. So the impatient behavior[23] is enough to guarantee the existence of a Pareto optimal allocation. In Brown and Lewis (1981), it is proved that the Mackey continuity of preferences implies impatience on the part of the consumers. The result of Araujo cited above and the latter ones, show that if the dual system considered is (l_∞, l_1), the Mackey topology is the strongest topology for which all upper semi-continuous preference is impatient.

1.3.2 Topology and the existence of equilibrium prices

In this section we will show that the existence of Walrasian equilibria is strongly related with the topology considered. That is, the existence of equilibrium is a property dependent on the dual system considered in $\mathcal{E}$.

Example 3 The exchange economy with utility functions defined by:

$$u_1(x) = \int_0^1 tx(t)dt; \quad u_2(x) = \int_0^1 (1 - t)x(t)dt$$

and endowments given by $w_1 = w_2 = 1/2\chi_{[0,1]}$, has no Walrasian equilibrium if the dual pair is $(L_p[0,1], C^1[0,1])$ and has an equilibrium when we consider the dual pair $(L_p[0,1], C[0,1])$.

In fact $p(t) = \max\{t, 1-t\}$ is the only one linear functional on $L_p[0,1]$, such that $x_i \succeq x$ implies $\langle p, x_i \rangle \geq \langle p, w \rangle, i = 1, 2$ where $x_1 = \chi_{[1/2,1]}$ and $x_2 = \chi_{[0,1/2]}$.

Notice that as $p \notin C^1[0,1]$, the allocation $(\chi_{[1/2,1]}, \chi_{[0,1/2]})$ is not a Walrasian equilibrium with respect to the dual pair $(L_p[0,1], C^1[0,1])$.

But $p \in C[0,1]$, so the allocation $(\chi_{[1/2,1]}, \chi_{[0,1/2]})$ is a Walrasian equilibrium with respect to the dual pair $(L_p[0,1], C[0,1])$.

In cases where we cannot prove the existence of a Pareto optimal allocation, some approaches to proving the existence of Walrasian equilibrium like the excess utility map originating in Negishi (1960) and pursued in Mantel (1974) and Arrow and Hahn (1971) and many others like Accinelli (1996), have no sense because they are strongly related with the existence of Pareto optimal allocations.

1.4 IS IT ENOUGH TO PROVE THE EXISTENCE OF EQUILIBRIUM?

A characteristic that distinguishes economics, especially GE, from other scientific fields is that the analysis of the equilibrium is the center of the discipline, others such as physics, ecology or evolutionary games, put comparatively more emphasis on the determination of dynamic laws of changes. To prove the existence of equilibrium is a necessary part of the story, but certainly it is not the end of the story. Having established the conditions under which an equilibrium is guaranteed to exist, new causes of worry appear: The cases in which the interpretations or possible applications of the Walrasian equilibrium concept as a mathematical object (see definition in Section 1.3), may be not clear enough, or not totally satisfactory from an economic viewpoint.

Existence of non-priced commodities As it is familiar for finite dimensional models, a price or a price system may be considered as a positive, continuous and linear form from L to R i.e. p is an element of $(R^l)^*$ which associates a commodity bundle $x \in R^l$ with its value $\langle p, x \rangle \in R$ expressed in monetary units. That is, a price is a functional (i.e. a real linear function on the commodity space) defined for each commodity. If the commodity space is a finite dimensional space, then every real linear space, like $(R^l)^*$ is isomorphic to R^l. That is, we can discover the properties of any n-dimensional real linear space by studying R^l. This is no longer true for infinite dimensional spaces, in these cases there is no canonical space.

It is well known that, for infinite dimensional spaces, continuity of a linear functional depends on the topology of the space and so on the dual system $\langle L, L^* \rangle$ considered. If we consider prices as continuous and positive functionals we need to impose restrictions on the dual systems, these restrictions are raised from economics. From a mathematical point of view, the restriction that implies to consider only topological vector spaces in which all positive linear functionals are continuous is not a strong restriction. Besides the above mentioned properties, we would like that all conceivable commodities were priced. However to require defined prices for each commodity conceivable, may be a very restrictive condition, because not every commodity is present in the market at any given time.

To clarify this topic consider the following example:

Example 4 Suppose that the economy is defined in a topological vector lattice L, on a measured space (S, μ) with topology τ and that the consumption set is $X_i \subset L$ for each individual i and that the utility possibilities set U is closed.

Let us define the set $L(w) = \{x \in L: |x| \leq \lambda w, \text{for some } \lambda > 0\}$, where w is the total endowment. Note that $L(w)$ contains all feasible consumption bundles. If we consider the restriction of the economy to $L(w)$, it is possible to obtain an allocation and a price of equilibrium, restricted to this set, see Mas-Colell and Zame (1991).

The search for equilibria in $L(w)$ is much easier than in L because in $L(w)$ with the norm given by:

$$\|x\|_\infty = inf\{\lambda > 0 : |x| \leq \lambda w\},$$[24]

the positive cone has not an empty interior.

If $L = L_\infty(S, \mu)$ the set of all essentially bounded measurable functions[25] on $\{S, \mu\}$ and w is bounded away from 0, then $L = L(w)$. In general, $L(w)$ is much smaller than L for instance, if $L = L_p([0, 1], \mu)$ with $1 < p < \infty, L(w) \subset L_p([0, 1], \mu])$. If $w = \mathbf{1}$ then, $L(w)$ is precisely $L_\infty(S, \mu)$, and as $L_\infty(S, \mu) \subset L_1(S, \mu)$, then $p \in L_\infty(S, \mu)^*$. Note that the economic activity of our agents is confined to the order interval $[0, w]$. So, we assign finite prices only to every feasible commodity bundle, that is commodities y having the property that $y = \lambda x$ for some commodity bundle $0 \leq x \leq w$ and for all $\lambda > 0$ but we do not assign finite prices to all elements of the consumption space i.e. some conceivable commodities in $L_\infty(S, \mu)$ may be left unpriced. However, as we will see below in some cases, it is possible to extend the prices from $L(w)$ to L.

Although the continuity on prices is a mathematical and methodological desideratum it is natural to ask about the continuity of p given the topology τ. In some setting, continuity of prices will not be a strong requirement.

In Yannelis and Zame (1986) it is proved that if preferences are F-proper[26] then prices p are continuous, and if $L(w)$ is dense in L (i.e. if and only if w is a quasi-interior point of L_+^*) then the price p has a unique continuous extension to all L.[27]

No clear interpretation for equilibrium prices concept The following example shows a case in which there is no natural interpretation for equilibrium prices in both economic and mathematical sense. As is well known, the supremum norm dual of $l_\infty = l_\infty(N, \mathcal{N}, \mu)$ is $ba(N, \mathcal{N}, \mu)$ (the space of the bounded additive measures) defined on N where N is the set of integers, $\mathcal{N}$, the set of all subsets in N and μ is the counting measure, i.e. $\mu(A)$ gives the cardinality of A. The countable additive elements of $ba(N, \mathcal{N}, \mu)$ are isomorphically equivalent with $l_1(N, \mathcal{N}, \mu)$, the set of absolutely summable sequences; it is possible to give for them a price representation. That is, for $p \in l_1(N, \mathcal{N}, \mu)$ and $z \in l_\infty$ the value of z is $\langle p, z \rangle = \Sigma_{t=0}^{\infty} p_t z_t$. However, there may exist linear functionals on l_∞ which are not identifiable with a price system in this way.

Example 5 Suppose an economy with consumption set contained in l_∞ and its preferences are continuous in a topology τ stronger than the Mackey topology

$\tau(l_\infty, l_1)$. In Araujo (1985), it was proved that there exists a τ – continuous linear functional $p \in l_\infty^*$ that is not in l_1.

Consider a one consumer economy, with $w = (1, 1, \ldots, 1)$ and utility function $u: l_\infty \to \Re$ defined by $u(x) = \lim inf\, x(t)$. This is a concave, monotone and norm continuous function, but it is not Mackey continuous. So, there is a price $p \in l_\infty^*$, such that $\langle p, x\rangle \geq \langle p, w\rangle$ whenever x is at least so good as w, i.e. $u(x) \geq u(w) = 1$. This p cannot belong to the subset of R^∞ consisting of all absolutely convergent series represented by l_1. To see this, consider $x_k \in l_\infty$ such that $x_k(t) = 0$ if $t < k$, and $x_k(t) = 2$ if $t \geq k$. Then $u(x_k(t)) = 2 > u(w) = 1$, but if $p \in l_1$ the $\langle p, x_k\rangle \to 0$ while $\langle p, w\rangle > 0$.

Then, how do we describe these equilibrium prices? In general, a measure may be uniquely separated into a countably additive measure and a measure of purely finite additives, this is the Yosida, a Hewith Decomposition Theorem, see Dubford and Schwartz (1958). Let us denote by $ba(2^N)$ the space of all signed charges of bounded variation on the σ-algebra 2^N of all subsets of N, the natural numbers, by $ca(2^N)$ the σ-additive signed measures of $ba(2^N)$, and by $pa(2^N)$ the *purely finite additive* signed measure, then $l_\infty^* = ba(2^N) = ca(2^N) + pa(2^N)$. The spaces l_1 and $ca(2^N)$ are isomorphically equivalent and analogously l_1^d the complementary set of the l_1 space in l_∞^*, and $pa(2^N)$. It can be proved that the elements of the set of purely finite additive measures are limit points of sequences e_n that assign mass one in $\{n\}$, thus there are measures zero-one, that is for each $A \subset N, \mu(A) = 0$ or $\mu(A) = 1$, see Aliprantis and Border (1994). Since in this case utility depends only on what happens at infinity, the fact that the equilibrium price is a measure that has all its mass concentrated in the infinite is not surprising from the economic point of view. Then we should not hope to find supporting prices[28] if the topology does not force prices to be in l_1.

Moreover, Araujo proved that to obtain prices in l_1, Mackey continuity of preferences in the dual system (l_∞, l_1) is a necessary condition. For stronger topologies, equilibrium "*prices*" could be in l_1^d which is, as we saw above, isomorphically equivalent to the set of purely finite additive measures, which do not have a clear economic intuition.

On the other hand, when the consumption set is included in L_∞ the Mackey continuity of preferences is not enough to yield prices in $L_1(S, A, \mu)$. We must require Mackey continuity, strictly monotone preferences, and that consumption sets coincide with the positive orthant $L_\infty(S, A, \mu)^+$, see Bewley (1973). Prices in $L_\infty(S, A, \mu)^* = \mathcal{M}(S, A, \mu)$ (the space of finite countably additive measures on the set S), that do not belong to $L_1(S, A, \mu)$ have no natural economic interpretation. (*$ba(S, A, \mu)$ is the bounded finitely additive measures in A which vanish on sets of μ measure* 0).

About the possibility of predictions An important question arises when we attempt to predict future states of an economy: is there uniqueness of equilibrium? If the equilibria set of an economy has exactly one element, we would have a complete explanation of the state of the economy in the Walrasian framework. Although global uniqueness requires very strong assumptions (Arrow and Hahn, 1971), generally this exigency is replaced by one of local uniqueness. The local uniqueness property guarantees the existence of a discrete set of equilibria; it is a property of interest because otherwise the slightest error of observation on the data of the economy might lead to an entirely different set of predicted equilibria, local uniqueness guarantees that in a sufficiently small neighborhood of the equilibrium price there is not another equilibrium price: then albeit locally the theory retains its predictive power. Using differential topology, G. Debreu has given a satisfactory answer to the question of local uniqueness for finite dimensional models (Debreu, 1970) that is, outside a small subset of the space of economies, every economy has a finite set of equilibria, i.e. the set of economies with local uniqueness of equilibria is typical.

An extension to infinite dimensional models whose utilities are separable using the Negishi approach is given in Accinelli (1996). In this work by means of the excess utility function the infinite dimensional problem is reduced to a finite dimensional one. Then using differential topology it is proved that generically, the set of Walrasian equilibrium is not empty and it is also proved that the local uniqueness of Walrasian equilibrium is a typical property. The excess utility function is a differentiable function $e: S^{n-1} \times (L_+)^n \to R^n$, such that

$$e_i(\lambda, w) = \int \nabla u_i(x_i(s, \lambda, w))(x_i(s, \lambda, w) - w_i(s))\, d\mu(s), \quad i = 1, 2, \ldots, n,$$

where S^{n-1} is the $n-1$ dimensional open simplex, $S^{n-1} = \{\lambda \in R^n : \Sigma_{i=1}^n \lambda_i = 1, \lambda_i > 0\}$, $(L_+)^n$ are n copies of the consumption space, and we symbolize by ∇u_i the gradient of u_i. We will say that a pair $(\lambda, w) \in S^{n-1} \times (L_+)^n$ *is an equilibrium if and only if* $e(\lambda, w) = 0$. In Accinelli (1996), it is proved that for each pair (λ, w) of equilibrium there exists a Walrasian equilibrium and vice versa.

Certainly, local uniqueness does not imply global uniqueness and then economies with the same utilities, endowment and moreover at the same state of the world may have a different behavior even in equilibrium. So, for inter-temporal economies with separable utilities, generically there is no uniqueness of the equilibrium path, thus economies with the same endowments and utility functions may have different kinds of evolution. This implies the impossibility to forecast at $t = 0$ the equilibrium path of an economic system.

This impossibility to forecast or characterize the future state of the economy does not depend on the precision with which we can observe the parameters (endowments) or the utility functions: it is a structural characteristic of the model. To prove the non-uniqueness of equilibrium it is sufficient to prove the existence of a singular endowment $w = (w_1, w_2, \ldots, w_n)$, i.e. an endowment such that the Jacobian of the excess utility function, being evaluated at any solution of $e_w(\cdot) = 0$ becomes a singular matrix. Although singularities can be considered as an atypical or negligible set, they are of interest to explain sudden changes in the economy. The existence of singularities can only be avoided with highly restrictive assumptions on the utility functions.

1.5 CONVEXITY, ECONOMY AND TOPOLOGY

As is well known, the convexity of preferences plays a crucial role in economics. In contrast to arbitrary preferences, convex preferences possess a number of remarkable properties of which we list some here:

- The selection procedure for choosing a commodity bundle from those available which maximizes an utility function can be viewed as an optimization program. In contrast to arbitrary real functions, quasi-concave utility functions guarantee the existence of a solution for this optimization program. It is well known that a quasi-concave utility function represents a convex preference.
- All topologies consistent with a given dual pair have the same closed and convex sets, see Schaefer (1966). So, if preferences are convex the set $H(y) = \{x \in X: x \succeq y\}$ i.e. the set of commodity bundles that a consumer prefers to another one, is the same in all compatible topologies.
- All these topologies have the same quasi-concave semi-continuous functions. Recall that f is quasi-concave if and only if the set $\{x: f(x) \geq \alpha\}$ is convex for each $\alpha \in \Re$. If preferences are convex, all these sets are closed and convex in every consistent topology with the dual system given then, in all compatible topologies, the semi-continuous and quasi-concave utility functions are the same.
- The second separation theorem[29] guarantees the existence of a supporting price if the set $H(y) = \{x \in X: x \succeq y\}$ is convex, provided that the interior of $H(y)$ is not empty. We would like to remind the reader that a price p is a supporting price for the commodity bundle y if $\langle p, x\rangle \geq \langle p, y\rangle$. The convexity of preferences implies the convexity of $H(y)$ and vice versa.

Convexity of preferences appears as the most serious assumption needed to prove the existence of equilibrium. If preferences are convex in the majority of cases as we saw before, some mild restrictions in the model allow us, following the methodology of functional analysis, to prove the existence of equilibrium. But without convexity of preferences it is no longer possible to follow this methodology, at least in its actual level of development, to prove the existence of Walrasian equilibrium.

1.6 EQUILIBRIA AND QUASI-EQUILIBRIA

As we have already shown, in many cases we must add some additional conditions to the model to guarantee the existence of equilibria, for instance endowments are strictly positive, or are a continuity of the utility functions. The quasi-equilibrium concept is weaker than the equilibrium concept but, in some models where the existence of equilibria is not guaranteed, it is possible, without adding new hypotheses, to prove the existence of a quasi-equilibrium. With the supportability and compactness of the utility possibility set, the existence of a quasi-equilibrium is guaranteed, see Mas-Colell and Zame (1991). In contrast to prove the existence of equilibrium, continuity and monotonicity of preferences are required.

If market prices are equilibrium prices they may be considered as a measure of the scarcity and, knowing equilibrium prices, each agent interacts with the market rather than with each other. To be a good signal, a system of prices must be, at least, clear about the possibilities that each agent has to obtain commodities in the market. These possibilities are restricted by his budget set. Quasi-equilibrium is a good signal in the sense of the above statement. The existence of a quasi-equilibrium allows each agent to know the commodity bundles he will be able to obtain in the market and which of them are out of his budget possibilities.

Let us now introduce the concept of quasi-equilibrium:

DEFINITION 2 Let there be an exchange economy in which $x = (x_1, x_2, \ldots, x_n)$ is an allocation and p is the price system, the pair (p, x) is a quasi-equilibrium if $\bar{x} \succeq_i x_i$ implies $\langle p, \bar{x} \rangle \geq \langle p, w_i \rangle$ for all agents i. For an equilibrium $\langle p, \bar{x} \rangle > \langle p, w_i \rangle$, so $\bar{x}$ is unreachable.

Two basic properties for a price p supporting a quasi-equilibrium allocation x are:

1 $\langle px_i \rangle = \langle pw_i \rangle$ for each i and
2 if one preference is monotone, then $p \geq 0$.

So, by the definition, either a quasi-equilibrium or an equilibrium price is a support for the allocation of x, in the sense that $\bar{x} \succeq_i x_i$ implies $\langle p\bar{x} \rangle \geq \langle px_i \rangle$.

The existence of an extremely desiderable[30] bundle for each consumer implies that a Walrasian equilibrium is necessarily a quasi-equilibrium, but the converse is not true: see a counter example in Section 1.6 in Aliprantis *et al.* (1989). Nevertheless for an exchange economy with strictly positive endowments w and continuous preferences, the quasi-equilibrium allocation is a maximal element in the budget set $B_i(p) = \{x \in L: \langle px \rangle \leq \langle pw_i \rangle\}$. If the quasi-equilibrium price is positive and $\langle p, w \rangle > 0$ then the quasi-equilibrium allocation is weakly Pareto optimal.

On finite dimensional models, the existence of a support price for a rational Pareto optimal allocation is a straightforward application of the convex separation theorems. Unfortunately, infinite dimensional spaces do not possess this property because, as we said in Section 1.2, convex sets in infinite dimensional spaces may have empty interiors (this is the case of the positive cone in L_p; $1 \leq p < \infty$). The property known as properness allows us to work in the absence of interior points in the positive cone; this concept appeared first as cone condition in Chichilnisky and Kalman (1980). The following definition is given in Mas-Colell (1986):

DEFINITION 3 Let E be a Riesz space on which τ is a linear topology. We say that the preference relation $\succeq$, defined on the consumption set $X \subset E$ is proper at x with respect to the vector v, if there is an open cone Γ_x at 0 containing v, such that $x - \Gamma_x$ does not intersect the preferred set $(\{x' \in X: x' \succeq x\})$ i.e. if $x' \succeq x$ then $x - x' \notin \Gamma_x$.

This property may be interpreted considering a bundle set v as extremely desirable, in the sense that the loss along the direction of v, of an amount $\alpha v, \alpha > 0$, cannot be compensated by an additional small commodity bundle.

The concept of properness is a restriction on preferences which could be used to prove that a Pareto optimum enjoyed the support property. This support property may be used as a step to prove the existence of a quasi-equilibrium in spaces where positive cones have empty interiors.

When preferences are convex, properness of $\succeq$ at x with respect to v is equivalent to the existence of a price $p \in E^*$ which supports the preferred set (the better than x set) and verifies that $\langle p, v \rangle > 0$.

A related notion was introduced in Yannelis and Zame (1986). We say that $\succeq$ is F-proper (F – for forward) at $x \in X$ if there is an open cone Γ_x at 0 containing v, such that $x + \Gamma_x \cap X \subset \{x' \in X: x' \succeq x\}$, i.e. if $z \in \Gamma_x$ and $x + z \in X$ then $x + z \succeq x$. In general, properness and F-properness are incompatible conditions, nevertheless both conditions are easy to check and hence have potential applications.

In Araujo and Monteiro (1988) it is proved that for economies with separable utilities, and in which $L = L_p[\mu] = \{f: X \to R: |f|^p$ is an integrable function$\}$ is the commodities space, properness is equivalent to the existence in the dual space L_q(where $1/p + 1/q = 1$) of the right hand derivative of $v(\cdot, s)$, (see equation (1.1)).

The following theorem is proved in Mas-Colell (1975). *If in a pure exchange economy preferences are uniformly* τ *proper*[31] *and the order interval* $[0, w]$ *is weakly compact, then the economy has a quasi-equilibrium.*

Moreover, if the total endowment is strictly positive and utilities are continuous functions, with the above hypothesis we obtain that the economy has a Walrasian equilibrium.

Existence of quasi-equilibrium does not require continuity of the utilities; the closedness condition of the utility possibility set (provided with the supportability of every weak optimum) is sufficient to guarantee the existence of a quasi-equilibrium, see Mas-Colell and Zame (1991). Remember that upper semi-continuity and quasi-concavity of utilities are implicit in the assumption that the utility possibility set is closed. Full continuity will be required to prove the existence of equilibria.

For economies with separable utilities and strictly positive endowments, the existence of quasi-equilibria follows from the weaker assumption that the properness property is satisfied only at initial endowments, or in some rational allocation, see Araujo and Monteiro (1989). The loss of working in such a way is to give up the original commodity space L and to work only with the subset $L(w) = \{x \in L: |x| \leq \lambda w;$ for some $\lambda > 0\}$. A quasi-equilibrium in $L(w)$ is not a quasi-equilibrium in the usual sense because not every commodity in L is priced.

An easy proof of the existence of the Walrasian equilibrium for economies with separable strictly quasi-concave and differentiable utility functions, using the K.K.M theorem and the excess utility function, is given in Accinelli (1994).

According to our above statement, in exchange economies, convexity of preferences is the more serious hypothesis to prove the existence of the equilibrium or quasi-equilibrium. Continuity of the utility function, strictly positive endowments and the separability of convex sets, may be in some cases avoided, but to prove the existence of equilibrium or quasi-equilibrium, the convexity of preferences is an unavoidable condition.

1.7 PRODUCTION ECONOMIES

In finite dimensional cases, the equilibrium analysis is technically more demanding for production economies than for pure exchange economies. In infinite dimensional models over the familiar difficulties, supportability of Pareto optima, compactness of the feasible allocations, there appear new ones specific to productive infinite dimensional economies.

Even in finite dimensional models, to guarantee the existence of equilibria we must consider some restrictions on the technological possibilities. Also in cases in which it is possible to prove its existence, the question of its efficiency appears as a problem with no trivial solution (in some cases it is possible to obtain an equilibrium allocation that is not Pareto superior).

The critical technological assumption regarding firms is that their production sets are convex. As it is well known this expresses the notion of constant or diminishing returns to scale. If we adopt the assumption of perfect competition this is sufficient to justify the same assumption in infinite dimensional economies. Convexity of the production set can be derived from the primitive concepts of *additivity and divisibility*. In models in which these hypotheses hold and under the classical hypothesis about the behavior of each agent and his consumption set, for finite dimensional models, the existence of equilibria follows as a corollary of a fixed point theorem, and its Pareto-optimality may be guaranteed. While the additivity assumption seems hard to reject, the divisibility assumption is much more debatable, both theoretically and empirically. Hence the main source of non-convexity appears related to a failure in this assumption. Non-convexity in many cases is a consequence of increasing returns to scale, see Mas-Colell (1987).

As it is well known firms with increasing returns to scale may behave as monopolies, and they could then settle prices, affecting the prices and the optimality of the possible equilibria.

In presence of non-convex technologies the identification between equilibrium and optimum will no longer hold. Thus the existence of equilibrium and the analysis of its optimality become very different questions.

When production sets are non-convex, prices can be understood as a regulation policy aiming at Pareto-efficiency. There is no way of efficiently allocating the resources through a price mechanism in the presence of increasing returns to scale: this aiming requires taking decisions with distributive impacts, then some consumer may feel that he is paying too much for the optimality.

Moreover the following discouraging result holds: each economy has a non-empty core, if and only if the aggregate production set is a convex cone, see Quinzii (1992).

The idea behind the core is the social stability. If there is an allocation in the core of an economy, a group of agents that can do better on their own does not exist. When the core is empty, the possibility of the intervention of some authority seems to be natural, and the core is empty without convexity in the aggregate production.

Now we will study production economies in the setting of infinite dimensional models, this extension is less straightforward than infinite dimensional setting. Formally we have the following definition of a production economy:

DEFINITION 4 A private ownership production economy is a set:

$$\mathcal{E} = \{X_i, w_i, u_i, Y_j, \theta_{ij}, \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, m\}$$

where:

1 $X_i \subset L$ is the consumption set, and L is a topological vector space. In L, it is defined a topology τ consistent with the dual system (L, L^*).

2 each consumer is characterized by his endowments w_i and by his utility function u_i.
3 there are m producers indexed by j, each of whom has a production set $Y_j \subset L$.
4 the real number θ_{ij} represents the share of consumer i to the profit of producer $j, 0 \leq \theta_{ij} \leq 1$ and $\Sigma_{i=1}^{n} \theta_{ij} = 1$, for all j. That is, the firms are owned by the consumers.

An allocation $(x, y) = (x_1, \ldots, x_n, \ y_1, \ldots, y_m) \in \Pi_i X_i \times \Pi_j Y_j$ is feasible if $\sum_{i=1}^{n} x_i \leq w + \Sigma_{j=1}^{m} y_j$, where w is the aggregated endowment.

In a private ownership productive economy, the *wealth* of each consumer is $\gamma_i(p) = \langle pw_i \rangle + \Sigma_{i=1}^{n} \theta_{ij} \langle pY_j \rangle$, where p is the vector of prices.

To prove the existence of an equilibrium for this kind of model, we must be careful with the problems that arise in an infinite dimensional pure exchange economy plus the new ones that appear with the introduction of the production sets.

A quasi-equilibrium for E is a feasible allocation (x, y) and a linear functional $p: L \to \Re, \ p \neq 0$, such that:

1 $\langle p, x_i \rangle \leq \langle pw_i \rangle + \Sigma_{i=1}^{n} \theta_{ij} \langle pY_j \rangle$, for all i.
2 $\langle p, y_j \rangle = max \ pY_j$ for all j.
3 If $z \succeq_i x_i$ then $\langle p, z \rangle \geq \langle pw_i \rangle + \Sigma_{i=1}^{n} \theta_{ij} \langle pY_j \rangle$, for $i \in (1, 2, \ldots, n)$.

Moreover if $z \succ_i x_i$ implies $\langle p, z \rangle > \langle pw_i \rangle + \Sigma_{i=1}^{n} \theta_{ij} \langle pj_j \rangle$, for $i \in (1, 2, \ldots, n)$, also we say that (x, y, p) is an equilibrium.

To prove the existence of equilibrium in infinite dimensional models, the boundedness assumptions that are typically used in finite dimensional problems to obtain compactness of the feasible allocations are not enough, see Section 1.1. Nevertheless, for a production economy with a symmetric Riesz dual pair $\langle E, E^* \rangle$, if all production sets Y_j are order bounded from above, each feasible set is weakly compact, see Aliprantis *et al.* (1989), and the existence of equilibrium can be proved, this result is essentially due to Bewley (1973). In some cases, to prove the existence of an equilibrium, the compactness of the feasible sets is directly assumed, see for instance Mas-Colell and Zame (1991).

The supportability problem disappears if we suppose that the production set is a non-empty positive cone, for instance l_∞^+ the set of all bounded real sequences. However, as in pure exchange economies, in production economies the problem of the meaning of the equilibria prices appears. We remind that a price is a linear functional on the dual space, in this case $(l_\infty^+)^*$ the space of all bounded finitely additive measures. Moreover, Mackey continuity and monotone preferences are not enough to prove the existence of equilibrium. We need to admit, in addition, that each set Y of technically feasible production program as is a weak-star closed set in the Mackey topology $\sigma(l_\infty, l_1)$, see Bewley (1991). Nevertheless, these conditions are not enough to find prices in l_1 (Bewley, 1991).

1.7.1 Properness condition in production economies

For production sets in which the positive cone has an empty interior, the failure of supportability may entail the non-existence of quasi-equilibria. Once again, the concept of properness appears as a good substitute to the Hahn–Banach theorem.

A production set Y contained in a locally convex space is uniformly proper whenever there exists a vector $w > 0$ and a convex neighborhood W of zero, such that the convex cone $\Gamma = \{\alpha(w + x): \alpha > 0, w \in W\}$ satisfies $(z + W) \cap \{y \in Y: y^+ \geq z^+\} = \emptyset$ for all $z \notin Y$, where $y^+ = sup\{y, 0\}$.

In Araujo and Monteiro (1993) it is shown that in many cases, including $L_p, 1 \leq p \leq \infty$, and in which measures are defined on a compact set, it is possible to prove the existence of an equilibrium with economic meaning, that is, an equilibrium price in L_1. To obtain these results the following hypotheses were stated:

- For each firm, the technological set Y_j is a convex Mackey closed subset.
- Y_j is a pointwise Mackey proper production set.
- The allocations set is bounded.
- Concerning the consumer, preferences are norm continuous, consumption spaces are pointwise proper[32] and endowments are strictly positive.

Pointwise proper is a weaker condition than the uniform properness condition (considered in Mas-Colell (1986)), but in Araujo and Monteiro (1993) the original commodity space is given up and only the feasible set is considered, and so there might exist a non-priced commodity.

In the above cited work the existence is proved of a general extended equilibrium for separable Banach lattices with order norm continuous E, see Peresini (1967). To prove the equilibrium existence P.K. Monteiro shows that there exists a linear bijection $\theta: L_\infty \to E$.

We shall see some examples to clarify the above statements.

Example 6 Suppose that $\mathcal{E} = \{X_i, \succ_i, Y_j, \theta_{ij}, i = (1, 2, \ldots, n); j = (1, 2, \ldots, m)\}$ is an economy with commodity space $E = L_p, p < \infty$ or $E = \mathcal{M}(\Omega)$ where $\mathcal{M}(\Omega)$. Suppose that:

- Preferences are convex and norm continuous; and X_i is a closed and convex set of E_+. Preferences on X_i are norm proper, that is for each $x \in X_i$ there exists $v \in E$ and U_x a neighborhood of zero such that $x' \succeq x$ for $x' = x + tv - tz, z \in U$, and $t \geq 0$.
- On the producers side Y_j is closed and convex, $0 \in Y_j, Y_j - E_+ \subset Y_j$. Y_j is a pointwise proper production set, i.e. for each $y_j \in Y_j$ there exists, $v \in E_+$ such that $h = y - tv + tz$ where $t > 0$ and $z \in U$ (a neighborhood of zero) is such that $h^+ \leq y^+$ then $h \in Y_j$.
- The set of feasible allocations restricted to $K(w) = \cup_{r>0}[-rw, rw]$, is bounded in $K(w)$. That is, there exists $b \in K(w)$ such that for all feasible allocations $(x_1, x_2, \ldots, x_n; y_1, y_2, \ldots, y_m), x_i \leq b$, and $y_j \leq b$.

Then $\mathcal{E}$ restricted to $K(w)$ has an equilibrium whose prices are in L_1, see Araujo and Monteiro (1993).

Contrary to intuition, the worst case to prove the existence of an equilibrium is the L_∞. Recall that the positive cone in L_∞ has a non-empty interior.[33] As we have already shown, to prove the existence of equilibrium prices in L_∞, we have to assume that preferences are Mackey continuous, and this assumption implies impatient behavior on the part of the agents, see Section 1.3, which leaves many interesting preferences outside the equilibrium theory. On the other hand, as the above example shows in $L_p, 1 \leq p < \infty$ or M one does not need special assumptions on continuity of preferences (such as some kind of weak continuity), norm continuity is enough. To prove the existence of quasi-equilibrium in L_∞, see Bewley (1973).

1.8 CONCLUSIONS

We wish to begin this last section with Plato's remark on the duplicate cube problem, that seems particularly apt for our discussion: "It must be supposed, not that the god specially wished this problem solved, but that he would have the Greeks desist from war and wickedness and cultivate the muses, so that, their passions being assuaged by philosophy and mathematics, they might live in innocent and mutually helpful intercourse with one another."

In General Equilibrium Theory there are two main questions, one of them is the problem of the existence of the equilibrium and the other one, in the cases in which the equilibrium exists, is about its properties and interpretations, in the first place the question related with its efficiency and in second place the question about its predictive possibilities. Again, the question is:

Is the knowledge of the existence of equilibrium enough to know the behavior of an economy?

As is well known General Equilibrium Theory does not have a dynamical representation in the sense of Dynamical Systems Theory. Nevertheless it is possible to prove, for inter-temporal models, the existence of an equilibrium manifold, and in this manifold to describe "equilibrium paths" and to show the possible future behavior of the economy once the economy is on one of its possible equilibrium paths. This representation does not follow from endogenous dynamic laws, it is predetermined by the endowments as functions of time.

However the existence of singular economies, i.e. economies with endowments for which zero is a singular value of the excess utility function, implies the existence of neighborhoods of them, where the existence is possible of abrupt changes between equilibrium path. Such neighborhoods could be the regions where small unanticipated shocks break away from the equilibrium path. So the possibility of foresight is lost near a singularity.

How to characterize an equilibrium by intrinsic dynamical properties, and how to give sense to the concept of *evolution* are open challenges for Equilibrium Theory.

Maybe that a totally satisfactory solution might be unreachable by the current mathematical and economic theory, and although "the assurance of existence of an equilibrium means that our notion of equilibrium passes the logical test of consistency it can hardly be the end of the story;"[34] perhaps a beginning.

NOTES

1 The non-separable L_∞ is a typical example.

2 Let X be a set and $\succeq$ a preference relation on X. If $F \subset X$ and for all x in X there are y, z in F with $y \succeq x$ and $x \succeq z$ we say that F bounds X. If F can be taken countable, we say that $\succeq$ is *countably bounded.*

3 Moreover, every linear functional $x^* \in L^*$ attains its supremum on the unit ball of L if and only if L is a reflexive Banach space (Ciranescu, 1990).
4 This claim is truthful in a completely metrizable locally solid Riesz space.
5 We will define this concept in Section 1.3.1.
6 Nonetheless if we assume endowments strictly positive, and monotone preferences, support prices are τ-continuous in the ideal generated by the total endowment w.
7 In particular if $x_n \rightarrow x, \sigma(X, X')$ in a reflexive Banach space X and $x'_n \rightarrow x', \sigma(X, X')$ implies $\langle x_n, x'_n \rangle \rightarrow \langle x, x' \rangle$ then X is finite dimensional.
8 Dubford and Schwartz (1958).
9 A locally convex space L is said to be semi-reflexive if $L = (L^*)^*$. We note that this property depends only on the duality (L, L^*), and hence is shared by all or by none of the locally convex topologies on L that are consistent with (L, L^*). (Semi-reflexivity and reflexivity agree for normed spaces.)
10 A binary relation like a preference is monotone if $x \geq y$ then $x \succeq y$ and it is convex if $x \succeq z$ and $y \succeq z$ then $\alpha x + (1-\alpha)y \succeq z$, where $0 \leq \alpha \leq 1$.
11 That is infinite marginal utility at zero.
12 If in addition w is in the quasi interior of L_p^+ then equilibrium price extends to a continuous price on all of L and is an equilibrium price for the original economy.
13 This concept will be defined in Section 1.5.
14 A pair $\langle E, E^* \rangle$ is a symmetric Riesz pair if and only if $\langle E^*, E \rangle$ is a Riesz pair.
15 The space l_∞ may be interpreted as all time sequences of bounded consumption plans, and l_1 as the space of the price vectors $p_j(t)$ satisfying $\Sigma_t |p_j(t)| < 1$.
16 Semi-reflexivity is a property that depends only on the duality (L, L^*), and hence is shared by all or none of the locally convex topologies on L that are consistent with (L, L^*).
17 Recall the following alternative characterization of semi-reflexive space: (L, τ) is semi-reflexive if and only if every bounded subset of L is contained in a $\sigma(L, L^*)$ compact set, then it is possible to consider economies like $\mathcal{E}$ with linear utilities without Pareto optimal allocations.
18 For example the following function is myopic: $u(x) = \Sigma_{t=1}^{\infty} \delta^{t-1} x_t^{\nu}, \delta, \nu \in (0,1)$.
19 Nevertheless the weak topology is really different from a strong one. To see this consider the sequence $\{e_i\}_{i=1}^{\infty}$, in the l_2 space, where e_i is defined by $e_{ij} = 0$ if $i \neq j$ and equal to one otherwise, $j = \{1, 2 \ldots\}$. From the Riesz Representation theorem it follows that for all linear functional on l_2, there exists an element $a \in l_2$ such that $f(e_i) = \langle e_i, a \rangle = a_i$, then $f(e_i) \rightarrow 0$ and so the weak convergence follows. However, this sequence does not converge in the norm topology.
20 This claim follows from the fact that Mackey upper semi-continuity of a quasi-concave function implies weak upper semi-continuity, then lim $sup_\alpha u_i(x_\alpha) \leq u_i(x)$ for each net x_α weakly convergent to x. Then for a feasible x if $u_i(x_\alpha)$ converges to $\eta_i, \eta = (\eta_1, \ldots, \eta_n) \in$ U.
21 For each allocation x such that $u(x)$ belongs to U consider $\mathcal{C}_x$ the set of all comparable allocations with x. Let us now consider the non-decreasing sequence $u(x_\alpha) \geq u(x)$ in $\mathcal{C}_x$. As the utility possibility set is a bounded real set, then closedness implies compactness of this set, then there exist $z \in \mathcal{U}$, such that $u(x_\alpha) \uparrow z$, consider now the feasible allocation y such that $u_i(y_i) = z_i$. This is an upper bound for the order given by preferences in the sequence x_α. Then by the Zorn lemma, there exists a maximal element in $\mathcal{C}_x$, this is a Pareto optimal allocation.
22 A consumer displays an impatient behavior if for any x, y, and z if x is preferred to y then x is preferred to $y + \bar{z}$ where $\bar{z}$ is defined by $z_{nt}^- = 0, 1 \leq t \leq n$ and $z_{nt}^- = z_{nt}$, for $t < n$.
23 In terms of Growth Theory, impatience is equivalent to the fact that consumers discount the future.
24 By $|x|$ we denote $sup(x, 0) + sup((-x), 0)$.
25 A function is essentially bounded if the essential supremun of its absolute value is finite. The essential supremum is defined by ess sup $f = \inf\{r : \mu\{s \in S : f(s) \geq r\} = 0\}$.
26 This concept will be defined later in Section 1.5.
27 In a Riesz space $(E, E^*), x \in E_+$, is a quasi interior point if $\langle x, x' \rangle > 0$ for each $0 < x' \in E^*$. A quasi-interior point is also called strictly positive, written $x \gg 0$.

28 A price p in an exchange economy is said to be a supporting price if $x \succeq y$ implies $\langle p,x\rangle \geq \langle p,y\rangle$.
29 Let A, B be non-empty, disjoint convex subsets of a locally convex space, such that A is closed and B is compact. There exists a closed real hyperplane strictly separating A and B (Schaefer, 1966).
30 Recall that a vector v is said to be an extremely desirable bundle if $x + \alpha v \succ x$.
31 A preference is uniformly proper if we can choose the same properness cone in each $x \in L^+$.
32 A set X is pointwise proper if for all $x \in X$ and if (v_{ix}, U_{ix}) are properness constants then $x + v_{ix} \succ x$.
33 Only $C(K)$ the space of continuous function on a compact set K and L_∞ has this property.
34 (Mas-Colell *et al.*, 1995).

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2 Learning in intertemporal equilibrium models and the sunspot case

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When there exists multiplicity of equilibria in intertemporal equilibrium models, the Arrow–Debreu model is insufficient to explain theoretically which of them will prevail. This is an important fact, because many of these equilibria are not efficient and a policy-maker would like to know which of the equilibria is reached by the agents in the economy. In this work we describe several types of learning rules that agents can use to update the expectations and we analyze the convergence of the actual dynamics generated by these learning rules. The actual dynamics can converge to steady states, cycles or stochastic equilibria (sunspot equilibria).

2.1 INTRODUCTION

Multiplicity of equilibria in economic models is one of the main concerns of theoretical macroeconomists. From their point of view, the existence of infinitely many equilibria brings some problems with the properties of the Walrasian equilibrium about efficiency, since many of these equilibria can be Pareto-improved by coordinating actions or expectations between agents; but in macroeconomic models this coordination is not always possible.

For applied macroeconomists the indeterminacy of equilibrium is also a problem because this phenomenon raises a serious difficulty for the confidence of macroeconometric estimates of some parameters of the economy.

The main question is: If there exists multiplicity of equilibria, which of them (if any) will the economy choose? Is there any natural selection of these equilibria? In his seminal paper, Lucas (1986) suggested that any reasonable learning rule must lead the economy to rational expectations equilibrium, where perfect coordination between economic agents exists. Many attempts were proposed in order to obtain convergence to this equilibrium. The most important works were developed by DeCanio (1979), Bray (1982), Marcet and Sargent (1989) among others. Sargent (1993) has a wide explanation of how the hypothesis of bounded rationality (which implies that agents use different types of learning procedures) in macroeconomics can produce convergence to rational expectations equilibria.

Although the convergence to non-stochastic rational expectations equilibrium is proved by all authors above, there also exist results of convergence to rational expectations equilibrium where expectations are not single valued (for example, convergence to truly stochastic or sunspot equilibrium). Woodford (1990) and Araujo and Maldonado (2000) provide robust examples of this convergence. Finally we have in the literature about learning processes that some reasonable learning rules can lead the economy to intertemporal paths that are neither deterministic nor stochastic rational expectations equilibrium; these paths are called "learning equilibria" in the literature (Brock and Hommes, 1997; Bullard, 1994) and they are not completely studied in their theoretical foundations.

The goal of this chapter is to give the state of the art in the literature of learning rational expectations equilibria in macroeconomic models. We will describe the main findings in this area and discuss the problems that were not solved yet. Special attention is taken in learning truly stochastic equilibrium (sunspot equilibrium) which is an important issue for policy-makers since this sort of equilibrium can be Pareto-improved by coordinating expectations. It is worth noting that in the literature of learning processes there exist many results applied to game theory but this is not the aim of this chapter; a good reference for this is Fudenberg and Levine (1998).

This work is divided in the following Sections: In Section 2.2, we present the framework where we will consider the main results of convergence of actual dynamics to non-stochastic rational expectations equilibrium. In Section 2.3, is defined the concept of sunspot equilibrium in non-stochastic models and we will show the results of convergence to this equilibrium under reasonable learning rules; also we will discuss the existence of learning equilibria. In Section 2.4, the conclusions are given.

2.2 LEARNING NON-STOCHASTIC EQUILIBRIUM

In this section, we will discuss some learning rules that allow convergence to deterministic rational expectations equilibria. In all this work we will consider the following framework: An economy is defined by its intertemporal equilibrium equations $\tilde{Z}(x_t, \mu_{t+1}) = 0$, where $x_t \in R^L$ is the vector of the state variable values at period t, μ_{t+1} is the probability distribution of the state variable values at period $t+1$ and $\tilde{Z} : X \times \mathcal{P}(X) \to X$ is the function that determines the temporary equilibria of the economy ($X \subset R^L$ is the state variable set). The $\tilde{Z}$ function incorporates characteristics of the economy like preferences, endowments, technology, governmental policies, etc. The framework defined above is called the intertemporal one-step forward looking model and it can be interpreted like this: "if agents have expectations for the next period state variable given by the probability distribution μ_{t+1}, then x_t is the current value of the state variable that equilibrates the markets." Overlapping generations models with one sector have this structure and if we put more sectors with lagged production we can obtain models with lagged variables (models with memory).

If we suppose that agents have single point expectations (it means that the probability distribution for the next period is concentrated in a single point) then we can obtain the deterministic dynamics given by the function $Z(x_t, x^e_{t+1}) := \tilde{Z}(x_t, \delta_{x^e_{t+1}})$ (δ_z is the Dirac measure at z). With this assumption the equation $Z(x_t, x^e_{t+1}) = 0$ can be written as $x_t = \phi(x^e_{t+1})$. When there exists perfect prevision $x^e_{t+1} = x_{t+1}$, the temporary equilibrium equation is $x_t = \phi(x_{t+1})$. For this reason the function ϕ is called *the backward perfect foresight map.*

An intertemporal equilibrium with perfect prevision is a sequence of state variable values $(x_t)_{t\geq 0}$ such that for all t, $Z(x_t, x_{t+1}) = 0$. The hypothesis of perfect prevision is made for finding some equilibria which eventually agents will attain. Examples of perfect foresight equilibria are:

1 A *steady state* is an $\bar{x} \in X$ such that $Z(\bar{x}, \bar{x}) = 0$.
2 A *k-cycle* is a vector $(x^1, \ldots, x^k) \in X^k$ such that $Z(x^i, x^{i+1}) = 0$ for all $i = 1, \ldots, k-1$ and $Z(x^k, x^1) = 0$.

Sometimes economists suppose the rational expectations hypothesis in its strongest form in order to justify that once they reach these equilibria, the agents will not leave them because they do not make mistakes systematically. The theoretically right way to analyze this would be to describe the set of all equilibria for the given structure and define criteria for selecting some of them. Obviously this has a high mathematical difficulty that until now has not been solved.

Let us give an example to explain the structure just considered. Suppose that we have a simple overlapping generations (OLG) model where the technology is linear and in each period two kinds of agents (young and old) co-exist in the same proportion. Each young agent supplies y units of labor to produce the unique perishable good in the same quantity (since the technology is linear) and each old agent consumes c units of the good. The agents hold M units of fiat money and the stock of money in circulation is constant for all periods. The utility function is represented by $U(c, y)$, where y is the labor supply when the agent is young and c is the consumption when the agent is old. If $(p_t)_{t\geq 0}$ is a sequence of prices for the good in this economy, then each agent will solve:

$$\max U(c_{t+1}, y_t)$$

such that:

$$p_{t+1}c_{t+1} = p_t y_t.$$

Now if in period t the next period price is a random variable the agent's problem will be:

$$\max E\left[U\left(\frac{p_t}{p_{t+1}} y_t, y_t\right)\right],$$

where the expected value is taken with respect to μ_{t+1}, the probability measure of p_{t+1}. In this case the first order condition and the equilibrium equation $M/p_t = y_t$ gives us the following $\tilde{Z}$ function:

$$\tilde{Z}(p_t, \mu_{t+1}) = E_{\mu_{t+1}}[U_c(M/p_{t+1}, M/p_t) + U_y(M/p_{t+1}, M/p_t)],$$

or in terms of output $\tilde{Z}(y_t, \nu_{t+1}) = E_{\nu_{t+1}}[U_c(y_{t+1}, y_t) + U_y(y_{t+1}, y_t)]$, where ν_{t+1} is the probability measure induced in y_{t+1} by the random variable p_{t+1}.

For these models many authors proved the existence of different types of equilibria and sometimes the co-existence of them. If there exists multiplicity of equilibria, the natural question is: which of these equilibria will the economy reach in fact? The rational expectation hypothesis cannot help us to answer this question, because it proposes that once the agents reach the equilibrium (whatever it be) they will not abandon it, so any of these equilibria are equally valid.

One way to justify the prevalence of an equilibrium is that it can be the result of some iterative process of learning that agents can consider when they are taking their decisions. That is one of the several criteria proposed by Guesnerie (1993) and we will discuss it for the equilibria previously studied.

2.2.1 Convergence to steady states or cycles

The classical least squares method used for forecasting future values of the state variable revealed itself as a very strong method for obtaining convergence to rational expectations equilibria (Marcet and Sargent, 1989). This method is based on the Ljung stochastic recursive algorithm (Ljung and Soderstrom, 1983). However, the required hypotheses for the equilibrium are rather restrictive, and convergence to some rational expectations equilibrium is guaranteed for all initial values of the state variable close to the equilibrium.

Another classical learning rule is the adaptive learning process (or error learning). This is based on the prevision formulated from the errors made in past periods. Explicitly, let $\tilde{Z}$ depend on the expected value of the future state variable; it means that the equilibrium equation is given by:

$$Z(x_t, x_{t+1}^e) = 0. \tag{2.1}$$

An adaptive learning rule based on the error made p periods before is:

$$x_{t+1}^e = x_{t-p} + \alpha(x_{t-p}^e - x_{t-p}), \quad \alpha \in (0, 1). \tag{2.2}$$

More sophisticated adaptive learning processes consider the gain α depending on t and converging to zero. Evans and Honkaphoja (1995) used such a type of learning rule based on the Ljung stochastic recursive algorithm again for learning cycles in stochastic models.

The stability of some equilibrium under this learning rule is analyzed from the convergence of the dynamical system defined by equations (2.1) and (2.2). Guesnerie and Woodford (1991) showed that the local uniqueness of a perfect foresight cycle is a sufficient condition for obtaining convergence of the learning dynamics. Maldonado (2000) showed the equivalence between local uniqueness and stability of the cycle for the learning dynamics.

2.2.2 The principle of uncertainty

At this point, we would like to comment on a seminal contribution that makes a link between the influence of expectations in an economic structure and the ability for learning some parameters of the economy. Grandmont (1998) made a unified study of different types of learning rules applied to structural equations with lagged variables ($Z(x_{t-1}, x_t, x^e_{t+1}) = 0$, in fact the linearization of this) and the main result he found was what he called "the principle of uncertainty": "If the influence of the expectations in the structural equations is strong and the agents are uncertain about the local dynamics of the system (and thus, they are ready to extrapolate a wide range of regularities out of past deviations) then the actual dynamics (generated by (2.1) and (2.2)) is local unstable." Let us illustrate briefly the result.

Consider the structural equation $Z(x_{t-1}, x_t, x^e_{t+1}) = 0$ and the learning rule (or the expectation function) considered by the agents is given by $x^e_{t+1} = \psi(x_t, x_{t-1}, \ldots, x_{t-L})$. The interaction of these two equations will give the actual dynamics, namely the dynamics implicitly defined by:

$$Z(x_{t-1}, x_t, \psi(x_t, x_{t-1}, \ldots, x_{t-L})) = 0 \Rightarrow x_t = W(x_{t-1}, \ldots, x_{t-L}).$$

Suppose that there exists an interior steady state $\bar{x}$ and our learning rule is ready to detect it ($\psi(\bar{x}, \ldots, \bar{x}) = \bar{x}$). The following parameters will define the local dynamics around the steady state:

$$b_1 = \frac{\partial Z}{\partial x_{t-1}}; \qquad b_0 = \frac{\partial Z}{\partial x_t}; \qquad a = \frac{\partial Z}{\partial x_{t+1}};$$

$$c_j = \frac{\partial \psi}{\partial x_{t-j}}; \qquad j = 0, \ldots, L.$$

All the derivatives are evaluated at $(\bar{x}, \ldots, \bar{x})$. The characteristic equation that describes the actual local dynamics around the steady state is:

$$Q_W(z) = b_1 z^{L-1} + b_0 z^L + a \sum_{j=0}^{L} c_j z^{L-j}.$$

This is the composition of two characteristic equations: the first one for the perfect foresight dynamics $Q_F(z) = b_1 + b_0 z + a z^2 = a(z - \lambda_1)(z - \lambda_2)$ and the other for the learning process dynamics $Q_\psi(z) = z^{L+1} - \sum_{j=0}^{L} c_j z^{L-j} = \Pi_k (z - \mu_k)$. The interpretation of λ's is simple; they determine the stability of the dynamics under perfect prevision. The roots of the characteristic polynomial of the expectation function are less intuitive. They determine the ability of the learning rule for detecting trends from deviations of the steady state and/or cycles (see Grandmont and Laroque, 1986). So a being large enough (expectations have strong effects on the structural equations) amounts to a high value of λ (the perfect foresight dynamics is unstable); and if some μ has a norm greater than one then the agents are rather uncertain about the local dynamics close to the steady state. With all these interpretations, the following theorem proves the principle of uncertainty.

THEOREM 1 *Suppose that there exists $\mu_1 < 0 < \mu_2$, different from any λ.*

1. *Let n_F and n_W be the number of real roots of the polynomials Q_F and Q_W that lie outside the interval $[\mu_1, \mu_2]$. Then n_W is odd if and only if n_F is even.*
2. *Suppose that $\mu_1 \leq -1$ and $\mu_2 \geq 1$. If $[\mu_1, \mu_2]$ contains in its interior all the perfect foresight characteristic roots that are real then the polynomial Q_W has a real root that satisfies either $r < \mu_1$ or $r > \mu_2$.*

Proof: Let $P(z) = \Pi_n(z - r_n)$ be a polynomial. It is easy to verify that:

$$\text{If } r \in R \text{ is such that } P(r) \neq 0 \quad \text{then} \quad [P(r) < 0 \Leftrightarrow \sharp\{r_i / r_i > r\} \text{ is odd}].$$
$$\text{If } r, s \in R \quad \text{then} \quad [(-1)^N P(r)P(s) < 0 \Leftrightarrow \sharp\{r_i / r_i \notin [r, s]\} \text{ is odd}].$$

1. Let $A = (-1)^L Q_W(\mu_1)Q_W(\mu_2)$ and $B = Q_F(\mu_1)Q_F(\mu_2)$. Since $Q_W(z) = z^{L-1}Q_F(z) - aQ_\psi(z)$ and $\mu_1 < 0 < \mu_2$ then $A = (-1)^L \mu_1^{L-1} Q_F(\mu_1)\mu_2^{L-1}Q_F(\mu_2) = -Q_F(\mu_1)Q_F(\mu_2) = -B$. Also, n_W is odd if and only if $A < 0$, therefore $Sign(A) = (-1)^{n_W}$. Analogously $Sign(B) = (-1)^{n_F}$. Since A and B have opposite signs we obtain that n_W is odd if and only if n_F is even.
2. In this case $n_F = 0$, so n_W is odd therefore there exists a root of Q_W with modulus greater than one.

Part 2 of the theorem above illustrates the principle of uncertainty: if agents are rather uncertain about the local dynamics close to the steady state (it means, the least μ is lower than -1 and the greatest μ is greater than 1) and expectations have a strong effect on the structural equations (it means that a is high so the λ's are small) then the actual dynamics is locally unstable (it means the characteristic roots of Q_W are greater than one). In the same paper, the converse is also proved: If (i) the influence of expectations is weak in the structural equations or (ii) traders are unable to, or have decided in advance not to, extrapolate any locally divergent trend out of small past deviations from equilibrium, then the actual local dynamics is stable.

2.3 LEARNING IN SUNSPOT EQUILIBRIUM MODELS

In Section 2.2, we analyzed the possibility of learning some non-stochastic equilibrium. Now we will show some learning processes that allow to learn truly stochastic equilibrium. These types of equilibria are usually called *sunspot equilibrium* because in most of the models the uncertainty arises from stochastic extrinsic events. In the following subsection we describe what the sunspot equilibrium is and give some existence results of these equilibria.

2.3.1 Definition and existence of stationary sunspot equilibrium

Following the structure presented in Section 2.2, we are going to define the sunspot equilibrium (SE).

DEFINITION 1 A sunspot equilibrium (SE) for the economy defined by $\tilde{Z}$ is a pair (X_0, Q), where $X_0 \subset X$ and $Q : X_0 \times \mathcal{B}(X_0) \to [0, 1]$ is a transition function[1] such that:

(i) There exists $x_0 \in X_0$ such that $Q(x_0, .)$ is not a Dirac measure; and
(ii) for all $x \in X_0, \tilde{Z}(x_0, Q(x_0, .)) = 0$.

It is not difficult to see why it is named "sunspot equilibrium". Suppose that agents relate the realization of some extrinsic event with the state variable of the economy (this relationship is called the "theory" of the agents) and such extrinsics follow a transition rule. If inducing this transition rule to the state variable of the economy throughout the theory results in a stochastic process which is an equilibrium, then we will have a sunspot equilibrium. In fact, our definition above just includes the theory of the agents.

When the transition function has an invariant probability measure μ,[2] we will have a stationary sunspot equilibrium (SSE) with stationary measure μ.

2.3.1.1 Existence of SSE from an indeterminate steady state

This is one of the most popular arguments for obtaining the existence of SSE. An indeterminate steady state is a stationary state that admits an (uncountable) infinite number of perfect foresight equilibria arbitrarily close to it. Many reasons can be presented for obtaining this type of multiplicity (Chiappori and Guesnerie, 1991; Benhabib and Farmer, 1994; Benhabib and Perli, 1994; Boldrin and Rustichini, 1994). The existence of infinite number of perfect foresight equilibria will allow to "randomize" among them in order to obtain stochastic equilibria. In fact Chiappori *et al.* (1992) gave a general proof of this for the structure we are considering. Woodford (1986) proved the same statement for economies with lagged variables. In the following we are going to give a simple sketch of the proof for the case of one-dimensional state variable.

The condition for indeterminacy of the steady state is that the forward perfect prevision map (defined as the map ψ such that $Z(x, \psi(x)) = 0$ for all x) must be a contraction around the steady state. It allows the existence of a continuum of equilibria in any neighborhood of the steady state. So a steady state $\bar{x}$ is indeterminate if and only if the matrix $B = (\partial_0 Z(\bar{x}, \bar{x}))^{-1} \partial_1 Z(\bar{x}, \bar{x})$ has at least one eigenvalue outside the unit disk.

A hypothesis usually considered in this type of model is the linearity of the function $\tilde{Z}$ with respect to the probability measure; this allows us to write:

$$(L)\ \tilde{Z}(x, \mu) = \int_X Z(x, y)\ \mu(dy),$$

so if the transition function is defined by a Markov chain $M = [m_{ij}]$, where $m_{ij} = Prob[\tilde{x}_{t+1} = x_j | \tilde{x}_t = x_i]$ then:

$$\tilde{Z}(x_i, M_i) = \sum_{j=1}^{N} m_{ij} Z(x_i, x_j).$$

Here, M_i represents the probability measure with support $(x_1, \ldots, x_N)$ and probabilities $(m_{i1}, \ldots, m_{iN})$.

Then the first result of existence of SE is the following:

THEOREM 2 *(Chiappori et al. (1992)) If condition (L) is satisfied, the steady state $\bar{x}$ is indeterminate and B has no eigenvalue with modulus one then for any neighborhood of $\bar{x}$ there exists stationary sunspot equilibrium with finite support in such a neighborhood.*

The proof for the linear case is as follows: If we suppose $Z(x,y) = Cx + Dy$ $(B = -C^{-1}D)$ then an SE with finite support is a set $\{x^1, \ldots, x^K\}$ and a Markovian matrix $M = [m_{ij}]$ such that:

$$
\begin{aligned}
&Cx^k + D(m_{k1}x^1 + \cdots + m_{kK}x^K) = 0, \quad \forall k = 1, \ldots, K,\\
&\Rightarrow X = M \oslash BX, \quad \text{or} \quad (I - M \oslash B)X = 0,
\end{aligned}
$$

where $X = [x^1, x^2, \ldots, x^K]'$ and $\oslash$ are the tensorial products. So an SE will exist if and only if the matrix $M \otimes B$ has a unitary eigenvalue. Since B has an eigenvalue with modulus greater than one and the eigenvalues of $M \oslash B$ are the product of the eigenvalues of M and B, we can always chose a matrix M with an eigenvalue being the inverse of the eigenvalue of B with modulus greater than one. Then an SE exists. The proof for a non-linear case is based in a construction of a bifurcation of the field defined by the equilibrium equations in the steady state.

2.3.1.2 Existence of SSE from regular cycles

Azariadis and Guesnerie (1986) related the existence of cycles with the existence of sunspot equilibria with finite support. Let (x_1, x_2) be a cycle or periodic equilibrium of order 2 $(Z(x_1, x_2) = Z(x_2, x_1) = 0)$, then we have:

$$
\tilde{Z}(x_1, (x_1, x_2; 0, 1)) = 0 \quad \text{and} \quad \tilde{Z}(x_2, (x_1, x_2; 1, 0)) = 0.
$$

Let $F(x, x', \alpha, \beta) = (\tilde{Z}(x, (x, x'; \alpha, (1-\alpha))), \tilde{Z}(x', (x, x'; (1-\beta), \beta)))$, so $F(x_1, x_2, 0, 0) = 0$, we will say that (x_1, x_2) is a regular periodic equilibrium if:

$$
D_{(x,x')}F(x_1, x_2, 0, 0) = \begin{pmatrix} \partial_0 Z(x_1, x_2) & \partial_1 Z(x_1, x_2) \\ \partial_1 Z(x_2, x_1) & \partial_1 Z(x_2, x_1) \end{pmatrix}
$$

is a non-singular matrix. In such a case there exist functions $x(\alpha, \beta), x'(\alpha, \beta)$ defined in a neighborhood of $(\alpha, \beta) = (0, 0)$ such that:

$$
F(x(\alpha, \beta), x'(\alpha, \beta), \alpha, \beta) = 0
$$

for all (α, β) in such a neighborhood, but it means that there exists an SSE with support close to $\{x_1, x_2\}$ and a Markov matrix:

$$
\begin{pmatrix} \alpha & 1-\alpha \\ 1-\beta & \beta \end{pmatrix}.
$$

This result is generalized in the following theorem.

THEOREM 3 *If $(p_1, \ldots, p_k)$ is a regular k-cycle for the deterministic model then there exists an SE with support close to $\{p_1, \ldots, p_k\}$.*

2.3.1.3 Existence of SSE from chaotic deterministic dynamics

The last argument for the existence of SSE that we will discuss was developed by Araujo and Maldonado (2000). They showed that when the economy exhibits complex deterministic dynamics, it is possible to construct SE with an (uncountable) infinite support and with a stationary measure which is absolutely continuous with respect to the Lebesgue measure. In the study of dynamical systems there exist at least two types of "chaotic" maps; in that work was used the ergodic version of chaos; roughly speaking, a map has ergodic chaos if there exists an absolutely continuous invariant measure for the map. This implies that almost all trajectories generated by the iterations of the map visit any interval with a fixed frequency.

It is worth noting that all the above arguments for existence of SE (and others we did not present) showed the existence of a finite SE (i.e. with a finite support) or the support is implicitly determined. In Araujo and Maldonado's work not only the sunspot is explicitly constructed but there is a numerical method for computing the invariant probability measure (its density). Let us discuss a simple example in order to understand the main result of them.

Let us consider the simple overlapping generations (OLG) model described in Section 2.2 with the following specification: The labor supply $y \in X = [0, 1]$ and the agents have the utility function: $U(c, y) = 4c - 2c^2 - y$, defined on $[0, 1] \times [0, 1]$. From the first order conditions and the equilibrium equation we obtain under perfect prevision the following dynamical system:

$$y_t = 4y_{t+1}(1 - y_{t+1}),$$

or $y_t = \phi(y_{t+1})$, where ϕ is the logistic map.

It is well known that ϕ has ergodic chaos; it means that it admits an invariant probability measure which is ergodic and absolutely continuous with respect to the Lebesgue measure, i.e. $\exists \mu \in \mathcal{P}(X), \mu \ll \lambda(\lambda$ is the Lebesgue measure) such that $\mu(\phi^{-1}(A)) = \mu(A)\ \forall A \in \mathcal{B}(X)$ and if $A \in \mathcal{B}(X)$ is such that $\phi^{-1}(A) = A$ then $\mu(A) = 0$ or $\mu(A) = 1$.

Let us consider $f : [0, 1] \to [0, 1/2]$ and $g : [0, 1] \to [1/2, 1]$, local inverse functions of ϕ. Since $Z(x, f(x)) = Z(x, g(x)) = 0\ \forall x$ and the (L) property holds, it is easy to see that:

$$Q(x, .) = \frac{1}{2}\delta_{f(x)}(.) + \frac{1}{2}\delta_{g(x)}(.) \quad \forall x \in Supp(\mu)$$

is an SE. Also it is easy to verify that $\int Q(x, A)\mu(dx) = \mu(A)$ for all $A \in \mathcal{B}([0, 1])$ therefore the SE is stationary with respect to the measure μ. The following theorem gives the extension of this result to more general dynamics.

THEOREM 4 *Let $\phi : X \to X(X \subset R^n_+)$ be the backward perfect foresight function associated to the stochastic excess demand function $\tilde{Z}$ (i.e. the function that satisfies $Z(\phi(x), x) = 0, \forall x$). Suppose that the (L) property holds. If:*

(i) There exists a partition $(A_i)_{i=1}^N$ of X with non-empty interior and $\phi : A_i \to \phi(A_i)$ is a diffeomorfism $\forall i = 1, \ldots, N$ with inverse φ_i.

(ii) There exists $\mu \in \mathcal{P}(X)$, $\mu \ll \lambda$ and ϕ-invariant.

Then there exists a set C with $\lambda(C) > 0$ such that the transition function:

$$Q(x, .) = \sum_{i=1}^{N} \frac{d\mu \circ \varphi_i}{d\mu}(x)\delta_{\varphi_i(x)}(.)$$

is an SSE on the set C and stationary measure μ.

Let us make some remarks on the theorem above. First, the function ϕ can be interpreted as the function that gives the current state that rationalizes some future state value taking for sure. Second, the partition in condition (*i*) says that in a perfect prevision world, for each current state there exist N future states that give a temporary equilibrium; so we can think that in an uncertain world agents will make a randomization of these future states. Finally, if the randomization is made using the gains given in definition of $Q(x, .)$ we will obtain an SSE with invariant probability measure μ. Condition (*ii*) is a technical one that can be verified for different values of the parameters of the model. (See Araujo and Maldonado (2000), Section 2.4.)

2.3.2 Learning stationary sunspot equilibrium

There exist in the literature two explicit results about convergence to stationary sunspot equilibrium; however many authors conjecture this convergence from results of convergence to non-stochastic equilibrium as a "natural" extension. The point is that convergence to deterministic equilibria has two facts that make it simpler: (1) a deterministic equilibrium for a non-stochastic model (it means a model with no intrinsic stochastic variable) is just based on the fundamentals of the economy. So the learning rule for obtaining convergence does not need to incorporate any extrinsic noise; and (2) the most popular learning processes (adaptives, recursives, Bayesian) when applied to non-stochastic models are sufficient for learning deterministic equilibria. However, the existence of SE raises the question: is there any reasonable and robust learning process which converges to the SE?

In this subsection we will present two results about convergence to SSE. The first one is due to Woodford (1990) who considered the following model. An OLG model with linear technology and an asset with stochastic real return R_{t+1}. With this, agents will maximize

$$E_t[u(n_t R_{t+1}) - v(n_t)],$$

where u is the utility of the next period consumption, v the "disutility" from labor, n_t the labor supplied in period t and the expected value is taking conditional to information until period t. The first order condition for consumers is:

$$v'(n_t) = E_t[R_{t+1}u'(n_t R_{t+1})].$$

There is a constant supply of fiat money M so the consumption satisfies $c_{t+1} = M/p_{t+1}$ (p_t is the price of the unique good in period t); and the labor supply must satisfy $n_t = M/p_t$. Finally, $R_{t+1} = c_{t+1}/n_t = p_t/p_{t+1} = n_{t+1}/n_t$. From this, the equation for temporary equilibrium is:

$$n_t v'(n_t) = E_t[n_{t+1}u'(n_{t+1})].$$

An SSE is a stationary process $(\tilde{n}_t)$ which is truly stochastic. A (deterministic) stationary equilibrium is an n^* such that $v'(n^*) = u'(n^*)$ and the idea for obtaining an SE from this is to make a stochastic perturbation (which is a bifurcation) of an adequate function in the point n^*. Woodford uses the existence argument of SSE with finite support given by Azariadis (1981). That sunspot consists in m, different states $(n^1, \ldots, n^m)$ and for each of them a probability distribution π_{ij} such that:

$$n^j v'(n^j) = \sum_{k=1}^{m} \pi_{ij} n^k u'(n^k).$$

If we define the function $F : R^m \to R^m$ as $F_j(\mathbf{n}) = n_j^{-1} \sum_{k=1}^{m} \pi_{ij} n_k u'(n_k) - v'(n_j)$ then a zero of this function, where at least two of the states are different, is an SE. Azariadis and Guesnerie (1982) gave a sufficient condition for it.

Proposition 1 *If* $n^*v''(n^*) + n^*u''(n^*) + 2u'(n^*) < 0$ *then there exists an SSE.*

Suppose that there exist preference shocks so that the disutility of labor is $v(n_t) - \tilde{\epsilon}_t n_t$ where $(\tilde{\epsilon}_t)_t$ is an i.i.d sequence of random variables (independent of the sunspot process) with bounded support, zero mean and variance σ^2. The value of ϵ_t is only observed after the choose of n_t. Then the first order condition for the consumer problem is:

$$v'(n_t) = E_t[R_{t+1}u'(n_t R_{t+1}) + \epsilon_t].$$

The expectation is referred to the subjective probability distribution of $(R_{t+1}, \tilde{\epsilon}_t)$. If consumer observes the j sunspot then the probability distribution of $(R_{t+1}, \tilde{\epsilon}_t)$ is G_j and the consumer chooses

$$n_j = \text{Arg}\max \int (u(nR) - v(n) + \epsilon n)\; dG_j(R, \epsilon).$$

The learning procedure, which is based on the "stochastic approximation" algorithm of Robbins and Monro (1951), is as follows: Starting with the estimate $\hat{n}_{jM}$ after M drawings from the G_j distribution ($M \geq 0$) and with the additional observation (R_{M+1}, ϵ_M) the new estimate is:

$$\hat{n}_{j(M+1)} = \hat{n}_{jM} + h(M+1)^{-1}[R_{M+1}u'(R_{M+1}\hat{n}_{jM}) - v'(\hat{n}_{jM}) + \epsilon_{m+1}], \qquad (*)$$

where the constant $h > 0$ indicates the effect of each new observation. That rule is a sort of gradient rule: If the term in brackets is greater than zero (it means that a labor supply greater than $\hat{n}_{jM}$ is desired) then the new estimate must increase. With this Woodford proves the following.

PROPOSITION 2 *Under hypothesis of proposition above (i.e. SE exists) the learning rule* $(*)$ *converges with probability 1 to the true value* n^j.

Another type of SSE can be learned using adaptive learning rules. In Theorem 4, we showed the result of Araujo and Maldonado (2000) on the existence of SSE when the deterministic dynamics is complex. They also showed a learning rule that converges to this sunspot equilibrium. Let us describe their findings. Suppose that the backward perfect foresight map $\phi : [0, a] \to [0, a]$ is a unimodal map and there exists an invariant measure μ for it which is absolutely continuous with support $[0, a]$. Then for all $x\lambda-$ a.s. the empirical measures $\{\mu_N\}_{N\geq 1}$ converge in the weak topology to μ. Since $\mu_N = 1/N \sum_{t=0}^{N-1} \delta_{\phi^t(x)}$, it is easy to see that:

$$\mu_N \circ f = \frac{1}{N}\sum_{t=0}^{N-1} 1_{X_1}(\phi^t(x))\delta_{\phi^{t+1}(x)}.$$

Here $X_1(X_2)$ is the interval where ϕ is increasing (decreasing) and $f(g)$ is the inverse of ϕ on $X_1(X_2)$. The learning process is as follows: Let us suppose that the economy has the state variable value $x_0 = z$ and the agents want to update (or formulate) their expectations about the next period.

2.3.2.1 Passing from deterministic to stochastic equilibrium

First, we will consider that the economy was following a deterministic path $x_0 = z, x_{-1} = \phi(z), \ldots, x_{-(N-1)} = \phi^{N-1}(z), \ldots$. Then the agents know two possibilities for the next period state value: $f(z)$ ("pessimistic" prevision) and $g(z)$ ("optimistic" prevision) and they want to know the probabilities they must assign in each case. Define the following sequence of regular partitions: $(\pi_n)_{n\geq 1}$ where $\pi_n = \{t_0 = 0, t_1 = 1/2^n \ldots t_{2^n} = 1\}$. If $z \in J_k^n = [k/2^n, k + 1/2^n]$ then $\mu_N(J_k^n)$ is the frequency of the N-orbit $\{z, \phi(z), \ldots \phi^{N-1}(z)\}$ in the interval J_k^n and $\mu_N \circ f(J_k^n)$ is the frequency of the points in the orbit $\{\phi(z), \ldots, \phi^N(z)\}$ which are in J_k^n and by the perfect foresight policy remain in X_1. The chance of the pessimistic prevision is defined by:

$$\frac{\mu_N \circ f(J_k^n)}{\mu_N(J_k^n)} \quad \text{if} \quad f(z) \in J_k^n \quad \text{and} \quad \mu_N(J_k^n) \neq 0$$

and zero in other cases. This is a sort of Bayesian learning, where the probability of the state $f(z)$ is given by the conditional probability of staying in the pessimistic region (X_1) given that the current state is in a small interval containing z (the current state). When the number of observations (N) is large we can approximate this chance (using the ergodic theorem) by:

$$c_k^n = \frac{\mu \circ f(J_k^n)}{\mu(J_k^n)}.$$

Then for each partition π_n we can define the simple function $R_n(z) = \sum_{k=1}^{2^n} c_k^n 1_{J_k^n}(z)$ for all z in $[0, a]$ as the map which associates the chances of pessimistic previsions. Therefore, the learning rule for the given partition is defined by:

$$Q^n(z,.) = R_n(z)\delta_{f(z)}(.) + (1 - R_n(z))\delta_{g(z)}(.). \qquad (*)$$

The following theorem shows that if the norm of the partition goes to zero then this rule converges to the stationary sunspot equilibrium.

THEOREM 5 *If ϕ is a unimodal map, μ is a ϕ-invariant measure, f and g are the local inverses of ϕ, and Q^n is defined by $(*)$ then for each $A \in \mathcal{B}(X)(Q^n(.,A))_{n\geq 1}$ converges to $Q(.,A)\mu$-a.s. and in $\mathcal{L}^1(\mu)$.*

Sketch of the proof It is sufficient to prove that $(R_n)_{n\geq 1}$ converges to $R = d\mu \circ f/d\mu$. Let $\mathcal{F}_n$ be the σ-algebra generated by π_n. Then we have that $R_n = E[R|\mathcal{F}_n]$. Since $R \in \mathcal{L}^1(\mu)$, the Lévy's "upward" theorem (see e.g. Williams, 1991) allows to conclude that $R_n \to E[R|\mathcal{B}(X)] = R$ almost surely and in $\mathcal{L}^1(\mu)$. (See details in Araujo and Maldonado, 2000.)

2.3.2.2 Learning the stochastic equilibrium

Now the following question arises: If the economy is in this SSE, how can the agents learn it? In the proof of Theorem 5 the key fact was the convergence of the empirical measures μ_N to the measure μ when they are constructed from the deterministic observations. Now we want to know if the empirical measures constructed with the stochastic observations also converge to μ. If the measure μ is Q-ergodic then the histograms provided by the stochastic observations will converge to the histogram constructed throughout the measure μ, namely: If $(\tilde{x}_t)_{t\geq 0}$ is the Markov process generated by (Q, μ) the Birkhoff theorem in the stochastic version gives us:

$$\frac{1}{N+1}\sum_{t=0}^{N} \delta_{\tilde{x}_t}(.) \to \mu(.)\mu - a.e.$$

in the weak topology (see Kifer, 1986). So we just have to verify if μ is Q-ergodic; this is made in the following theorem.

THEOREM 6 *With the hypotheses of Theorem 5, if μ is absolutely continuous and ϕ-ergodic then μ is Q-ergodic.*

Proof Since μ is Q-invariant, it is sufficient to prove that if $A \in \mathcal{B}(X)$ is such that $Q(x, A) = 1_A(x)$ for x μ-a.e. then $\mu(A) = 0$ or $\mu(A) = 1$. Let $\alpha(x) = d\mu \circ f/d\mu(x)$; by Theorem 4 $\alpha(x) \in (0, 1)$. By hypothesis:

$$\alpha(x)\delta_{f(x)}(A) + (1 - \alpha(x))\delta_{g(x)}(A) = 1_A(x), x\ \mu - a.e.$$

If $x \notin A$ then $f(x) \notin A$ and $g(x) \notin A$, then $x \notin f^{-1}(A) \cup g^{-1}(A)$; so $\phi(A) \subset A$.

If $x \in A$ then $f(x) \in A$ and $g(x) \in A$, then $x \in f^{-1}(A) \cap g^{-1}(A)$; so $A \subset \phi(A \cap X_1) \cap \phi(A \cap X_2)$.

Therefore $\phi(A) \subset A \subset \phi(A \cap X_1) \cap \phi(A \cap X_2)$; from this it is easy to conclude $A = \phi^{-1}(A)$, i.e. A is ϕ-invariant, then $\mu(A) = 0$ or 1 because μ is ϕ-ergodic. Q.E.D.

Theorem 6 and the observation above imply that if the learning rule $(*)$ is based on the histograms constructed with the stochastic observations, it will converge to the SSE (C, Q). Note that the support of this learning rule (the support of the invariant measure) has positive Lebesgue measure. This fact must be contrasted with the Woodford's learning rule which converges to a finite sunspot equilibrium.

2.3.2.3 Learning equilibria

All the statements above defend the convergence to rational expectations equilibrium. They claimed local or global convergence as a result of a specified and reasonable learning procedure. However, we can describe a simple learning rule that applied to a simple linear model generates a dynamics that never converges to any equilibrium and presents systematic errors that never vanish. Suppose that the structural equation is given by $Z(x_{t-1}, x_t, x^e_{t+1}) = ax_{t-1} - x_t + bx^e_{t+1} = 0$. Agents have an autoregressive model as the perceived law of motion for the state variable, namely $\tilde{x}_{t+1} = \beta x_t + \tilde{\epsilon}_t$, so the expected value of the state variable is $x^e_{t+1} = \beta_t^2 x_{t-1}$, where $\beta_t = x_{t-1}/x_{t-2}$ is the estimate of β in period t. The actual law of motion of the state variable is the result of replacing the percieved law of motion into the structural equation. It results:

$$\frac{x_t}{x_{t-1}} = a + b\left(\frac{x_{t-1}}{x_{t-2}}\right)^2 = F\left(\frac{x_{t-1}}{x_{t-2}}\right),$$

where the F function is the logistic map. Dynamical systems say that there exist a large set of parameters (a, b) where the trajectories of the ratios x_t/x_{t-1} will eventually fill a non-degenerate interval without converging to any point. Then the actual dynamics does not converge to any rational expectation equilibrium[3] and the motion of the actual ratio of the state variable shows that agents make (with this learning rule) mistakes that will never disappear. This trajectory is called a *learning equilibrium* and was also studied by Bullard (1994) and Brock and Hommes (1997).

2.4 CONCLUSIONS

In this survey, we discussed the main difficulties that multiplicity of equilibrium may arise in macroeconomic models. Essentially, the problem is that the Walrasian equilibrium approach does not give a selection criterion of any of these equilibria. The bounded rationality of economic agents requires that they use learning rules in order to formulate expectations about future states and these learning procedures lead the economy to one or another equilibrium.

First we discussed conditions on the structural equations of the economy and learning rules that allow convergence to deterministic rational expectation equilibrium. One result that unifies all these conditions is the Grandmont's "*principle of uncertainty*", that relates the sensitivity of expectations in the economy with the ability of the learning rule for extrapolating trends out of small past deviations from equilibrium.

We also presented robust learning procedures that lead the economy to sunspot equilibrium. This is an important fact that must be analyzed deeply in order to eliminate inefficiencies of these equilibria.

Finally, we show that some learning rules can generate actual dynamics that do not converge to any rational expectation equilibrium, but they keep the motion bounded. These "learning equilibria" may support forecasting errors that would never vanish.

So the open questions that this type of work raises are: Is there a "natural" learning rule for a given model? Is there any learning rule that includes all the relevant rules? How can a policy-maker do for passing from one equilibrium to another? Can learning equilibria explain some real disequilibrium with systematic errors? All these questions can lead researchers to obtain alternative explanations to the weak rationality assumption.

NOTES

1 A transition function is a $Q : X_0 \times \mathcal{B}(X_0) \to [0,1]$ such that: (1) For all $x \in X_0, Q(x, .)$ is a probability measure defined in the Borel sets containing in X_0; and (2) For all $A \in \mathcal{B}(X_0), Q(., A)$ is a measurable function.

2 For all Borel set A we have $\mu(A) = \int Q(x, A) d\mu(x)$.

3 Evans and Honkapohja (1986) gave a complete characterization of solutions for linear models like the one given in this example.

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3 Disequilibrium and stability*

Franklin M. Fisher

3.1 INTRODUCTION

In a conference about general equilibrium theory (GE), it seems appropriate to step back and consider just what it is that one expects such a theory to accomplish. There are, in fact, several reasons to be interested in general equilibrium theory:

1 Microeconomics provides a powerful set of theories about the choices of individual agents and the behavior of particular markets. Without general equilibrium theory, however, there would be no guarantee that such particular theories could all hold at the same time. We could not be sure that it is generally possible for all agents to carry out their plans or, equivalently, for all markets to clear at the same time. This question is settled by existence theory. The fact that positions of general equilibria exist under a wide set of conditions means that there is not a lacuna at the center of microeconomic analysis, forcing us to reformulate the theories of the household, the firm, and the competitive market.
2 The central set of propositions that economists have to offer the outside world – propositions that are, in a real sense, the foundations of Western capitalism – comprises the two Welfare Theorems. These theorems elucidate the relations between general competitive equilibria on the one hand and Pareto-efficiency on the other. They underlie all the looser statements about the desirability of a free-market system. These propositions are also well understood and firmly founded.
3 The third thing that one wants from general equilibrium theory is essentially a positive theory of value. We want to know how prices are set, and partial theory cannot tell us that in a satisfactory way. More importantly, we want to know how, starting with the possibly incompatible plans of individual agents, the economy moves to positions in which those plans have become compatible. Merely studying positions of compatibility cannot answer this question, and this is where general equilibrium theory becomes unsatisfactory or incomplete, even if its practitioners often fail to realize it. The present chapter is concerned with what we do know and do not about this problem.

* This chapter is a revised version of "The Formation of Economic Magnitudes: Disequilibrium and Stability" published in Cartelier (1990) and as Chapter 1 of Fisher (1999). My most extensive and formal discussion of the topics covered is Fisher (1983).

Modern economic theory is overwhelmingly a theory of equilibrium. It analyzes positions from which there is no incentive to depart, positions at which the plans and expectations of economic agents are mutually compatible. It is almost silent on the question of how such positions get reached, on how economic magnitudes get formed if they do not already happen to be in equilibrium.

Equilibrium analysis is an elegant and powerful tool, providing considerable illumination of the way in which real economies operate. But the total concentration on equilibrium, now characteristic of formal economic models, runs the serious risk of misunderstanding the basic insights of economics itself. Thus, the proposition that competitive industries earn no profits in long-run equilibrium is an important (if elementary) theorem. To take this to mean that competitive industries *never* earn profits is not only wrong, it is to lose sight of the fundamental role that profits and losses play in the allocation of resources when demand or technology changes. The proposition that competitive equilibria and Pareto-optima are closely related is a basic insight. The policy prescription that (under the conditions of the two Welfare Theorems) government interference with a competitive system is bound to be inefficient requires more than this, however; it requires the assurance that competitive economies are close to equilibrium most of the time. That assurance cannot be provided by only examining the properties of equilibria.

Nor are such issues restricted to microeconomics. To take a leading modern example, the statement that agents will eventually learn about and act on systematic profit opportunities is an appealing assumption. The proposition of the rational expectations literature that agents always instantaneously understand the opportunities thrown up by an immensely complex and changing economy is breathtakingly stronger. That proposition begs the question of how agents learn and of the role that arbitrage plays in the formation of economic magnitudes. To take an older example, the proposition that, under some circumstances, there can exist underemployment equilibria was the major contribution of the Keynesian literature. To show that the economy can tend toward such equilibria is a much harder proposition, requiring analysis of dynamic, disequilibrium behavior.

Indeed, such dynamic, disequilibrium analysis is always required if we are truly to have a satisfactory theory of value. Certainly, if the economy does not spend most of its time near equilibrium, disequilibrium analysis is the only useful kind. Even if equilibrium is the usual case, however, disequilibrium analysis is indispensable. For one thing, only such analysis can provide the assurance that our equilibrium theories are consistent; if equilibrium is the usual case, we need to know why. Further, only analysis of the dynamic path that a stable system follows in disequilibrium can tell us to which of several possible equilibria that system will go. This is a matter of considerable importance, not only because multiplicity of equilibria is the rule rather than the exception, but also because, as we shall see, the analysis of disequilibrium shows that the dynamic behavior involved often changes the equilibrium that is eventually reached.

There are two fairly common mistakes that must be avoided in considering such matters. First, one must not confuse the fact that the economy will move away from positions that are not equilibria with the much deeper and unproven proposition that the economy always converges to equilibrium (let alone the proposition that it spends most of its time near equilibrium). In more specific terms, the fact that agents will seize on profitable arbitrage opportunities means that any situation in which such opportunities appear is subject to change. It does not follow that profitable arbitrage opportunities disappear or that new opportunities do not continually arise in the process of absorbing old ones.

The second mistake is the belief that such problems can be avoided by redefinition of terms so that there is no such thing as disequilibrium. For example, the non-clearing of markets by prices is sometimes said not to be an example of disequilibrium because agents form queues, with the length of the queue being determined by the shadow price of time as well as by money prices. This may be a valuable way to think about what happens when markets fail to clear, but it reformulates rather than solve that question (What happens to money prices? How do the queues themselves disappear over time?). Certainly, there is a sense in which the disequilibrium behavior of any given system can be represented as the equilibrium behavior of a larger system in which the original one is embedded. To say this, however, is only to say that there is some definite outcome out-of-equilibrium in the smaller system. To insist that therefore there is no such thing as disequilibrium is to rob the term "equilibrium" and all equilibrium analysis of meaning. For if "equilibrium" is to be a useful concept in analyzing a particular system, then one must contemplate the possibility of points that are not equilibria of that system. The fact that such points can be represented as equilibria in some larger system does not change this.

If equilibrium analysis is to be justified, the crucial question that must first be answered is one of stability. That question in its most interesting and general form is as follows. Suppose an economy is made up of agents who understand that they are in disequilibrium and perceive and act on profit opportunities. Does the action of those agents lead the economy to converge to equilibrium, and, if so, to what sort of equilibrium? I shall refer to this as the "key question" of stability analysis.

It is important to note, however, that while stability of competitive general equilibrium is perhaps the only disequilibrium question addressed in a long literature, that literature has seldom addressed the key question directly. Rather, as we shall see, writings on the stability of general equilibrium have only recently endowed agents with much perception. Instead, agents have been supposed to make their plans as though disequilibrium did not exist, and the interaction of those plans has been modeled only as an afterthought at best.

Why should this be? The answer may be related to the phenomenon of concentration on equilibrium and to the distaste or at least disinterest with which many theorists regard the stability literature. Economic analysis is extremely powerful when considering the optimizing behavior of the individual agent. It is comfortable with positions in which the plans of those agents are mutually compatible. It must break untrodden ground to describe what happens when this is not so. This means modeling both the way in which trade takes place when agents' plans cannot be completely fulfilled and how agents react to frustration. Neither aspect can be properly done by considering equilibrium behavior.

3.2 TÂTONNEMENT AND ITS FAILURE

As already indicated, however, the study of stability has historically been marked by the failure to model out-of-equilibrium behavior as more than an afterthought. That was particularly true of the development that characterized the first 20 years or so of the subject – the study of tâtonnement.

It was P.A. Samuelson (1941) who took the first crucial step in the study of stability. Reacting to a suggestion of J.R. Hicks (1939) that "perfect stability" might be defined in terms of demand curves that slope down after various prices are allowed to adjust, Samuelson pointed out that there could be no study of stability without an explicit dynamic model. He assumed that price-adjustment takes place out-of-equilibrium by prices moving in the direction indicated by the corresponding excess demands,[1] an assumption that can be written in its general form as

$$\dot{p}_i = H^i(z_i(p)) \quad i = 1, \ldots, c \tag{3.1}$$

where there are c commodities, subscripted by i, p is the vector of prices, $z_i(p)$ the excess demand for commodity i when prices are p, and the $H^i(.)$ continuous and sign-preserving functions. (A dot over a variable denotes differentiation with respect to time.) Samuelson proposed the study of equation (3.1) as the *only* out-of-equilibrium adjustment mechanism.

Models of this type are known as "tâtonnement" models. They suffer from the obvious lack of reality of the assumption that only prices adjust out-of-equilibrium, with agents constantly recontracting rather than trading (let alone consuming and producing). Yet that assumption (which goes nicely with the fictitious Arrow–Debreu world in which all markets open and close at the dawn of time) may not be the most troublesome one for purposes of understanding disequilibrium behavior. Since price adjustment equations such as (3.1) are also characteristic of the later, non-tâtonnement literature, it is worth discussing this in detail.

Whose behavior does equation (3.1) represent? It cannot reflect directly the behavior of the individual agents whose demands are to be equilibrated. Indeed, we now see a central conundrum: In a perfectly competitive economy, all agents take prices as given and outside of their control. Then who changes prices? How do sellers know when demand or costs rise so that they can safely raise prices without losing all their customers? At a formal level such questions are deep ones.

It only begs the price-adjustment question to say (as is often done) that equation (3.1) reflects the behavior of an "auctioneer" whose job it is to adjust prices in such a way.[2] Most real markets do not have such specialists. Those markets that do have them are such that the specialist is rewarded for his or her endeavors. To understand where and how such price-setting takes place requires analysis of how markets equilibrate. That cannot be done by adding equation (3.1) as an afterthought, nor is it likely to be done satisfactorily in the tâtonnement world where only prices adjust and there are no consequences to remaining in disequilibrium.

The fact that there are no such consequences provides some justification for the way in which the behavior of the agents themselves is treated in tâtonnement models. Disequilibrium never enters the dreams of those agents; they construct their excess demands as though prices are fixed and unchanging and as though their desired transactions will in fact take place. Since nothing happens until prices have adjusted to equilibrium (assuming that ever occurs), agents have nothing to gain by being more sophisticated about what is really happening.

Tâtonnement models, then, do little about the two basic facets of disequilibrium behavior. They model the out-of-equilibrium interaction of agents in terms of price adjustment only, without any basis for such an adjustment mechanism. Further, since such an unsatisfactory adjustment mechanism does not permit agents to find their plans frustrated in any meaningful sense, there is no analysis of the way in which the agents react to such frustration.

Despite these defects, the analysis of tâtonnement was the exclusive subject of the first 20 years or so of the stability literature (roughly 1940–1960). This is understandable when one recalls that the subject was then in its infancy. Perhaps because the adjustment process in equation (3.1) seems the simplest case and perhaps because, even so, until the late 1950s major results seemed very hard to come by, no serious attention seems to have been paid in this period to the underlying defects of the model. What is more surprising is the casual view still sometimes encountered that stability analysis necessarily means the study of tâtonnement. Perhaps partly because of the obvious defects of the tâtonnement model and partly because of the total collapse of the tâtonnement effort in 1960, that casual view tends to be accompanied by a disdain for the entire subject of stability.

As just indicated, however, the late 1950s seemed a time of considerable promise for tâtonnement results. This was largely because of the introduction of Lyapounov's Second Method into the economics literature, rather than because of the attractive nature of the tâtonnement model itself.

Following Samuelson's introduction of equation (3.1), the literature (which was not voluminous) concentrated on the question of whether this equation was locally stable. Essentially, this is the question of whether equation (3.1) tends to converge to a rest point (a point at which $p = 0$, here identical with a Walrasian equilibrium) if it begins close enough to that rest point. Such concentration on local properties seemed natural, for it allowed linear approximation, and the properties of autonomous linear differential equations are completely known.

Less understandable, save in historical, terms was the early concentration on the relations between local stability of equation (3.1) and the conditions for Hicksian "perfect stability" – an attribute that, as already mentioned, has nothing directly to do with stability at all. Those conditions – the alternation in sign of the principal minors of the Jacobian of the excess demand functions – were shown by Samuelson (1941, 1947) and L. Metzler (1945) to be equivalent to the local stability of equation (3.1) on the very strong assumption that all goods are gross substitutes (excess demand for any good goes up when the price of any other good increases).[3]

Since the alternation of the principal minors is not a particularly interpretable property, the Samuelson–Metzler results are properly to be regarded as a lemma rather than a theorem, but it was a long while before any further progress was made. That was done independently by F.H. Hahn (1958) and T. Negishi (1958). Each of these authors realized that the economic structure of the problem could be further exploited and each showed – Hahn using Walras' Law and Negishi the homogeneity of degree zero of the excess demand functions – that the gross substitute assumption itself implied the Hicks conditions on the principal minors and hence the local stability of equation (3.1).

This quite neat contribution was eclipsed, however, by the really big development of the late 1950s, the introduction of Lyapounov's Second Method.[4] This was done in a pair of papers by Arrow and Hurwicz (1958) and Arrow *et al.* (1959).

Lyapounov's Second Method works as follows. Continuing with equation (3.1) as an example of a differential equation, suppose that there exists a function, $V(p)$, which is continuous, bounded below, and decreasing through time except at rest points of equation (3.1). The existence of such a function, called a "Lyapounov function," implies that equation (3.1) is *quasi-stable*, that is, that every limit point of the time-path of p is a rest point. If that path can be shown to remain in a compact set, then p approaches the set of rest points. If, in addition, rest points are locally isolated or unique given the initial conditions, then equation (3.1) is a *globally stable* process; it converges to some rest point no matter where it starts. (Recall that the rest points of equation (3.1) are Walrasian equilibria.)[5]

This powerful tool was used by Arrow *et al.* to demonstrate the global stability of tâtonnement under apparently different strong restrictions on the excess demand functions. The first such restriction was that of gross substitutes, thus completing the early literature. Unfortunately, as we now realize, both this and nearly every other restriction considered were special cases of the assumption that the Weak Axiom of Revealed Preference applies to *market* demand functions – a very strong restriction indeed. As a result, Arrow *et al.*'s conjecture, that tâtonnement is *always* stable given only those restrictions (such as Walras' Law) that stem from the basic assumptions of microeconomic theory, was a bold one indeed.

In fact, that conjecture is wrong. H. Scarf (1960) quickly provided a counter-example of an exchange economy with non-pathological consumers in which equation (3.1) is not stable. As we know now from the work of H. Sonnenschein and others, that example implies the existence of an open set of economies for which a similar result holds.[6] Indeed, so far as anything useful is known, it appears to be that stability rather than instability of tâtonnement is a special case.

Scarf's counter-example was thus of major historical importance. Its true analytical importance today, however, is not often realized. Scarf did not show that stability analysis was guaranteed to be unfruitful. (Indeed, as we shall see, a very fruitful development immediately began in the early 1960s.) Rather Scarf showed that *tâtonnement* would not generally lead to stability. This means that the facile proposition that disequilibrium is cured by fast-enough price adjustment is not generally true (although, of course, it may be true in special circumstances).

If price adjustment alone is not sufficient to guarantee stability, however, then equilibrium economics must rest on the assumption that quantities also adjust. While, as we shall see, such an assumption does indeed lead to more satisfactory stability results, it has a major consequence. When trade takes place out-of-equilibrium (and even more when disequilibrium production and consumption occur), the very adjustment process alters the equilibrium set.

This is easily seen even within the simplest model of pure exchange. In such a model, the equilibrium prices and allocations depend on the endowments. If trade takes place out-of-equilibrium, those endowments change. Hence, even if the trading process is globally stable, the equilibrium reached will generally not be one of those corresponding to the initial endowments in the static sense of the Walras correspondence.

Rather the equilibrium reached will be path-dependent, dependent on the dynamics of the process taking place in disequilibrium.

If such effects are large, then the popular enterprise (ironically led by Scarf himself (1973)) of computing points of general equilibrium from the underlying data of the economy is quite misleading. The points computed by such algorithms are the equilibria corresponding statically to the initial endowments of the economy. They are not the equilibria to which the economy actually tends, given those endowments. Hence such algorithms make dangerous predictive (or prescriptive) tools.

More important than this, the principal tool of equilibrium analysis – comparative statics – is called into question. Displacement of equilibrium will not be followed by convergence to the new equilibrium indicated by comparative statics. Rather it will be followed by a dynamic adjustment process which, if stable, generally converges to different equilibrium. While general comparative-statics results are not plentiful in general equilibrium analysis, the foundation for such results, even in a partial equilibrium setting, has become shaky.

Out-of-equilibrium effects may, of course, be small. But we have no reason to believe that they are. The failure of tâtonnement means that we cannot escape by assuming that quantity-adjustment effects are negligible relative to price effects. The doubtful project of tacking anonymous price adjustment on to an equilibrium model is known to be a failure. Further progress requires more serious attention to what happens out-of-equilibrium, and we see that what happens out-of-equilibrium can have a serious effect on equilibrium itself.

3.3 TRADING PROCESSES: THE EDGEWORTH PROCESS

The failure of tâtonnement, however, does not imply the failure of stability analysis, and the early 1960s saw the beginning of a more fruitful development. Not surprisingly perhaps, that development involved a closer look at out-of-equilibrium behavior.[7] In particular, while equation (3.1) remained the equation supposedly explaining price adjustment, trade was now allowed to take place out-of-equilibrium, and some thought was given to the specification of trading rules. The resulting models were called "non-tâtonnement" processes, but as that name is not particularly descriptive, I prefer to call them "trading processes."

Trading processes made only a modest concession to realism in allowing trade to take place out-of-equilibrium. Households (the original models concerned only pure exchange) were permitted to trade endowments out-of-equilibrium, but no consumption could take place until equilibrium was reached. Indeed, the pre- and post-equilibrium situations were unnaturally separated, for equilibrium involved an exhaustion of trading opportunities with previously planned consumption then allowed but with trade already over. This was perhaps an inevitable development, given the dominance of the Arrow–Debreu model of general equilibrium in which markets for all present and future goods clear at the beginning of time, but can be considered only as a first step in the analysis of the disequilibrium behavior of actual economies.

As already observed, the price-adjustment equation (3.1) was retained in trading processes. The task then was to specify the adjustment equations describing changes in endowments. Here, there quickly developed one restriction common to all models (in one form or another). This was the assumption that trade at constant prices cannot increase an agent's wealth, since goods of equal value must be exchanged. I shall refer to this as the "No Swindling" assumption.

That progress might be made by considering trading processes becomes apparent when one realizes that the No Swindling assumption alone implies that any Lyapounov function that works in tâtonnement also works for trading processes in a pure exchange. Essentially this is because, with prices constant, trade in endowments cannot change any household's ordinary demand for any commodity, since wealth will be unaffected. While such trade can certainly change a particular household's *excess* demand for the commodity traded by changing its actual stock, such effects must cancel out in pure exchange when summing over households. Hence trade in endowments does not change aggregate excess demands, and those demands only move with prices. It follows that if such movement is consistent with a Lyapounov function when only prices move, then it is still consistent when trade in endowments is permitted.

This is an interesting result, incorporating both some consideration about out-of-equilibrium behavior and the properties of the underlying theory of the consumer. Surprisingly, it shows that stability proofs will generally be no harder for trading processes (in pure exchange) than for tâtonnement. Unfortunately, this does not get us very far, since we know that such proofs are usually not available for tâtonnement. Further specification of trading processes beyond the No Swindling assumption is required if real progress is to be made.

Such specification took the form of two alternative assumptions about the way trade takes place. The first of these, the "Edgeworth process," was introduced by H. Uzawa (1962) (see also Hahn, 1961b); the second, the "Hahn process" (named by Negishi, 1962), made its first published appearance in a paper by Hahn and Negishi (1962). Each of the two processes involves what turns out to be a deceptively simple and appealing assumption about out-of-equilibrium trade.

The basic assumption of the Edgeworth process is that trade takes place if and only if there exists a set of agents whose members can all increase their utilities by trading among themselves at the then ruling prices. With some complications stemming from the possibility that initial prices may not permit any such trade, it is easy to see that at least quasi–stability must follow. This is because, for each agent, the utility that would be achieved were trade to stop and the endowment then held to be consumed must be non-decreasing and strictly increasing if that agent engages in trade. Hence the sum (or any other monotonic function) of such utilities must be non-decreasing and strictly increasing out-of-equilibrium. The negative of the sum can then be used as a Lyapounov function.

This is very neat, but problems emerge when one begins to think hard about the basic assumption involved. In the first place, it is easy to construct examples in which the only Pareto-improving trades that are possible involve large numbers of agents. Indeed, the only upper bound on such constructions (other than the number of agents) is the number of commodities itself. Since we wish to deal with models in which all present and future goods are involved, that upper bound cannot be an effective one. Hence the assumption that trade must take place if such a Pareto-improving possibility exists places a massive requirement on the information flow among agents.[8]

A somewhat deeper problem lies in the other part of the Edgeworth process assumption. Since trade is voluntary, it seems very natural to assume that trade only takes place when the agents engaging in it are all made better off. Once one considers the possibility of moving from trading processes in the direction of what I have referred to above as the "key question," however, the usefulness of this assumption in the form employed in the Edgeworth process becomes very doubtful.

The "key question" is that of whether the economy is driven to equilibrium by the behavior of arbitraging agents taking advantage of the opportunities thrown up by disequilibrium. But speculating agents can certainly engage in trade not because they believe that their utility will be directly increased by each trade but because of the sequence of trades they expect to complete. An agent who trades apples for bananas in the hope that he or she can then make an advantageous trade of bananas for carrots may not care for bananas at all. More realistically, agents sell goods for money, not because they expect happily to consume the money they receive but because they expect to use the money to buy something else. The basic assumption of the Edgeworth process, however, is that every individual transaction is utility increasing – that agents would gain from each leg of a transaction even if trade were to stop so that later legs could not be completed. Whether the fact that individuals engage in trade because they *expect* to gain can be used to extend the Edgeworth process to cover multi-part transactions is not known and seems doubtful.

One cannot avoid this problem if one wishes to examine the serious out-of-equilibrium behavior of agents who have non-naive expectations. The fact that the economy is not in equilibrium means that some expected trades may not materialize. In turn this means that agents who expected to gain from such trades will be disappointed. As a result, they may very well regret having taken past actions – actions they would not have taken had they realized what was to occur.

This phenomenon is not restricted to speculative actions. If one considers the extension of the analysis of trading processes to permit out-of-equilibrium production and consumption, one encounters a similar difficulty with the extension of the Edgeworth process. Both consumption and production involve technically irreversible acts – the consumption of goods or the transformation of inputs into outputs. If those acts are taken on mistaken expectations about later occurrences – either later prices or the ability to complete later transactions – then they will sometimes be regretted. It is hard to accommodate in a model whose Lyapounov function depends on agents always having non-decreasing utilities.

3.4 THE HAHN PROCESS

The second of the two important trading processes, the Hahn process, places a much less severe informational requirement on trades than does the Edgeworth process. In the Hahn process it is supposed that goods are traded in an organized way on "markets." (How such markets get organized is a question for a different level of analysis.) It is assumed that prospective buyers and sellers of a given good can find each other and trade if they desire to do so – indeed, in some versions (Fisher, 1972), this is taken to define what is to be meant by a "market."

Naturally, out-of-equilibrium, it can and often will happen that prospective buyers and sellers of a given good cannot all complete their planned transactions in that good. There may thus be unsatisfied sellers or unsatisfied buyers. The principal assumption of the Hahn process is that markets are "orderly," in the sense that, *after trade*, there are not both unsatisfied buyers and unsatisfied sellers of the same commodity. Only on one side of a given market are agents unable to complete their planned transactions.

This assumption can easily be seen to lead in the direction of a stability proof. Trade is supposed to take place instantaneously or outside of time relative to the rest of the process, and we look only at post-trade situations. Since markets are orderly, after trade, any agent with unsatisfied excess demand for apples, say, finds that there is aggregate excess demand for apples. Since equation (3.1) is retained as the price adjustment equation, the price of apples must be rising. Similarly, any agent with unsatisfied excess supply for bananas finds that there is aggregate excess supply for bananas. Then the price of bananas must be falling, unless that price is already zero. Since anything an agent wants to buy and cannot buy is becoming more expensive, and any non-free goods that an agent wants to sell and cannot sell is becoming cheaper, any agent with either unsatisfied excess demand or unsatisfied excess supply of non-free goods is becoming worse off. In slightly more formal terms, the agent's *target* utility – defined as the utility that the agent would get if he or she completed all planned transactions – is non-increasing and strictly decreasing if the agent's plans are frustrated.[9] It follows that the sum of such utilities over agents (or any monotonic function of the utilities of individual agents) will serve as a Lyapounov function, decreasing except in equilibrium when all agents can complete their planned transactions.

This shows the quasi-stability of the Hahn process. If one either assumes or proves boundedness of the prices, it is possible to show global stability, since expenditure minimization and the strict quasi-concavity of indifference curves imply that all limit points must be the same.

It is important to understand the difference between the Lyapounov functions of the Edgeworth and Hahn processes. In the Edgeworth process, the utilities that increase out-of-equilibrium are the actual utilities that agents would obtain if trade ceased and they had to consume their endowments. In the Hahn process, the utilities that decrease out-of-equilibrium are the target utilities that agents expect to get by completing their transactions at current prices. In effect, out-of-equilibrium, those expectations are not compatible; agents jointly expect more than can be delivered. As the Hahn process goes on, agents revise their expectations downward until they do become mutually compatible and equilibrium is reached.

Of course, since the two processes are quite different, it will sometimes happen in the Hahn process that trade leads to a decrease in the utility that an agent would get if that were his or her last trade. This is not a defect, however. Indeed, as can be seen from our earlier discussion of the Edgeworth process, such a property is desirable, since we want to focus on ultimate plans, not myopic desires as the reason for trade.

Moreover, continuing to look ahead toward the "key question" and more realistic models, the Hahn process has another desirable feature that the Edgeworth process lacks. Since the Lyapounov function of the Hahn process involves declining target utilities, it should be fairly easy to accommodate the decline in utility that occurs when an irreversible consumption or production action is taken and later regretted. This turns out to be the case (Fisher, 1976a, 1977).

Before we can properly get to such matters, however, we must deal with an underlying problem. The basic assumption of the Hahn process, that markets are "orderly" in the sense described, cannot be reasonably maintained without deeper consideration. The problem at issue can be seen by considering the following example.

Suppose that there are at least three commodities, apples, bananas, and croissants. Suppose that, at non-zero current prices, before trade, apples and bananas are in excess supply and croissants in excess demand. Suppose further that some agent, A, owns only apples and wishes to trade for bananas. Suppose that another agent, B, wishes to sell bananas and buy croissants, but does not wish to sell bananas for apples. Then even though A and B can meet each other, no trade between them will take place at current prices. This means that, post-trade, there can perfectly well be agents with an unsatisfied excess demand for apples and also agents with an unsatisfied excess supply of apples. The apple market in this example is not "orderly," and such situations cannot be ruled out merely by supposing that agents can find each other readily.

This problem appears first to have been recognized in the modern literature by R. Clower (1965), who pointed out (in a different context) the need to sell before one can purchase. But a homely example comes readily to hand.[10] A familiar English nursery rhyme states:

> Simple Simon met a pieman going to the fair.
> Said Simple Simon to the pieman, "Let me taste your ware."
> Said the pieman to Simple Simon, "Show me first your penny."
> Said Simple Simon to the pieman, "Indeed, I haven't any."

This is a clear example of a Hahn–process economy in crisis. Markets are sufficiently well organized that willing buyers and willing sellers can meet. Indeed, in the rhyme, the prospective buyer and seller of pies meet on their way to the marketplace (the "fair"). Nevertheless, no trade takes place because the buyer has nothing to offer the seller that the seller is willing to accept.

The case of Simple Simon, however, points up one possible way to think about this problem. It does so by introducing an element so far conspicuously lacking from stability analysis. The pieman does not ask Simple Simon for apples or bananas or croissants; instead he asks for money, and the time has plainly come to consider the introduction of money into stability analysis.

Indeed, that introduction cannot be long delayed in any case. Aside from the Simple Simon problem under discussion and the use of money in the intermediate stages of arbitrage transactions, one cannot get beyond pure exchange without introducing it. This is for a reason that, interestingly, does not apply in equilibrium.

Firms, unlike households, are usually assumed to maximize profits. Suppose that some firm produces a large excess supply of some commodity, say toothpaste. Out-of-equilibrium, even with toothpaste in aggregate excess supply, the price of toothpaste can be positive. If that price is high enough, and if there is no standard medium of exchange in which profits are measured, the toothpaste producing firm may regard itself as making a positive profit, *even though it sells no toothpaste*. This means that the firm's inventory of toothpaste need not be offered for sale, so that the excess supply of toothpaste will have no effect on the price.[11] Only by insisting that profits be measured in a common medium of exchange (and a common unit of account) can we ensure that firms producing commodities other than the exchange medium have an incentive to sell those commodities. This makes money indispensable.

The introduction of money into Hahn–process models was begun by Arrow and Hahn (1971). They assumed that one of the commodities, "money," plays a special role in that all transactions must involve it. They then assumed that agents first formulate "target excess demands" – excess demands constructed by maximizing utility functions subject to budget constraints in the usual way – but that these must be distinguished from "active excess demands," constructed as follows. If an agent has a negative target excess demand for a given commodity, then that agent wishes to sell it. Since commodities can be offered for sale whether or not the supplier has any money, active excess demand in such a case is assumed to equal target excess demand. On the other hand, positive target excess demands cannot generate offers to buy unless they are backed up by money, so Arrow and Hahn assumed that the agent allocates his or her available money stock over the goods for which he or she has a positive excess demand. This leads to the assumption that any good for which the agent has a positive target excess demand is also one for which that agent has a positive active excess demand, with the active excess demand never exceeding the target one (agents do not offer to buy more than they really want and always make a positive offer for anything they want). It is active, rather than target demands that are assumed to obey the orderly markets assumption and unsatisfied aggregate excess active demand that is assumed to affect prices according to equation (3.1).

With this in hand, Arrow and Hahn were able to isolate the Simple Simon problem by assuming that no agent ever runs out of money. If this assumption holds, then it is easy to see that the Hahn–process stability proof goes through in much the same way as before. Prices change in the direction indicated by unsatisfied aggregate active demands; unsatisfied individual active demands have the same signs (post-trade) as the corresponding aggregate demands; finally, unsatisfied individual target demands have the same signs as the corresponding unsatisfied individual active demands. Hence target utilities are still decreasing out-of-equilibrium.

As already indicated, the introduction of money permits the introduction of firms, and this was done in Fisher (1974).[12] Firms are assumed to be subject to the orderly markets assumption, but to maximize profits which they ultimately distribute to their shareholders. Shareholders expect to spend those profits. Because of the orderly markets assumption, any firm that cannot complete its planned transactions must revise its forecast of profits downward. Households then find their target utilities decreasing both because of the direct influence of the orderly market phenomenon on their own transactions and because of the declining fortunes of the firms they own. The sum of household utilities can thus again be used as a Lyapounov function. While boundedness is now a more complex matter, a global stability proof follows nicely from it, employing both profit maximization on the part of firms and expenditure minimization on the part of households to show that all limit points are the same. Money and the target-active excess demand distinction are handled as before.

This is a pretty story, and one that can even be extended to permit out-of-equilibrium production and consumption, as indicated above (Fisher, 1976a, 1977). But the difficulties are all too apparent.

The role of money in this model is very much an afterthought. Agents plan their target excess demands as though they were in equilibrium. In so doing, they take no account of the cash constraint imposed by the institutional structure. Instead, they allocate their money stocks to their positive excess demands as though any cash difficulty will necessarily be only temporary, so that ultimately target transactions will be completed.

That naivete is also reflected in the assumption that agents make a positive offer for every good for which they have a positive target excess demand. So long as we remain in an Arrow–Debreu world where all markets open and close at the dawn of time, this may not matter. Once we begin to be serious about disequilibrium, however, and to permit consumption and production to take place before equilibrium is reached, it matters a lot. It is not reasonable to suppose that agents facing a liquidity crisis always allocate funds to all demanded commodities. Some of those commodities may not be needed for years, while others may be required for near-term consumption.

And of course the afterthought method of allocating cash is related to the most obvious difficulty. The Simple Simon problem has not been solved, but merely well defined. It is still necessary to assume that agents never run out of money. This may be hard to swallow in any case; it is particularly unpalatable when agents make their money-allocation plans as though their planned sales would always materialize.

In the same connection, the time has come to remember how awkward the price-adjustment assumptions are in all these models. We are not dealing with a case in which agents, faced with impending cash shortages when planned sales do not occur, can lower their prices. Rather, we are still in a world in which price is set anonymously, and sellers who might benefit from lower prices are just out of luck.[13]

In other respects as well the model is less than satisfactory. Money is assumed to be a commodity entering the utility function. This is required in order to ensure that agents wish to hold money in equilibrium, avoiding the "Patinkin problem" (Patinkin, 1949, 1950, 1965). But that problem arises because equilibrium in this Arrow–Debreu world means a cessation of trading opportunities. If equilibrium had the more natural property of involving the carrying out of previously planned transactions at previously foreseen prices, then the transactions motive for holding money would not disappear. Yet such a version of equilibrium requires agents to care about the timing of their transactions.

In several ways, then, the defects of the more sophisticated Hahn process models point the way toward possible progress. In one way or another, those defects are all related to the fact that the agents in such models (as in all the models considered so far) pay very little attention to the fact that the economy is in disequilibrium. They go on believing that prices will not change and that transactions will be completed. Disequilibrium behavior and phenomena are modeled at best as an afterthought. Plainly, the difficulties encountered cannot be solved in such a context. A full disequilibrium model is required and must be built if we are to address the "key question" of whether arbitraging actions drive the economy to equilibrium.

3.5 TOWARDS A FULL DISEQUILIBRIUM MODEL

So far as I know, the only attempt to examine the stability question in the context of a full disequilibrium model in which consumption and production take place out-of-equilibrium and agents consciously act on arbitrage opportunities is that of my book (Fisher, 1983; see also Stahl and Fisher, 1988). As will be seen that any attempt to answer the "key question" cannot be considered truly successful, but there is, I think, much to be learned from it and from its inadequacies.

I begin by considering a problem of only moderate importance which nevertheless exemplifies the need for dropping equilibrium habits of thought when thinking about disequilibrium problems. This problem arises when one allows consumption and production to take place out-of-equilibrium.

It is common, correct, and necessary to regard commodities consumed or produced at different dates as different commodities even if they are physically indistinguishable. In the Arrow–Debreu world where nothing ever happens until equilibrium is reached, this does not matter; a commodity with a different date is just a different commodity traded on a different market and with its own price. If consumption or production takes place out-of-equilibrium, however, then commodity dates take on a new significance. Only currently dated commodities can be consumed or produced; future commodities can only be traded. Hence, allowing disequilibrium consumption or production means allowing some commodity dates to be passed before equilibrium is reached. Since there can only be trading in current or future commodities, but no trading in "pasts," this means that trading in some commodities becomes impossible as the adjustment process unfolds.

To see why this creates a difficulty, consider the following example. For simplicity, assume that commodities are dated by year. At midnight on 31 December 2007, trade in 2007 toothpaste ceases. Since we are out-of-equilibrium, this can mean that there are agents who cannot buy as much 2007 toothpaste as they had planned. Since they must now make do with a different amount than planned, this can cause a discontinuity in their behavior.

An obvious solution to this difficulty presents itself, however. Assume that toothpaste is a durable good (a somewhat different analysis applies to pure perishable commodities). Then, at midnight on 31 December 2007, 2007 and 2008 toothpastes are perfect substitutes. Our agent may not be able to buy the 2007 toothpaste he or she planned, but this will not create any discontinuity, since 2008 toothpaste can be purchased instead.

The problem cannot be made to go away so easily, however. Since 2007 toothpaste is a different commodity from 2008 toothpaste, the two commodities have different prices. If those prices do not coincide at midnight on 31 December 2007, then discontinuity is still a real possibility.

It is very tempting to reply that the two prices *must* coincide at that time, because the two commodities are then perfect substitutes. That temptation must be resisted. The proposition that the prices of perfect substitutes must coincide is an *equilibrium* proposition. It rests on the argument that arbitrage will erase any difference between the prices. But that the working of the arbitrage is what a full stability model is supposed to be about. We cannot, in a disequilibrium framework, simply assume that the arbitrage will be successful by the time the crucial hour arrives.

There is an important sense, however, in which this difficulty is more apparent than real. That difficulty stems from the treatment of the markets for 2007 and 2008 toothpaste as wholly distinct, with prices set anonymously according to some rule such as equation (3.1). In fact, this is unlikely to be the case. Instead, the same firms that sell 2007 toothpaste are also likely to sell 2008 toothpaste and to quote prices for both. Similarly, dealers specializing in wheat futures are unlikely to deal in futures for only one date. But if the same seller (or, more generally, the same dealer) quotes prices for both 2007 and 2008 commodities, then he or she will have an active interest in making sure that those prices come together at midnight on 31 December 2007, since otherwise arbitrage at the dealer's expense will be possible.

There are three lessons to be learned from all this. First, one cannot think about disequilibrium problems using only equilibrium habits of thought. Certain issues that seem not to matter in equilibrium can matter quite a lot out of it. Second, the farther one gets into serious disequilibrium analysis, the less satisfactory is the assumption of anonymous price adjustment. Third, disequilibrium considerations have something to do with the institutional structure of transactions and the way in which markets are organized – subjects on which no work has been attempted in the disequilibrium context, but which are crucial if we are ever to gain a satisfactory understanding of the formation of economic magnitudes.[14]

Such subjects, however, are truly difficult, for they involve analysis of what happens when agents interact and their plans do not mesh. It is far easier to consider how those plans get formulated, and the analysis of Fisher (1983) does this at some length, producing a number of results on the way in which agents plan to take advantage of the arbitrage opportunities they see thrown up by changing prices. In the course of so doing, the positive cash assumption of Arrow and Hahn becomes far less arbitrary, since agents now optimize their planned transactions, paying attention to their money stock. Interestingly, it emerges that one reason for trading in the shares of firms is because anticipated dividend streams permit liquidity transfers from one period to another, and, out-of-equilibrium, such transfers may be needed.

Such arbitraging actions come principally from allowing agents to expect prices to change. But allowing agents to be conscious of disequilibrium means more than this; it also means allowing them to realize that their transactions may be limited in extent. So long as we retain anonymous price adjustment, we must suppose that such constraints are regarded as absolute. This has led to a literature on the analysis of equilibria under such circumstances – so called "fixed price equilibria."[15]

More interesting for the study of true disequilibrium is what happens when we allow agents to believe that they can alter the constraints they face by making price offers. Consider, for example, the case of a seller who believes that the amount that can be sold at a given price is limited. If the seller also believes that a lower price will bring more sales, then the constraint expresses expected sales as a function of price and becomes an ordinary, downward-sloping demand curve. In this case, the seller will only refrain from offering a lower price for the usual reason in the analysis of monopoly: a lower price must be given on all units to be sold, and marginal revenue will fall short of marginal cost.

This leads to a number of interesting problems. First, there is the distinct possibility in such cases that equilibrium will be non-Walrasian. Specifically, the economy can be stuck in a position where agents believe they face binding transaction constraints and do not attempt to get round them with price offers because they believe that it would be unprofitable to do so. In macroeconomics, this can be regarded as a version of the original Keynesian question as to underemployment equilibrium. Hahn (1978) shows that it can happen with the beliefs of the agents rational in some sense.

Second, the crucial question of whether an equilibrium is Walrasian or non-Walrasian becomes the question of whether perceived monopoly power vanishes in equilibrium. This is not a question that can be answered by only analyzing equilibria; it pretty clearly depends on the experiences agents encounter on the way to equilibrium (assuming that some equilibrium is reached). In this regard, it is interesting that, as Fisher (1983) shows, there is a relation between the nature of the equilibrium and the question of whether or not liquidity constraints are actually binding therein. Only where perceptions of monopoly power remain (and change over time in certain ways after equilibrium is reached) will the equilibrium be non-Walrasian and cash remain a problem.[16]

Whether or not a given equilibrium is Walrasian, however, some clarification of the role of money is achieved. We saw above that the equilibria of trading processes (or of tâtonnement models, for that matter) were merely an exhaustion of trading opportunities. In a full model, such as the one under discussion, transactions do not cease in equilibrium; rather, equilibrium involves the carrying out of previously made optimal plans involving planned transactions at correctly foreseen prices. This means that the transactions demand for money does not disappear in equilibrium. While money in this model is an interest-bearing asset (so that there is no explanation for equilibrium holding of non-interest-bearing money), this explains why agents hold that asset rather than others bearing the same rate of interest in equilibrium, even though money itself enters neither utility nor production functions.

3.6 DYNAMICS AND STABILITY IN A FULL MODEL

All this is very interesting, but it says little about what happens when agents interact out-of-equilibrium and plans are frustrated. What can be said about such interactions and about the "key question" of whether they lead to stability? Alas! it is here, as already indicated, that the analysis under discussion produces less than satisfactory answers.

We have already seen that one cannot retain the old anonymous price adjustment equation (3.1) left over from tâtonnement days. Individual price adjustment is essential. But how does such price adjustment take place? The answer suggested above is that prices are set optimally depending on perceived monopoly (or monopsony) power. That is all well and good, but it does not take us very far. How do such perceptions get formed and change? How do institutions arise determining which agents make price offers and which choose among offers? Out-of-equilibrium, where offers and acceptances will not all match, how does partial matching take place?

On these crucial questions, Fisher (1983) offers relatively little guidance. Rather, price movements, like all other movements in the model, are assumed to be restricted by a vague but strong restriction called "No Favorable Surprise" (NFS). To understand that restriction, and the motivation for it, requires us to step back for a moment and consider the purpose of stability analysis.

Real economies are subject to a succession of exogenous shocks. The discovery of new products, new processes, new sources of raw materials, new demands, and new ways of organizing production are, as emphasized by J. Schumpeter (1911), the driving forces of economic development and growth. It is unreasonable to suppose that such Schumpeterian shocks are all foreseen and can be incorporated as part of equilibrium. Rather, equilibrium analysis, if it is useful at all, is so because the economy rapidly adjusts to such shocks, approaching a new equilibrium long before the next shock occurs.

The role of stability analysis, then, is to analyze the question of whether such adjustment in fact takes place. This means analyzing the part of the Schumpeterian model occurring after the initial innovation, when imitators enter and act on the profit opportunities they see. What I have called the "key question" can be interpreted as the question of whether such action does in fact lead the system to absorb a given Schumpeterian shock. Evidently, then, the first task of stability analysis is to answer this question on the assumption that further Schumpeterian shocks do not occur.

There is more to it than this, however. In a full model, where agents form their own expectations, there is the possibility that agents will perceive Schumpeterian opportunities that do not exist. If such agents have the resources with which to back their perceptions, equilibrium will at least be postponed. The entrepreneur who believes that he or she can profitably build a better mousetrap and who has the money to invest will affect the economy even if the world does not in fact beat a path to the door. Stability implies that such occasions disappear, at least asymptotically, and no stability proof in a complete model can succeed without either proving or assuming that this happens.

The basic first step in an adequate analysis of stability as a full attack on the "key question," therefore, is the weak one of showing that arbitrage leads to equilibrium if no new unforeseen opportunities arise. This is the assumption of "No Favorable Surprise." More precisely, NFS assumes that agents are never surprised by the unforeseen appearance of new, favorable opportunities causing them to deviate from previously formed optimal plans if those plans are still feasible. In other words, any plan now optimal is assumed to have been feasible a short time ago. Useful new opportunities (technological change, for example) must be foreseen at least a short time before agents actually change plans so as to act on them.

It is not hard to see that, as in the Hahn process which is a special case, NFS implies that agents' target utilities are declining out-of-equilibrium. While agents can be doing quite well in a foreseen way (including taking advantage of foreseen technological progress), any abrupt departures from what was expected must mean declines in utility (if they matter at all). With this in hand, a global stability proof can be made to follow, although the details are technically complex and require a number of non-primitive assumptions on the dynamics involved.

The problem with this is that NFS itself is not a primitive assumption, either. It is all very well to argue as above that one must exclude further exogenous Schumpeterian shocks in examining stability. It is far stronger to rule out the favorable opportunities that may suddenly arise in the course of adjustment to an existing shock.

Evidently, this difficulty arises precisely because we have no good model of how agents interact in reacting to disequilibrium. This causes us to be unable to describe exactly how endogenous surprises do or do not arise and makes NFS a somewhat unsatisfactory assumption.[17]

Like earlier models, then, the analysis of Fisher (1983) is only partially successful. It is strongest when dealing with the plans of individual agents or with equilibrium. It is weak when considering how those plans interact when they cannot all be fulfilled and how agents then change their expectations. While it succeeds in doing away with anonymous price adjustment, it tells us very little about how prices are in fact set. We still have much to learn about the theory of value.

To acquire such learning, serious modeling of disequilibrium is required. If we are ever to understand how resources are allocated, how consumption and production are organized, how prices come to be what they are and the role that they play, we must examine disequilibrium behavior. Among other things, this means examining the ways in which agents change their expectations when their plans are frustrated. Obviously, such questions cannot be begged by using equilibrium tools (In particular, the assumption of rational expectations can tell us nothing at all about how disequilibrium works.). We cannot simply examine positions in which economic magnitudes happen to be such that there is no tendency to change. To understand the workings of the "Invisible Hand" it is not enough to understand what the world looks like when the "Invisible Hand" has nothing to do.

The present state of general equilibrium theory must therefore be regarded as unsatisfactory or incomplete when it comes to the provision of a positive theory of value. That, in itself, does not vitiate the conclusions of general equilibrium theory. Still less does it validate those of alternative theories. But it leaves us with a lot to do.

NOTES

1 If price is zero and excess demand negative, the price is assumed to remain zero. I generally ignore this complication in what follows.

2 The auctioneer may have been invented by J. Schumpeter in lectures at Harvard and was probably introduced into the literature by Samuelson. Despite the fact that the construct is often referred to as the "Walrasian auctioneer," it does not appear in the work of L. Walras (who did, however, suppose that prices adjust in the direction indicated by excess demands). Interestingly, F.Y. Edgeworth wrote (1881: 30): "You might suppose each dealer to write down his *demand*, how much of an article he would take at each price without attempting to conceal his requirements; and these data having been furnished to a sort of market-machine, the *price* to be passionlessly evaluated." I am indebted to P. Newman for this reference.

3 Years later, D. McFadden (1968), writing in the Hicks *Festschrift*, showed that the Hicks conditions imply global stability of equation (3.1) on very strong assumptions about relative speeds of adjustment in different markets.

4 Lyapounov (1907). Lyapounov's "First Method" for proving stability is the explicit solution of the differential equations involved, an alternative never available at the level of generality of the stability literature.

5 The limit point, however, generally depends on the initial conditions. For a more extended discussion as well as exact statements and proofs, see F.M. Fisher (1983). Note that G. Debreu

(1970) has shown that local isolation of equilibria is true almost everywhere in the appropriate space of economies given certain differentiability assumptions.

6 Sonnenschein (1972, 1973), Debreu (1974) and R. Mantel (1976) show that the basic assumptions of economic theory do not restrict the excess demand functions except by continuity, homogeneity of degree zero, and Walras' Law. Since Scarf's example shows that such restrictions do not imply stability of equation (3.1) and since properties such as the signs of the real parts of the eigenvalues of the Jacobian matrix of equation (3.1) are continuous, instability must hold on an open set.

7 The first paper to suggest (by example) that there might be considerable pay-off in a closer look at the adjustment process appears to have been that of Hahn (1961a), which considered specialization of equation (3.1) instead of restrictions on excess demands as a way of making progress in tâtonnement. (See also Kagawa and Kuga, 1980.)

8 Let there be n agents and $c \geq n$ commodities. With the exception of agent n, let agent hold only commodity i and desire only commodity $i + 1$. Let agent n hold only commodity n and desire only commodity 1. Then the only, Pareto-improving trade involves all n agents. The problem is quite similar to that involved in coalition formation in the theory of the core, and D. Schmeidler has shown (privately) that, if $c \leq n$, the existence of some Pareto-improving trade implies the existence of such a trade for no more than c agents. P. Madden (1978) proves that the existence of a Pareto-improving trade implies the existence of a Pareto-improving bilateral trade, provided that every agent always has a positive amount of every commodity, but such a condition cannot be reasonably expected to hold. But, when the positive-amount assumption is relaxed, the number of agents required can be much larger and the construction of Edgeworth-Process trades far more difficult. See Fisher (1989) and Tsai (1993).

9 With the exception of disposing of free goods. It is tiresome to have to constantly repeat this, and I shall not always do so hereafter.

10 I apologize for using again the same light-hearted example that I have already employed on two previous occasions (Fisher, 1976b: 14, 1983: 33). It is so apt as to be irresistible.

11 The device of assuming that the firm distributes toothpaste dividends to its stockholders hardly seems satisfactory.

12 A parallel introduction of firms into the Edgeworth process was accomplished by F.M.C.B. Saldanha (1982).

13 Some progress can be made here. Fisher (1972) provides a model in which goods are identified by the dealers who sell them. In such a model, the orderly markets assumption is essentially trivial, since there is only one agent on the supply side of any "market." Since prices are set by suppliers (with buyers searching for low prices), they can be adjusted when planned sales do not occur and cash is low. But there are plenty of other difficulties with such a model. See M. Rothschild (1973).

14 For work on transaction arrangements in general equilibrium, see D. Foley (1970) and Hahn (1971).

15 While such circumstances are sometimes referred to as "disequilibrium," they are not properly so-called, since what is involved is non-Walrasian equilibrium, rather than dynamic adjustment. See A. Drazen (1980) for a survey of the literature.

16 Incidentally, this shows a connection between the two great revolutions in economic theory of the early 1930s – the Keynesian revolution in macro-theory and the introduction of imperfect and monopolistic competition in micro-theory.

17 There is at least one other problem with NFS. The agents in the model being described have point expectations and no subjective uncertainty. (They are all economists – often wrong but never uncertain.) It is an open question as to whether there exists a version of NFS that is both palatable and strong enough to produce a similar stability result when subjective uncertainty is permitted.

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4 Statistical equilibrium in economics

Method, interpretation, and an example*

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Statistical equilibrium is a short-run, temporary equilibrium model of market exchange which replaces the Walrasian picture of the market in equilibrium as a budget hyperplane defined by equilibrium relative prices with a scalar field of transaction probabilities. Statistical equilibrium synthesizes the classical notion of competition as a market with a large number of traders with the idea of liquidity limited by traders' need to find actual counterpart transactors. From an economic point of view, statistical equilibrium is the feasible Pareto-improving multilateral transaction that can be achieved in the largest number of distinct ways. On the assumption that all Pareto-improving transactions are equally probable, the statistical equilibrium can be characterized in terms of entropy maximization as a Gibbs distribution in which the transaction probability of any transaction is proportional to its value at equilibrium absolute entropy prices. The statistical equilibrium approximates, but does not in general achieve, Pareto-efficiency. A possible interpretation of the statistical equilibrium is as the statistical outcome of a sequence of identical repeated markets, in which the Gibbs probabilities are interpreted as flows of transactions per unit time. The phenomenon of market arbitrage is examined in the context of a simple model of an asset market, in which the statistical fluctuations of outcomes in each period represent shifts in the arbitrageur's capital position.

4.1 INTRODUCTION

Markets are decentralized, disorderly and spontaneously organized social interactions from which well-defined average prices emerge. But our everyday experience of markets is that actual transaction prices vary over any time scale, so that the realization of average price in a large number of transactions is at most a tendency. In seeking an abstract representation of markets, both Walras and Marshall resorted to the mathematical simplification of representing the statistical distribution of transaction prices by its mean, presumably with the idea that the equilibrium statistical distribution of transaction prices is highly concentrated, so that the mean would closely approximate the actual distribution.

* These notes were prepared for the XII Workshop on "General Equilibrium: Problems, Prospects and Alternatives" 4–11 July, 1999, at the Certosa di Pontignano, Siena, Italy. The calculations and graphics were created in *Mathematica* 3.0. The example analyzed was developed in conversations with Perry Mehrling.
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This simplification has drawbacks which lie at the heart of many unresolved theoretical problems having to do with the conceptualization of market disequilibrium and the process by which a market reaches equilibrium. An adequate market theory should account for the emergence of equilibrium from disequilibrium. In real markets this process is the formation of the equilibrium price distribution, so that both equilibrium and disequilibrium states of the market are most conveniently described statistically. But the method of replacing the statistical distribution of prices by its mean leads to the self-contradictory concept of a "disequilibrium price", a uniform price which has somehow emerged from the market, but is not an equilibrium. Attempts by market participants to trade at a single disequilibrium price cannot by definition be consistent. A more straightforward notion would be a "disequilibrium transaction distribution" describing the actual transactions consummated in the market even out-of-equilibrium.

There are two apparent paths around this dilemma that retain the notion of a single disequilibrium price, neither of which adequately resolves the contradiction. One path allows for actual transactions to take place at disequilibrium prices, with the idea that the failure of some transactors to achieve their desired trades will provide the forces required to adjust the price to its equilibrium. The difficulties encountered in carrying out this program arise from the fact that there are many different ways to model what transactions actually occur in disequilibrium, and the ultimate equilibrium achieved depends critically on which particular path of transactions is assumed to occur. Thus allowing trade at disequilibrium prices defeats the original aim of the theory of markets, which was to show how equilibrium is uniquely and stably determined by transactors' willingness to trade. The second path, proposed by Walras, rules out actual trade at disequilibrium prices. But this in turn leads to serious unresolved questions, since it is precisely through transactions that markets find their equilibrium. Walras attempted to resolve this problem by introducing the fictional "auctioneer", who has some of the attributes of real world market makers: the auctioneer "cries out" prices, offering to buy and sell any quantities at those prices, but is conveniently relieved of the obligation to make good on his offers until equilibrium prices are actually found at which the auctioneer's net trade is zero. But real world market makers have to make good on their contracts whether they have managed to find "equilibrium" or not; it is presumably through their disappointments that the market gropes its way to equilibrium.

The theory of statistical equilibrium proposes to replace the representation of the market as a single system of relative prices at which agents believe they can make any transactions with a representation of the market as a scalar probability field over possible transactions. The individual trader comes to the market with an "offer set" of acceptable and feasible transactions, and the actual outcome is the realization of the probability field over this offer set. Rather than submitting a definite transaction request, as in the Walrasian image, the trader "dips" the offer set into the transaction flux represented by the market, and sees what comes up. Since the possibility of no trade is always in the offer set, there is always a chance that the trader will come up with no transaction, remaining unemployed, for example, if a worker, or with excess capacity or unsold inventory if a firm.

The conceptualization of the market as a probability field over transactions implies statistical fluctuations in the experience of identical traders entering the market at any time and in the experience of any individual trader over time. Thus the market can be in statistical equilibrium without requiring that each trader be individually in equilibrium, as Walras and Marshall presume. The magnitude of these statistical fluctuations is a characteristic of the market and its equilibrium state, a dimension of economic experience that is perforce neglected by any theory that replaces the price distribution with its mean.

Statistical equilibrium treats the phenomena of competition among many traders in a market and the liquidity or depth of the market as aspects of the market equilibrium itself. Competition among traders gives rise to the emergence of an average price and a regular equilibrium probability distribution of transaction prices, but that distribution implies that some trades are "harder" or less likely to occur than others, reflecting the liquidity of the market and the requirement for each trade to find a counterpart.

The statistical equilibrium probability distribution of transactions in a market is determined by the offer sets of the agents. These offer sets, in the case of durable commodities or investments, must depend on agents' expectations about the future. From this point of view, statistical equilibrium is a form of *temporary equilibrium*, in which agents' expectations are taken as given. In the language of the Classical political economists, statistical equilibrium concerns "market prices", actual transactions prices that fluctuate in time and space, rather than "natural prices" derived from underlying costs of production around which market prices gravitate. Statistical equilibrium thus rules a particular time scale, which may be peculiar to individual markets, depending on the rate at which new information arrives that changes traders' offer sets.

4.2 ECONOMIC FOUNDATIONS

People have understood for a long time that market transactions are driven by the interaction of traders in possession of assets or commodities seeking to improve their situation through exchange. Such traders can conceive of transactions that would improve their situation. As in conventional equilibrium models, we can model the range of things to be traded as a "commodity space", Z, typically a finite (m-dimensional) Euclidean space. A transaction $z = (z_1, \ldots, z_m) \in Z$ is represented by a point in this space, with the interpretation that negative components of z represent commodities given up and positive components commodities acquired in the transaction. In this representation, the origin, 0, represents no trade. We represent the idea of willingness and ability to trade as a finite (for the moment) "transaction set", $A \ni 0$ characteristic of the trader entering the market. The transaction set reflects the trader's endowments, preferences, information, and expectations, and is a primitive of the theory. The transaction set represents a limited degree of control on the part of the trader over the outcome of the market transaction: the trader can certainly avoid some transactions by refusing them, but cannot guarantee the achievement of any particular transaction in the offer set. Since the origin is always a point in the offer set, the trader may not achieve any transaction at all. (The assumption that the transaction set is finite is convenient for the development of the analysis, but can be replaced by the condition that transaction sets are bounded below.)

To make matters definite, suppose that the market consists of n traders denoted by $i = 1, \dots, n$ of r different types, denoted by $k = 1, \dots, r$, where n_k traders are of type k, each with the same offer set A^k. The type of trader i will be represented by the function $k(i)$.

A *market transaction* $\zeta = (z_1, \dots, z_n)$ will specify what transaction each trader achieves. A *feasible market transaction* clears the market in the sense that

$$z_i \in A^{k(i)}, \quad i = 1, \dots, n \tag{4.1}$$

and

$$\sum_{i=1}^{n} z_i = 0. \tag{4.2}$$

Thus for a market transaction to be feasible, each trader must be given a transaction (possibly 0) in her offer set, and the sum of all the transactions must be zero. (With a finite number of agents and finite offer sets, it might not be possible to clear the market exactly except by assigning every trader the transaction 0, so that some degree of approximation must be assumed in equation (4.2). Market clearing can be approached more closely as the number of agents of each type becomes larger.). In writing the condition for market clearing as a sum over all the agents, we implicitly allow for multilateral trade, just as the definition of Walrasian equilibrium does.

The set of feasible market transactions is typically very large, once the number of types, number of traders of each type, and number of points in the offer sets become moderately large. There are many different ways of assigning traders to transactions in their offer sets that clear (or approximately clear) the market. The principle of voluntary market exchange in and of itself is not sufficient to determine the market transaction.

In the economics literature this indeterminacy is addressed by a variety of ad hoc devices. Walras' auctioneer is one such device, as is Edgeworth's recontracting, which leads to the Walrasian equilibrium in the limit as the number of traders of each type becomes large. Traders may be matched randomly in bilateral trades, or trading posts set up to organize trade in particular commodities. The statistical equilibrium method takes a different tack, assuming, in the absence of specific information about the patterns of transactions, that all feasible market transactions are possible, and indeed that all are equally probable (The assumption of equal likelihood is not as restrictive as it might seem at first because of the flexibility of the offer sets. For example, we can represent the preference of agents for some regions of the transaction space by putting a larger number of points in the offer set in those regions.). The transactions dynamic appears in statistical equilibrium as a "black box" that can reach any feasible market transaction.

If ζ is a feasible market transaction, any permutation of the transactions of ζ within types is also a feasible market transaction, since all the traders of a given type have the same offer sets, and the permutation of transactions does not change the total. Furthermore, such a permutation, though it affects the fate of individual traders, leaves the statistical distribution of traders of each type over their offer set unchanged. From a statistical point of view, it is only this distribution that is of economic significance in any case. For any market transaction ζ, whether it is feasible or not, we can define the statistical *type distribution* of traders of any type k over their common offer set, h_ζ^k:

$$h_\zeta^k(z) = \frac{\#\{i|k(i) = k \text{ and } z_i = z\}}{n_k}. \tag{4.3}$$

Thus the type distribution $h_\zeta^k(z)$ is the histogram that counts the fraction of agents of type k assigned the transaction $z \in A^k$ in the market transaction ζ. Since the offer sets are finite, the type distributions h^k are non-negative vectors that sum to unity. Given a type distribution h^k, the mean transaction of traders of type k will be

$$\bar{z}_{h^k}^k = \sum_{z \in A^k} z h^k(z).$$

Two market transactions, ζ and ζ' that differ from each other only through permutation of transactions within types will have the same type distributions. A *market distribution* $h = (h^1, \ldots, h^r)$ is a collection of type distributions, one for each type of trader. A *feasible market distribution* is a market distribution h that satisfies:

$$\sum_k \bar{z}^k = \sum_k \sum_{z \in A^k} z h^k(z) = 0. \tag{4.4}$$

As we have seen, each market distribution can arise from many different market transactions. The *multiplicity* of a market distribution is the number of different market transactions that correspond to it. For any type k, the multiplicity $W(h^k)$ of the type distribution h^k can be calculated from combinatorial principles, remembering that $0! = 1$, to be:

$$W(h^k) = \frac{n_k!}{\Pi_{z \in A^k}(n_k h^k(z))!}. \tag{4.5}$$

Since the permutations of traders of different types can be carried out independently, the multiplicity of the market distribution $h = (h^1, \ldots, h^r)$, $W(h)$ is:

$$W(h) = \Pi_k W(h^k). \tag{4.6}$$

The more "spread-out" the agents are in their offer set, the more multiplicity of a type distribution increases. If, for example, all the agents are assigned to the same point, the multiplicity is one, since permuting the agents among the transactions does not change it at all.

From the statistical point of view, market distributions of higher multiplicity are more likely to occur than market distributions of lower multiplicity. The *statistical market equilibrium* is the market distribution of highest multiplicity. The investigations of statistical physics show that the relative likelihood of the statistical equilibrium compared to any other distribution, even those that are very similar, grows very rapidly as the number of agents of each type becomes moderately large.

4.3 THE MATHEMATICAL FORMALISM OF STATISTICAL EQUILIBRIUM

J. Willard Gibbs discovered the remarkable fact that it is possible to compute the statistical equilibrium distribution in a tractable form. The key to this formalism is the use of Stirling's Approximation

$$\ln(n!) \approx n\ln(n),$$

where ln is the natural logarithm, to estimate $\ln(W)$, the logarithm of the multiplicity. The negative of the logarithm of the multiplicity of a type distribution h^k is the number of traders of type k, n_k multiplied by the *entropy* of the type distribution, $S(h^k)$, where:

$$S(h^k) = -\sum_{z\in A^k} h^k(z)\ln(h^k(z)). \tag{4.7}$$

In calculating entropies, $0\ln(0) = 0$. The logarithm of the multiplicity of the market distribution h, is just the sum of the logarithms of the multiplicities of the type distributions:

$$\ln(W(h)) = \sum_k n_k S(h^k). \tag{4.8}$$

To find the feasible market distribution of maximum multiplicity, we can maximize the entropy over the choice of the h^k, subject to the constraints $h^k(z) \geq 0$, $\sum_{z\in A^k} h^k(z) = 1$ and equation (4.4), which defines the constrained maximization problem:

$$\max_{h^k(z)\geq 0} \sum_k n_k S(h^k) \tag{4.9}$$

subject to

$$\sum_{z\in A^k} h^k(z) = 1, \quad k = 1,\ldots,r$$

$$\sum_k \left(n_k \sum_{z\in A^k} z h^k(z)\right) = 0.$$

Since the entropy is a strictly concave function, and the constraints are linear, this problem always has a unique, interior solution, characterized by shadow prices on the m commodities, *entropy prices*, $\pi = (\pi_1,\ldots,\pi_m)$. In this solution $h^k(z) \propto \exp(-\pi z)$, where πz is the dot product of the vectors π and z.

Thus the likelihood of any transaction for any type is governed by the same entropy prices, which reflects the fact that all the traders are indeed linked in the same market. The likelihood of a transaction is inversely proportional to its entropy value (or cost) at the entropy prices. If entropy prices are positive, transactions which absorb a large amount of commodities are relatively unlikely. The likelihood of more costly transactions declines exponentially with their entropy cost.

While all the types experience the same entropy prices in statistical equilibrium, their actual distribution over transactions depends in a critical way on their offer sets. The normalizing factor for the probabilities that apply to a particular type k is the *partition function*, $Z^k(\pi) = \sum_{z \in A^k} \exp(-\pi z)$. (The partition function is the Laplace transform of the indicator function of the offer set.) Thus the actual statistical equilibrium probabilities for type k are the Gibbs distribution:

$$h^k(z) = \frac{e^{-\pi z}}{Z^k(\pi)}. \tag{4.10}$$

The logarithm of the partition function, $Z(\pi)$, for the whole economy is the weighted average of the logarithms of the partition functions for each type:

$$\ln(Z(\pi)) = \sum_k \frac{n_k}{n} \ln(Z^K(\pi)). \tag{4.11}$$

The dual of the maximum entropy problem, equation (4.9), is to minimize the logarithm of the economy-wide partition function over the entropy prices. The log partition function is a strictly convex function, so that this problem always has a unique interior solution in the entropy prices.

The log partition functions completely summarize the statistical equilibrium, since the negative of their derivatives with respect to the entropy prices give the moments of the type distributions:

$$\bar{z}^k = \frac{-\partial \ln(Z^k(\pi))}{\partial \pi}. \tag{4.12}$$

Similarly, the covariance matrix for the type distribution is the matrix of second order partial derivatives of the negative of the log partition functions with respect to the entropy prices.

The second derivative of the log partition function also represents the Jacobian matrix of the average demand system $\bar{z}^k(\pi)$ with respect to the entropy prices, and is always negative semi-definite. Statistical excess demand functions are always "well-behaved", (for example, downward sloping in own entropy price) and do not exhibit the income effects that complicate Walrasian equilibrium analysis.

The computation of statistical equilibrium is straightforward given the partition functions that define the market. The computation of closed form approximations to the partition functions themselves is often not so easy, though it is always possible to reach exact numerical results through brute-force computation.

It is important to understand that the convexity of the constraints in the maximum entropy programming problem does not depend on the convexity of the offer sets. In fact, since the offer sets are finite, they are never convex. Entropy prices clear the market by distributing agents over their offer sets, rather than moving agents to optimal commodity bundles in their consumption sets, and thus effectively "convexify" the economy.

4.4 WHAT IS A MARKET?

The statistical equilibrium entropy prices define a scalar field of probabilities over the commodity space. Traders of different types face the same equilibrium probability field, but their transactions outcome depends on their offer sets. Statistical equilibrium is characterized by the emergence of well-defined entropy prices. The fact that traders of different types face the same probability field over transactions reflects the fact that they are participating in the same market. (If trade between two economies was impossible, for example, they would in general have different equilibrium entropy prices. The establishment of trade would lead to the emergence of a single equilibrium entropy price system, subject to the usual problems of transactions costs.)

Statistical equilibrium distributes traders of a given type over actual transactions through the realization of the probability field over their offer sets. This implies that different traders of the same type realize different transactions prices. The market thus introduces *horizontal inequality* among traders of the same type. This is a striking difference between statistical equilibrium and Walrasian equilibrium. In Walrasian equilibrium, the assumption that no transactions occur until the equilibrium price system is established implies that traders with the same endowments, preferences, and access to technology will choose the same final transaction, so that all inequality in the Walrasian market appears to be the result of differences in endowments, preferences, or access to technology.

Since the origin, representing no trade, is always in the offer set, some proportion of traders of each type will fail to transact in statistical equilibrium. Unemployment in labor markets, excess capacity among firms, vacancies in rental markets, and similar phenomena, are a direct implication of statistical equilibrium.

Furthermore, the fact that different traders experience different transactions' prices implies that statistical equilibrium does not exhaust all the potential Pareto-improving transactions in the economy. Thus statistical equilibrium approximates, but does not achieve, Pareto-efficiency. A further implication is that statistical equilibrium is compatible with the possibility of arbitrage. (We will pursue this issue in the example discussed later.)

All these implications flow from the reconceptualization of the market as a probability field over transactions, rather than as a system of equilibrium relative prices at which agents believe they can make transactions of any size. In statistical equilibrium, it becomes increasingly difficult to make transactions of larger magnitude in the sense of a higher entropy cost. The formalism of entropy maximization does not "like" to give a trader a high entropy cost transaction because such transactions reduce the degrees of freedom in the assignment of other traders in their offer sets. In real world terms, the decreasing likelihood of high entropy cost transactions is a reflection of the limited liquidity of all markets, since every transaction must have an actual counterpart in the release and absorption of the commodities by other traders. (This point will also be central to the example discussed below.)

Thus the statistical equilibrium picture of a market is a chaotic process that tends to explore all feasible patterns of market transactions. To speak in anthropomorphic terms, the market is "trying" to make the economy as *disorderly* as is possible. What lends coherence and a degree of rationality to market transactions is the behavior of individual agents in refusing transactions outside their offer set. This rather weak form of "choice" is enough of a constraint to give rise to well-determined entropy prices.

To a physicist schooled in thermodynamics, an "equilibrium" system is one that has a well-defined temperature. The formalism of the statistical mechanical foundations of thermodynamics is identical to the maximum entropy programming problem studied in the last section. In the simplest thermodynamic models there is only one "commodity" to be allocated among the molecules of the system – energy. The entropy price of energy, usually denoted β, is the inverse absolute temperature of the system. The emergence of well-defined entropy prices from this point of view is the hallmark of equilibrium.

The emergence of some type of price system in commodity exchange is also, reasonably enough, the precondition for a quantitative science of economics. The existence of prices at which bundles of disparate commodities can be valued underlies the suspicion of a discoverable quantitative orderliness to economic interactions on which economic theory and measurement rest.

The absolute magnitudes of the entropy prices that characterize statistical equilibrium carry significant information about the equilibrium state. When entropy prices are large in magnitude, the exponential Gibbs distributions over the transactions space decay very rapidly, and the variance of transactions over any type of agent will be small. In the limit, as entropy prices become very large, the Gibbs distribution degenerates and puts effectively its whole weight on the lowest value point in the offer set. The statistical fluctuations of price in a market are direct indicators of the entropy prices, just as statistical fluctuations of energy and momentum in a thermodynamic system are direct indicators of temperature.

The two great theoretical traditions in economics, the Classical Political Economy which reaches its apogee in the work of Smith, Ricardo, and their critic Marx, and the Marginalist Revolution of Jevons, Menger, Walras, Edgeworth, and Marshall which flowered into Neoclassical Economics, each have their characteristic and different insights into the source and meaning of prices. Classical political economy acknowledged the statistically fluctuating character of market prices, and sought to explain their orderliness in terms of natural long-run cost prices yielding equal profit rates at a given technology. Marginalist economics saw short-run equilibrium prices as reflecting relative scarcities of desirable goods, and tried to explain them as the shadow prices arising from the attempt of the market to maximize the utility of individual agents subject to short-run resource constraints. From a statistical equilibrium point of view the persistent paradoxes and contradictions that dog the marginalist theoretical project arise from its attempt to attribute a superhuman utility-seeking benevolence to impersonal markets. Like the human agents whose struggles to survive and prosper constitute it, the market values commodities. But it values them for an entirely different reason, because larger quantities of commodities allow a more disordered state of higher entropy. The great insight supporting the marginalist point of view is the "law of supply and demand", the tendency for price to fall as the supply of a commodity increases. The entropy seeking market actually offers a more consistent explanation of this phenomenon than a general equilibrium (GE) of utility-seeking agents, in which scarcity is confounded through income effects with distribution.

Since human decisions are situated in time, however, the scarcity of commodities in utility terms is conditioned on the future uses and the usefulness of commodities that human beings imagine at any point in time. The marginalist explanation of scarcity prices is thus a theory of temporary equilibrium conditional on agents' imagined futures. Statistical equilibrium is also a temporary equilibrium explanation, since the offer sets of agents, which shape the equilibrium price system, depend on the imagined future in the same way. Whether a theory of this imagined future can be supplied by the natural prices of the Classical political economists in an era of rapid and accelerating technical change remains an open question.

4.5 PROBLEMS OF INTERPRETATION OF STATISTICAL EQUILIBRIUM

Unfamiliar abstractions require some getting used to before "economic intuition" (which is often described as whatever the economist learned in the first course in microeconomic theory) begins to function smoothly to link the abstraction to real experience and data. In this section, I address some of the issues that have puzzled students and readers of earlier papers (including Sergio Parrinello and Graciela Chichilnisky) about the interpretation and limits of statistical equilibrium. Curiously, many of these questions are actually identical to interpretational problems with Walrasian equilibrium that have never been satisfactorily resolved in many economists' minds, but appear with fresh urgency in the context of statistical equilibrium.

4.5.1 Time and operationalization

Equilibrium theories are static theories that purport to describe the state of a system which we know to be capable of motion in equilibrium. One of the deeper puzzles of the philosophy of science is why we make such great strides in understanding by applying static reasoning to dynamic systems. Markets exist inextricably in time and are inherently dynamic and changing. How can statistical equilibrium be interpreted in terms of observations of market transactions that occur in real time?

There are probably several possible approaches to answering this question, but here I will put forward one simple and, I believe, workable way to operationalize the theoretical concepts of statistical equilibrium. We interpret a market as a series of repeated interactions of agents with given offer sets. Thus every day, or week (or, in the case of financial markets, perhaps, every hour) new transactions occur among agents who appear with essentially the same offer sets. These agents may or may not be the *same* agents that appeared in the previous iteration of the market. For example, markets for financial assets are constituted by agents appearing with a desire to buy or sell the asset in question, and with an offer set reflecting their private preferences and information. In some contexts, such as buying and selling houses, it is most likely that a particular agent who makes a transaction will not return to the market on the next iteration, or indeed for many iterations. In a case like this, the interpretation requires that a fresh set of agents appear with essentially the same offer sets. In other contexts, like the market for foodstuffs, it may well be that many or most of the agents who transacted in the last iteration return in the next iteration (because they get hungry).

In this interpretation, it is natural to view the transactions' probabilities that arise from the statistical equilibrium formalism as *transaction rates* per unit time.

The equilibrium probability distributions governing actual transactions can be interpreted, from this point of view, both as governing the realization of time series and cross-sectional data arising from the market. One "testable implication" of the statistical equilibrium approach is that the statistical fluctuations in these two types of data should be consistent.

The idea that agents appear in the market in iteration after iteration with the same offer sets points to the need to establish the appropriate time scale on which empirical observations are relevant in any given market. The arrival of new information, new technologies, the evolution of preferences, and the discovery of new resources all will change the agents' offer sets and lead to a new statistical equilibrium. Like any equilibrium methodology, statistical equilibrium explanation will be useful only in situations where these underlying parameters move significantly more slowly than the market processes that bring about equilibrium. Thus there are inherently at least two time scales to be distinguished in any application of statistical equilibrium: the time scale on which transactions approximate the statistical equilibrium distributions (which physicists refer to as the "relaxation" of the system into equilibrium), and the time scale on which the underlying parameters determining the statistical equilibrium evolve. These two time scales will be different in equity or commodity markets and in food or housing or labor markets.

4.5.2 Adjustment to equilibrium

Exactly how does the relaxation to statistical equilibrium take place? This question arises naturally from the desire to specify the exact dynamic process of which the statistical equilibrium is the static representation.

Unfortunately the example of statistical physics suggests that this question is very difficult to answer. The difficulties seem at first to be largely practical and operational, but deeper philosophical issues inevitably arise as well. The success of statistical methods is based on the idea that the underlying dynamics are subject to determinate and knowable laws. For example, statistical thermodynamics purports to represent the evolution of systems of molecules which are also subject to the ordinary laws of mechanics. The motivation for moving to the statistical level of explanation is in the first instance simply the computational impossibility of solving the myriad mechanical equations that govern the motion of molecules in a cubic meter, say, of gas, and the practical impossibility of measuring the momenta of each molecule at a particular moment to determine the boundary conditions that determine the solutions to those equations. The deeper philosophical issue, however, is that the equations of mechanics are structurally time-reversible, while the evolution of statistical systems always moves to higher entropy, and is irreversible.

Without trying to unravel the puzzles about the arrow of time, the practical motivation for treating the market exchange as a "black box" is just as strong in economics as it is in physics. It is possible to imagine many stylized pictures of real economic exchanges, such as random matching models, trading posts, agents arrayed on lattices, and so forth. Each of these models has some claim to be a plausible representation of some aspects of certain real world market transactions, but none of them can plausibly represent the full richness of market transactions we actually experience and partially observe. The search for an "underlying micro–micro model" of the transactions' process that would give support to statistical equilibrium may be long and hard.

This is not an entirely satisfactory situation, but perhaps it is no worse than the methodological situation in statistical physics, and somewhat better than the fictional device of the auctioneer which is the best that the Walrasian equilibrium theory has come up with. The Walrasian theory requires the introduction of an entirely separate, imaginary, theoretical tâtonnement time into the theoretical structure in which the market can find its equilibrium prices. Relaxation to statistical equilibrium, on the other hand, like relaxation to equilibrium in physical thermodynamic systems, takes place conceptually in real time, even if it is very difficult to specify exactly how.

The mathematically convenient way to introduce more structure into the statistical equilibrium framework is in the form of further constraints on the feasible market transactions. This type of constraint, however, always takes the form of a constraint on some moment of the distribution characterizing equilibrium, just as the feasibility constraint requires that the first moment of the distribution, its mean, be equal to zero. It is not easy to translate intuitions about the mechanics of actual transactions (which agents meet which other agents in what order, for example) into such moment constraints. On the other hand, if observation or analysis suggests the relevance of further regularities of the moments of the probability distributions, it is fairly straightforward to introduce additional moment constraints into the maximum entropy formalism.

4.5.3 Bilateral exchange

Connected with the search for a micro–micro foundations of transactions theory is the desire to decompose market equilibrium (whether statistical or Walrasian) into specific bilateral transactions. This tendency arises from our concrete experience, especially as customers in retail markets, of market exchanges as bilateral transactions, and from the legal requirement that all transfers of property be between identifiable individual owners.

Unfortunately, it is generally impossible to satisfy this ambition in any equilibrium model that writes the market clearing condition as a summation of the individual net transaction vectors. This way of writing feasibility, common to the Walrasian and statistical approaches, implicitly allows for multilateral transactions. In real economic markets, this conceptual gap is bridged by the existence of retailers, wholesalers, brokers, arbitrageurs, and market-makers who, while abiding by the legal requirement that all transactions be bilateral, effectively allow the formation of multilateral coalitions of traders. As individuals, we make bilateral purchases from our retail grocer, but the grocer's economic function is to allow a large number of customers to act as a multilateral coalition in the market.

Statistical equilibrium tightens the theoretical screws in this area for the following reason. When two agents withdraw their endowments from the market to engage in bilateral trade, the exact transaction that they arrive at is completely irrelevant to the entropy achievable through the distribution of the remaining traders over their offer sets. From the entropy point of view all that matters is the withdrawal of the two traders and their offer sets from the market, because it reduces the combinatorial possibilities for the other traders. A consequence of this observation is that the exact transaction made in any isolated bilateral exchange is completely indeterminate on statistical grounds. This underlines the point made earlier that the essence of the market is the simultaneous connection of all the participants through the market clearing constraint, which exposes them all to the same probability field of transactions in equilibrium.

4.5.4 Infinite offer sets

The exposition above was based on the assumption that offer sets are finite. This assumption is convenient to simplify the mathematical proofs, and reduce the conceptual complications of the maximum entropy formalism to a minimum. Since the number of points in an offer set may be extremely large, we might hope that we could adequately approximate any real world situation with finite offer sets. Once we understand the structure of the Gibbsian distribution, we see that this hope is largely well-founded. The point is that the Gibbsian distribution gives vanishingly small probabilities to transactions that have large entropy values, so the equilibrium is practically insensitive to whether or not we include these points in the offer set, and to how many of them there may be. This line of argument implies that statistical equilibrium will be well-defined as long as the offer sets are sufficiently bounded below, so that there is a *lower* bound to the entropy value of points in the offer sets at any non-negative system of entropy prices.

Once this point is clear, we see that there is no obstacle to allowing offer sets to contain continua, as long as there are well-defined and convergent prior measures on the continua. In this case the sums in the constraints on the maximum entropy formalism have to be replaced by integrals with respect to these measures, but the formalism goes through without change.

Some lower-boundedness of the offer sets, however, is necessary in order for the maximum entropy state to be well-defined. There is no statistical equilibrium of a market consisting of buyers and sellers of a single commodity for money in which buyers are willing and able to purchase any quantity of the commodity at a reservation price above or equal to the reservation price of sellers who are also willing and able to sell any quantity of the commodity. (Under these circumstances there would be no Marshallian equilibrium, either, since the Marshallian demand schedule would lie uniformly above the Marshallian supply schedule.) In this type of situation, the set of feasible market distributions is unbounded, and the maximum entropy program has no solution. Entropy can always be increased by dispersing the buyers and sellers more and more widely on their unbounded offer sets. This example shows, however, that while lower bounds on all offer sets is a sufficient condition for the determination of a statistical equilibrium, it is not necessary. Statistical equilibrium will be well defined in this type of market if just one of the types, say the sellers, has an offer set bounded below.

While the mathematical formalism is robust to the introduction of continua into offer sets, we should be wary of the economic implications of this type of assumption. Again, the example of statistical physics can provide some hints. Classical physics, which assumed a continuum of time, space, and energy levels, turned out to be both incompatible with the observed quantum phenomena and to imply paradoxes that can be resolved only with the abandonment of the continuum. Similarly, the promiscuous introduction of continua as offer sets into economic models will lead to some disconcerting results. For example, in a labor market where the offer sets of workers are subsets of the transaction space that contain the origin and are bounded below, but not above, and the prior measure is, say, exponential, the statistical equilibrium will always predict zero unemployment, because the measure of the origin as a point is an infinitesimal. It might be possible to mend this by regarding some neighborhood of the origin with a finite measure, in which wages and hours of work are both very small, as effectively equivalent to unemployment, but the need to make such an adjustment underlines the sensitivity of particular models to the assumptions made about the structure of the offer set.

4.5.5 Non-uniform offer set probabilities

At first glance, the assumption of a uniform prior measure over the finite offer sets implied by the maximum entropy formalism as developed here seems highly restrictive. This would be particularly true if we thought of the offer sets as subsets of some regular lattice. But the formalism does not say anything about the actual location of the points in the offer set, and thus provides effectively unlimited modeling flexibility. If we have reason to believe that agents are more likely to be found in some regions of the transaction space than in another, we can reflect this prior information by putting more points of the offer set in these regions. In fact, nothing in the formalism stops us from counting the same transaction several times in the offer set. Such non-uniform distributions of the offer set points correspond in the limit as the number of points in the offer set grows without bound to non-uniform continuous prior distributions. An interesting question is whether non-uniform prior probability distributions might be a way of operationalizing the idea of trader preferences within the theory.

In fact the primitive idea of an offer set already contains within it a non-uniform prior probability, since it effectively puts a zero prior probability on all transactions outside the offer set, and a positive probability on transactions in the offer set. It is precisely this non-uniformity that gives rise to the determinacy of the statistical equilibrium. As we have seen, there is no statistical equilibrium on unbounded infinite offer sets, because without lower bounds on the supply of at least some commodities, entropy can always be increased by dispersing the traders more widely across their offer sets.

4.5.6 Repeated statistical equilibrium and Pareto-efficiency

The statistical equilibrium fails to achieve Pareto-efficiency because it represents a finite intensity of trade over a finite time period. We could, however, consider a scenario in which a group of traders with conventional "well-behaved" utility functions come to a market with their sets of utility-increasing transactions from their endowments as their offer sets, reached statistical equilibrium in a "round" of the market, and then returned to the market with the changed endowments that resulted from the first round of trade, and reached statistical equilibrium again. It is not hard to see that the iteration of this process will asymptotically lead to a Pareto-efficient allocation (at which the common vector of marginal utilities is asymptotically proportional to the entropy prices). Each of the rounds of trade exhausts some of the potential Pareto-improving exchanges, leaving fewer opportunities for trade, so that the entropy prices necessary to clear the market will asymptotically go to infinity.

While this process leads asymptotically to a Pareto-efficient allocation, it does not lead in general to the Walrasian equilibrium, because in each round of trade the agents make transactions at disequilibrium prices, and the resulting income effects will affect the final allocation.

4.5.7 Endogenous offer sets

Theories of "rational expectations" have led economists to think of equilibrium not just in terms of market clearing, but also in terms of informational consistency. If we imagine a given agent repeatedly entering a market in statistical equilibrium, it is tempting to suppose that she will alter her offer set in order to optimize her market outcome given the probabilities that govern transactions in the market equilibrium. This idea gives rise to the concept of *endogenous offer sets*. As opposed to the scenario described in the last section, each of the rounds of this market start with the same endowments.

If we endow the trader with a utility function, we could, taking the entropy prices in the market as a given, evaluate the expected utility of any arbitrary offer set, and thus order the offer sets. In order to make the maximization of expected utility in this context a well-defined problem, however, certain restrictions have to be placed on the available offer sets.

For example, if we permitted the trader to choose her offer set without the requirement that she put an atom of probability at the origin to represent the risk of failure to transact, then the problem is not well-defined, since the trader will choose offer sets that give her an arbitrarily large expected utility. Furthermore, we have to put a lower bound on the atom of probability at the origin, or on the agent, by adding more and more points to the offer set. This could lower her risk of failing to transact to zero, and again achieve arbitrarily large expected utilities.

If we set up the problem to avoid these pitfalls, for example, by allowing the agent to construct the offer set by assigning a finite number of points to the transaction space, with the requirement that one be placed at the origin, then it is a well-defined optimization problem.

Suppose a trader faces a market characterized by entropy prices π. She has a von Neumann-Morgenstern utility function, $u(.)$, defined over transactions, normalized so that $u(0) = 0$. (Remember that the origin represents no trade, or the endowment point.) She can construct her offer set from $s + 1$ points, which we might think of probabilistically as market "chances", so that $A = \{z_1, \ldots, z_{s+1}\}$, with the convention that $z_{s+1} = 0$, to represent the restriction that she must take some risk of failure to trade.

Given her choice of points in the offer set, her partition function will be $Z(\pi|A) = 1 + \sum_{j=1}^{s} e^{-\pi z_j}$, and her expected utility is:

$$E(u|A) = \frac{\sum_{j=1}^{s} e^{-\pi z_j} u(z_j)}{Z(\pi|A)}$$

(Remember that $u(0) = 0$ by convention.) The first-order conditions for maximizing the expected utility with respect to the position of any point, say, z_1, is:

$$\frac{\partial E(u|A)}{\partial z_1} = \frac{-\partial \ln Z}{\partial z_1} E - \frac{e^{-\pi z_1}}{Z} (\pi u(z_1) - \frac{\partial u(z_1)}{\partial z_1}) = 0$$

Since $\partial Z/\partial z_1 = -\pi e^{-\pi z_1}$, this equation can hold only if

$$\frac{\partial u(z_1)}{\partial z_1} \propto \pi$$

Since the expected utility is symmetrical in the points in the offer set, the trader maximizes her expected utility by putting all her points in the same place, aiming for a transaction where her marginal utility vector will be proportional to the entropy prices.

4.5.8 Statistical and Walrasian equilibrium

The results of the last section suggest that there may be a sense in which Walrasian equilibrium can be viewed as an asymptotic approximation to statistical equilibrium. Traders in a market in which population, endowments, preferences, and technology have been unchanged for a long time might start to adapt their offer sets to maximize their expected utility. This will lead them to concentrate all their chances to trade on single points. But as the offer sets change in this way, so will the statistical equilibrium entropy prices. The reasoning of the last section implies that the traders will eventually concentrate all their actual transactions chances on points where their marginal utilities are proportional to the entropy prices, and thus on a single hyperplane. Thus asymptotically all agents of the same type who actually transact will achieve the same transaction, as in Walrasian equilibrium. Market clearing requires that the sum of all the transactions of different types of agents be zero, so the hyperplane on which all the transactions asymptotically concentrate will be oriented by the asymptotic entropy prices, and pass through the origin, which represents the endowment points. These are the properties that define a system of equilibrium Walrasian relative prices.

When offer sets become degenerate, as they would in this model, the entropy prices have to become very large to clear the market. In the limit of complete adaptation, the entropy prices would have to be arbitrarily large in magnitude (corresponding to a zero "temperature" of the market), but it seems likely that they would converge to a direction vector, representing the asymptotic relative prices. This type of scenario would explain why the Walrasian equilibrium determines only the relative prices of commodities.

There remains a further discrepancy between the asymptotic statistical equilibrium in this scenario and the Walrasian equilibrium, which is the fact that some traders will be assigned to the origin in the statistical equilibrium, even asymptotically. Since the hyperplane representing the asymptotic entropy prices goes through the origin as well as the transactions for each type of trader, the probability of failure to trade is equal to $1/(s+1)$. Walrasian equilibrium has no such built-in "frictional" inefficiency. But if we also allow the number of chances in the offer set, $s+1$, to increase without limit, this asymptotic inefficiency will also become negligible.

These considerations suggest that it may be possible to establish rigorously that Walrasian equilibrium is the asymptotic outcome of a process in which endogenous offer sets adapt to statistical equilibrium entropy prices, and where the number of chances to transact in any period become very large. Walrasian equilibrium is not unique, whereas statistical equilibrium for given offer sets of traders is unique. But the adaptation process sketched here allows offer sets to change over time, giving rise to a dynamical process which may have multiple equilibria, each corresponding to a Walrasian equilibrium in markets with multiple Walrasian equilibria.

4.6 AN EXAMPLE: STATISTICAL EQUILIBRIUM AND ARBITRAGE IN A SIMPLE ASSET MARKET

In this section, I develop an example of statistical equilibrium in a simplified asset market to clarify the methods of statistical equilibrium modeling, the interpretational issues discussed above, and the types of results that are possible. The example arose from conversations with Perry Mehrling.

4.6.1 Primitive demand and supply

For simplicity, let us consider a market for a single financial asset, such as a bond or treasury bill. The "primitive traders" of the market will be represented by a number of sellers, each of whom comes to the market ready to sell a fixed amount of the asset, normalized to be one unit, for a reservation price p^s or something higher, and an equal number of buyers each of whom comes to the market with a fixed amount of money, normalized to be one unit, ready to spend it on the asset at a reservation price p^b or lower. These primitive buyers and sellers are not able to trade on both sides of the market (presumably because they have no inventory of money, in the case of the sellers, or of the asset, in the case of the buyers). In order for there to be a positive level of actual transactions, we assume $p^s < p^b$.

The offer sets describing these two primitive agents appear drawn together in Figure 4.1.

The partition functions for buyers and sellers are:

$$Z^s(\pi_a, \pi_m; p^s) = 1 + \frac{e^{\pi_a - p^s \pi_m}}{\pi_m}$$

$$Z^b(\pi_a, \pi_m; p^b) = 1 + \frac{e^{-(\pi_a/p_b) + \pi_m}}{\pi_a}$$

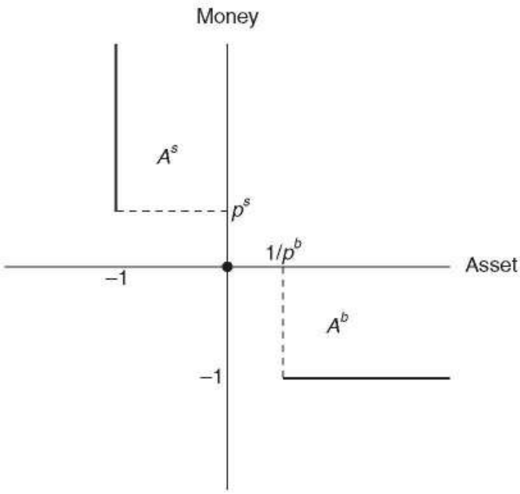

Figure 4.1 The offer set of sellers is the origin together with the vertical line segment above the point $(-1, p^s)$; the offer set of buyers is the origin together with the horizontal line segment to the right of the point $(1/p^b, -1)$.

The origin contributes the 1 to the partition functions, and the second terms are the integrals of $e^{-\pi_a a - \pi_m m}$ over the respective line segments.

The mean excess demand vector for any agent is the derivative of the negative of the logarithm of the partition function with respect to the entropy prices. For example, the mean excess demand function for sellers is:

$$x^s(\pi_a, \pi_m; p^s) = -\frac{e^{\pi_a}}{e^{\pi_a} + \pi_m e^{p_s \pi_m}}, \frac{e^{\pi_a}(1 + p_s \pi_m)}{\pi_m(e^{\pi_a} + \pi_m e^{p_s \pi_m})}.$$

Working out the statistical equilibrium for the general case of arbitrary reservation prices is somewhat tedious, but much easier if we assume that $1/p^b = p^s = p$, because in this case the market is completely symmetrical between the two commodities. (In order for there to be any trade we have to take $p < 1$.) From the consideration of symmetry, we see that the statistical equilibrium entropy prices must satisfy $\pi_a = \pi_m = \pi$. Under this assumption, we can calculate the market excess demand for any π by summing the excess demands for the sellers and the buyers.

$$x(\pi; p) = \frac{e^{\pi}(1 - (1 - p)\pi)}{\pi(e^{\pi} + e^{p\pi}\pi)}, \frac{e^{\pi}(1 - (1 - p)\pi)}{\pi(e^{\pi} + e^{p\pi}\pi)}.$$

Because of the symmetry assumptions, the excess demands for the two assets are identical functions of π. The level of π that clears the market is:

$$\pi(p) = \frac{1}{1 - p}.$$

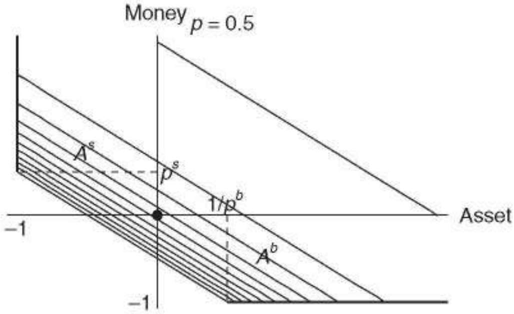

Figure 4.2 Statistical equilibrium in a symmetrical market for two assets when $p = 0.5$. The diagonal lines represent the iso-probability loci for the equilibrium Gibbsian distribution.

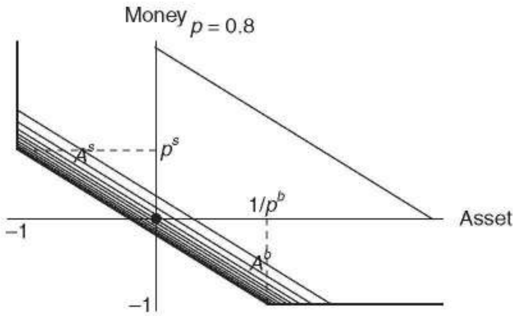

Figure 4.3 Statistical equilibrium in a symmetrical market for two assets when $p = 0.8$. The diagonal lines represent the iso-probability loci for the equilibrium Gibbsian distribution.

As p approaches 1, there are fewer opportunities for mutually advantageous transactions, and the statistical equilibrium entropy price has to rise toward infinity to clear the market. Figures 4.2 and 4.3 show the iso-probability loci of the equilibrium Gibbs distribution for $p = 0.5$ and $p = 0.8$.

In statistical equilibrium some traders succeed in finding a trade, and some do not. The sellers who do transact, for example, sell one unit of the asset in exchange for a transaction price m where $m - p$ is distributed exponentially with parameter $1/(1 - p)$. Since the mean of this distribution is $1 - p$, the mean of the actual transaction price is always 1, regardless of the parameter p. This makes sense, since every seller who trades sells one unit of the asset, and every buyer who trades pays one unit of money, so that in order for the market to clear, the mean excess demand of sellers who actually transact has to be $(-1, 1)$ and the mean excess demand of buyers who actually transact $(1, -1)$. The variance of the price, sellers actually receive is $(1 - p)^2$, as is the variance of the inverse price buyers actually receive, and does depend on p. In this simple model the standard deviation of transaction prices, $1 - p$ is equal to $1/\pi$, and is a direct measure of the entropy price.

The probability of not transacting (on either side of the market) can be calculated from the partition functions to be $1/(1 + e(1 - p))$ and thus also depends on p. When $p = 1$, there is no trade at all, and in the extreme case $p = 0$, in which sellers will accept and buyers will pay any non-negative price, the fraction of traders who fail to transact reaches a minimum of $1/(1 + e)$. A typical trader thus experiences two risks in entering the market, a finite chance of failing to transact at all, and an uncertain price in the event of actually transacting.

4.6.2 A small arbitrageur

The statistical equilibrium in this market fails to achieve Pareto-efficiency, because some potentially mutually advantageous transactions fail to be executed, and there is dispersion in actual transactions' prices. There is thus an opportunity for arbitrage (which cannot be exploited by the primitive buyers and sellers because each can transact only on one side of the market).

Let us introduce a single arbitrageur who has access to inventories of both the asset and money in the form of capital. This arbitrageur sees the same probability field over transactions defined by the statistical equilibrium π as the primitive traders. Since the arbitrageur can trade on either side of the market, she is in a position to buy cheap and sell dear. This type of arbitrage offer set is illustrated in Figure 4.4.

Again, in order to simplify the calculations, assume that the arbitrageur's bid and asked prices, q^b and q^a, are symmetrical in the asset and money, so that $1/q^b = q^a = q > 1$. The arbitrageur's partition function is:

$$Z^a(\pi_a, \pi_m; q) = 1 + \frac{1}{-\pi_a + q\pi_m} + \frac{1}{q\pi_a - \pi_m}.$$

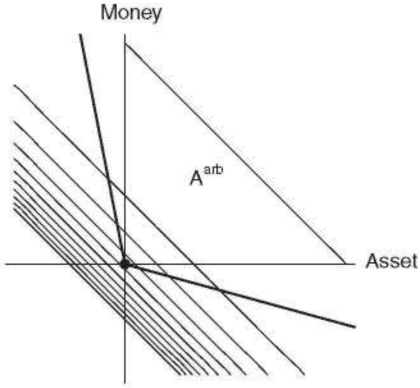

Figure 4.4 An arbitrageur "fishes" the transactions probability field by standing ready to buy or sell any quantity of the asset, but at a bid price that is lower than her asking price. Her offer set thus consists of the origin, to represent a failure to transact, and two rays, one representing the bid price, and the other the asking price. The mean transaction for this offer set yield is strictly positive in both the asset and money.

The 1 here represents the possibility of no trade at the origin, and the other two terms are integrals along the bid and ask price rays.

The mean transaction for this partition function in the statistical equilibrium with $\pi_a = \pi_m = \pi$ is:

$$\left(\frac{(1-p)^2}{(1-p)+(q-p)}, \frac{(1-p)^2}{(1-p)+(q-p)}\right).$$

Thus the arbitrageur, by buying cheap and selling dear, can expect to make a pure profit in the asset and money. Of course, this profit depends on the arbitrageur's having the capital to make good on her transactions.

The probability distribution governing the arbitrageur's position is:

$$\frac{q-1}{(1-p)+(q-p)} e^{\frac{q-1}{1-p}m} \quad \text{for } m < 0$$

$$\frac{q-1}{(1-p)+(q-p)} e^{\frac{q-1}{1-p}\left(-\frac{m}{q}\right)} \quad \text{for } m > 0.$$

Taking the market, as characterized by the parameter p as given, the arbitrageur has to choose q, which determines the bid-ask spread with which she enters the market. The larger the bid-ask spread, the lower the arbitrageur's risk of exceeding her capital limits, but the lower her expected return, measured, for example, as the expectation of the sum $a + m$. The reason for this tradeoff is that a higher bid-ask spread, while it increases the arbitrageur's expected profit conditional on transacting, reduces the probability of transacting by even more, and thus reduces the overall expected return. On the other hand, a small bid-ask spread exposes the arbitrageur to a larger risk of a statistical fluctuation in her position which will exceed her capital. Figure 4.5 visualizes this tradeoff.

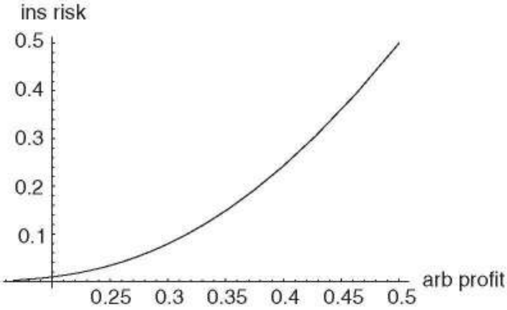

Figure 4.5 The arbitrageur's return, measured as the expectation of $a + m$, is plotted against the probability that $m < -1$, representing a capital constraint, as q varies from 1 to 3, facing a market with $p = 0.5$.

4.6.3 An arbitrage industry

In the last section, the analysis was carried out on the assumption that the individual arbitrageur was "too small" to affect the statistical equilibrium of the market. We can also study the impact of a significant sector of arbitrageurs on the statistical equilibrium.

When a significant number of arbitrageurs enter a market, they will add a strictly positive mean demand at any entropy price, which will imply excess demand at the previous equilibrium entropy price. This will tend to raise the entropy prices for the commodities traded until the excess demand is eliminated. The effect of the rise in entropy prices is to squeeze a surplus out of the primitive traders, by depressing the mean selling price and raising the mean buying price for them.

Arbitrage in statistical equilibrium leads to a market with a lower variance of transaction prices for primitive traders, that is, to a more "efficient" market. In this respect, arbitrage performs one of the functions Walras attributed to his "auctioneer". But real market arbitrageurs have to risk real capital, and sustain the risk of insolvency from statistical market transaction fluctuations, vicissitudes that the Walrasian auctioneer never experiences.

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5 Savings, investment and capital in a system of general intertemporal equilibrium

Pierangelo Garegnani†

5.1 INTRODUCTION

1. The criticism of the neoclassical theory based on the inconsistency of the concept of a 'quantity of capital' has been met from the orthodox side essentially with the claim that contemporary reformulations of the theory do not rely on any such concept.[1] The present chapter is intended to show that the claim is unfounded and that the deficiencies of the concept undermine the reformulations no less than they do the traditional versions.[2]

In Section 5.2 we shall introduce for this purpose, the very simple model of intertemporal equilibrium which Hahn put forward in 1982 to counter what he took to be the 'neo-Ricardian' critique. This model will allow us to bring out the decisions to save and to invest of any 'year' which are implied in an intertemporal general equilibrium (GE).[3] In Section 5.3 we shall then define what can be described as the 'general-equilibrium saving-supply schedule' and the 'general-equilibrium investment-demand schedule' for such a 'year'. The detailed determination of those schedules – which may be left aside at a first reading – has been postponed to Appendix I and to the Mathematical Note attached to the chapter. Section 5.4 will examine the general information which the schedules provide about the behaviour of the system, while Section 5.5 will deal with the effects on investment demand of changes in techniques and in consumption outputs as intertemporal prices change.

The above will enable us to approach in the final section the question of how a 'quantity of capital' enters intertemporal equilibrium. That will involve pointing out first how misleading can the idea be that the adjustments in intertemporal consumptions (i.e. in decisions to save and invest) raise no more problems than adjustments in contemporary consumption do.[4] Whereas the latter imply a shift of resources between

† The present chapter is a revised version of the one contributed to H. Kurz (2000). The revision has entailed the addition of an Appendix II and the shift to an Appendix I of the material previously in Section IV. I thank for useful comments the participants at several seminars held in Italy and elsewhere, where the ideas contained in this chapter have been discussed since 1992. Thanks are due in particular to Profs. R. Ciccone, J. Eatwell, J. Geanakoplos, G. Impicciatore, H. Kurz, F. Petri, B. Schefold, F. Serrano, D. Tosato and Dr F. Ravagnani. Special thanks are owed to Dr M. Tucci and Prof. M. Angrisani for help on the mathematical parts of the chapter (see Dr Tucci's Mathematical Note at the end of the chapter). I am grateful to Luisa Milanese for her assistance through the numerous versions of this chapter.

the respective *contemporary* productions and can be activated *directly* by the disequilibrium prices, the analogous disequilibrium in intertemporal prices (own rates of interest), due, for example, to excess savings, can only adjust the respective intertemporal productions *indirectly*, through the intermediate link of an incentive given to entrepreneurs to change methods of production and/or relative consumption outputs, so as to increase the 'amount of means of production' relative to labour and other primary factors employed in the economy in that year. The corresponding additional investment is indeed what should, on the one hand, absorb in the production of the capital goods, the resources of time $(t-1)$ set free by the additional savings of t and, on the other, increase the productivity of primary factors in $(t+1)$, $(t+2)$, etc. to provide for the future increased consumption which the savers have planned. It will then be seen how the impossibility of measuring that 'amount of means of production' independently of distribution entails that no such 'increase' in means of production needs to follow from the competitive fall of the prices of commodities of early, relative to those of later dates (fall of the respective own rates of interest) caused by excess savings. The conclusion will thus be that treating under the same heading intertemporal and contemporary consumptions can obscure, but not do away with the differences between the two cases – the "quantity of capital" re-emerging essentially unchanged in its relevance, and in its deficiencies, for the determination of the equilibria. Indeed in proportion to their value, heterogeneous capital goods are for savers perfectly substitutable means of transferring purchasing power over time, so that savers' decisions about capital goods will refer to that 'quantity'[5], which will accordingly have to be implicitly or explicitly present in the system like that of any other good on which individuals exert their demand and supply decisions.

Appendix II completes the chapter by examining some flaws in the mathematical argument Professor Hahn has conducted in his 1982 article. Those flaws will allow bringing out some misunderstandings which appear to have seriously hindered communication between the two sides in the course of the capital controversies.

Our analysis of general equilibrium will be conducted by analytical instruments, other than excess demands generated by treating *all* prices as independent variables and used since Hicks (1939). As indicated, we shall use 'general equilibrium demands and supplies' of particular commodities or factors, assuming that all markets other than the specific ones on which we focus our attention are in equilibrium.[6] An equilibrium in the particular market considered will then imply an equilibrium of the whole system. The advantage of these instruments is the possibility they offer to trace the effects of peculiarities of that market on the general equilibrium and its properties. Thus we shall here centre on those commodity markets which constitute the savings–investment market, and study the effects of the phenomenon of 'reverse capital deepening' which directly affects such markets. The reader is thus asked for some effort in entering a less familiar method of analysis, which however, we hope, may turn out to allow for some novel results and for a better economic grasp of key phenomena affecting a general intertemporal equilibrium. In particular, the reader should try to take these unfamiliar instruments on their logic, and resist the temptation to translate them too quickly into the language with which he is more familiar.

5.2 DECISIONS TO SAVE AND INVEST IN A SYSTEM OF INTERTEMPORAL GENERAL EQUILIBRIUM

2. To have a first, bird's eye view of the ground we shall travel, it might be useful briefly to focus our attention back on the traditional versions of the theory. We need to consider the seeming contradiction between the varying physical capital underlying the demand function and that underlying the supply function for capital,[7] the single 'factor' characterising those versions.[8] For the sake of a definite example we may refer to Wicksell's "Lectures" (1901) where a 'quantity of capital' demanded, expressed as a value in terms of consumption goods, is equalised to the economy's endowment of it (loc. cit. vol. I, 204–205).

The seeming contradiction lies in the fact that, whereas in the demand schedule the physical capital which the quantity *K demanded* at each interest rate should express is that corresponding to the techniques and outputs most profitable at such a rate and changes with it, the physical capital making up the *supply*, or endowment of *K* is the stock in existence in the economy and will of course generally differ physically from the unknown one of the equilibrium to be determined. Thus, while in equilibrium 'quantity' demanded and supplied of 'capital' are equal, the two apparently refer to altogether different aggregates of capital goods.

However, clearly, this contradiction is only apparent: what is in fact meant in the supply schedule of 'capital' is that the physical form of the stock appropriate to the equilibrium position will be assumed by the existing stock *over a period of time* as, each 'year', a part of the capital goods in existence has to be replaced and a corresponding proportion of the labour force is set free to be re-equipped by investing the gross savings of the year.[9]

The implications of this for us here are important. The demand and supply schedules for 'capital' (the fund) envisaged in Wicksell and the other traditional writers for their equilibria, were in fact intended to analyse forces supposed to operate *through* the demand for gross investment, and the supply for gross savings (the flows). The attention was concentrated on the *fund* (capital) rather than the *flow* (savings–investment) concepts, in order to analyse that key case of substitution of factors in a purer form, undisturbed by the monetary and other phenomena which would have interfered when dealing with a savings–investment market. Now, once that is made clear, it should also be clear that all the phenomena traditionally treated by means of the 'quantity of capital', and therefore the 'quantity of capital' itself, cannot be absent in the new intertemporal versions of the theory, where each 'year' will of course entail investment and savings.[10]

Our task now will therefore be, first of all, to render explicit the savings supply and investment demand which pertain to each 'year' in the equations of general intertemporal equilibrium: this will be done in this section, leaving for the next the presentation of the method we shall use for analysing their changes as prices vary in the intertemporal system.

3. A very simple model will suffice for that purpose. Assume an economy with two goods only, *a* and *b*, each being both a consumption and a (circulating) capital good. The economy lasts two 'years' in all, $t = 0$ and $t = 1$, indicated by their initial moments 0 and 1. Production therefore occurs in a single cycle for $t = 0$, with all outputs becoming available at the end of that 'year' (a second production cycle in $t = 1$ would make no sense, because it is completed when the economy ceases to exist). As usual, all markets occur at 'moment' zero, so that the prices P_{a1} and P_{b1} of commodities a_1 and b_1 available for the year $t = 1$ are discounted to moment zero, when they are quoted together with the prices P_{a0} and P_{b0} of the spot commodities a_0 and b_0, and with the wage W.

We may at first suppose that one method only is known for producing each of the two commodities (this assumption will be abandoned in Section 5.5); l_a, a_a, b_a and l_b, a_b, b_b are the corresponding production coefficients, which for simplicity we shall assume to be all strictly positive. The methods are of course assumed to be 'viable', i.e. capable of producing a surplus over the mere replacement of the means of production.

We shall then have the following equilibrium relations:

$$\begin{cases} P_{a1} = l_a W + a_a P_{a0} + b_a P_{b0} \\ P_{b1} = l_b W + a_b P_{a0} + b_b P_{b0} \end{cases} \tag{5.1e}$$

$$P_{b1} = 1 \tag{5.2e}$$

$$\begin{cases} A_0 \geq D_{a0} + (a_a A_1 + a_b B_1), & \text{if sign} > \text{applies, } P_{a0} = 0 \\ B_0 \geq D_{b0} + (b_a A_1 + b_b B_1), & \text{if sign} > \text{applies, } P_{b0} = 0 \\ L \geq l_a A_1 + l_b B_1, & \text{if sign} > \text{applies, } W = 0 \\ A_1 = D_{a1} & \\ B_1 = D_{b1} & \end{cases} \qquad \text{(E)} \quad (5.3e)$$

In system (E), equations (5.1e) are the usual competitive price relations for the products a_1 and b_1,[11] while equation (5.2e) chooses b_1 as the numéraire. The first two relations (5.3e), on the other hand, regard the supply of commodities a_0 and b_0, provided by endowments A_0 and B_0, and the demand for them, given by consumptions D_{a0} and D_{b0}, plus investment $(a_a A_1 + a_b B_1)$ and $(b_a A_1 + b_b B_1)$, respectively; the third relation regards the demand and supply of labour, while the remaining two express the utilisation of the two outputs A_1 and B_1 for (only) the consumptions D_{a1}, D_{b1}.

System (E) thus has eight relations, only seven of which are independent, with seven unknowns: i.e. the four prices, the wage and the two outputs A_1 and B_1. Beyond the first test of consistency given by these numbers, the enquiry into the existence and character of the solutions of (E) will be part of the analysis we intend to conduct by means of the above-mentioned general equilibrium savings–supply and investment–demand schedules (see system (F) par. 7).

It may now be important to observe first that we have here simplified the system by ignoring the possibility of storing the two goods between $t = 0$ and $t = 1$, thus 'transforming' a_0 into a_1, and b_0 into b_1 – a simplification which does not affect the limited conclusions aimed at here, but the implications of which will be recalled below when necessary.[12] It is this simplification which justifies the assumption that both commodities are produced, and that, therefore, relations (5.1e) hold with a strict equality sign (cf. also Appendix II par. [iv]).

A second observation may be in order about system (E). The choice of b_1 as numéraire in equation (5.2e) entails that the variables P_{a0} and P_{b0} emerge from (E) as the relative prices P_{a0}/P_{b1} and P_{b0}/P_{b1} which involve commodities of the two different dates, and which we shall accordingly call 'intertemporal relative prices'. We shall distinguish them from 'contemporary relative prices', e.g. P_{a0}/P_{b0}, since we shall find that the properties of the two sets differ in important respects.[13]

4. We can now come to the decisions to save and to invest implied in system (E) for each of the two years' life of the economy. Indeed, some readers might have been surprised by our reference in par. 3 to savings distinguished by year in a context of intertemporal equilibrium – where all contracts are made in an initial 'moment', and therefore all income is received and disposed off in that single 'moment'; and furthermore it will be disposed in consumption only (if 'final' capital is zero). However, reflection shows that *outputs*, including of course capital goods, have to flow out year by year, and accordingly the *incomes* making up the prices of those outputs must also be distinguishable by year, together with their savings component.

The fact that, given the two years' life of the economy, production only makes sense in $t = 0$, entails investment and savings will also only make sense for year $t = 0$. The aggregate decisions to invest I_0 of that period are the value of two physical components I_{a0} and I_{b0}, consisting of the parts of the two initial stocks A_0 and B_0 which are used as the means for the production of a_1 and b_1, and have already appeared in the first two relations (5.3e). We thus have

$$I_0 = (a_a A_1 + a_b B_1)P_{a0} + (b_a A_1 + b_b B_1)P_{b0} = I_{a0}P_{a0} + I_{b0}P_{b0}. \tag{5.4}$$

Similarly gross savings S_0 will be part of the social gross income Y_0[14] of $t = 0$ – which, unlike the income Y_1 of $t = 1$, will not be the counterpart of a social gross product, and will consist instead of the initial stocks, A_0 and B_0. Thus S_0 will be expressed as the following difference between the gross income Y_0 and the aggregate consumption G_0 in year $t = 0$:

$$\begin{aligned} S_0 = Y_0 - G_0 &= (A_0P_{a0} + B_0P_{b0}) - (D_{a0}P_{a0} + D_{b0}P_{b0}) \\ &= (A_0 - D_{a0})P_{a0} + (B_0 - D_{b0})P_{b0} = S_{a0}P_{a0} + S_{b0}P_{b0}, \end{aligned} \tag{5.5}$$

where the physical components of the aggregate saving decisions S_0 are distinguished by S_{a0} and S_{b0} and where the equilibrium magnitudes of system (E) imply $S_{a0} = I_{a0}$, $S_{b0} = I_{b0}$ and therefore $I_0 = S_0$.[15]

As for the year $t = 1$, we shall have

$$S_1 = Y_1 - G_1 = (L_0W + S_0) - (D_{a1}P_{a1} + D_{b1}P_{b1}),$$

where $(L_0W + S_0)$ is the value of the gross social product from the income side, and where, however, the last two equations (5.3e) stating that the entire output of $t = 1$ is consumed, entail $I_1 = 0$[16] and $S_1 = 0$.[17]

5. It may now be of interest to note how, in the individual 'wealth equations', relating to the entire lifetime of the economy, the savings of each year disappear (contributing perhaps to the misleading view that the problems raised by savings and investments disappear leaving place to a question 'not any different from [...] choosing commodities today').[18]

The equation in question is in fact simply the sum of the *yearly* individual budget equations of the kind just seen, and in that sum the savings on the 'expenditure side' of the budget equation for any year t, reappear on the 'income side' for $(t+1)$, and must therefore cancel out with the latter (the exception being any savings of the final year of the economy which will of course be zero, if terminal capital is to be zero).

Thus, for example, the yearly budget equations of an individual in our two-year economy can be written as follows, where the small letters y_0, s_0, l_0, a_0, b_0 stand for the individual's yearly consumption, gross savings and initial endowment respectively:

$$\begin{cases} (y_0 =)a_0P_{a0} + b_0P_{b0} = g_0 + s_0 \\ (y_1 =)l_0W + s_0 = g_1. \end{cases} \tag{5.5a}$$

In summing equations (5.5a), the s_0's cancel out and we are left with

$$(y_0 + y_1 =)a_0P_{a0} + b_0P_{b0} + l_0W = g_0 + g_1,$$

where the terms after the first equality sign constitute the 'wealth equation'.

Non-zero gross savings and investments being possible in our model only for $t = 0$, we shall henceforth simplify our notation by dropping the zero deponent from the savings and investment variables.

5.3 THE GENERAL-EQUILIBRIUM SCHEDULES OF SAVINGS-SUPPLY AND INVESTMENT-DEMAND

6. Our task is now to bring out how the savings and investment decisions of equations (5.4) and (5.5) vary with prices and can accordingly affect the equilibria of the system. This is what will be done here by means of the two constructs mentioned already: 'the general-equilibrium investment-demand schedule' and 'the general-equilibrium savings-supply schedule'.

The two schedules will be obtained from the relations of system (E) by (i) treating one of the two own rates of interest of period $t = 0$, say r_b, as the independent variable;[19] (ii) waiving the equality between I and S implied in (E).[20] This requires first of all, the introduction of the definitory equation

$$r_b = (P_{b0}/P_{b1}) - 1. \tag{5.6}$$

The release of condition $I = S$, on the other hand, allows for either $S_a \neq I_a$, or $S_b \neq I_b$, or both, and therefore a difference between what we may now call the total demand of a_0 given by $A_0^D = D_{a0} + I_a$ (cf. the R.H.S. of the first of the relations (5.3e) in par. 3) and its total supply $A_0^S = A_0$, which can also be expressed as $A_0^S = D_{a0} + S_a$ (cf. equation (5.5), par. 4) – and similarly for the total demand and supply of b_0.

7. The result is seen in system (F) below where,

1 the two unknowns A_0^D, B_0^D replace the data A_0 and B_0 in the relations (5.3e) which now, in their form (5.3f), with an equality sign only define the two total demands;
2 the data A_0, B_0, re-labelled as A_0^S, B_0^S, appear instead in the relation (5.5f) defining savings;

and where the unknowns I and S constitute the points of the two schedules corresponding to the given level of the independent variable r_b:

$$\begin{cases} P_{a1} = l_a W + a_a P_{a0} + b_a P_{b0} \\ P_{b1} = l_b W + a_b P_{a0} + b_b P_{b0} \end{cases} \tag{5.1f}$$

$$P_{b1} = 1 \qquad (5.2f) \quad (F)$$

$$\begin{cases} A_0^D = D_{a0} + (a_a A_1 + a_b B_1) \\ B_0^D = D_{b0} + (b_a A_1 + b_b B_1) \\ L \geq l_a A_1 + l_b B_1 \quad \text{if inequality, then } W = 0 \\ D_{a1} = A_1 \\ D_{b1} = B_1 \end{cases} \tag{5.3f}$$

$$I = (a_a A_1 + a_b B_1) P_{a0} + (b_a A_1 + b_b B_1) P_{b0} \tag{5.4f}$$

$$S = (A_0^S - D_{a0}) P_{a0} + (B_0^S - D_{b0}) P_b \qquad (5.5f) \quad (F)$$

$$r_b = (P_{b0}/P_{b1}) - 1 \tag{5.6f}$$

$$A_0^D / B_0^D = A_0^S B_0^S \tag{5.7f}$$

All markets are here assumed to be in equilibrium except those of savings and investments, i.e. as we saw, the markets where saved and investible quantities of a_0 and b_0 are traded.[21] System (F) in fact implies equilibrium

1 in the market for labour (see the respective relation in (5.3f));
2 in the markets for commodities a_1 and b_1 (see the last two equations in (5.3f));
3 in the markets of a_0 and b_0 for consumption (see the inclusion of D_{a0} and D_{b0} in equation (5.5f)).

However, if we exclude equation (5.7f) to be presently discussed, system (F) has eleven relations, ten of which are independent, containing eleven unknowns (the five prices; the two outputs A_1, B_1, the two aggregate quantities demanded A_0^D, B_0^D and, finally, I and S).[22] Were it not for equation (5.7f), system (F) would therefore possess the one degree of freedom which we could expect since, essentially, we replaced with the two new unknowns A_0^D and B_0^D, the single unknown P_{b0} of (E) which becomes a given in (F) in the shape of the given $r_b = P_{b0} - 1$ of equation (5.6f).

Before discussing that degree of freedom, and its closure by means of (5.7f), we may, however, re-write the equations (5.3f), (5.4f) and (5.5f) in the more transparent form which we shall frequently use in what follows.

$$\begin{cases} A_0^S = D_{a0} + S_a \\ B_0^S = D_{b0} + S_b \\ A_0^D = D_{a0} + I_a \\ B_0^D = D_{b0} + I_b \end{cases} \tag{5.3f$'$}$$

$$I = I_a P_{a0} + I_b P_{b0} \tag{5.4f$'$}$$

$$S = S_a P_{a0} + S_b P_{b0}. \tag{5.5f$'$}$$

8. In fact, the economic meaning of the degree of freedom we would have in (F) but for equation (5.7f) is quite simple. We have aggregated all decisions to invest into the single magnitude I, but nothing has been specified about the *physical composition* of the investment flows of Schedule I: this is what is done by means of equation (5.7f) which in fact fixes I_a/I_b jointly with D_{a0}/D_{b0} since A_0^D/B_0^D is a weighted average of those two ratios.

That physical composition cannot, however, be specified arbitrarily. Our use of the I and S schedules in order to analyse the properties of system (E) imposes two requirements. The first and stricter requirement is that when $S = I$, the aggregate demands of a_0 and b_0 should also be equal to the respective supplies. And the same correspondence should hold for possible 'extreme' equilibria at the level $r_{b\,\min}$ with $S > I$, or with $W = 0$, for $S < I$ (see par. 14 below, points I and IV respectively, and Appendix I). This will in fact be the case when the proportion A_0^D/B_0^D in which the two commodities are there 'demanded' are the same as the proportion A_0^S/B_0^S in which they are supplied, as is imposed by equation (5.7f) (cf. parr. 15–16). The second, less strict, requirement is that the proportion A_0^D/B_0^D should reflect a non-unplausible out-of-equilibrium behaviour of the economy. And equation (5.7f) seems to provide a description of an out-of-equilibrium behaviour as plausible, it seems, as any equally general condition (cf. parr. 17–18).

9. There remains a rather technical point we need to consider in order to complete our account of system (F). It concerns the consistency of the system with the sum of the individual budget equations underlying it (Walras's identity) and the often supposed impossibility of a disequilibrium confined to a single market, such as we have assumed in (F).[23] However, that impossibility would follow only if the individuals could spend for the commodities available in $t = 1$ according to the total income which they would derive from selling exactly the quantities of a_0 and b_0 they wish to sell at the going prices (i.e. A_0^S, B_0^S in the aggregate, if we include in the demand the consumption by owners) and realise the corresponding savings for $t = 0$, but that is just what *cannot* happen when $S \neq I$. When, on the other hand, the purchasing power for $t = 1$ originating from the savings S_0 is appropriately 'adjusted' to what the going level I would allow them to sell of a_0 and b_0 – and this is what we have assumed in (F) – the contradiction disappears and system (F) is consistent.[24]

The 'adjustment' in expenditure we assume here, when compared with the more usual procedure of admitting disequilibrium in at least one further market, has the advantage of being compatible with a constancy in the employment of labour as r_b varies, thus providing a more transparent basis for deducing the shapes of the S and I schedules. It also allows for a simpler and perhaps better than any equally general representation of the out-of-equilibrium behaviour of the system, in the sense just mentioned that households failing to sell part of their A_0^S and B_0^S resources because of excess savings can hardly exert excess demand on the commodities of $t = 1$.[25]

10. Some preliminary observations concerning our method of analysis may in fact be useful at this point. The general equilibrium nature of the two schedules and condition (5.7f) entail that any equilibrium in the market they represent is also an equilibrium of the system and that the converse is true (parr. 15–16). The properties of the general equilibrium relevant for its uniqueness and stability, and their dependence in particular on the circumstances of the savings and investment market then become visible in a form not unlike that of a partial equilibrium problem. It is in this way that in Sections 5.4–5.6 the two schedules will render visible a cause of multiple and unstable equilibria which does not appear to have yet been sufficiently analysed in the literature. That cause is the way in which investment changes as intertemporal prices change (see Figure 5.1).

Possibilities of non-uniqueness and instability of the equilibria have in fact been in the foreground of current general equilibrium literature. However, those possibilities seem to have been investigated in what we may describe as a mainly negative and economically unspecific way. The attention has been focused, that is, either on the impossibility of establishing uniqueness and stability under the more general premises of the theory, or on some sufficient, rather than necessary conditions for such properties.[26] Similarly, the efforts seem to have been concentrated on systems of pure exchange, or of production without capital – this, despite the fact that the implications of reverse capital deepening and reswitching which had been pointed out for the traditional versions of neoclassical general equilibrium should have alerted scholars to the possible implications of those phenomena for intertemporal theory (cf. par. 2).

Thus, with regard to the *causes* of multiplicity and instability of the equilibria, recent literature does not seem to have added substantially to what, owing to a more simplified, but also better focussed analysis, had been known, since Walras, Marshall or Wicksell[27], about income effects and their causes. The paradox seems then to be that general conclusions about those properties that are at times presented as drastically negative for neoclassical theory[28] have in effect favoured the comparatively comfortable, but unwarranted belief that the difficulties in question all have their origin in income effects – with which the theory has, after all, long managed to co-exist – and thus have had little dissuasive effect on the actual practice of the profession[29].

11. Our general equilibrium demand and supply schedules may be seen as part of an attempt to remedy this situation by tackling again such central properties of the equilibria in the way Marshall, Walras or Wicksell approached them, that is by starting from specified economic conditions susceptible of causing the difficulties in order to arrive at their consequences for the equilibrium. This has led to an analysis of production with capital, and to focussing on the savings–investment market – on which reverse capital deepening impinges directly – in order to ascertain how those phenomena can affect the properties of a general intertemporal equilibrium (Figure 5.1).

A word of caution must now be added concerning our application of the method of general-equilibrium demand and supply schedules. Just because of its greater specificity, these tools of analysis bring to light questions which, apparently buried in the mathematical formalism previously used, require now[30] definite answers. Where possible, those answers have been attempted here, however provisionally. At other times the questions have been treated by referring to specific, more manageable subcases which do not however alter the generality of the *negative results* the paper is concerned with, since the sub-case is part of the general case[31]. It may incidentally be stressed that when sub-cases have been resorted to, they have been specified so as to grant more favourable conditions to the theory. In particular, care has been taken to avoid mixing the cases of non-uniqueness, instability or zero factor prices emerging here, with the altogether different cases which might be due to income effects.

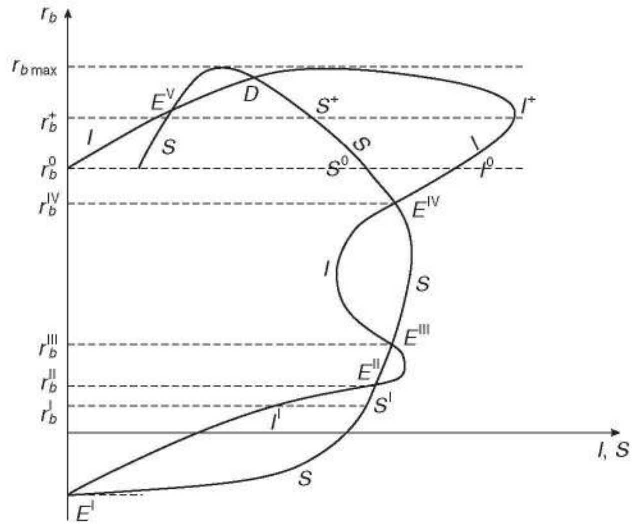

Figure 5.1 Possible shape of investment demand I and savings supply S: points E indicate intertemporal general equilibria in the model.

12. Passing now from the method to the content of our argument, the reader may ask why we have introduced the *aggregate* savings and *aggregate* investment of system (F) in order to discuss a system (E) which was in fact formulated independently of any such aggregates. The answer will of course have to come from what follows in this chapter. However, we have indicated already how the total demands of a_0 and b_0 which we find on the right-hand side of the first two relations in systems (5.3e), or (5.3f), are in fact made up of two quite heterogeneous elements each: the consumption demands D_{a0}, D_{b0} (which we assumed to be always satisfied along the I and S schedules) and the investment demands I_a and I_b. Now, the investment demands are ruled by principles that are totally different from those which govern consumption demands: hunger can be satisfied by corn, and not by coal; but desire for future income, the motive of the demand for capital goods from savers, can surely be satisfied by tractors, as well as by looms or any of the thousands of other capital goods, whichever of them offers a higher rate of return. It can here be indifferently satisfied by a_0 or b_0 lent for production. In fact, as we shall see in par. 26, different capital goods are perfect substitutes for the savers.[32]

Now, in view of the different principles thus regulating investment demands as distinct from consumption demands, and in view above all of the perfect substitutability of heterogeneous capital goods for the saver, but of course not of consumption goods for the consumer – it does *prima facie* stand to reason that the separation of the two kinds of demand and, then, the aggregation of the capital goods demanded for investment, might lend transparency to the workings of the system.

13. In par. 8 we mentioned 'stability' among the properties of the equilibrium which might be inquired into by means of our two schedules. That implies that the schedules should be applied to discuss adjustments to the equilibrium. Although only hints of that analysis will be contained in the present chapter (parr. 17–18), we should perhaps clarify what can be meant by 'adjustments' and 'stability' here, in a context of dated equilibria.

In fact, as I have argued elsewhere (1976: 38), an analysis of stability, capable of fulfilling its traditional role of ensuring 'correspondence' between theoretical and observable magnitudes, has to be founded on the possibility of a sufficient repetition of transactions on the basis of approximately unchanged data. If a tendency to equilibrium could be established on that basis, it could also be generally supposed that disequilibrium deviations would tend to compensate each other, letting the equilibrium levels emerge as some average of observable levels, and be capable, therefore, of providing some guidance to reality. Essentially, the question in this respect would be to allow for a time setting in which

> fitful and irregular causes in large measure efface one another's influence so that . . . persistent causes dominate value completely.
>
> (Marshall, 1949: p. 291)

That meaning of the positions of the economy to which theory refers its variables, and the corresponding notion of its stability, appear in fact to have been the unanimously accepted basis of economic analysis from Adam Smith, and before, until comparatively recent decades. At those earlier times, however, such a necessary repetition of markets on approximately unchanged data could be grounded in the conception of a normal position of the economy, a sufficient persistency of which was ensured by the uniform rate of return on the supply prices of the capital goods and by the corresponding adjusted composition of the capital endowment – which in neoclassical theory depends on the conception of the capital endowment as a single magnitude.[33] If the abandonment of the corresponding notion of equilibrium (fundamentally different, recall, from that of 'stationary' or 'steady states') was by itself sufficient to undercut *in fact* the previous meaning of an analysis of stability, by imposing data too impermanent to allow for a sufficient repetition of markets, the 'dating' of the equilibria has jettisoned it even in principle by excluding repetition as such.[34] This appears to leave in some obscurity the precise significance of present-day analyses of stability, quite independently of their negative results.

Our present critical purpose seems, however, to exempt us from entering further into the question and allows us to adopt the formal way out generally taken when the concern are still variables determinable by the equations of general equilibrium, and not the essentially indeterminable variables of a path-dependent equilibrium. This formal way out is, of course, that of 'recontracting', or of 'tâtonnenment', as it has come to be named with a misleading reference to Walras.[35] In the modem fictitious theoretical world into which we shall enter by means of that assumption, the repetition of transactions – thus in fact admitted to be essential for an analysis of stability – is supposed to take place in some initial 'moment', or period before the actual time, measured out by 'dated' equilibria, has rendered such repetition impossible.

14. Though the model is simple, the discussion of system [F] and the determination of the schedules become complex as soon as we wish to go beyond the mere formal demonstration of the existence of solutions of system (F) – for which see the Mathematical Note at the end of the chapter – and we attempt to gain an understanding of their properties and economic meaning. We have accordingly placed that discussion in an Appendix and shall here confine ourselves to listing the conclusions we shall use in the rest of the chapter, references being given to the Appendix for the supporting argument.

I. When we suppose, as is done above, that neither commodity can be stored, the economically meaningful interval of r_b extends from $r_{b\,\min} = -1$, for $P_{b0}/P_{b1} = 0$ (Appendix I par. [1]) to a level $r_{b\,\max}$, the highest among those for which $W = 0$ (see point IV below).

II. Changes in the intertemporal relative price $r_b = [(P_{b0}/P_{b1}) - 1]$ need not affect in one direction rather than the other total relative demands of the two contemporary commodities a_0 and b_0, i.e. the ratio $(D_{a0} + I_a)/(D_{b0} + I_b)$, and therefore the relative contemporary price P_{a0}/P_{b0} necessary to satisfy equation (5.7f). Making then the further reasonable supposition that such a contemporary relative price will not be much affected by changes in r_b, we assume that the intertemporal price P_{a0}/P_{b1} moves in the same direction as P_{b0}/P_{b1} and hence r_b in the interval $r_{b\,\min} < r_b < r_b^+$, in which $W > 0$. This suffices to ensure a unique solution in an interval $r_{b\,\min} < r_b < r_b^0$ where $r_b^0 \leq r_b^+$ is, as we shall see under point IV below, the minimum level for which we find $W = 0$ (Appendix I, parr. [5]–[7]).

III. Two other important consequences follow from that assumption. The first is that the monotonic relation between P_{a0}/P_{b1} and P_{b0}/P_{b1} entails a decreasing relation between W and the intertemporal price P_{b0}/P_{b1} and hence r_b over the whole interval $r_{b\,\min} < r_b < r_b^+$, along what we shall call 'the main branch' of the relation between r_b and the unknowns in (F), in particular between r_b and $I * S$. That 'branch' is of course where, as we saw under point II, we have unique solutions, but only up to r_b^0 and not for $r_b^0 \leq r_b \leq r_b^+$ where a continuum of positions for $W = 0$ will exist (see point IV below). The second important consequence of the monotonic direct relation between the intertemporal prices P_{a0}/P_{b1} and P_{b0}/P_{b1} is that r_a will move in the same direction as r_b allowing us to refer unambiguously to a rise or fall of the own interest rates (Appendix I parr. [8]–[9]).

IV. Should labour continue to be supplied at $W = 0$ – as would generally be necessary for ensuring the continuity of the functions and the existence of solutions of system (E) (ibid. par. [10]) – then at that zero level of W we shall find a continuum of solutions of (F) for levels of L^D of labour demanded in the interval $0 \leq L^D \leq L^S$, and for the corresponding levels of r_b in the interval $r_b^0 \leq r_b^+ \leq r_{b\,\max}$ where, with r_b^0 as the minimum such level, $r_{b\,\max} \geq r_b^0$ is the maximum one (ibid. par. [6]; see also Figure (5.2)).

V. The minimum level $r_{b\,\min} = -1$ corresponding to the *intertemporal* price $P_{bo}/P_{b1} = 0$ does not entail, as one would perhaps expect, that b_0 is a free commodity so that, for example, also $P_{b0}/P_{a0} = 0$. On the contrary, there are reasons which force us to admit that, as $P_{b0}/P_{b1} \to 0$, the *contemporary* relative price P_{b0}/P_{a0} will generally tend to a positive and finite level, with all commodities available in $t = 0$ having zero price in terms of all commodities available in $t = 1$ (ibid., parr. [7] and [12]).

VI. As for the shape of the two schedules (see Figure 5.1) what we can say generally is that: (i) the choice of b_1 as our numéraire entails, as just said, that for $r_b = r_{b\,\min} = -1$ both P_{a0} and P_{b0} are zero, so that both the S and I schedules intersect the vertical axes at $r_b = -1$; (ii) as we saw under point III a 'main branch' of the S and I schedules will exist in the interval $r_{b\,\min} \leq r_b \leq r_b^+$: along it W will decrease from $W_{\max}$ for $r = r_{b\,\min}$ down to zero for r_b^+; (iii) the two schedules will then continue beyond r_b^+ in order to represent the continuum of positions indicated under point IV above, with the I schedule finally reaching the vertical axis with $L^D = 0$ (ibid. par. [14]); (iv) it does not seem however that anything general can be said about the overall shape of the schedules in the intermediate interval $0 \leq r_b \leq r_b^0$ except for the single-valued character of the schedules seen under point II and the bias towards rising S and I schedules due to our choice of b_1 as numéraire, and therefore of P_{a0} and P_{b0} rising as r_b rises (a bias which is of course innocuous as far as the properties of the equilibria are concerned, which only depend on the ratio S/I at each level of r_b); (v) in particular, the impossibility of supposing a necessarily falling I schedule is not however due to that bias. Nor is it due to the fact that alternative techniques have not yet been considered: possibilities of substitution between labour and means of production are already present in the system because of consumer choice between a_1 and b_1. The essential reason why a falling I schedule cannot be assumed will be seen below (par. 19) and is the same as for the phenomenon of reverse capital deepening, familiar from the traditional analysis. No confusion should in fact be caused by the presence of two consumption goods which could conceivably engender a rising I schedule because of income effects: as we make clear by one of the assumptions on which our argument is based (Assumption (IIIa), ibid. par. [5]), our conclusions are independent of any income effects.

5.4 THE REPRESENTATION OF THE INTERTEMPORAL SYSTEM

15. What we must now see, therefore, is how the two schedules can aid our understanding of the behaviour of system (E). In this and the next paragraph we shall see how the schedules can represent the equilibria of the system and then, in the following two, we shall consider the information they can provide on out-of-equilibrium behaviour.

A 'position' (F) of the system (i.e. the solution of (F) for a particular value of r_b) will also be an equilibrium (i.e. a solution of E) when the first two relations in (5.3e) of par. 3, concerning the aggregate demand and supplies of a_0 and b_0 happen to be satisfied: all other relations of (E) are in fact already present in (F). Leaving aside at first the case of 'extreme' equilibria occurring, that is, for $r_{b\,\min}$ or for $W = 0$, it can be asserted that when the system is in equilibrium the two schedules S and I intersect, and that the converse is also true.

As for the first proposition, when r_b is in that intermediate interval, or also in the upper interval $r_b^+ \geq r_b \geq r_b^0$ along the 'main branch' of the S and I functions (point III in par. 14), and P_{a0}, P_{b0} are accordingly strictly positive, any solution of (E) entails that the first two relations in (5.3e) of par. 3 above are satisfied with an equality sign,[36] i.e.

$$A^S = A^D \quad \text{and} \quad B^S = B^D \tag{5.3a}$$

and hence (see relations (5.3f′) in par. 7)

$$I_a = S_a, \qquad I_b = S_b \tag{5.3a}$$

and

$$I_a P_{a0} + I_b P_{b0} = S_b P_{b0} + S_a P_{a0}, \quad \text{i.e.} \quad S = I. \tag{5.3b}$$

The general equilibrium of the system thus entails an intersection of the two schedules.

As for the converse proposition, when we have an intersection of the schedules in that same interval (see Figure 5.1, par. 11), equation (5.3b) is satisfied and therefore, after adding $D_{a0}P_{a0} + D_{b0}P_{b0}$ to both sides, we obtain

$$A_0^D P_{a0} + B_0^D P_{a0} = A_0^S P_{a0} + B_0^S P_{b0} \tag{5.3c}$$

Indicating then, by the constant γ, the common value or the ratios appearing on the two sides of equation (5.7f), we may write equation (5.3c) above as follows:

$$A_0^S P_{a0} + \gamma A_0^S P_{b0} = A_0^D P_{a0} + \gamma A_0^D P_{a0}; \text{ i.e. } A^S(P_{a0} + \gamma P_{b0}) - A^D(P_{a0} + \gamma P_{b0}) \tag{5.3d}$$

from which $A_0^S = A_0^D$ and hence, from equation (5.3c), $B_0^S = B_0^D$, thus fulfilling all relations (5.3e) in system (E), and ensuring that we are in a general equilibrium position.

16. Turning now to the representation of possible 'extreme' equilibria of the system, we may note that equilibria in the upper interval $r_b^0 \leq r_b \leq r_{b\max}$ (see Figure 5.2) will also be shown by intersections of the two schedules, but the converse proposition will not be true. Since different (F) positions may correspond to the same level of r_b, an intersection between I and S may occur in that interval at a point representing an (F) position on the I schedule, and a different one on the S schedule. The intersections representing equilibria have then to be traced by checking whether the I and S points of the intersection pertain to the same (F) position. This will be possible in the diagram because, starting, for example, from points like I^+ and S^+ (see Figure 5.2), the two schedules will go through exactly the same values of r_b in exactly the same sequence: 'couples' of I and S points corresponding to the same (F) position can therefore easily be singled out.[37] The reasoning conducted in the preceding paragraph will then apply to those points of intersections which represent the same (F) position on both schedules.

As we proceed to $r_{b\min}$, at the opposite extreme, although the zero intertemporal prices P_{a0}, P_{b0} yield $S = I = 0$, we shall generally have definite non-zero physical quantities I_a, I_b, S_a, S_b (par. [13] in Appendix I). We need first of all to note here that the assumption about both goods being scarce for consumption in $t = 0$

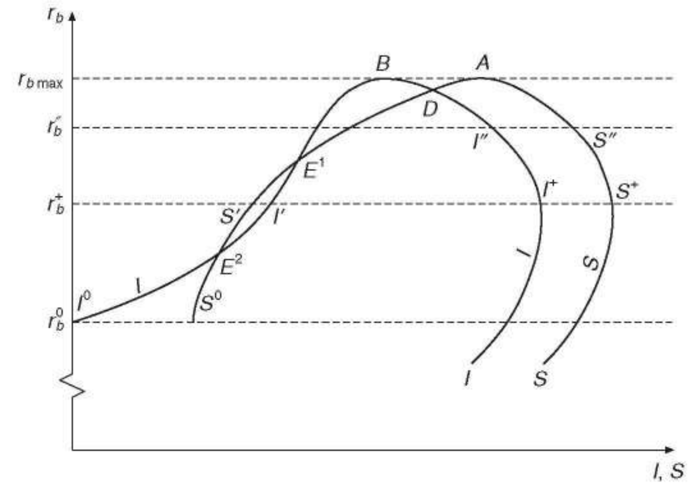

Figure 5.2 The upper reaches $I^+ I^0$ and $S^+ S^0$ of the I and S schedules representing positions (F) for $W = 0$ characterised by different levels of labour employment and therefore of the investment required to equip them.

as well as in $t = 1$ (see Assumption (ii) in par. [3] of Appendix I) entails that the position (F) for $r_b = -1$ is the one characterised by the 'collective' zero intertemporal price of all commodities available in $t = 0$ relative to those of $t = 1$, which we mentioned under point VI of par. 14 and discussed in parr. [7] and [12] of Appendix I.

Now, that (F) position is an equilibrium when, as $r_b \to r_{b\min}$, $S_b > I_b$. At that position savings would generally exceed investment when expressed in contemporary prices by means of either a_0 or b_0. Then, by a reasoning analogous to the one conducted in par. 15 for equation (5.3c) and (5.3d) we could in fact conclude $S_a > I_a$; $S_b > I_b$ and therefore we have an 'extreme equilibrium' with $A_0^S > A_0^D$; $B_0^S > B_0^D$. When however $S < I$ as $r_b \to r_{b\min}$ no equilibrium will generally exist at $r_{b\min}$.[38]

17. While thus representing the equilibria of the system, the two schedules can, as we said, provide elements for a discussion of its out-of-equilibrium behaviour.

Suppose first an (F) position for $r_b = r_b'$ in the interval $r_{b\min} < r_b' < r_b^0$, such that $S' > I'$ (see Figure 5.1, par. 11). As we just saw, the inequality $S > I$ entails $A_0^S > A_0^D$ and $B_0^S > B_0^D$. It would then seem natural to suppose an 'initial' reaction in the markets for a_0 and b_0, more directly affected by the disequilibrium, which would occur *before* adjustments can take place in connected markets: in our case under the reasonable assumption of excess supply for both commodities, that 'initial' competitive reaction could only be a fall of intertemporal prices P_{a0}/P_{b1} and P_{b0}/P_{b1}. However, the connected markets will then tend to adjust, so that we may envisage an out-of-equilibrium behaviour in the recontracting dominated by movements centring on the two general equilibrium schedules. In this respect equation (5.7f), assuming a proportionate change of the algebraic excess demands of a_0 and b_0, seems to be as reasonable an assumption as can be made at a general level: it may indeed be taken to represent a condition of 'even flexibility' of the price system, in the sense of allowing for the excess demands of a_0 and b_0 to change in the same proportion. Our critical aim strengthens on the other hand the legitimacy of assuming that the dominant out-of-equilibrium movement will be along the schedules: if deficiencies of the demand and supply apparatus result under that assumption, they would seem to be all the more plausible when obstacles to the adjustments to equilibrium are also considered in the connected markets, which the schedules assume instead to be broadly kept in equilibrium.

Then, as we start moving along the schedules, the 'initial' fall of both P_{a0}/P_{b1} and P_{a0}/P_{b0} in response to the assumed excess savings will result in a movement downward along the schedules with a fall of *both* own rates r_a and r_b (see point III in par. 14). This result can indeed be taken to be general – largely independent, that is, of our assumption about the monotonic rise of P_{a0}/P_{b1} with P_{b0}/P_{b1} (cf. point II in par. 14, and par. [8] in Appendix I).[39]

Similarly to what could be argued in any partial equilibrium use of the schedules – we have then elements for arguing a tendency e.g. toward equilibrium E^{III} in Figure 5.1 of par. 11, where the I schedule cuts the S schedule from above and where therefore any given initial position in the interval $r_b^{III} < r_b < r_b^{IV}$, r_b can be expected to fall, and to rise for any r_b in $r_b^{II} < r_b < r_b^{III}$. A tendency away from equilibrium can instead be argued when, as we move from left to right, I cuts S from below (see E^{II} and E^{IV} in Figure 5.1).[40]

18. As we turn to 'extreme' values of r_b, we may note that if we happened to have $S > I$ in the proximity of $r_{b\,\min} = -1$, the fall of r_b, which we can assume in the presence of $S > I$, would imply competitive recontracting to tend to the equilibrium with the zero intertemporal (but not contemporary) prices of both a_0 and b_0 we saw in par. 16.[41]

As we could expect, some novel problems are met when we shift our attention to the upper extreme for $r_b^0 \leq r_b \leq r_{b\,\max}$. We may leave aside the 'main branch' of the S and I schedules (where with $L = L^D$ and $W > 0$ we have all the conditions for the out-of-equilibrium behaviour considered in par. 17). In all other (F) positions of that interval, we shall have excess supply of labour with $L > L^D$ and a zero wage. The I schedule will tend to finally extend leftward so as to reach the vertical axis for the (F) position corresponding to $L^D = 0$ (cf. par. [14] in Appendix I) and, as we just saw in par.16, any equilibrium will be shown by the two schedules intersecting, and intersecting for the same (F) position.

Outside any such equilibria, if in the given position (F) we have $I < S$ (see e.g. points I'' and S'' in Figure 5.2) it would be natural to suppose that the excess savings, i.e. the excess supply of a_0 and b_0, will cause the 'initial' fall of both P_{a0}/P_{b1} and P_{b0}/P_{b1} we admitted for intermediate levels of r_b.

However with $W = 0$ and the available excess supply of labour making it possible to expand outputs, that temporary 'initial' fall of the prices of a_0 and b_0, relative to those of their outputs a_1 and b_1 would make it profitable for entrepreneurs to raise outputs, and as adjustments occur we would tend to move along the I schedule towards the right in the direction of an increase of the labour employment L^D and therefore of the investment required to equip that labour. Prices P_{a0} and P_{b0} will have to move in the opposite direction and so may therefore do r_a and r_b. Unless an equilibrium were to be met in the process, that increase of L^D would continue until full labour employment is reached in position (F^+) for $r_b = r_b^+$. Excess savings and the consequent persisting 'initial' fall of P_{a0}/P_{b1} and P_{b0}/P_{b1} would then result in a positive wage W, and a fall of r_b: the (F) positions would become those already discussed in par. 17, characterised by $W > 0$.

In the case in which, in that same upper interval of r_b and for $W = 0$, we instead had $I > S$ in the (F) position – as exemplified by points I' and S' in Figure 5.2, for the same level r_b'' of r_b of the just discussed position (F) with excess savings – then, an opposite process of decreases of employment L^D and investment would have to be expected. It would lead leftwards along the I schedule to an equilibrium which would then have to exist. As we just recalled, investment I has in fact to change continuously down to zero, while S also changes continuously, though generally without reaching zero. Then, with I starting to the right of S and having to pass finally to its left while going through exactly the same levels of r_b, it is inevitable that the two schedules will cross – as exemplified by E^2 to the left of the points I' and S' in Figure 5.2.

5.5 ALTERNATIVE TECHNIQUES AND THE INVESTMENT DEMAND

19. System (F), like system (E) which generates it, still rests on the assumption that only one method of production is available for each commodity. It is time to drop that assumption and consider the existence of several alternative methods for each commodity, all sharing the properties we mentioned in par. 3. We may call a set of 'methods of production', one for the commodity in question and one for each of its (direct and indirect) means of production, 'the system of production' or 'technique of production' of the commodity. Here, the 'technique' or 'system' of production of the commodity will accordingly include two 'methods of production', one for the commodity and one for the other commodity as means of production of the former. Thus one 'technique' for producing a, will also be a 'technique' for producing b, and we may therefore refer to techniques $i = 1, \ldots, n$ without mentioning the commodity they refer to.

Despite our assumption that all alternative methods of production require the same three factors, there is no assurance that marginal products, even of the discontinuous kind, will exist.[42] For the determination of the method that would be the cheapest at current prices and would therefore be chosen by competitive entrepreneurs, we must therefore resort to the more general approach we find in Sraffa's *Production of Commodities*: namely, comparing the expenses for producing the commodity by the alternative methods. However, at each level of r_b the comparison can only be done in terms of prices P^i_{a0}, P^i_{b0} and the wage W^i holding for the particular technique i 'in use' – meaning here by 'in use' that the technique is the one whose adoption we assume to be generally planned at the stage reached by the recontracting.[43] Maximisation of entrepreneurial profits will then entail that the recontracting proceeds to any method for each commodity which happens to be cheaper at those prices.

A question which is well known from the 'traditional', non-intertemporal assumptions then arises, about whether the *order* of the alternative methods of production of the commodity as to cheapness, might not itself change with the technique 'in use': with the possibility of either endless switching between techniques, or of the technique finally adopted depending on the one initially 'in use'. Our critical intent, however, will again allow us to grant the assumptions most favourable to the theory criticised and therefore to assume what has been demonstrated to be true under the traditional assumptions: that the order of cheapness of an alternative method is the same, whichever the technique in (planned) use at the given level of r_b.[44] We can thus suppose that entrepreneurs' choice will always arrive at one and the same method(s) for each of the two commodities, so that at any given r_b the cheapest technique(s) or 'system(s) of production' can be uniquely determined, together with the corresponding series of the prices, the outputs, and the I and S quantities, where the plurals above take care of the possible co-existence at some r_b of two (or more) methods for the same commodity and hence of the technique(s) differing from i by the method of that single commodity which will then entail the same wage and prices for the given level of r_b.

20. We can therefore proceed to reporting below the family (Fi) of systems of equations defining the two schedules under the assumption of a multiplicity of techniques of production $j = 1, 2, 3 \ldots$. Each member of that family is a system of equations like (F) defined in par. 6, but applied now to the technique i which happens to be the one no dearer than any other at the given level of r_b. Thus, to any level r_b in its relevant interval there will correspond a system (Fi) containing, as well as the relations (F) pertaining to the technique i adopted, as many quadruplets of relations, (5.8fi) and (5.9fi), as there are alternative 'techniques' or 'systems of production' $j \neq i$. The first two equations (i.e. 5.8fi) reckon the production expenses of a_1 and b_1 with the respective methods j. The second couple of relations, namely (5.9fi), states that no method j for producing each of the two commodities is cheaper than the method pertaining to the technique i 'in use'.[45]

$$\begin{cases} P^i_{a1} = l_{ai}W^i + a_{ai}P^i_{a0} + b_{ai}P^i_{b0} \\ P^i_{bi} = l_{bi}W^i + a_{bi}P^i_{a0} + b_{bi}P^i_{b0} \end{cases} \qquad (5.1\text{fi})$$

$$P^i_{bi} = 1 \qquad (5.2\text{fi})$$

$$\begin{cases} A_0^{Di} = D^i_{a0} + (a_{ai}A^i_1 + a_{bi}B^i_1) \\ B_0^{Di} = D^i_{b0} + (b_{ai}A^i_1 + b_{bi}B^i_1) \\ L \geq l_a A^i_1 + l_b B^i_1 \\ A^i_1 = D^i_{a1} \\ B^i_1 = D^i_{b1} \end{cases} \qquad (5.3\text{fi})$$

$$I^i = (a_{ai}A^i_1 + b_{bi}B^i_1)P_{a0} + (b_{ai}A^i_1 + b_{bi}B^i_1)P_{b0} \qquad (5.4\text{fi}) \qquad (\text{Fi})$$

$$S^i = (A_0^{Si} - D^i_{a0})P^i_{a0} + (B_0^{Si} - D^i_{b0})P^i_{b0} \qquad (5.5\text{fi})$$

$$r_b = (P^i_{b0}/P^i_{b1}) - 1 \qquad (5.6\text{fi})$$

$$A_0^{Di}/B_0^{Di} = A_0^{Si}/B_0^{Si} \qquad (5.7\text{fi})$$

$$\begin{cases} P^i_{aj1} = l_{aj}W^i + a_{aj}P^i_{a0} + b_{aj}P^i_{b0} \\ P^i_{bj1} = l_{bj}W^i + a_{bj}P^i_{a0} + b_{bj}P^i_{b0} \end{cases} \quad \text{any } j \neq i \qquad (5.8\text{fi})$$

$$P^i_{a1} \leq P^i_{aj1}, \qquad P^i_{b1} \leq P^i_{bj1} \quad \text{any } j \neq i, \qquad (5.9\text{fi})$$

where i at the exponent indicates that the variable in question is calculated under the assumption that technique i is being planned for use at the given r_b, whereas j at the deponent indicates the alternative technique to which there pertains the variable in question, coefficient of production or price based on such coefficients (no deponent has been given to the same variables pertaining to technique i). The equality signs in (5.9fi) take care of the possible co-existence between technique i and other techniques for same levels of r_b, when the equality sign will apply to both the relations (5.9fi).

Correspondingly, also system (E), determining the equilibrium of the system, should now be written in the form of the following family of systems (Ei) allowing for alternative techniques:

$$\begin{cases} P^i_{a1} = l_{ai}W^i + a_{ai}P^i_{a0} + b_{ai}P^i_{b0} \\ P^i_{b1} = l_{bi}W^i + a_{bi}P^i_{a0} + b_{bi}P_{b0} \end{cases} \tag{5.1ei}$$

$$P^i_{b1} = 1 \tag{5.2ei}$$

$$\begin{cases} A_0 \geq D^i_{a0} + [(a_{ai}A_1 + a_{bi}B_1)] & \text{if } >\text{, then } P_{a0} = 0 \\ B_0 \geq D^i_{b0} + [(b_{ai}A_1 + B_{bi}B_1)] & \text{if } >\text{, then } P_{b0} = 0 \\ L \geq l_{ai}A_1 + l_{bi}B_1 & \text{if } >\text{, then } W = 0 \\ D_{a1} = A_1 \\ D_{b1} = B_1 \end{cases} \tag{5.3ei}$$

(Ei)

$$\begin{cases} P^i_{aj1} = l_{aj}W^i + a_{aj}P^i_{a0} + b_{aj}P^i_{b0} \\ \qquad\qquad\qquad\qquad \text{any } j \neq i \\ P^i_{bj1} = l_{bj}W^i + a_{bj}P^i_{a0} + b_{bj}P^i_{b0} \end{cases} \tag{5.8ei}$$

$$P^i_{aj1} \leq P^i_{aj1}, \qquad P^i_{b1} \leq P^i_{bj1} \qquad \text{any } j \neq i, \tag{5.9ei}$$

where, as for (E) in par. 3, the relations corresponding to equations (5.4fi), (5.5fi), (5.6fi) and (5.7fi) do not need to appear, $S = I$ being implied in (Ei).

21. The main question which the existence of alternative methods of production raises for us here is the changes in the investment requirements I due to changes in the cheapest technique as r_b varies. We might perhaps expect that owing to those changes (as well as to those of the relative outputs A_1/B_1 already determined in system F) the schedule I would generally show a negative slope. However, such an expectation has no better foundation for the present investment–demand schedule than it had for the capital–demand schedule in the 'traditional' setting.

A simple line of reasoning seems sufficient to show this. As has been pointed out,[46] the roots of reverse capital deepening, as well as those of the re-switching of techniques, lie in the effect of changes in distribution (rate of profits) upon the *relative value* of the alternative sets of capital goods required by the processes of production which are being compared – whether such processes are alternative methods of production for the *same* consumption good, or the methods for two different consumption goods. In the traditional, non-intertemporal setting, it is the changing relative value of such two sets of capital goods that can make a more 'capital-intensive' technique become more profitable, or a more capital-intensive consumption good fall in price, as the interest rate rises. And it is that same change in the relative value of the alternative sets of capital goods that can bring about 're-switching' among alternative techniques. Now, the same variability of the relative value of alternative sets of capital goods is clearly present in an intertemporal setting.

To see the thing in more definite terms, let us consider first the case of alternative techniques as distinct from that of competing consumption goods. From equations (5.8fi) and (5.9fi) we may see how, at the given level of r_b, the choice between the cheapest technique i and any other alternative technique j differing, say, by the method of production of a alone, hinges on the following relative costs of the two methods:

$$\frac{P^i_{ai1}}{P^i_{aj1}} = \frac{l_{ai}W^i + (a_{ai}P^i_{a0} + b_{ai}P^i_{b0})}{l_{aj}W^i + (a_{aj}P^i_{a0} + b_{aj}P^i_{b0})}, \tag{5.10}$$

the method of technique i for a_1 being more profitable than the j one at the given r_b when $P^i_{ai1}/P^i_{aj1} < 1$. Defining now

$$C^i_{ai} = a_{ai}P^i_{a0} + b_{ai}P^i_{b0},$$
$$C^i_{aj} = a_{aji}P^i_{a0} + b_{aji}P^i_{b0},$$

where the C^i_a's are the respective capital expenses estimated at the given level of r_b and for i prices. We may then write the relative production expenses (5.10) of the two methods as follows:

$$\frac{P^i_{ai1}}{P^i_{aji1}} = \frac{l_{ai}W^i + C_{ai}}{l_{aj}W^i + C^i_{aj}}. \tag{5.10a}$$

With (C^i_{ai}/l_{ai}) and (C^i_{aj}/l_{aj}) as the respective ratios of capital to labour in the (direct) production of a_1, we then have

$$\frac{P^i_{ai1}}{P^i_{aj1}} = \frac{W^i + (C^i_{ai}/l_{ai})}{W^i + (C^i_{aj}/l_{aj})} \cdot \frac{l_{ai}}{l_{aj}}. \tag{5.10b}$$

Assume now, without loss of generality, that $C^i_{ai}/l_{ai} > C^i_{aj}/l_{aj}$. Should the C^i_a's be measured so that they are independent of changes in W, clearly P_{ai1}/P_{aj1} could only fall as W rises (and r_b falls: point III, par. 14). Any changes of method could only be in favour of the more "capital intensive" one (from j to i in our case). Since however the C^i_a's and therefore the key ratio C^i_{ai}/C^i_{aj} are not so independent, the rise of W (fall of r_b) need *not* entail the fall of P^i_{ai1}/P^i_{aj1} we might have expected: a sufficient rise of C^i_{ai}/C^i_{aj} may well make P^i_{ai1}/P^i_{aj1} rise, and not fall as W^i rises. This means that the rise of W may well result in the less capital-intensive method j becoming the more profitable one of the two, and therefore being adopted.

The same change of C^i_{ai}/C^i_{aj} in the relative value of the sets of capital goods of the two alternative processes of production may entail, as can be shown by replacing P^i_{aj1} with P^i_{bi1}, that the less 'capital-intensive' consumption good b_1 may become cheaper relative to a_1 as W rises (r_b falls) so that regular substitution in consumption has "perverse" effects on factor demands. Hence the freedom with which we were able to draw the shape of the I schedule in our Figures 5.1, 5.2 or 5.3.[47]

5.6 SOME CONCLUSIONS

22. We can now use the schedules to see some properties of a general intertemporal equilibrium and thereby how 'capital' as a single 'quantity' enters intertemporal equilibrium. The question may usefully be approached by showing how misleading is the widespread idea is that savings and investment in an intertemporal equilibrium raise no more problems than do relative demands for contemporary commodities and can therefore be subsumed under a single theory of consumer choice.[48]

It is of course true that if we assume no capital to be left at the final date, the (gross) savings at t must consist of demand for consumer goods at future dates $(t + \tau)$, at the expense of demand for the same or other consumer goods at t. It is then equally true that any excess of saving decisions over investment decisions at t must necessarily take the form of an excess supply of consumer goods at t, and an excess demand for them at future dates. Thus, imagine that initial recontracting had brought the economy to the position ($\hat{F}$) of quantities and prices which the system (Fi) associates with the

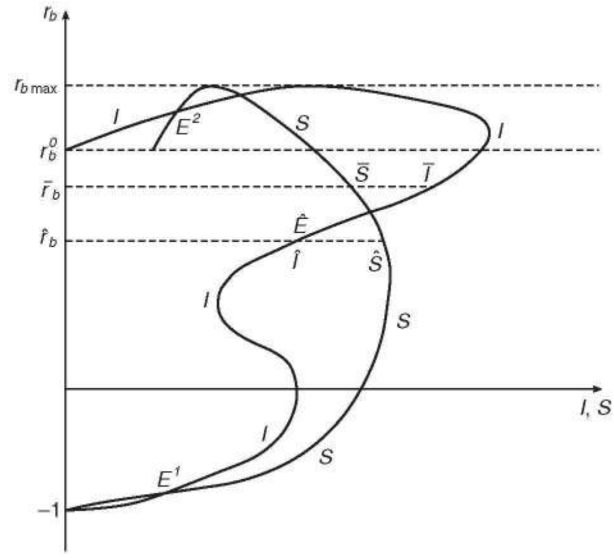

Figure 5.3 Starting from $\hat{r}_b$, equilibrium would only be found at E^1, with a negative rate of interest, just as starting from $\bar{r}_b$ the equilibrium would be reached at E^2, with W = 0 and labour unemployment.

interest rate $\hat{r}_b$ (see Figure 5.3), and suppose that $(\hat{F})$ would coincide with an equilibrium $(\hat{E})$ except for a positive excess of savings $\Delta S = (\hat{S} - \hat{I})$, $(\hat{S})$ and $(\hat{I})$ being estimated at the prices of $(\hat{F})$.

From the households budget equations in (Fi) for $\hat{r}_b$ we obtain

$$\Delta\hat{S} = -(\hat{P}_{a0}\Delta D_{a0} + \hat{P}_{b0}\Delta D_{b0}) = \hat{P}_{a1}\Delta D_{a1} + \hat{P}_{b1}\Delta D_{b1}, \tag{5.5b}$$

where the ΔD's indicate the differences in the respective quantities demanded between positions $\hat{F}$ and $\hat{E}$ (where savings would have been equal to $\hat{I}$); which, for simplicity, we have supposed to be *both* negative in $t = 0$ and positive in $t = 1$.[49] Undoubtedly, equation (5.5b) *looks* similar to that holding in the case of contemporary commodities, should $(\hat{F})$ have failed to be an equilibrium simply because of an excess demand of b_0 relative to a_0, giving

$$-\hat{P}_{a0}\Delta_{a0} = \hat{P}_{b0}\Delta D_{b0}. \tag{5.5c}$$

23. However, the analogy between equations (5.5b) and (5.5c) remains at the surface of the two phenomena and hides a basic difference between them which emerges as soon as we consider the adjustments which should lead to a new equilibrium and therefore the forces warranting it in the two cases. That basic difference can be best brought out if, for a moment, we extend our two-year model to the three years (−1), (0), (1), with the commodities a_0 and b_0 accordingly coming from production in $t = -1$ by means of L_{-1} labour and A^S_{-1}, B^S_{-1} initial stocks.

Now for the contemporary commodities of equation (5.5c), the question of achieving a neighbouring equilibrium will be the comparatively simple one of shifting the labour and means of production of $(t - 1)$ freed by $(-\Delta D_{a0})$ to producing ΔD_{b0} and no obvious obstacle stands in the way of achieving that as a consequence of the competitive rise of P_{b0}/P_{a0}, which would plausibly follow from initial competitive bidding in the situation.

The position is entirely different in the intertemporal (savings/investment) case of equation (5.5b). Obviously, it will *not* be possible to shift the labour and the means of production of period $t = -1$, set free by the reduced consumption of $t = 0$, to *directly* producing the increments ΔD_{a1}, ΔD_{b1} of equation (5.5b): the labour and means of production of $t = -1$ are not those of $t = 0$, which *can* directly produce ΔD_{a1} and ΔD_{b1}. Even less will it be possible to devote to the direct production of ΔD_{a1} and ΔD_{b1} any of the labour and means of production of $t = 0$, which *could* directly produce them, but unlike those of $t = -1$ are assumed to be already fully employed. No competitive rise of P_{b1}/P_{b0} plausibly following from the relative rise of consumption demands in $t = 1$ can achieve either of those two feats.

How, then, can we raise the $t = 1$ outputs and consumptions and, moreover, do so at the expense of the consumptions of $t = 0$, as required by the excess savings of equation (5.5b)? The answer clearly remains that of traditional, non-intertemporal theory. This change of relative outputs over time can only be achieved by raising the gross productivity of the already fully employed labour L_0, by means of an increase, in some sense, of the quantity of means of production cooperating with it. It is a question, that is, of producing in $t = -1$ quantities ΔI_a and ΔI_b while decreasing production by quantities ΔD_{a0} and ΔD_{b0}, and then using the increments of investment with the constant quantity of labour L_0 to produce increments ΔD_{a1} and ΔD_{b1} of consumption. But, and here comes the essential point, those increments of investment can only be motivated by the rise of the intertemporal prices of a_1 and b_1, relative to b_0 and a_0, i.e. by the fall of the interest rates: no question of the increments of investment being caused *directly* by the additional consumptions ΔD_{a1}, ΔD_{b1} entailed in the savings of equation (5.5b) – contrary to the case for the contemporary consumptions of equation (5.5c).

The idea that savings and investment in intertemporal equilibrium cause no more problems than relative demands for (contemporary) commodities should rather be turned upside down into the one that intertemporal equilibrium raises the problem of savings and investment in basically the same terms as traditional equilibrium does.

24. The problem then is of course that what the capital controversies taught about 'capital reversing' and 're-switching' in traditional equilibrium – and has been confirmed for the present context in par. 19 – entails that the rise of the intertemporal prices $P_{i,t+1}/P_{i,t}$, $(i = a, b)$, i.e. a fall of the own rates of interest $r_{i,t}$ might fail to provide the entrepreneurs with a motive for that increase 'in some sense' of I_t which is required for the *intertemporal* adjustments in consumption. The result might then be the striking one that, however small the initial excess savings, the theory could force us to admit movement to an equilibrium with drastic changes in wages and prices (cf. in Figure 5.3 above, the equilibrium E^1 to which there would be a tendency starting from the position $\hat{F}$ for $\hat{r}_b$ in our original two-year model). Further, if the position $\hat{F}$ with excess savings happened to be that for $\hat{r}_b < r_b''$ in Figure 5.1 (par. II, p. 125) the theory would force us to admit a tendency to negative interest rates if not to the zero intertemporal prices P_{a0} and P_{b0} for our economy with non-storable goods.[50] And as indicated in par. [12] of Appendix I such zero intertemporal prices far from being a result of satiety, would mean that the attempt of some individuals to take care of even more acute scarcities in $t = 1$ runs counter to the inability of the market forces, as envisaged in the theory, to transfer consumption from $t = 0$ to $t = 1$.[51]

Alternatively we might find an equilibrium in which it is W_0 which has to become zero[52], with the attending labour unemployment, when the initial contracting had brought to a level r_b for which $\bar{I} > \bar{S}$ (see in Figure 5.3), the equilibrium E^2 to which there would be a tendency when the economy happened to start from $r_b = \bar{r}_b$.[53]

25. The above difference between the cases of contemporary and intertemporal consumptions allows us to begin seeing how the concept of a quantity of capital enters intertemporal general equilibrium. What we have analysed in parr. 23–24 is no less and no more than the process of substitution between labour and "capital", the single factor, which underlies the determination of distribution between wages and profits in the traditional versions of the theory, exemplified by our reference to Wicksell's *Lectures* in par. 2. The process was there viewed as involving the entire capital and labour endowments, whereas here it has been dealt with from the viewpoint of flows and not funds. But if 'capital' was what Wicksell and traditional theorists dealt with for their determination of wages, interest and prices, then – as indeed we foresaw in par. 2 – 'capital' is also what we have dealt with when studying the forces that should correct a surplus or deficit of savings over investment, thus determining the prices, in particular the own interest rates, of our simple intertemporal equilibrium and their properties.

Thus we duly found there the central relevance of an inverse relation between the own interest rates and the 'amount' of means of production, i.e. of capital per worker in the economy: as we saw, only if that inverse relation holds, can entrepreneurs operate adjustments towards plausible equilibria in intertemporal consumptions. And that inverse relation is of course the same which was required to ensure adjustments to plausible equilibria in the market for 'capital' and the other factors in the traditional version of the theory. Also the consequences of the impossibility of measuring that 'amount', i.e. 'capital', in terms independent of distribution are then essentially the same as in the traditional versions of the theory. Also the consequences of the impossibility of measuring that 'amount', i.e. 'capital', in terms independent of distribution are then essentially the same as in the traditional versions of the theory. As we recalled in par. 21, that impossibility undermines the inverse relation between interest rates and capital intensity and therefore the uniqueness, stability and overall plausibility of the resulting equilibria.[54]

No important difference is on the other hand made by the fact that in the intertemporal versions the inverse relation concerns a set of own rates (intertemporal relative prices) rather than the traditional single interest rate. A tendency of the own rates to move in the same direction can be argued and, more importantly, any contrasting movements among the own rates can be seen to be due to intratemporal phenomena hardly relevant for intertemporal adjustments (cf. parr. [7]–[9] in Appendix I).

26. This re-emergence of the neoclassical need for a 'quantity of capital' in intertemporal equilibria goes back ultimately to the basic fact that the demand for capital goods obeys principles altogether different from those governing the demand for consumption goods (par. 12). Whereas the latter comes from preferences that are specific to the goods demanded, the former results from savers' preferences that are non-specific to the individual capital goods, and are only specific with regard to the *aggregates* of them. This is lucidly expressed by Walras when he introduces savings in his general equilibrium as the demand of the particular commodity which he calls 'perpetual future income',[55] with a price of its own which is the reciprocal of the (effective) interest rate. That of course means that, in proportion to their value, capital goods are perfect substitutes for the saver as means of transferring income in time.[56]

The question of a 'quantity of capital' in neoclassical theory is indeed no more than an application of Jevons's 'law of indifference' concerning the single competitive price and hence a single magnitude by which we must refer to any commodity which individuals, in this case wealth holders, treat as homogeneous. Just as one chooses according to the principle of the minimum price between alternative sources of, e.g. "corn" – so one chooses between the alternative sources of 'perpetual net income', the different capital goods, according to the minimum price of such 'future income' – i.e. the maximum of the effective net rates of return. Capital goods are no more distinct commodities for savers than physically homogeneous "corn" from different farms is for its consumers.

In the case of capital goods, this basic fact is however obscured by a second fact, which mixes the market for the single commodity 'future income' with a second layer of markets and where, furthermore, a quite different kind of substitutability emerges among capital goods. Unlike homogeneous corn where the allocation of consumers' demand among different merchants or farms is theoretically uninteresting beyond the application of Jevons's law of the tendency to a single price, the allocation of the demand for future income among potential sources of supply involves nothing less than the entire theory of production – in order to determine the rentals of the several capital goods and their ratio to the respective supply price. This gives central visibility to that allocation and substitutability in production which, however, is ultimately no more relevant for an explanation of the rate of interest, or system of own rates of interest (intertemporal relative prices), than the analogous allocation of the aggregate demand for corn among the several farmers is for the explanation of the price of corn. Just as it is only the *aggregate* demand and the supply of corn that is relevant for determining the price of corn, so it is only the demand and supply of aggregates of capital goods that is relevant for the determination of the rates of interest.

27. The presence *at some stage* of the theory of a quantity representing aggregates of capital goods is therefore as inevitable for the neoclassical determination of prices on the basis of the demand and supply decisions of individuals,[57] as is the presence of the quantity of each consumption good. Individual demand and supply decisions about capital goods are ultimately taken in terms of that 'quantity' just as the similar decisions about corn consumption are taken in terms of the quantity of corn. It is on a single commodity 'capital' and not on individual capital goods that savers' preferences operate, whether in the traditional 'fund' context, or in the intertemporal 'flow' context.

The traditional versions of the theory, with their equilibrium condition of a uniform effective net rate of return on the supply prices of the capital goods, entailed already taking care of that single commodity at the level of the endowment of factors,[58] and those authors did so by expressing the endowment of capital *directly* as a single 'quantity of capital', its allocation in a capital-goods vector being then an unknown of the system. The abandonment of that traditional long-period notion of the equilibrium – not to be confused, recall, with a stationary or steady state[59] – and of its specific condition of a uniform effective rate on capital good supply prices has meant getting rid of the single commodity 'perpetual net income' and of its bearer, 'homogeneous capital', but only at the cost of assuming away, at the level of the factor endowments, the perfect substitutability of capital goods for the saver, and Jevons's indifference law with it: no surprise, then, for the methodological problems which are raised by today's pure theory.[60]

However, those high methodological costs may have been borne in vain. They have been borne, that is, for what may turn out to have been getting rid of the obviously inconsistent notion of a 'quantity of capital' at the level of the demands and supplies of the *immediately visible* factors of production, and therefore at the level of the endowments, in order to have it re-enter the theory at the *less immediately visible* but theoretically equivalent level of investment-demand and savings-supply.

APPENDIX I: THE DETERMINATION OF THE *I* AND *S* SCHEDULES

[1] In par. 7 of the text[1] we had a first check of the consistency of system (F) by counting independent relations and unknowns. The existence of economically meaningful solutions for the unknown prices and quantities of (F) in the economically relevant interval of r_b, $r_{b\,\min} \le r_b \le r_{b\,\max}$ to be presently specified, is demonstrated in the Mathematical Note at the end of this chapter. However, an intuitive account of the demonstration is necessary for a better understanding of the argument which we have conducted, and of the assumptions we have introduced.

We start by noting the lower limit of the relevant interval of values of r_b. Due to our assumption that no storage is possible for either commodity,[2] the 'intertemporal price' P_{b0}/P_{b1} can fall to zero and equation (5.6f) of par. 7 will accordingly give

$$r_{b\,\min} = -1. \tag{5.6b}$$

The specification of the upper limits of r_b where $W = 0$ will, on the other hand, require a better acquaintance with the properties of system (F) and we shall come to it in par. [6] below.

[2] As we proceed to the intermediate range of r_b, equations (5.2f) and (5.6f) of system (F) in par. 7 of the text allow us to write the second price equation (5.1f) as follows:

$$1 = l_b W + a_b P_{a0} + b_b(1 + r_b), \tag{5.1a}$$

where, having used equation (5.2f), P_{a0} is now in effect the *intertemporal* price P_{a0}/P_{b1} as, of course, is $P_{b0} = (1 + r_b)$.

Two implications of equation (5.1a) are of interest here.

(a) It is only for $b_b(1 + r_b) \le 1$, i.e. for $r_b \le (1 - b_b)/b_b$ that we may have non-negative values of *both* W and P_{a0}. Considering also $r_{b\,\min}$ from equation (5.6b), we may therefore begin to restrict our attention to the interval

$$-1 \le r_b \le (1 - b_b)/b_b \tag{5.6c}$$

of our independent variable, where $(1 - b_b)$, and hence $(1 - b_b)/b_b$, are evidently positive whenever the method of production of b is viable (par. 3 in the text).

(b) Equation (5.1a) above will then entail that for any r_b in the interval (5.6c), non-negative levels of W require P_{a0} to stay in the interval.

$$0 \le P_{a0} \le [1 - b_b(1 + r_b)]/a_b \tag{5.1b}$$

Given, then, r_b and P_{a0} in the respective intervals (5.6c), (5.1b), equation (5.1a) will determine the corresponding non-negative level of W. Consequently the first of equations (5.1f) determines P_{a1}. Thus the entire series of prices and the wage will be uniquely determined by equations (5.1f), (5.2f) and (5.6f), *once P_{a0} is given*, besides r_b.

To that unique series of prices and the wage, there will correspond the quantities demanded expressed by the functions $D_{a0}, D_{b0}, D_{a1}, D_{b1}$. The amount of total savings S in equation (5.5f) with its physical components S_a, S_b of (5.5f) will then be determined as well. The methods of production of the two commodities, being given, the same will be the case for the amount of investment I of equation (5.4f) and its physical components I_a, I_b. Such quantities need be neither single-valued, nor continuous functions of P_{a0}.[3] It will however follow common practice and not be unduly restrictive to make the following assumption:

Assumption (i) *Given r_b in the interval* (5.6c), *the quantities demanded $D_{a0}, D_{b0}, D_{a1}, D_{b1}$, and hence the aggregate quantities demanded A_0^D and B_0^D are single-valued, continuous functions of P_{a0} in the interval (5.1b).*[4]

[3] Thus, then, given a level of r_b in the relevant interval, any level of P_{a0} in the interval (5.1b) will entail a ratio $\delta = A_0^D/B_0^D$, which will generally differ from the ratio $\gamma = A_0^S/B_0^S$, equality with which is imposed by equation (5.7f). As we then change P_{a0} in the interval (5.1b), there are three possibilities (cf. Figure 5.4 in the Mathematical Note to this chapter).

(a) At one or more levels of $P_{a0}, \delta = \gamma$. Equation (5.7f) will be satisfied and we shall have a solution of (**F**) for each of those values of P_{a0}.
(b) Over the entire interval (5.1b) of $P_{a0}, \delta < \gamma$. This will mean that at the given level of r_b it will be impossible to use a_0 in as high a proportion to b_0 as that in which we find the two commodities in the endowment. This means then that a_0 would be a free good in an equilibrium occurring at that level of r_b, when, that is, $A_0^D = A_0^S$.
(c) Finally, over the entire relevant interval of P_{a0}, we shall have $\delta > \gamma$. This is the case symmetrical to (b), in which b_0 cannot be *used* in as high a proportion to a_0 as B_0^S/A_0^S (in as low a proportion as A^S/B^S) and b_0 would be a free good in an equilibrium occurring at that particular level of r_b, which would then have to be $r_b = -1$, because of $P_{b0} = 0$.

Cases (b) and (c) would exclude an economic solution of system (**F**) as formulated in the text, with *equalities* in the first two relations (5.3f), but the simple economic rationale of the cases, i.e. the non-scarcity of either a_0, or b_0 when equality between demand and supply were to be achieved for the other, indicates that a solution could be ensured by a slight formal modification of (**F**).[5] However, that would complicate the exposition and risk obscuring the main points we wish to bring out, which are altogether independent of that kind of excess supplies. We shall therefore leave aside cases (b) and (c), as on the other hand we implied already by assuming two (scarce) goods in our model, for period $t = 0$, no less than for $t = 1$. We may therefore render explicit the following assumption:

Assumption (ii) *In the interval $r_{b\,\min} \le r_b \le r_{b\,\max}$, in which $W \ge 0$, and for $r_{b\,\max}$ to be defined below* (par. 6), *the commodities a_0 and b_0 can be used in the proportion A_0^S/B_0^S in which they appear in the endowment.*

We may then conclude that, for any level of r_b in the interval between $r_{b\,\min}$ and the level $r_{b\,\max}$ to be defined, at least one level of P_{a0} will exist satisfying equation (5.7f), and thus solving system (**F**).

[4] The key role of the price P_{a0} in the solution of (F) at any relevant level of r_b calls for a specification, since there are two aspects of that price which play a different role in system (F): an ‘intertemporal’ aspect and a ‘contemporary’ one.

Having been expressed in terms of the numéraire b_1 of equation (5.2f), P_{a0} possesses an 'intertemporal' aspect, the one which we have chosen here to measure by r_b (i.e. P_{b0}/P_{b1}), the independent variable of our system (F). P_{a0} however also possesses a 'contemporary' aspect, which may be expressed by $\pi = P_{a0}/P_{b0}$, a new variable. Indeed:

$$P_{a0}/P_{b1} = (P_{a0}/P_{b0})(P_{b0}/P_{b1})$$

i.e.

$$P_{a0} = \pi P_{b0} \tag{5.2f$'$}$$

In par. [3], with r_b a given, variations of P_{a0} were in fact variations only of the contemporary relative price π. The intertemporal role of P_{a0} will instead be prominent when considering the joint variation of P_{a0} and P_{b0} with r_b, as we shall presently do in Assumption (iii) (see also par. [7]).

[5] The generally made Assumption (i), and the confirmation of the two scarce-good nature of our model by means of Assumption (ii), are sufficient to ensure the existence of a solution for (F), but not its uniqueness: income effects may cause the same ratio A_0^S/B_0^S to occur for more than one value of $\pi = P_{a0}/P_{b0}$, at a given level of r_b. We may however make the following assumption:

Assumption (iii) *The value of $\pi = P_{a0}/P_{b0}$ satisfying equation* (5.7f) *together with the remainder of system (F) is unique for each level of r_b in the interval $r_{b\,\min} < r_b < r_b^0$, where r_b^0 is the lowest level of r_b for which $W = 0$. Further P_{a0} will be a continuous increasing function of P_{b0} i.e. of r_b, in the interval $r_{b\,\min} < r_b < r_b^+$, where $r_b^+ \geq r_b^0$ is the level at which that joint rise of P_{a0}* and *P_{b0} will have to cease, having brought to $W = 0$ in equation (5.1a) (par. [2] above).*

By excluding more than one level of π for which the ratio of the total demands $(D_{a0} + I_a)/(D_{b0} + I_b)$ of a_0 an b_0 can satisfy equation (5.7f), the first part of Assumption (iii) excludes income effects which in $t = 0$ might make the ratio A_0^D/B_0^D decrease through decreases in the consumption ratio D_{a0}/D_{b0}, as P_{a0}/P_{b0} falls; it also excludes similar effects in $t = 1$ causing D_{a1}/D_{b1} to change so as to make I_a/I_b fall rather then rise as $\pi = P_{a0}/P_{b0}$ falls.[6] Since here we wish to exclude income effects, we may proceed and make the following:

Assumption (iii(a)) A monotonic inverse relation is assumed between D_{a0}/D_{b0} and P_{a0}/P_{b0} as well as between D_{a1}/D_{b1} and P_{a1}/P_{b1}.

Assumption (iii(a)) only has the purpose of clarifying that the results which we shall reach as to multiplicity and instability are altogether independent of income effects. The assumption will have no consequences on our argument beyond those following from the weaker Assumption (iii).

On the other hand, as we shall see in par. [7], the direct relation between the two intertemporal relative prices P_{a0}/P_{b1} and P_{b0}/P_{b1}, postulated in the second part of Assumption (iii), amounts to simply supposing that the change in the *intertemporal* price r_b will cause no large dislocation in the *contemporary* relative demand D_{a0}/D_{b0}

and I_a/I_b, so that the relative contemporary price π need not undergo large changes in order to continue and satisfy (equation 5.7f).

We shall see in par. [8] some important implications of Assumption (iii) with respect to the relations between W and r_b, and between the 'own rates' r_a and r_b. But before coming to that, we must deal with the upper limit of r_b and thereby clarify the relation between the three levels $r_b^0, r_b^+, r_{b\max}$ which Assumptions (ii) and (iii) have already associated with $W = 0$.

[6] We saw that the monotonic rising relation between P_{a0} and r_b (i.e. P_{b0}) of Assumption (iii) comes to an end when, for $r_b^+ = P_{b0}^+ - 1$, W becomes zero in equation (5.1a). This zero wage will then permit labour unemployment in (F), and allow for a *continuum* of solutions of (F), one for each level of labour 'demanded' L^D in the interval $0 \leq L^D \leq L$. In the resulting continuum of solutions of (F), r_b can fall from r_b^+ down to the mentioned minimum level r_b^0 and/or rise up to a highest level $r_{b\max}$, giving $r_b^0 \leq r_b^+ \leq r_{b\max}$.[7] It also follows that in such a sub-interval of r_b, P_{a0} and all remaining unknowns in [F] will no longer need be single-valued functions of r_b (see Figure 5.2, in the text, par. 24). Since, on the other hand, using (5.2f′) of par. [4] equation (5.1a) of par. [2] becomes

$$1 = a_b\pi(1 + r_b) + b_b(1 + r_b), \tag{5.1c}$$

$\pi = P_{a0}/P_{b0}$ will have to change along that continuum of (F) positions, in a direction opposite to that in which r_b and hence P_{b0} vary.[8] Thus for $r_b^0 \leq r_b \leq r_b^+$, the increasing relation of Assumption (iii) between P_{a0} and P_{b0} (i.e. r_b) will only hold for the values of P_{a0} corresponding to what we shall call the 'main branch' of the function linking r_b with π and the remaining unknowns in (F) – the only branch, that is, in which $W > 0$.

[7] We may now come to what turns out to be perhaps the key relationship in the system, the one already considered in Assumption (iii) between the contemporary relative price $\pi = P_{a0}/P_{b0}$, and the intertemporal relative prices, represented here by r_b.

Thus, let us drop equation (5.7f) from system (F) for a moment, and see the likely effect on A_0^D/B_0^D of a fall of P_{a0}, which happened to be in strict proportion to that of P_{b0}.[9] By thus keeping constant π while r_b falls we aim to distinguish between the 'intertemporal' effects of that fall and the side-effects if any, that it may have on the relative demands of a_0 and b_0 – and hence, once equation (5.7f) is re-introduced, on the contemporary relative prices $\pi = P_{a0}/P_{b0}$ and P_{a1}/P_{b1}, as well as on the other own rate of interest, r_a (par. [8]).

Now, the proportionate fall of P_{a0}/P_{b1} and P_{b0}/P_{b1} might be thought to affect the decisions to save and invest in some definite direction, but no general reason appears to exist why the *ratios* D_{a0}/D_{b0} or I_a/I_b, and therefore the ratio $A_0^D/B_0^D = (D_{a0} + I_a)/(D_{b0} + I_b)$, should be affected in one direction rather than the other. It follows that, quite independently of any uncertainty due to the income effects which we chose to rule out by Assumption (iii(a)) in par. [4], we could not expect any definite sign in the change of π necessary to keep A_0^D/B_0^D at the A_0^S/B_0^S level imposed by equation (5.7f): π may, that is, move either way, or even alternate the signs of the change as r_b falls; and the same will be true for the other contemporary relative price P_{a1}/P_{b1}, controlled by price equations (5.1f). Therefore, unless the fall of r_b causes pronounced dislocations of relative demands in $t = 0$, a tendency can be supposed for the contemporary price π not to change drastically as r_b falls and, therefore, for the intertemporal price P_{a0}/P_{b1} to follow the other intertemporal price P_{b0}/P_{b1} (*i.e.* r_b) in its fall. This tendency of the two 'intertemporal' prices to fall or rise together is at the basis of our Assumption (iii).

An important and perhaps surprising implication of what we have just said should now be noted for future reference. We saw that no reason exists why as r_b falls towards $r_{b\,\min}$, the relative contemporary price $\pi = P_{b0}/P_{a0}$ should move in one direction rather than the other. However, the own interest r_b falling towards the minimum level of (-1) holding when the commodities are not storable is in fact the intertemporal price P_{b0}/P_{b1} of b_0, falling towards zero. It therefore appears that in system (F), the assumption of a zero level of the *intertemporal* price P_{b0}/P_{b1} of b_0 does *not* entail a zero level of its *contemporary* price P_{b0}/P_{a0}, contrary to what we would expect from a commodity which is becoming 'free' in the generally accepted sense – when a tendency to zero of the price of the commodity in terms of one scarce commodity (b_1 in this case) would entail a tendency to zero of its price in terms of *all* other scarce commodities (like a_0 here) whether of the same, or of another, date. We shall return on this basic point in par. [12].

[8] After seeing the grounds on which Assumption (iii) rests we may now turn to two important implications of that assumption. The first is that, as shown by equation (5.1a) of par. [2] the monotonic direct relation between P_{a0} (i.e. P_{a0}/P_{b1}), and r_b (i.e. P_{b0}/P_{b1}) entails a monotonic inverse relation between r_b and W in the interval between $r_{b\,\min}$ and r_b^0. The same is true for any level in the upper interval $r_b^0 \leq r_b \leq r_b^+$, limitedly, however, to the 'main branch' of the functions linking r_b to π and the other unknowns of the system (par. 6).

The second implication of Assumption (iii) is that the monotonic increasing relation between the intertemporal prices P_a and P_b there specified, also entails a monotonic increasing relation between r_a and r_b so that over that interval the two own rates of interest will always move in the same direction.[10]

[9] It is indeed time for some more general considerations on the relationship just referred to between own rates of interest. The considerations will refer to our simple model, but appear susceptible of generalisation.

There seems indeed to be no reason why r_a should always move in the same direction as r_b when the latter changes in system (F). The relation between the two rates ensured by arbitrage is given by:

$$1 + r_a = \frac{P_{a0}}{P_{a1}} = \frac{P_{b0}}{P_{b1}}\frac{P_{b1}}{P_{a1}}\frac{P_{a0}}{P_{b0}} = (1 + r_b)\left[\frac{P_{a0}/P_{b0}}{P_{a1}/P_{b1}}\right], \tag{5.6a}$$

where the factor within square brackets shows, as we would expect, that the proportion between the two rates of $t = 0$, established by arbitrage, compensates the disadvantage which those, who lend in terms of the commodity whose relative price falls between time $t = 0$ and time $t = 1$, would otherwise have: thus if it is a that falls, the factor in question is larger than 1, and r_a is correspondingly larger than r_b, and *vice versa*.

The two rates might therefore *move* in an opposite direction over a limited interval of r_b[11] but only if the change in r_b were to modify considerably the variation in the relative price of the two commodities over the year: where the stress falls on 'modify' rather than 'variation', since a constancy in the price variation would allow the two rates to move, not only in the same direction, but also in strict proportion. However, the variation in question is governed by the technical conditions of production expressed in price equations (5.1f) and its change is not therefore likely to be large unless the change in r_b affects very strongly the ratios between the three factor prices P_{a0}, P_{b0}, W and, besides, large differences exist in the proportion in which the three factors are used in producing the two goods.

[10] We have so far been mainly concerned with the determination of the two schedules at intermediate levels of r_b. Completion of the analogous determination at the 'extreme' values of $r_{b\min} = -1$, and $r_b^0 \leq r_b \leq r_{b\max}$, for $W = 0$, raises some additional problems connected with the zero levels of W or P_{b0} which those extremes imply.

We may turn first to the case of $W = 0$ along the corresponding upper range of r_b (par. [6]), and have a closer look at the supply of labour in that range. It has often been noted that there is no reason why, as the wage *approaches* zero, the supply of labour should also approach zero. If wages are the only income available to the worker for survival, the supply of labour can easily be imagined to *increase*, rather than decrease, as the wage gets close to zero (a quarter pound of daily bread is better than nothing).[12] The situation only changes when the wage actually *reaches* zero, and supplying labour no longer makes any sense for the worker. It would thus seem reasonable to envisage a *discontinuity* in the supply of labour at a zero wage or close to it, where a jump to zero would presumably occur from the high level to which the supply might tend as the wage tends to zero.

Such a *discontinuity* would, however, have the undesirable consequence of eliminating the certainty of the existence of at least one economically significant solution to a system of general equilibrium: in particular the system would no longer admit the zero wage solutions should the supply of labour happen to exceed demand at all strictly positive wages. We have therefore followed what seems to be generally supposed: i.e. a continuity of the supply of productive resources even at zero prices for their services and, therefore, that the quantity of labour made available at that price is the limit to which that quantity tends as the wage tends to zero[13], which in the system (F) is the given supply L. This assumption – which we have in fact used already, when referring to a continuum with different levels of labour employment at a zero wage in the range $r_b^0 < r_b < r_{b\max}$ (par. 6) – and the irrational behaviour it would entail, should however be kept in mind when, in what follows, the shape of the I schedule will show the possibility of equilibria in the interval $r_b^0 \leq r_b \leq r_{b\max}$.

[11] As we proceed now to the analogous question concerning savings, i.e. the supply of a_0 and b_0 as capital goods, it is similarly possible to envisage a state of acute scarcity for individuals whose only way to survive in $t = 1$ are stocks of (non-storable) a_0 and b_0. Savings may then well increase in physical terms as both *intertemporal* prices P_{b0}/P_{b1} and P_{a0}/P_{b1} tend to zero (on that joint tendency to zero of all intertemporal prices for commodities available in $t = 0$ cf. par. [12]), with savers discontinuously annulling their useless savings as those prices actually reach zero, and $r_b = r_a = -1$.[14] We shall however assume continuity in the supplies S_a and S_b of a_0 and b_0 as capital goods, so as to ensure the existence of at least one general equilibrium for the system. We shall therefore find at zero intertemporal prices, the levels of physical savings to which S_a and S_b tend as r_b tends to (-1).

[12] As a second preliminary to considering more closely the determination of the extreme points of the I and S schedules, we must now examine the meaning of the zero price P_{b0}/P_{b1} we assume when referring to the point in the schedules for $r_{b\,\min} = -1$. This zero intertemporal price presents an interest which goes beyond its direct importance, confined as that is in system (F), to the case in which no commodity in the economy can be stored. As we noted in par. 7, where we first came across it, that zero price does not designate b_0 as a free commodity in the generally accepted sense of having a zero price also in terms of *any* other scarce commodity besides b_1. On the contrary we saw that there is no reason why b_0's *contemporary* price $\pi = P_{b0}/P_{a0}$ should be zero when P_{b0}/P_{b1} is such.

The two kinds of prices, contemporary and intertemporal, have in fact two quite different meanings. The intertemporal price P_{b0}/P_{b1} expresses the conditions at which the commodity can be transferred over time – and thus reflects merely the scarcity or abundance of savings relative to investment needs. The contemporary price $\pi = P_{b0}/P_{a0}$ reflects instead the scarcity of b_0 relative to a_0 *in* $t = 0$, and by setting $r_b = -1$ we did not assume anything in particular about it. This contemporary relative scarcity is largely dependent on the endowments A_0^S and B_0^S, which remain exactly the same all along the, I and S schedules and up to $r_{b\,\max}$.

Indeed our Assumption (ii) in par. 2, that a_0 and b_0 can always be used in the given proportion A_0^S/B_0^S, has excluded the possibility of either b_0 or a_0 having a zero *contemporary* relative price whether at $r_{b\,\min}$ or at any other level of r_b. And what we just saw in par. 11 entails that both b_0 and a_0 might become *increasingly* scarce for consumption in $t = 0$, as r_b approaches its minimum level, with the corresponding *definite scarcity of* b_0 *relative* to a_0, expressed by $\pi = P_{b0}/P_{a0}$. What is *assumed* not to be scarce the moment in which we give the value of (-1) to our independent variable r_b is only the purchasing power which people seek to transfer from $t = 0$ to $t = 1$, *relative* to the only possibility to collectively do so in that economy: lending capital goods a_0 and b_0 to producers, who transform them into a_1 and b_1: but for that to happen a_0 and b_0 must be demanded *for production* and can therefore be in excess supply, independently of their scarcity in consumption in $t = 0$.

This nature of the zero price of b_0 at $r_b = -1$ comes more fully into light when we observe that arbitrage will then entail zero intertemporal prices of all commodities dated $t = 0$ in terms of any other dated $t = 1$. Were it not so anybody wishing to exchange b_0 for b_1 could do so via a_1 (if $P_{b0}/P_{a1} > 0$),[15] or by first getting a_0 and then b_1, and this either directly (if $P_{a0}/P_{b1} > 0$), or again through a_1 (if $P_{a0}/P_{a1} > 0$).

[13] We are now finally able to complete our discussion of the solutions of (F) at the lower and upper extremes of the possible levels of r_b. With respect to $r_{b\,\min} = -1$, we know that, given the continuity assumed in par. 11 for the supply of physical savings, solutions of (F) will exist there, no less than for the remaining interval $r_{b\,\min} < r_b \leq r_{b\,\max}$.[16] We are therefore legitimised to extend to that point Assumption (iii) about the uniqueness of the solution of (F).

The peculiarity of the diagrammatic representation of this point is that the zero price we find there for both commodities a_0 and b_0, of which I and S physically consist, will make the two schedules converge to zero as r_b reaches (-1) (see Figure 5.1, in the text, par. 11). The positive, finite contemporary relative price of the two commodities available in $t = 0$ (par. 7 and 12) entails however that, should we choose to measure S and I by taking a_0 or b_0 as the numéraire, we would find there two non-zero separate points S and I, with S_a, S_b being both smaller or both larger than I_a, I_b, the sign of the inequality being the one indicated by the relation between S and I as r_b approaches $r_{b\,\min}$ sufficiently (cf. par. 15 in the main text).

[14] As we proceed to the determination of the I and S points of the schedules at the opposite extreme, we find the continuum of (F) positions we outlined in par. [6]. That continuum, we said, will correspond to levels of labour employment L^D between zero and L, with the corresponding changes of the investment required to equip that variable amount of labour.[17]

We may look at the continuum by starting at $r_b = r_b^+$, where W reaches zero after falling monotonically from its maximum level as r_b rises from $r_{b\,\min}$ up to r_b^0, and then along the 'main branch' of the function linking I, S and the other unknowns of (F) with r_b (see Assumption (iii), par. [5]). Beyond r_b^+, the I schedule will however sooner or later turn left as it will have to extend to touch the vertical axis as the investment demand falls to zero together with the amount of labour L^D to be equipped by means of that investment (see Figure 5.2 in par. 16 of the text). Thus, starting from the value r_b^+, the schedule I may either rise or fall as it moves right or left, to finally reach the vertical axis on the left, having its highest and lowest points at, respectively, $r_{b\,\max}$ and r_b^0, each or both of which may or may not coincide with r_b^+.[18]

The levels of S will similarly change as r_b varies in that interval, and the schedule may also extend either right or left as it rises or falls together with the I schedule, starting from point S^+ for r_b^+. No reason however exists why it should tend to zero as L^D tends to zero (see again Figure 5.2 in the text):[19] at the corresponding points, savings remain those effected out of the endowments A_0^D and B_0^D: wages being zero, the different levels of labour employment do not affect the income out of which savings are decided.

APPENDIX II: TWO NOTES ON HAHN ON 'THE NEO-RICARDIANS'

[A] Two meanings of the uniform rate of return on capital and some mathematical flaws in Hahn's (1982) argument

[i] What follows is not a belated commentary on, or an answer to, Hahn's (1982) article on the so-called 'neo-Ricardians'. That misleading article had essentially two aims: the first was to claim that Sraffa and his 'followers' were only concerned with a 'Special case' of neoclassical theory; the second was to answer their critique, centred on the notion of a 'quantity of capital', by contending that the notion plays no role in neoclassical theory.

Now, the first claim has been answered time ago by explaining that in the classical theory of Smith and Ricardo which Sraffa aimed to revive, the idea of a distribution of the social product based on the substitutability of 'productive factors' and therefore on the determining role of 'factor endowments' is absent; so that Sraffa's contribution cannot be based on the assumption of the capital endowments of Hahn's 'Special neoclassical case'.[1]

As for the second contention the answer is in the main text above, where it is argued in detail how a 'quantity of capital' is implied, together with its deficiencies, in an intertemporal equilibrium. It is also contended there that a reference to that 'quantity' in neoclassical theory is rendered ultimately inevitable by the fact that capital goods are for savers perfectly substitutable means of acquiring future income and savers' decisions are accordingly taken with respect to that single 'quantity' (parr. 26–27 in the text).

[ii] What follows in this Appendix is therefore confined to two specific points intended to clarify some misunderstandings that have seriously marred the 'capital controversies' so far. The first is a widespread confusion between two quite different kinds of uniform rate of return on capital: a uniformity of own commodity interest rates, and that of effective rates of return on the supply prices of the capital goods. This confusion, causing the flaws we shall see in Hahn's (1982) mathematical argument, is an expression, I shall argue, of the obscuration of the change which has occurred in the notions of equilibrium during the last few decades and of its deeper causes: the change, that is, sometimes perceived as the 'Formalist revolution' in neoclassical theory.[2]

The second point, considered in Section B, regards instead a peculiarity implicit in the new notions of equilibrium, which Hahn again expresses in a particularly candid way when assuredly denying that the 'quantity of capital' has any relevance in neoclassical pure theory. The peculiarity consists, essentially, of regarding physical compositions of the capital endowment the economy moves away from, as equivalent, in defining non-stationary equilibria, with physical compositions the economy tends to.

[iii] In Hahn (1982) a key role is played by the distinction between a 'General' neoclassical case and a 'Special' one, where the latter is characterised by a uniformity of the rate of return on capital. The distinction echoes in fact one drawn by critics early in the capital controversies (par. [vii]), between the contemporary versions of neoclassical theory and the traditional ones based, as the latter were, on the above condition of a uniform effective rate of return on the supply prices of capital goods, and on the resulting 'adjusted' physical composition of the 'capital stock' (n. 2 in the text).

Hahn's model is the one we adopted for our chapter, and his 'General' neoclassical case coincides with our system **[E]**[3] but for two elements. The first is the exclusion of the inequality signs, and associated zero prices, from the first three relations of our system [3e][4] (par. 3 in the text), regarding factor demands and supplies. The missing inequality signs rule out, besides the possibility that one of the factors be non-scarce in the generally accepted sense, the less obvious possibility of 'extreme equilibria' arising from excess savings (point III in par. 14 of text).

[iv] Of more interest to us here is however a second shortcoming of Hahn's formalisation. It is his supposition that 'both goods are produced' (ibid., 364), which he does not motivate, as we did (par. 3 of the text), by the assumption that the two goods are non-storable: in fact the whole issue of the storability of commodities, hardly separable from that of an intertemporal equilibrium, is absent in Hahn (1982). Now, in that unspecified form, Hahn's assumption is in conflict with the fact that the two goods are also capital goods, and for durable or storable capital goods, the possibility of demand prices below supply prices, and therefore temporary non-production of them, is the logical entailment of a system, like Hahn's, based on a given physical composition of the initial capital endowments[5] (e.g. the so-called 'Hahn problem' about adjustments to equilibria rests entirely on some capital goods not being produced, and their prices being therefore free to move *below* the respective supply prices so as to adjust to price expectations). The result is that when applied to ordinary and therefore durable, or storable capital goods, Hahn's 'General neoclassical case' falls into an overdeterminacy which, curiously enough, is a neat replica in terms of present-day intertemporal general equilibria of the overdeterminacy to be found in Walras's originary general equilibrium system.

The question here is ultimately that of the uniform effective rate of return on the *supply prices* of the capital goods. Demand prices below the respective supply prices for capital goods offering a potentially lower rate of return, is indeed the way in which arbitrage achieves a uniformity of returns on the *demand prices* of the capital goods. The moment in which such prices are equalised with the respective supply prices – in which, that is, production of all capital goods is assumed[6] – also is the moment in which we are introducing in the equations the traditional equilibrium condition of the uniformity of rates of return on the supply prices, postulated by Walras in common with all his contemporaries and successors up to recent decades. But in Walras that traditional condition was in conflict with a capital endowment given in its physical composition, and not as the abstract 'quantity of capital', susceptible of assuming any physical form, of his contemporaries and successors – thus causing what I have elsewhere referred to as 'Walras's inconsistency'.[7]

Now, in Hahn's intertemporal context, with price changes considered in the equilibrium equations, a uniform *effective* rate of return necessarily takes the form of rates of interest which *differ* according to the commodity in which the loan is expressed (like $r_a \neq r_b$ in our chapter). This *effectively* uniform rate of interest, and therefore of return on the *demand prices* of capital goods, is introduced in the equations when we refer to $n-1$ discounted prices for n dated commodities, thus implying arbitrage through triangular intertemporal exchanges. It becomes therefore a uniform rate of return on the *supply prices* of the capital goods, the moment in which these goods are assumed to be produced and, accordingly, their prices are equalised with the supply prices. And this is just what is done by Hahn in his 'General' case, when he assumes that both goods are produced. The condition thus introduced cannot however be generally satisfied in his context, any more than it could in Walras's original system, since the physical composition of the capital endowment is as given in Hahn as it was in Walras.

This flaw of Hahn's treatment, we said, is curious. Walras's inconsistency had been acknowledged by Hahn in *e.g.* an earlier (1975) paper of his and indeed, as we shall see in par. [vii], is the very one he intends to show to affect the 'Special case' and to have been overcome in the 'General case', which we now see to be instead affected by it (the 'Special case' being affected by an overdeterminacy of quite different origin, cf. par. [v]). The contradiction is strictly connected with the confusion between two kinds of uniformity of rates of return we mentioned at the beginning, and to which we can now turn.

[v] The confusion emerges as we proceed from Hahn's 'General' neoclassical case to his 'Special' case, and note a third flaw in Hahn's mathematical argument. He obtains the 'Special' case by adding to the relations of the General case that of a uniformity of the two *own interest rates*:

$$r_a = r_b \quad \text{and therefore} \quad \frac{P_{a0}}{P_{a1}} - 1 = \frac{P_{b0}}{P_{b1}} - 1 \quad \text{or} \quad P_{a0}/P_{a1} = P_{b0}/P_{b1}, \qquad \text{[I]}$$

i.e. the condition of a constancy $P_{a0}/P_{b0} = P_{a1}/P_{b1}$ of the relative price of the goods over the two periods (see equation [21] in Hahn, 1982: 364).

Now, uniformity (I) of the *own commodity interest rates* is quite a different matter from the uniformity of the *effective rates of return on the supply prices of the capital goods* referred to by the critics, to whom Hahn is there intending to answer (par. [vii]). However, he analyses the former *as if* it were the latter: in particular, he incorrectly attributes the overdeterminacy of his 'Special' case to the cause pointed out by critics for Walras's overdeterminacy, i.e. the given proportions between the stocks of the several capital goods in the endowment.[8]

That Hahn's interpretation of his own 'Special' case is incorrect can, on the other hand, be easily seen as soon as we extend from two to three years the life of the economy of his model (clearly, Hahn does not intend his distinction between 'General' and 'Special' neoclassical cases to be valid only for a two-year economy). We would then have *two* additional unknown prices P_{a2}, P_{b2}, but *three* new equations: the two additional price relations of type (1e) of par. 3 in the text, and the equation $P_{a1}/P_{a2} = P_{b1}/P_{b2}$ relating to the uniformity of the own rates in $t = 1$. Thus the total of excess equations in the 'Special' case now adds up to *two*, against which the *single* additional unknown B_0 or A_0/B_0 proposed by Hahn is of no avail. Indeed the physical composition of the initial capital stock has little to do with a constancy of relative prices, i.e. with the *kind* of uniformity of returns Hahn states in equation (I) for his 'Special' case.[9] This uniformity would depend, if anything, on the *size* rather than the *physical composition* of the capital endowment: it would be the size of that endowment, relative to that of labour and other original factors, that might ensure the incomes, and therefore the savings, of a stationary or steady-growth economy with its constant relative prices.[10]

[vi] In fact both in his interpretation of the 'Special' neoclassical case, and in his assumption that all capital goods are produced (par. [iii]), Hahn takes as one:

(a) the uniformity of the own commodity rates of interest of equation (I), due to *a constancy of relative prices over time*;
(b) the traditional uniformity of the effective rate of return on the supply prices of the capital goods, *the result of an adjusted physical composition of the capital stock* (par. [i]),

where uniformity (b) is independent of (a) since, as shown for Hahn's 'General' case (par. [iv]), it is quite compatible with changes in equilibrium relative prices over time, and therefore with the divergent own commodity rates ruled out by uniformity (a).

And Hahn takes (a) as if it were (b) not only when, as we saw, he ascribes the overdeterminacy caused by equation (I) and uniformity (a) to the proportions between the two stocks as if it were due to (b): he does so also when the traditional uniformity (b), being unaccompanied by (a), unlike what happened in the old theorists referred to by the critics (par. [vii]), is unwittingly introduced in a 'General case' intended to show those critics how one can do *without* that very traditional uniformity (b).

We may also note here incidentally that Hahn appears to misinterpret the divergence of own commodity rates of interest when he writes: 'The crudest empirical observations will convince one that there is no unique rate of profits to be observed in the economy: Do we conclude from *that* that competition is functioning badly? Answer: No. Consult any general equilibrium text' (1975: 361), thus apparently referring to divergences of own commodity rates. However, what empirical observation can reveal is divergent profit rates for different businesses, compared therefore in terms of money or some other *common numéraire*. But then, that divergence has nothing to do with divergences between own commodity rates, which would show by reckoning in *different numéraires* the same rate of profits (when, of course, the relative prices of those numéraires change over the year).

[vii] Unambiguous confirmation of the Hahn (1982) confusion about uniform rates, as well as of the intent we attributed in par. [iii] to his (1982) distinction between 'General' and 'Special' neoclassical cases, can in fact be obtained from the already quoted earlier (1975) paper of Hahn. He referred there to results advanced early in the capital controversies by the critics, who had pointed out how the contemporary attempt to surmount the inconsistency of the notion of a 'quantity of capital' by taking the capital endowment as a given physical vector entailed falling into the alternative inconsistency of Walras's originary general equilibrium. This was the case unless uniformity (b) was dropped, together with the entire traditional notion of equilibrium, as is done in contemporary pure theory – thus undermining, it was further argued by those critics, the persistency of the equilibria necessary for a correspondence between theoretical and observable variables.[11]

Accordingly, in (1975), Hahn begins by reporting statements by Harcourt (1975) to the effect that in the 'Walrasian model' it is not possible to have a 'theory of the rate of profits' [read: to determine a uniform rate of return on capital of type (b) above] and that

> to get a uniform rate of profit Wicksell (and others) had to work with Robinsonian leets [homogeneous capital]
>
> (Hahn, 1975: 360–361).

To this Hahn objects that the 'Walrasian model' *can* deal with 'the uniform rate of profit' by treating it as just 'an extremely specialised case' (Hahn, ibid.).

The passage is of some interest in itself because Hahn acknowledges there those early critical results of the capital controversies, and in particular: (i) the inconsistency between 'uniform rate of profit' and 'Walrasian model' and, therefore, (ii) the role of 'Robinsonian leets' (homogeneous capital) for 'Wicksell and others' in allowing for a uniform rate of profit. However, surprisingly enough, those two points are not mentioned in the (1982) paper, apparently devoted to answering the 'neo-Ricardian' critique on capital of which they evidently were central elements.[12]

What is of specific interest to us now is however the confirmation which the (1975) article gives of the confusion about uniformity of rates of return, and of the intent of the 'Special' neoclassical case. With respect to the first point Hahn describes as follows the 'specialisation' of the 'Walrasian model' yielding Wicksell's 'uniform rate of profit':

> General Equilibrium Theory is general and so we can discuss the equilibrium of an economy whatever its initial conditions, *e.g.* outfit of goods inherited from the

> past. For most such specifications it will not be the case *that the equilibrium price of a good for future delivery in terms of the same good for current delivery will be the same for all goods* (360, our italics).

But the uniformity of the rate of profits of 'Wicksell (and others)', i.e. uniformity (b) above, is quite different from 'the equilibrium price of a good for future delivery in terms of the same good for current delivery [being] the same for all goods', quite different, that is, from the uniformity (a) of own rates to which Hahn refers here.

(We may incidentally note how the above straight identification of Wicksell's uniform rate of profits with a uniformity of own rates of interest which is simply a constancy of relative prices, provides the key to what probably misled Hahn and other authors into confusing the two uniformities. In Wicksell, as in Walras, as in all theory up to recent decades, the persistence of the traditional equilibrium ensured by uniformity (b) above allowed abstracting from changes in equilibrium prices, so that uniformity (b) took form (a) and the latter can therefore be easily mistaken for (b)).

But, as we said, the (1975) article also clarifies the intent of the (1982) 'Special neoclassical case'. We find in (1975) words like the following referred to the 'capital outfit' allowing for a 'uniform rate of profit'

> that a very extreme specialisation of a general model somehow shows the latter to be inapplicable requires the very summit of incomprehension (1975: 360)

where the contention clearly is that the homogeneous capital referred to by the critics was introduced by 'Wicksell and others' in order to deal with what was only 'a very extreme specialisation' of contemporary neoclassical theory, with its given physical capital endowments. The passage makes clear, were it necessary, that the 1982 'Special neoclassical case' is simply this (1975) 'extreme specialisation' to which Hahn believes he can reconduct all neoclassical theory prior to recent decades.

We shall return in Section B below on this idea of Hahn, according to which an equilibrium with a capital stock of adjusted physical composition, and to which the economy therefore tends, is seen as just an extreme specialisation of 'equilibria' from which the economy would tend to depart.[13]

[viii] However what has been of interest for us here has been above all to trace through Hahn (1975, 1982) a widespread misunderstanding[14] which, during the capital controversies, has contributed to obscuring a point of considerable importance: namely the causes and implications of the drastic change which has occurred in mainstream pure theory in the wake of Hicks (1939). Uniformity (b), I submit, has played a central role in that change which, slow at first in gaining acceptance, rapidly established itself after the early phase of the capital controversies. For the reasons we saw, uniformity (b) had in fact to be abandoned when, after the failure of the attempts of Jevons, Böhm-Bawerk, Wicksell and others at an 'average period of production', it began to be admitted that capital could only be consistently measured in terms of a set of quantities. This also meant abandoning the neoclassical version of the traditional notion of a normal or 'natural' position of the economy on which economic thinking had relied since its inception in order to achieve correspondence with observable variables (cf. e.g. par. 13 in the text; also below in this section).

Now, this disappearance of uniformity (b) with Hicks (1939) and subsequent work seems to have occurred without any clear recognition of its implications or even clear acknowledgement of the disappearance itself. Uniformity (b) could indeed be mistaken for the uniformity of the rate of return on the demand prices of the capital goods which was of course retained in the system by letting capital goods prices fall below their supply prices, but had altogether different implications. What appears however to have been decisive in preventing the issue from emerging has been, I submit, the confusion with uniformity (a), for which the disappearance of (b) from a non-stationary equilibrium has tended to be identified with the simultaneous disappearance of uniformity (a) owing to intertemporal pricing.

The obfuscation of condition (b) and therefore of the basic reasons of the change which pure theory has undergone with the adoption of the 'Walrasian' capital endowment as a capital goods vector in Hicks (1939) – the reasons, that is, of the 'Formalist revolution' in neoclassical theory as we saw it has been called[15] – has indeed made it possible to continue viewing that change in terms of Hicks' (1939) more explicit argument, namely the dependence of present individual decisions, and therefore of equilibrium, upon future prices: as if the persistence of the traditional equilibrium, made possible by uniformity (b), had not always been viewed as the basis for abstracting from such a dependence and from the radical difficulties it entails at the level of general theory. The introduction of dated prices, I submit, should indeed be seen as a *consequence* of the new equilibria and their 'fleeting' character (Marshall, 1920: 443 (543)) rather than a *cause* of the change over to them.[16]

(Before concluding on the question, it may be interesting to provide an example of how some contemporary authors find it difficult to grasp the traditional concept of a normal position, even in the neoclassical form it took in Marshall, J.B. Clark, Pigou, etc. An author like Christopher Bliss reconstructs as follows that concept in a writer as clear as Wicksell. We are given, Bliss describes (1975: 115), two economies 'in semi-stationary growth' such that 'the citizens are determined to hold capital goods of a specified value in terms of the *numéraire* no matter what equilibrium comes about' and such that 'in economy 2 the citizens [hold] a higher *numéraire* value of capital'. Wicksell's question then is, Bliss reports, 'how will the economies differ with regard to the total product produced' – an exercise, he then comments, which is 'extremely contrived' because 'no convincing specification of the demand for capital would lead to a totally inelastic demand for a particular numéraire value' (Bliss, 1975: 115). To unravel this description we need three observations. First, if it were a question of semi-stationary economies (i.e. economies in steady growth: Bliss, 1975: 86–87) which is not the case for Wicksell (cf. e.g. 1901, Part III) then the steady growth condition would determine the amount of capital per head in the economy[17] and there would be no problem of 'citizens holding capital goods of a specified value' – (a question of a 'totally inelastic' *supply* rather than demand, unlike what Bliss writes). Second, the point of those equilibria in Wicksell was not 'to hold capital goods of a specified value.... no matter what equilibrium comes about', but rather that a sizable increase in 'capital' (the single quantity, of course) would take time, and the capital in existence could be assumed constant over the period required for the equilibrium variables to emerge through a compensation of deviations (e.g. Marshall [1920]: 443–543). Third and last, surely Wicksell's priority in the alleged 'comparison' between the two economies would have been the respective rates of interest rather than 'the total product produced' (see e.g. Wicksell, 1934: 157, 162): indeed what Bliss describes as a 'comparison' between 'two economies' is simply, in Wicksell as in the other traditional neoclassicists, the construction of a (general-equilibrium) demand schedule for 'capital' (the mirror image of one for labour) to be then matched with the already mentioned supply. So Bliss's 'extremely contrived exercise' is essentially the basis of the neoclassical theory of the distribution between capital and primary factors, in the form in which it gained acceptance at the end of the nineteenth century and it dominated before recent decades: the lack of any wide resonance of the works of a Walras or Pareto at the time, when compared with those of Marshall, Jevons, Wicksteed, J.B. Clark or Pigou, etc., suggests that it was perhaps thanks to the roots it took in the shape of that 'contrived exercise' that neoclassical theory has been able to survive the 'Walrasian' form which, for the reasons we saw, it was forced to adopt with the 'Formalist revolution' of recent decades.)

[B] Capital endowments and 'History'

[ix] It is not surprising that in his 1982 reply to the 'neo-Ricardians' on capital, Hahn should overlook the role of the notion of 'capital' as a single factor in lending credence to the idea of a generalised substitutability between 'factors of production'. This oversight is common to most contemporary pure theory (though not to applied theory) although it is transparent that, contrary to what would be required by the substitutability between *physical factors* postulated in the theory, alternative methods of production, or methods for alternative consumption goods, generally differ by the *kind* of capital goods employed, rather than by their proportions.[18]

What is more surprising is that in (1982) no trace should be found of the purely theoretical role acknowledged in (1975), which 'capital' played for 'Wicksell (and others)' in allowing for a 'uniform rate of profit' (see quotation in par. [vii]). It is owing to the latter oversight that in (1982: 354) Hahn can write

> it seems to me impossible (as a matter of intellectual history) to maintain that the *possibility* of perfect capital (or labour) aggregation is a neo-classical doctrine

which would otherwise have raised the problem of how 'Wicksell (and others)' could refer to an aggregate capital ('Robinsonian leets') while failing to maintain its 'possibility'. But Hahn may have been betrayed here by a lack of transparency in his compact language.

[x] Hahn however moves then from intellectual history to logic, and argues the irrelevance of the concept of a quantity of capital in neoclassical theory as follows

> Arguing in a circle is not the problem [...]. The point is much simpler [...] In general, the neoclassical equilibrium can be found given the vector of endowment which may have, say, 10^8 components. It would be surprising if there were a single number [a 'quantity' of capital C] which gives the same information as the 10^8 dimensional vector. (1982: 369)

where the point is simple, but appears to be the opposite one: namely that a 10^8 dimensions capital endowment gives *excessive* information. Indeed C would be required in order to express the capital endowment according to the perfect substitutability of capital goods *for savers*, the consequent uniform rate of return and adjusted physical composition of the endowment allowing for a persistence of the equilibrium sufficient for the traditional correspondence between theoretical and observable variables – along the lines Hahn had indeed acknowledged in 1975 to be those of Wicksell (par. [vii]). And it is of course with this needed *C* that the 'arguing in a circle' comes in.

[xi] Hahn's clear-cut belief in the uselessness of the notion of a quantity of capital in neoclassical theory may however be of use now for looking from a particular angle to what can be seen as a basic deficiency of the contemporary equilibria. In 1975, after acknowledging that 'to get a uniform rate of profits' 'Wicksell (and others)' resorted to 'Robinsonian leets', Hahn continued as follows

> But all they had to do (writing before v. Neumann they cannot be blamed for not doing so) is [. . .] simply assume that *history has given us the appropriate outfit* (1975: 360, our italics).

Reflection however shows that the 'appropriate [capital] outfit' has nothing more to do with 'history' than have e.g. the particular outputs of consumption goods allowing for equality between prices and expenses of production: both sets of quantities clearly result from assuming a competitive process and not from an accident of history. Just as those consumption goods outputs can only be unknowns in a neoclassical demand-and-supply theory, so the same ought to be the case for the physical quantities constituting a given capital endowment. It was surely this, rather than any as yet missing von Neumann's theorems that made 'Wicksell (and others)' introduce capital as a single quantity.

Hahn's idea, that *any* physical composition of the initial capital endowment can be the basis of a neoclassical non-stationary equilibrium, seems thus to be ultimately not much better founded than that of taking as given in such a theory the outputs of consumption goods, and then arguing that it will be a 'Special' case when 'history' has yielded outputs allowing for prices equal to their production expenses. The necessary existence of stocks for capital goods may of course slow down, relative to many consumption goods, the competitive tendency to the equality between demand and supply prices, but surely it does not make of that tendency any less of an equilibrating tendency than the analogous one for consumption goods.

NOTES

1 For example, that is the basic contention in Prof. Hahn's article on the 'neo- Ricardians', 1982 (see Appendix II at the end of the chapter). See also "equilibrium theory (say of an intertemporal equilibrium) . . . does not need aggregate notions like capital" (Bliss, 1974, 117n). Similarly, Prof. Samuelson had written earlier:

> Repeatedly in writings and lectures I have insisted that capital theory can be rigorously developed without using any Clark-like concept of aggregate 'capital', instead relying upon a complete analysis of a great variety of heterogeneous physical capital goods and processes through time.
>
> (Samuelson, 1962: 193)

The claim seems to have been widely accepted also from the critically inclined side of the controversy (see e.g. Currie-Steedman).

2 The 'traditional' version of neoclassical theory is here understood to be that which has dominated neoclassical pure theory until comparatively recent decades, and is characterised by equilibria with the adjusted physical cmposition of the capital endowment imposed by the condition of a uniform *effective* rate of return on the supply prices of the capital goods (but see n. 8 below on today's frequent confusion between this concept of equilibrium and the quite different notion of a stationary or steady state). That equilibrium was however inconsistent with expressing as a vector of capital goods its given capital endowment (cf. Appendix II, par. (iv) below), and was in fact accompanied by a treatment of it as a single 'quantity of capital', which could change its 'form', so as to allow for the rentals of the several capital goods to come into line with the above uniform rate (e.g. Hicks, 1932, p. 20). Walras had been the outstanding exception in relying on a vector of capital goods but, sharing as he did that traditional notion of equilibrium, his treatment of capital was simply inconsistent. A return to Walras's conception of the capital endowment as a physical vector, accompanied however by the abandonment of the traditional notion of equilibrium, occured when, with Hicks (1939), the impossibility of consistently conceiving capital as a single magnitude began to be in fact admitted. That return characterises what we indicated above as the 'contemporary' versions of neoclassical theory.

The italicised word 'effective' by which we qualified the uniform rate of return was meant, on the other hand, to take care of the fact that the definition of that uniform rate will entail a non-uniformity of the own commodity rates of interest as soon as changes in relative prices over time are considered, as happens in the 'contemporary' versions of the theory. In the capital controversies, that inequality of own rates of interest, due to price changes, has indeed been often confused with the inequality of effective rates on the supply prices of the capital goods, due instead to the unadjusted physical composition of the capital endowment pertaining to Walrasian theory. The confusion, which has contributed considerably to the opacity of the capital controversies, is discussed in Appendix II below, with particular respect to the form it took in Hahn (1982).

3 We shall here be exclusively concerned with intertemporal equilibria based on complete 'futures' markets and leave aside 'temporary equilibria'. It should, however, be evident that if the 'quantity of capital' underlies the savings–investment decisions of intertemporal equilibria, that quantity will not be any less entailed in those of a 'temporary equilibrium'.

4 Cf. e.g. the view that when it is recognised that in any discussion of savings and investment ('intertemporal allocation'), the proper variables are 'today's prices for future goods' there is 'a perfectly consistent story that does not look any different from the story about choosing commodities today' (Arrow, 1989: 155).

5 Bliss has rightly claimed that 'capital cries out to be aggregated' (1975: 8). He does not however seem to have uncovered the reason of that "cry": the perfect substitutability of capital goods for savers.

6 Cf. par. 9 for consistency with the aggregate of budget equations ('Walras's law').

7 For these general equilibrium demand and supply functions of 'capital' in traditional neoclassical theory, cf. Garegnani (1970: 425). To develop analogous concepts applicable to intertemporal equilibria will, as we said, be a main aim of the present chapter.

8 Cf. par. 1, n. 2. In the course of the capital controversies, the 'traditional' concept of equilibrium, in which the capital endowment is a *given*, has often been identified with the altogether different concept of a "stationary" or "steady state" of the economy where it is instead an *unknown*. The confusion has been favoured by the fact that a price constancy is today generally explained in terms of "steady states". However, the traditional price constancy was a *direct assumption*, founded only on the *persistence* attributable to those equilibria – a result largely of the assumption of an adjusted physical composition of the capital endowment. That assumption and the persistence it attributed to the equilibrium had the advantage of cutting through the difficulties which the dating of equilibrium variables entails: from the arbitrariness of the initial instant, to that of the final horizon, or to the meaning of a stability for such dated equilibria. The assumption had however the decisive disadvantage that, in neoclassical theory, adjusted physical composition of capital and uniform

effective rate of return on the supply prices of capital goods entailed conceiving the capital endowment as a single magnitude (n. 2 above).

9 Cf. the frequent use among those authors of expressions like 'free' or 'fluid' or 'floating' capital, as opposed to 'invested' or 'fixed' or 'sunk' capital e.g. Jevons (1957: 242–244); Marshall (1920: 62, 341); Wicksell (1893: 156; 1934: 145, 234; 1935: 192).

10 Indeed, under our present assumptions of circulating capital only, and of yearly production cycles, the demand for gross investment and the supply of gross savings for the year would coincide with the demand and supply of 'capital' of the traditional theories (for a more detailed examination of the connection between the two notions, see Garegnani (1978: 352.)). We may take this occasion to note how the relationship between demand for investment and demand for 'capital' has often been obtained by referring to the demand for capital at a lower rate of interest, and by then spreading the 'net investment' required in order to bring the capital stock to that level, over some given time period of adjustment. However, such a procedure either reflects a turnover period of aggregate capital, in which case our argument provides a foundation for it, or is arbitrary, as it overlooks the fact that even under constant technical conditions, capital accumulation generally entails changing most kinds of capital goods and not adding new capital goods to those in existence. Capital accumulation can only be generally conceived as one process with the replacement of the existing physical capital.

11 It may be interesting to notice here the form which these price relations assume with undiscounted prices (indicated by the small letters)

$$p_{a1} = l_a w + (a_a p_{a0} + b_a p_{b0})(1 + r_b)$$
$$p_{b1} = l_b w + (a_b p_{a0} + b_b p_{b0})(1 + r_b),$$

similar to that of the traditional price equations, except of course for the different prices applied to the same good according as it appears as input or as output; for the connected dependence of the level of the uniform effective rate of profit upon the chosen numéraire cf. n. 2 above and passim (for the relation between these undiscounted prices and the discounted ones cf. par. 21, n. 47).

12 It may thus be interesting to note that, had we considered the possibility of storage, system (E) would have needed to be modified by replacing the last two equations in (5.3e) with the following relations:

$$\begin{cases} D_{a1} = A_1 + T_a(1 - \alpha_a) \\ D_{b1} = B_1 + T_b(1 - \alpha_b), \end{cases} \tag{5.3e$'$}$$

where T_i is the quantity of commodity i stored ($i = a_0, b_0$).

Further, equations (5.1e) in (E) would have to be modified by introducing an inequality sign

$$P_{i1} \leq l_i W + a_i P_{i0} + b_i P_{i0}, \tag{5.1e$'$}$$

and the following relations would also have to be added

$$P_{i0} \geq P_{i1}(1 - \alpha_i) \tag{5.1e$''$}$$

indicating the price P_{i0} low enough to make it convenient to provide the commodity of period $t = 1$ by the storage of that of period $t = 0$, at a cost of storage here assumed to consist of a *physical* wastage $\alpha_i < 1$. Now, the cases will be three, depending on T_i relative to D_{i1}

1 if $T_i = 0$, the relation (5.1e$'$) for commodity i will hold with an equality, and (5.1e$''$) generally with an inequality;
2 if $0 < T_i < D_{i1}$, i.e. the demand of i in $t = 1$ is satisfied by storage only in part and therefore the needs of i in $t = 1$ will be partly satisfied by storage and partly by production, both the respective relations in (5.1e$'$) and (5.1e$''$) will hold with an equality sign;

3 if finally $T_i = D_{i1}$ relation (5.1e″) will be satisfied with an equality sign, (5.1e′) being generally satisfied with an inequality.

Conditions (5.1e″) will on the other hand allow determining the two new unknowns T_a and T_b.

13 An example is provided already by equations (5.1e″) in n. 12 which establish a constraint on 'intertemporal' prices which has no substantive correspondent for 'contemporary' prices. We shall also see (point IV in par. 14, and parr. [7] and [12] in Appendix I) that the principle for which the zero price of one commodity in terms of one scarce commodity, entails a zero price in terms of *any other* scarce commodity does not appear to apply to intertemporal prices.

14 It should be noted that, contrary to general usage, we need to include in the gross investment, and hence in both 'gross' social product and 'gross' savings, the replacement of circulating means of production (the only means of production of our model).

15 From the first two equations (5.3e) we obtain $I_{a0} = A_0 - D_{a0}$, and $I_{b0} = B_0 - D_{b0}$; and since in relations (5.5) we have $S_{a0} = A_0 - D_{a0}$; $S_{b0} = B_0 - D_{b0}$; we obtain $I_{a0} = S_{a0}$; $I_{b0} = S_{b0}$ and, finally, $I_0 = S_0$. We have here assumed the relevant relations (5.3e) to be equations. Should the inequality sign apply in any of the two, the corresponding price would be zero and the 'excess savings' in that commodity would not affect the value equality, $I_0 = S_0$.

16 It may be asked why the income L_0W is being excluded from Y_0 in equation (5.5) and is included instead in Y_1. However, 'yearly' production cycles, as distinct from continuous production, force us to distinguish between the period in which the participation of resources to production has occurred (in the present case $t = 0$) and the period in which the corresponding income must be supposed to accrue, *if* the equality between the *social income* and value of the *social product* is to be maintained. This does not preclude wages being 'advanced' in $t = 0$, but that would be out of the savings of capitalists in $t = 0$, unlike what we have assumed here in equation (5.5).

17 The relations we are describing are in the nature of accounting identities and would hold whether the economy is in equilibrium or out of it – whether, more generally, they refer to *realised* savings and investment or, instead, to *decisions* to save and invest under some *a priori* specified, hypothetical circumstances. The latter is the case in equations (5.4) and (5.5), where we have applied those relations to the equilibrium quantities of system (E), just as it will be the case when we shall apply them to the partly different hypothetical circumstances of system (F) of the next paragraph, implying equilibrium in some markets and not in others.

18 See the quotation in par. 1, n. 4.

19 We have referred to two own rates of interest for period $t = 0$. As is well known, when the relative price of the two goods changes from $t = 0$ to $t = 1$, arbitrage will impose a lower nominal interest rate for loans in terms of the good, say a, whose relative value rises from $t = 0$ to $t = 1$, in order to compensate the advantage its lender would otherwise have relative to lenders of b, and so as to realise what is *in effect* a uniform rate of interest. This rate takes then a numerical expression dependent on the numéraire, not unlike what happens for commodity prices (cf. n. 2 and Appendix I, par [9]).

20 Cf. n. 8, 15. The nature of these two constructs can perhaps be more easily grasped when we realise that they follow the simple logical procedure which underlies, in an elementary textbook, the representation of, say, the demand for labour, when the quantity demanded L^D is directly derived from the marginal product of the labour employed with the given supply λ of land, the only other productive factor. At any point along that demand schedule, the following equations will therefore hold:

$$Q = f(L^D, \lambda); \qquad w = f_{L^D}(L^D, \lambda); \qquad \rho = f_\lambda(L^D, \lambda); \qquad L^S = \text{constant}$$

The wage w is the independent variable, leaving four unknowns in the four equations, i.e. the corn output Q, the rent rate ρ and the quantities demanded and supplied L^D and L^S of labour, where for simplicity we have supposed the factor supplies L^S and λ to be rigid. At any relevant level of w, equilibrium will hold in the remaining two markets: for corn, where Q is equal to the corn expenditure ($L^D w + \lambda\rho$) from the owners of the two factors, and for land, where the supply λ is fully employed. The two schedules

$$L^D(w) \quad \text{and} \quad L^S = \text{constant}$$

resulting as w varies, will therefore be 'general equilibrium schedules' in the sense meant in the text: general equilibrium will hold when equilibrium holds in the labour market, i.e. when either $L^D = L^S$, or $L^D \langle L^S$ with $W = 0$, or $L^D \rangle L^S$ and the wage is at its maximum for which $\rho = 0$. We may note for future reference how that disequilibrium in a single market – that of labour – is here compatible with the sum of budget equations (Walras's law) because the labour income in the economy is taken to be L^Dw, i.e. that corresponding to the quantity L^D of labour *demanded*, and not to the endowment L^S: cf. par. 9 in this text for the similar problem in the savings–investment market.
We may also note how the schedules may clarify the economic rationale of the necessary existence of an equilibrium, once excess supplies of factors are admitted for a zero price of their services. Not only an *excess* supply $L^S > L^D$ permits an equilibrium for $w = 0$, but also an *excess demand* $L^S > L^D$ does so for w at its maximum, since that situation is, in fact, just the same case with land in the place of labour: *i.e.* excess supply of land with zero rent, when land used can be scaled down to employ no more than the labour supply so any relative position of the schedules entails equilibria. Once excess supply and zero prices are admitted, the existence of solutions can be seen as a rather clear implication of the aggregate of budget equations (and, of course, of the continuity of the functions). Uniqueness, stability and more generally plausibility of the "equilibria" are what the neoclassical demand and supply rested on and the principle of substitution was for.

21 See par. 9 (and the preceding footnote), for the 'adjustment in expenditures' which allows the disequilibrium to be confined to the single market of savings and investment.

22 The changes in methods of production which we might expect to occur along the schedules as r_b varies will be introduced in Section 5.5. It should however be noted that the schedules determined by (F) would already allow for the 'substitutability' between factors arising from consumer choice in $t = 1$.

23 The fact that the single market for I and S involves in effect the two markets, for a_0 and b_0, is irrelevant here.

24 The question is essentially the same as the one concerning the adjustment in the power to purchase 'corn' in the simple example of general equilibrium demand for labour of par. 6, n. 20 above. We implied that adjustment when we attributed an income only to the workers employed L^D. In the present model, let us indicate by D'_{a1}, D'_{b1}, the consumption demands in $t = 1$ resulting from the equations of consumer equilibrium on the usual hypothesis that they dispose of the income resulting at the given prices from all the resources they own. Summing the budget equations through consumers and the two 'years', we get, after some simple transformations,

$$(A_0^S - D_{a0})P_{a0} + (B_D^S - D_{b0})P_{b0} + LW = D'_{a1}P_{a1} + D'_{b1}P_{b1}. \tag{5.8a}$$

Clearly the L.H.S. of equation (5.8a) gives both the social *income* Y'_1, and the value of the purchases the individuals would carry out at $t = 1$, under the stated assumptions of complete sales of A_0^S, B_0^S, besides L. Then using relation (5.5) of par. 4, we have

$$Y'_1 = D'_{a1}P_{a1} - D'_{b1}P_{b1} = S + LW. \tag{5.8b}$$

On the other hand, the value Q_1 of the gross social *product* for $t = 1$, as it results from the system (F) by substituting for prices in accordance with equations (5.1f), is given by

$$\begin{aligned} Q_1 &= A_1P_{a1} + B_1P_{b1} = A_1(l_aW + a_aP_{a0} + b_aP_{b0}) + B_1(l_bW + a_bP_{b0} + b_bP_{b0}) \\ &= (l_aA_1 + l_bB_1)W + (a_aA_1 + a_bB_1)P_{a0} + (b_aA_1 + b_bB_1)P_{b0} \end{aligned}$$

and using equation (5.4f),

$$Q_1 = (I + LW). \tag{5.8c}$$

Thus the purchasing power $(S + LW)$ in equation (5.8b) would face commodities of the value $(I + LW)$ in equation (5.8c), and if we had used D'_{a1}, D'_{b1} in the system (F), the system would have been inconsistent. The 'adjustment' of purchases mentioned in the text can, on the other hand, be represented by the following equations, in which we indicate by D_{i1}, as distinct from D'_{i1}, the 'adjusted' purchases in $t = 1$ appearing in our system (F):

$$D_{i1} = D'_{i1}\frac{LW + I}{LW + S} \quad (i = a, b).$$

It follows that the 'adjusted' aggregate expenditure and income Y_1 in $t = 1$ is now given by

$$Y_1 = \sum_i D_{i1}P_{i1} = \frac{LW + I}{LW + S}\sum_i D'_{i1}P_{i1} = \frac{LW + I}{LW + S}(LW + S) = LW + I = Q_1$$

and consistency has been brought back into the system (F). We may note that, by definition, consumption purchases for $t = 0$ remain unchanged, i.e. $D_{a0} = D'_{a0}$, $D_{b0} = D'_{b0}$, and so does therefore S the amount of the decisions to save in $t = 0$.

25 Apparently less plausible is the behaviour assumed in the opposite case of $I > S$, where the purchases for $t = I$ would have to *exceed* what is possible with the purchasing power obtained from the full sale of the A_0^S and B_0^S endowments. However, the extra purchasing power implied in our adjustment of expenditures may be taken to express the tendential relative rise of purchasing power available in $t = 1$ because of the excess demand in $t = 0$ and a resulting tendential rise in the prices of commodities a_0 and b_0 (in terms of which savings are effected) *relative* to those of a_1 and b_1.

26 Cf.: 'Unfortunately, necessary conditions are unlikely to be available' (Arrow, Hahn, 1971: 242).

27 See e.g. Marshall (1949: 109–110, 391n, 665); Walras (1954: 112–113); Wicksell (1934: 56–61).

28 See e.g. Kirman (1989).

29 Thus Hicks could conclude in 1939 about the stability of equilibrium in exchange, and with reference to income effects:

> It cannot indeed be proved a priori that a system of multiple exchange will be necessarily stable. But the conditions of stability are quite easy conditions, so that it is quite reasonable to assume that they will be satisfied in almost any system with which we are likely to be concerned.
>
> (Hicks, 1939: 72)

and he even thought that such a conclusion could be strengthened when introducing production (1939: 104).

30 See e.g. the distinction between 'intertemporal' and 'contemporary' relative prices in par. 3. (For its implications, cf. point V in par. 14 and par. 8 in Appendix I).

31 Essentially, the resort to a sub-case has occurred once, namely for the monotonic rising change of P_{a0} as r_b rises (cf. point **II** in par. 14 below).

32 They are not of course perfect substitutes in production.

33 See above, n. 2.

34 Thus in *Value and Capital*, 1939, where the new notions of equilibrium and the associated 'dating' of equilibrium variables were used for perhaps the first time in influential Anglo-American work, Hicks felt forced to assume that transactions carried out at non-equilibrium prices would have very little effect on the amounts transacted, so that equilibrium prices could be realised on his 'Mondays' (1939: 127–128: the stability of those equilibria was evidently taken for granted). However, both in that book and in much subsequent work in pure theory, it is not mentioned that the question in the preceding literature was that of a

compensation of deviations through repetition, and not that of the price actually hitting its equilibrium level. Thus Bliss (1975) refers to the 'Herculean programme' of constructing 'the complete theory of the behaviour of the economy out-of-equilibrium' necessary, in his view, in order to validate equilibrium 'as something that would be expected to be realised' apparently overlooking that repetition of transactions on unchanged data and not the truly Herculean 'complete theory etc.' had been the question – so long as one could rely on the traditional notion of equilibrium. We get however closer to the basic difficulty, when Bliss continues

> Furthermore, even if equilibrium were to be stable there might not be enough time within the space of a "week" for the prices to adjust to an equilibrium
>
> (Bliss, 1975: 28)

where Bliss sees the importance for the significance of the equilibrium of a sufficient lapse of time under approximately unchanged data. But, then, why not an equilibrium like the traditional one which would last more than a week, without falling in the limited relevance of the "steady state"? Here again (cf. n. 5 above) Bliss does not seem to ask himself the question, clearly relevant for the theory of capital to which he devotes his book (we shall indeed see in the Appendix II, par. viii, the very particular way in which he reconstructs the traditional equilibrium of Wicksell).

35 Walras introduced re-contracting only in the 4th edition of the *Elements* (1900). In the previous editions, the word *tâtonnement* had covered only the process of repetition of actual transactions, which in his view (confirmed up to the posthumous 4th 'definitive' edition of 1926):

> is perpetually tending towards equilibrium without ever actually attaining it (. . .) like a lake agitated by the wind where the water is incessantly seeking its level without ever reaching it.
>
> (Walras, 1954: 380)

Re-contracting with 'bons' (tickets) was introduced in the 4th edition of 1900 and exclusively in order to avoid considering the changes occurring in the capital stocks during the process of gravitation around the equilibrium (1954: 242 and Jaffè's collation note [h] pp. 582–583; Garegnani, 1960). Thus, it seems, recontracting was adopted in order to rigorously analyse a *real* repetition of transaction (in which, he thus implied, changes in the capital stocks over the relevant period would not make appreciable difference) and not in order to have a *notional* repetition of transactions when a real one was prevented even in principle by the dating of the equilibria.

36 The level $r_b > r_{b\min}$ entails $P_{b0} > 0$ and hence, by assumption (iii) par. [5], Appendix I, $P_{a0} > 0$.

37 Thus, should the schedules be representable as in Figure 5.2 below, intersection D would *not* indicate an equilibrium because, of the two successive (F) positions, (F′) and (F″), that we meet in sequence at $\bar{r}_b$ as L^D falls from the full employment level of I^+ and S^+, point D corresponds to (F′) on the I schedule, but to (F″) on the S schedule. Intersections E^1 and E^2 however are equilibria since each corresponds to the same (F) position on both schedules.

38 Except for the fluke case of $S_a = I_a$, and therefore, $S_b = I_b$ exactly achieved for $r_b = -1$, which would of course be compatible with both $S > I$ and $I > S$ in the proximity of $r_{b\min}$.

39 It would be correct to say that the cause of the possible rise of one own rate, call it r_a, as the other falls because of excess savings, is quite different from the excess savings, which cause the other to fall and would generally make for a fall of *both* rates. A fall of all consumption goods prices for $t = 0$, relative to those for $t = 1$ is in fact what we would expect from excess savings, i.e. excess supply of consumption goods *in general* in $t = 0$, and excess demand for them *in general* in $t = 1$. A rise of r_a as we move along the schedules because of excess savings can only result from a strong dislocation of contemporary relative prices for which the relative price of a to b were to rise for more between $t=0$ and $t=1$ than it did at a higher levels of r_b (cf. Appendix I par. [9]).

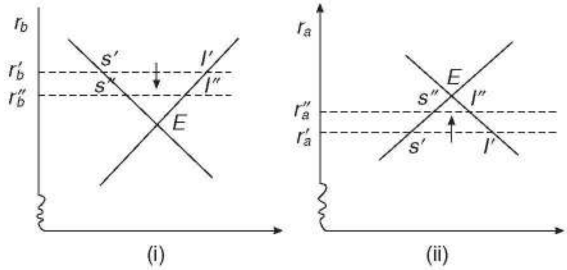

Figure A Should the own commodity rates r_a and r_b change in opposite direction in moving from a position (F′) of the economy with $S' > I'$ to a position (F″) with only r_b falling as indicated by the arrow in diagram (i) – then the spontaneous competitive movement in diagram (ii), with r_a on the vertical axis, would be upwards.

A seeming puzzle connected with opposite movement of the own rates should perhaps be sorted out at this point. Should r_a rise in the proximity of an equilibrium as r_b falls because of excess savings, would not a representation of the same (F) position with r_a in place of r_b on the vertical axis (see Figure A) lead to opposite conclusions about the stability of the equilibrium? The answer is of course negative since the *adjusted* spontaneous competitive change consequent to excess savings would, with an apparent paradox, be a rise and not a fall of the interest rate – owing to the change in the relative-prices factor of equation (5.6d in loc.cit. Appendix I). Figure A above illustrates the 'upside-down' representation of an equilibrium E and of the neighbouring positions (F′) and (F″) when r_a is put on the vertical axis.

40 As I remarked in (1991: 359) arguments about stability conducted like the one here, confined essentially to *signs* of changes, while finding ready confirmation in competitive behaviour, 'are by their nature quite general [and] render . . . possible equally general conclusions'.

41 No tendency would be there to the 'fluke' equilibrium mentioned in n. 38 when $S_a = I_a, S_b = I_b$ at $r_{b\,\min}$ but $S < I$ as $r_b \rightarrow r_{b\,\min}$.

42 Marginal products, whether of the discontinuous or the continuous variety, require that the available techniques be susceptible of being ordered so that they can be made to differ by the quantity of only one factor at a time. That, it seems, cannot generally be done when the factors are more than two: weighted averages of the different methods available which could give the above result will not make general economic sense, since it would be an exception when the methods entering such averages could co-exist.

43 A sufficient unanimity concerning the technique to be adopted has evidently to be assumed in order to let the corresponding prices emerge from the re-contracting. That unanimity has then to be replaced by a similar one concerning a second technique which had been found to be cheaper at those earlier prices, and so on and so forth. The way out of this conundrum is of course the further fiction of the auctioneer.

44 For the question in its traditional context, cf. Sraffa (1960, Chapter xii) and the subsequent literature referred to in Kurz and Salvadori (1995: 151).

45 Comparisons regard in fact methods of production of each commodity rather than techniques: and changes in technique, in particular those leading to the cheapest technique *i*, are best envisaged as resulting from changes of methods for one commodity at a time. However, referring to techniques, as above, rather than to methods, has no drawback here when one remembers that there will then be no one-to-one correspondence between method and technique, the same method appearing in several techniques differing from each other by the method of the remaining commodity.

46 Cf. Garegnani (1978–1979: I).

47 To make r_b appear explicitly in equation (5.10b) we should turn to the undiscounted prices, here indicated by the small letter p, (cf. n. 11 above). By itself, this change in the equations does not, of course, entail any change in the assumptions of the model (in particular in the assumption of complete future markets or of an 'initial' contracting period): it only requires that each price be notionally referred to the date of delivery of the good by taking as numéraire for it, the good b of the same date. Then the wage W and the undiscounted prices of a_1 and b_1, are numerically equal to the discounted ones because already expressed in terms of b of $t = 1$

$$p_{b1} = P_{b1} = 1; \qquad p_{a1} = P_{a1}; \qquad w = W.$$

As for the undiscounted prices of a_0 and b_0, using equation (5.6f) we have:

$$p_{b0} = 1 = P_{b1} = P_{b0}/(1 + r_b).$$

and since $P_{b0} = 1 + r_b$, we also have

$$p_{a0} = P_{a0}/P_{b0} = P_{a0}/(1 + r_b).$$

The two relations (5.1f) can then be written as follows:

$$\begin{aligned} p^i_{a1} &= l_{ai} W^i + (a_{ai} p^i_{a0} + b_{ai} p^i_{b0})(1 + r_b) \\ p^i_{b1} &= l_{b1} W^i + (a_{bi} p^i_{a0} + b_{bi} p^i_{b0})(1 + r_b) \end{aligned} \qquad (5.3\text{fi}')$$

and the first equation of each couple (5.8fi) is

$$p^i_{aj1} = l_{aj} W^i + (a_{aj} p^i_{a0} + b_{aj} p^i_{b0})(1 + r_b). \qquad (5.9\text{fi}')$$

Expressing now the undiscounted capital expenses as

$$\begin{aligned} c^i_{ai} &= a_{ai} p^i_{a0} + b_{ai} p^i_{b0} \\ c^i_{aj} &= a_{aj} p^i_{a0} + b_{aj} p^i_{b0}. \end{aligned} \qquad (5.10')$$

By transformations analogous to those operated in passing from equations (5.10) to (5.10b) in the text, we obtain

$$\frac{p_{ai1}}{p_{aj1}} = \frac{1 + \frac{c^i_{ai}}{l_{ai}} \cdot \frac{1 + r_b}{W^i}}{1 + \frac{c^j_{aj}}{l_{aj}} \cdot \frac{1 + r_b}{W^i}} \frac{l_{ai}}{l_{aj}}, \qquad (5.10\text{b}')$$

where the change in the p_{ai0}/p_{aj0} can now be seen to depend on the ratio $(1 + r_b)/W$.

48 For an example cf. the quotation in par. 1, n. 4.

49 Equation (5.5b) holds before the adjustments of purchasing power mentioned in par. 9.

50 Thus it seems incorrect to hold, with evident reference to reverse capital deepening and the reswitching of techniques, that

> it is only because we want to have some kind of geometric average, called the rate of interest, that we get some of these paradoxes.
>
> (Arrow, 1989: p. 155)

The paradoxes are present, whether we refer to the intertemporal prices which, as Arrow states there, are 'what we are really interested in', or to *the* single rate of interest in the case of the traditional equilibrium. Only the language for describing them changes (cf. n. 39 above).

51 It perhaps ironical that Keynes should have been incorrect when he wrote: 'If savings consisted not merely in abstaining from present consumption but in placing simultaneously a specific order for future consumption, the effect *might* indeed be different. For in that case the resources released from preparing for present consumption could be turned over to preparing for the future consumption' (Keynes, 1936: 210–211). In fact the order for future consumption could remain unfulfilled if the reaction of the interest rates failed to incentivate entrepreneurs to 'deepen' capital in the economy. However, the 'might' we have italicised indicates, perhaps, Keynes' doubt that his 'struggle of escape from habitual modes of thought' (1936: p. viii) could have gone further.

52 Cf. e.g. 'Certainly . . . we should not be much interested in an equilibrium with a zero real wage' (Arrow and Hahn, 1971: 354–355).

53 It may seem paradoxical that it should be *excess* investment that accompanies here labour unemployment. However in the model, excess investment is excess demand for the *given* stocks of a_0 and b_0, with whose owners labour competes in sharing the product available in $t = 1$ (see equation (5.1a) in Appendix I, par. [2]): zero wages due to deficient investment need at least three periods in order to emerge.

54 To a (1970: 422) statement of mine to the effect that the neoclassical explanation of distribution rests essentially on the premise that a fall of the rate of interest must increase the ratio of "capital" to labour in the economy, Chistopher Bliss reacted by writing "If by explanation of distribution the author means equilibrium theory (say of the intertemporal economy) then he has dreamed up this condition" (1975, 117). What we just saw in the text might now convince Bliss that dreams can indeed be prescient – of course if one regards as relevant for intertemporal "equilibrium theory" the possibility of zero wages or negative interest rates in its equilibria.

55 Walras (1954: 274 ff). Of course it is difficult to define 'perpetual future income' in an intertemporal equilibrium, rather than in the traditional long-period equilibrium of Walras's system (par. 1, n. 2). However, the essential nature of savings in neoclassical theory as demand for a homogeneous commodity 'future income' remains the same.

56 Walras's point incidentally explains why competitive arbitrage tends to achieve a uniform rate of return over any (however short) period by acting on the *demand price* of the capital good, and lowering it below its supply price when the rate of return on the latter is below that of other capital goods. But the prices of capital goods, like those of any product, tend to equality with the respective supply prices in the Marshallian long period, and Walras's single price will turn out to obtain on the *supply* prices of the capital goods as Walras and the present discussion imply.

57 The need for that 'quantity' is in fact strictly connected with neoclassical theory. The perfect substitutability among capital goods for wealth holders does not impose the notion of an independently measurable quantity of capital in the classical theories, where the rate of interest (profits) is not explained in terms of individual demand and supply decisions about 'factors of production' and is instead obtained, essentially, as a difference between product and wages determined separately from prices.

58 See n. 2 above.

59 Cf. n. 8 above.

60 Cf. e.g. par. 13.

NOTES FOR APPENDIX I

1 Paragraphs in this Appendix are numbered in square brackets to distinguish them from those of the main text.

2 Should b_0 be storable, its intertemporal price P_{b0}/P_{b1} could not fall below unity, or below $(1 - \alpha_b)$ when storage for a year implies costs α_b (cf. equations (5.1e″), n. 12 in the text), and r_b could not accordingly fall below zero, or $(-\alpha_b)$ respectively.

3 E.g. we cannot exclude that at some relative price P_{a0}/P_{b0} a consumer be indifferent between the two goods *within* certain limiting proportions of them: all quantities staying within those

limits would then be demanded at the corresponding single price. (This question, which concerns multiplicity of quantities demanded at given prices should of course not be confused with 'income effects', which may cause multiplicity of prices for the same quantities demanded: par. [5]).

4 In fact the rise of P_{a0} together with r_b (par. [5]) will lower the economically relevant upper limit of both r_b and P_{a0} below those of relations (5.6c) and (5.1b).

5 The device would consist of somewhat counterintuitively dissociating the total quantities A_0^U, B_0^U of the two commodities 'used' for consumption and investment, from the respective quantities 'demanded' A_0^D, B_0^D appearing in equation (5.7f). Provided the price of the commodity in question is zero, we could then let the quantity 'demanded' A_0^D or B_0^D exceed the respective one used A_0^U, or B_0^U, by allowing for inequalities in the corresponding relations [5.3f], where the R.H.S. expresses A_0^U, B_0^U, respectively, and in that way satisfy equation (5.7f).

6 A lowering of P_{a0}/P_{b0} would tend to lower the relative price P_{a1}/P_{b1} of the good requiring more of a for its production (provided the relative labour intensities in the production of a and b do not cause the variation of the wage to more than compensate that effect) and with the proportion consumed of that good increasing, the proportion I_a/I_b would also increase.

7 It might on the other hand seem that having in common $W = 0$, such a continuum of solutions of (F) would have in common also the contemporary relative price $\pi = P_{a0}/P_{b0}$ and the rest of the price system with it, so that a single level of $r_b = r_b^+ = r_{b\max}$, and the corresponding single set of prices, would hold over the whole *continuum* for $W = 0$. However zero total wages $L^D W$ mean that Q_1, the value of the product in $t = 1$ (par. 9, n. 24 in the text) along the *continuum*, is given by

$$Q_1 = S = D_{a1}P_{a1} + D_{b1}P_{b1}.$$

Now, if prices were not to change along that continuum, the *ratio* D_{a1}/D_{b1} would be constant, and so would then be the *ratio* I_a/I_b, though the *absolute* levels of D_{a1}, D_{b1} and therefore I_a, I_b would be falling as L_D falls (see the adjustment to the level of I of the purchasing power in $t = 1$, described in the main text: par. 9, n. 24). Since D_{a0} and D_{b0} would instead be constant also in their *absolute* amounts, it would follow that, A_0^D/B_0^D, a weighted average of D_{a0}/D_{b0} and I_a/I_b (see the first two equations (3f)), would generally be changing with L^D. This imposes a change in the contemporary price π (and hence the other prices) in order to continue to satisfy equation (5.7f).

8 It may be useful to note that r_b^0, although no greater than r_b^+, must be larger than $r_{b\min}$. Since $P_{b0} = 0$ would entail $P_{a0} = 0$ (par. [11]), equation (5.1c), valid for $W = 0$, is only compatible with $P_{b0} > 0$, and therefore with $r_b^0 > r_{b\min}$.

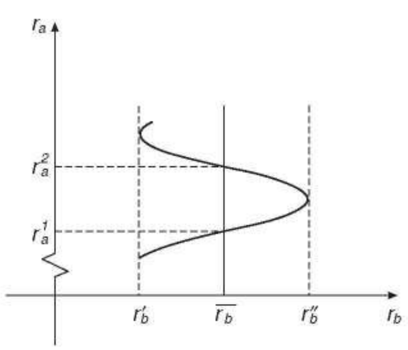

Figure B As the own rate r_b falls from r''_b to r'_b a rise or constancy of the own interest rate r_a, as depicted in the figure, is incompatible with the monotonic increasing relation between P_{a0}/P_{b1} and P_{b0}/P_{b1} of Assumption (iii) in par. 4.

9 Formally we replace equation (5.7f) with an equation P_{a0}/P_{b0} = constant, and treat A_0^D/B_0^D as an unknown.

10 Suppose that not to be so and thus r_a to fall as r_b rises in a sub-interval $r_b^I < r_b < r_b^{II}$ included in the overall interval $r_{b\min} < r_b < r_b^0$ (see Figure B). At least two distinct levels, r_a^1 and r_a^2, would have to correspond to any given level $\bar{r}_b$ in that sub-interval, since r_a must have initially risen from $r_{a\min} = -1$, together with r_b, as P_{a0}/P_{b1} rose monotonically from zero together with P_{b0}/P_{b1} (Assumption (iii); and par. 11 below). However, that same monotonic relation between P_{a0} and P_{b0} entails that the two levels P_{a0}^1 and P_{a0}^2 corresponding to r_a^1 and r_a^2 respectively, would be equal because they correspond to the single value $\bar{P}_{b0} = \bar{r}_b + 1$. But then $P_{a0}^1/P_{b0} = P_{a0}^2/P_{b0}$ would have in common the same unique series of prices (par. 2), and therefore $r_a^1 = r_a^2$, contrary to our premiss. The same monotonicity of the relation between P_{a0}/P_{b1} entails that r_a cannot remain constant as r_b changes: for that to happen two levels of r_b and therefore of P_{b0}, i.e. P_{b0}^1, P_{b0}^2 should correspond to the level $\bar{P}_{a0}$ holding for the given r_a.

11 This can only happen for a part of the relevant interval of r_b, since r_a has to rise from -1 as r_b does (see the preceding endnote).

12 A position which we find exemplified in Morishima (1964: 87) is to assume, at a zero wage, both a zero supply *and* the continuity of the supply schedule for labour. It might seem possible to reconcile this position with the question of 'survival' discussed in the text, by assuming a nearly horizontal segment joining the origin of the axes with a comparatively high level of labour supply for a wage close to zero (a *strictly* horizontal segment would instead be the graphic representation of the kind of continuity we are assuming). However, it is not easy to see why a worker who has been increasing his labour supply to try to survive, should abandon that purpose by progressively *decreasing* his supply when the wage is still positive: a discontinuity of supply would rather seem natural when the worker gives up hope of survival. Even more arbitrary seem to be other assumptions ensuring continuity, such as e.g. that of each individual being endowed with some quantity of each resource (Debreu, 1959: 19) *i.e.* of a quantity of some other resource whose price is bound to rise when that of the resource considered tends to zero, thus avoiding dependence on the latter for survival.

13 We are here referring to general-equilibrium-demand and supply schedules for labour, obtained from (E) by treating W as an independent variable, and using the third of relations (5.3e) to define the new unknown labour demanded L^D, with an adjustment in aggregate expenditure analogous to that we saw in par. 9, n. 24 in the text.

14 The discontinuity would result from savings suddenly disappearing, at $r_b = -1$ with a_0 and b_0 being used instead entirely for consumption in $t = 0$ at the 'contemporary' price $\pi = P_{a0}/P_{b0}$, ensuring relative consumption demands D_{a0}/D_{b0} compatible with (5.7f). It is also possible to envisage that, through consumption loans, wage income (reaching its maximum, as r_b and r_a fall towards their minimum) be spent in advance by increasing consumptions D_{a0}, D_{b0} of both goods as their intertemporal prices relevant here fall towards zero: this could engender negative gross savings in the lower range of the S curve, and savings could become zero at some level $r_b > r_{b\min}$.

15 This arbitrage condition like any arbitrage conditions entailed by indirect exchanges is implied when reducing the number of prices from that of all combinations of the n commodities taken two by two, down to $(n-1)$. Thus e.g. $P_{b0}/P_{b1} = 0$ entails that in $P_{b0}/P_{b1} = (P_{b0}/P_{a1})(P_{a1}/P_{b1}) = 0$, with $P_{a1}/P_{b1} \neq 0$, we should have $P_{b0}/P_{a1} = 0$. This, so to speak, 'collective' zero intertemporal price of commodities dated $t = 0$, in terms of commodities dated $t = 1$ confirms on the other hand that capital goods a_0 and b_0 are perfect substitutes for the savers acquiring them (parr. 25–26 in the text) so that the zero intertemporal price of one entails a zero price of the other. The reader may however wonder how it will be possible to discriminate in practice between, on the one hand, producers of b_1 or a_1, who would get b_0 and a_0 for free, and owners of a_0 and b_0 who should instead give *some* of their commodity in order to get the other.

16 Cf. par. 3. The only difference is that for $r_b = -1$, i.e. $P_{b0} = 0$, equation (5.2f′) of par. 4 gives the intertemporal price $P_{a0} = 0$.

17 Given the possible multiplicity of solutions of (F) in the interval $r_b^0 \leq r_b \leq r_{b\max}$, the determination of the (F) positions for $W = 0$ is most easily envisaged by the device of letting L^D (which is uniquely related to those positions) to take the role of independent variable when $W = 0$, so as to obtain the corresponding r_b and with it all the other unknowns of the (F) system for that level of L^D. In particular this will allow tracing the multiple (F) positions which may correspond to the same level of r_b for $W = 0$.

18 Assumption (iii) of par. [5], on the joint rise of P_{a0}/P_{b1} and P_{b0}/P_{b1} so long as $W > 0$, excludes that among the multiple solutions for $r_b^0 \leq r_b \leq r_{b\max}$, there may be any with a positive W, apart from those on the 'main branch' of the function relating $\pi = P_{ao}/P_{b0}$ and r_b.

19 Contrary to what one might think at first, the income from which those savings come, is likely to change little as L^D changes since, with $W = 0$, only the owners of the initial stocks A_0^S and B_0^S will have an income (cf. n. 7) – the adjustment in the expenditure for $t = 1$ described in par. 9 of the text then taking care of the changes in the physical social product for that period as L^D changes.

NOTES FOR APPENDIX II

1 See par. (v) about the capital endowments of Hahn's 'Special' case. Cf. Garegnani (1990b: 113–115) for an answer to Hahn's (1982) first claim. We may also note that in (1975: 361) Hahn finds 'incomprehensible' the 'neo-Ricardian' idea that 'distribution precedes value'. However, it was in (1951, xxx–xlviii) that Sraffa had advanced his interpretation of Ricardo and the old classical economists in terms of a wage determined by historical and institutional circumstances ('subsistence'), with the non-wage distributive variables accordingly resulting as a residue or surplus. This interpretation, where in an evident sense distribution (the determination of the real wage) precedes values, had indeed been expounded and variously discussed before, with, and after, Sraffa (1960), also with respect to its distinction from the alternative theory advanced by Joan Robinson in 1956 and her other works.

2 Cf. e.g. Hutchison (2000); Blaug (1999).

3 Cf. equations (22) in Hahn (1982: 364) corresponding to our equations (1e) of par. 3 in the chapter; his equation (17), p. 363, for our (2e); then (25), p. 365, for the first three of our relations (3e) and, finally, equations (23), p. 364, for the remaining two of relations (3e).

4 Cf. equations [25], in Hahn (1982: 363). Hahn does not, on the other hand, assume 'activities' of free disposal and justifying those equality signs.

5 Cf. n. 7 below. For the distinction between demand and supply prices of capital goods, cf. e.g. Walras's distinction between the 'prix de vente' and the 'prix de revient' of such goods, which Jaffé translates in his edition as 'selling prices' and 'costs of production' respectively (Walras, 1954: 271 ff.).

6 Of course obsolete capital goods are not supposed to be produced but neither can they be present in the physically specified endowment of an economy like that of Walras (1954) and Hahn (1982) where only the dominant methods of production appear. I owe to Professor Fabio Petri having attracted my attention to the point.

7 See Garegnani (1990a: 11–22); on the point in Walras's own texts, cf. also by the same author (1958) and (1960), Part II, Chs. II and III. In Appendix G of the (1960) book, I also noted how Pareto, who in the *Cours* (1896–97) had already been ambiguous as to whether all capital goods were being produced, finally abandoned in his *Manuel* (1906) any attempt to formalise the theory of capitalisation. (The overdeterminacy of Walras's system becomes evident if we refer to the extreme case of absolute redundancy in the endowment of one capital good, with its rate of return on the supply price being accordingly zero or negative, and obviously less than that of any scarce capital goods).

8 Cf. Hahn (1982: 365–366) where it is suggested that B_0 be treated as an unknown, while keeping A_0 as a given.

9 One may be curious about the reason why in Hahn's model letting the proportion between the two capital goods in the endowment be an unknown will instead ensure constancy of prices. That has to do with the peculiarity of a model which admits of one production cycle only. By starting from a sufficiently high stock A_0^S relative to stock B_0^S (see par. 6 in the text) and then lowering it sufficiently, we may generally continuously change P_{a0}/P_{b0} so that it rises from zero to infinity: i.e. from a_0 being a free good to b_0 being such. There must then be at least one proportion A_0/B_0 of the two stocks for which the curve described by the relative price P_{a0}/P_{b0}, with A_0/B_0 in the abscissae intersects the analogous curve described by the production prices P_{a1}/P_{b1} of equations (1e) of par. 3 in the text, thus giving $r_a = r_b$. However, as soon as a second yearly cycle of production is allowed for, and price equations must be satisfied also for P_{a2} and P_{b2}, the same relative prices could be maintained for a third year only if the economy happened to be stationary (or the technical conditions of production allowed for a labour theory of value).

10 Cf. e.g. Hicks (1939: 118) and Garegnani (1960, Appendix E). This of course does not detract from the fact that a constancy of relative prices will generally imply also an adjusted physical composition of the capital stock.

11 See the references in n. 7, and Garegnani (1976: 38–39). The results are reported, e.g. in Harcourt (1972: 170, 171), J. Robinson (1970: 338), Dobb (1973: 205 n).

12 Cf. par. (iv) and n. 8.

13 As just mentioned Hahn (1982) does not recall that this 'specialisation' was traditionally achieved by treating capital as a homogeneous substance so that the proportions between capital stocks were in fact unknowns, and not 'specialised data' which 'history' had happened to throw up (par. [xi]).

14 On this failure to distinguish the basic form (b) of uniformity of returns on capital characterising a normal position of the economic system from Adam Smith down to recent decades, from the uniformity (a) between own rates due to price constancy and, besides, from the short period uniformity imposed by arbitrage (and to be presently considered in this Appendix), see e.g. Christopher Bliss's *New Palgrave* article dedicated to the 'Equality of Profit Rates'. Under what he calls 'the rate of profit' Bliss does not appear to distinguish between the above short-period uniformity on the demand prices of the capital goods and the uniformity on the supply prices, the one traditionally indicated as the 'uniform rate of profit'. The uniform 'rate of profit' is instead counterposed to the divergence of own commodity rates of interest which, as in Hahn, accordingly appears to be taken as one with the divergence of effective returns on the supply prices of the capital goods which the critics had indicated as a shortcoming of contemporary theory.

15 Cf. n. 2.

16 Such an attentive reading of the relevant part of Hicks's *Value and Capital* shows, I believe, that the difficulty Hicks was trying to overcome was not that of the change of equilibrium prices over time: but rather that of the 'quantity of capital' – as, on the other hand, he indicated by the very title of his book (on the point, cf. Garegnani, 1976: 30–36; Milgate, 1979: 6–9).

17 Cf. n. 8 in the text and n. 10 here.

18 Possibilities of substitution would still enter the system as the physical composition of the stock can change by means of capital replacement: but then, as argued in the main text, we are back to 'capital' as a single quantity and its problems through the relation between gross investment and the interest rate.

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Mathematical note to chapter 5

Michele Tucci

THE MODEL

Let us take into consideration the following model (F):

$$\begin{cases} P_{a1} = l_a W + a_a P_{a0} + b_a P_{b0} \\ P_{b1} = l_b W + a_b P_{a0} + b_b P_{b0} \end{cases} \tag{5.1f}$$

$$P_{b1} = 1 \tag{5.2f}$$

$$\begin{cases} A_0^{D'} \geq D_{a0} + a_a A_1 + a_b B_1 \\ B_0^{D'} \geq D_{b0} + b_a A_1 + b_b B_1 \\ L \geq l_a A_1 + l_0 B_1 \\ D_{a1} = A_1 \\ D_{b1} = B_1 \end{cases} \tag{5.3f}$$

If in the first relation of (5.3f) the inequality sign holds, then $P_{a0} = 0$; if it holds in the second one, then $P_{b0} = 0$. However, as is specified in Paragraph **[3]** of Appendix I, Assumption (ii), it should be noted that such cases will not be taken into consideration in the economic discussion in the text. The inclusion in the present demonstration is due to the need for clarifying the mathematical passages.

If the inequality sign holds in the third relation of (5.3f), then $W = 0$.

$$I = (a_a A_1 + a_b B_1) P_{a0} + (b_a A_1 + b_b B_1) P_{b0}, \tag{5.4f}$$

$$S = (A_0^S - D_{a0}) P_{a0} + (B_0^S - D_{b0}) P_{b0}, \tag{5.5f}$$

$$P_{b1}(r_b + 1) = P_{b0}, \tag{5.6f}$$

$$\frac{A_0^D}{B_0^D} = \frac{A_0^S}{B_0^S}, \tag{5.7f}$$

where P_{a0}, P_{b0}, P_{a1} and P_{b1} refer to prices, W indicates wages, S and I represent savings and investments, respectively, A_1 and B_1 correspond to quantities of produced

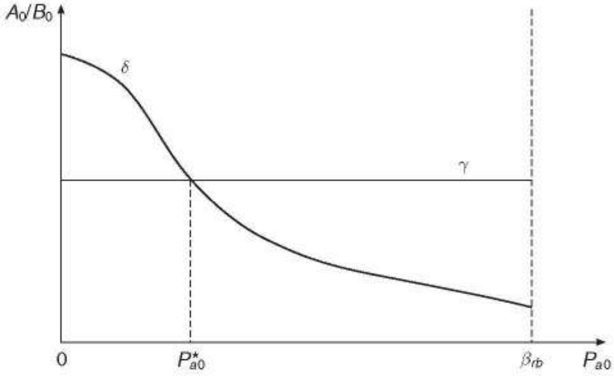

Figure 5.4 An example of case 1.

commodities, A_0^D and B_0^D specify demands, and A_0^S, B_0^S and L refer to endowments. The single-valued mappings D_{a0}, D_{b0}, D_{a1} and D_{b1} designate standard Walrasian demand functions, which are characterised by the following assumptions:

1 the set of independent variables in the functions are those indicated by the first five among the symbols specified above;
2 in the non-negative orthant of the independent variables, each demand function is positive and continuous.

For the sake of simplicity, we suppose that the technical coefficients are all strictly positive. Moreover, the usual vitality conditions will hold. Finally, r_b specifies the own interest rate of commodity b over period $t = 0$. The first eleven among the symbols listed above constitute the unknowns of the model, while r_b is exogenously defined.

Let us substitute equations (5.6f) and (5.2f) into the second equation of (5.1f), thus obtaining:

$$1 = l_b W + a_b P_{a0} + b_b(r_b + 1), \tag{5.8}$$

i.e. equation (5.1a) of Paragraph **[2]** of Appendix I.

Easy passages allow the following propositions to be derived from equations (5.1f), (5.2f), (5.6f) and (5.8):

(a) The quantities W and P_{a1} can be defined as functions of the single variable P_{a0}.
(b) Consider the interval $H_{r_b} = \{-1 \le r_b \le \max_{r_b}\}$, with $\max_{r_b} = (1 - b_b)/b_b$. Define $\beta_{r_b} = [1 - b_b(r_b + 1)' / a_b$. For every $r_b \in H_{r_b}$, we can determine an interval $H_{P_{a0}} = \{0 \le P_{a0} \le \beta_{r_b}\}$ such that, for every $P_{a0} \in H_{P_{a0}}, W \ge 0, P_{a1} > 0$.
(c) In the interval $H_{P_{a0}}$, the functions $W(P_{a0})$ and $P_{a1}(P_{a0})$ are continuous.
(d) If $-1 < r_b \le \max_{r_b}$, then $P_{b0} > 0$; if $r = -1$, then $P_{b0} = 0$.

Due to the income correction, which is specified in Paragraph 9 of the text, the third of (5.3f) is always satisfied with the equality sign, except in border solutions, which are examined in connection with Assumption (iii), Paragraph **[5]** of Appendix I, and in Paragraphs **[6]**, **[10]**, **[13]** and **[14]** of the same Appendix, where there will be a continuous set of solutions characterised by $W = 0$.

Let us assume that the equality sign holds in the first two relations of (5.3f). Substituting the last two equations of (5.3f) into the expressions on the right-hand side of the equality sign in the first two relations of (5.3f) and in equation (5.4f), we are able to define the variables A_0^D, B_0^D and I. Moreover, equation (5.5f) defines the variable S. In the interval $H_{P_{a0}}$, the four above-quoted expressions are continuous functions of the unique variable P_{a0}.

Define

$$\delta(P_{a0}) = \frac{D_{a0} + a_a D_{a1} + a_b D_{b1}}{D_{b0} + b_a D_{a1} + b_b D_{b1}}, \tag{5.9}$$

$$\gamma = \frac{A_0^S}{B_0^S} = \text{constant}. \tag{5.10}$$

In the interval $H_{P_{a0}}$, the function $\delta(P_{a0})$ is continuous.

Taking into consideration equation (5.7f), for every $r_b \in H_{r_b}$ one, and only one, of the following three sentences is necessarily true:

1 There exists $P_{a0}^* \in H_{P_{a0}}$ such that $\delta(P_{a0}^*) = \gamma$.
2 For every $P_{a0} \in H_{P_{a0}}, \delta(P_{a0}) < \gamma$.
3 For every $P_{a0} \in H_{P_{a0}}, \delta(P_{a0}) > \gamma$.

Figure 5.4 shows an example of case (1). Here, $P_{a0} \geq 0$ and the first two relations of (5.3f) are satisfied with the equality sign. Assumption (ii) in Paragraph **[3]** of Appendix I confines the main argument there to case (1) above. Therefore, the remaining two cases will be examined only for the sake of completeness.

In case (2), at the given level of r_b, commodity a_0, taken in the sum of both its consumption and investment uses, cannot be employed in as high a proportion to b_0 as the ratio γ in which it is found in the endowment. As a result, (F) can only admit solution if we allow the quantity A_0^D 'demanded' of equation (5.7f), to exceed the quantity used expressed by the R.H.S. of the first part of equation (5.3f), provided $P_{a0} = 0$ (cf. n. 5 to Appendix I).

In case (3) we have the case symmetrical to II, where b_0 cannot be used in as high a proportion to a_0 (i.e. a_0 in as low a proportion to b_0), as the ratio in which the two commodities are found in the endowment. The inequality sign of the second relation in (5.3f) will allow for a solution of (F) provided $P_{b0} = 0$, i.e. if r_b is set at $r_{b\,\min} = -1$. But no solution would exist if we set r_b, our independent variable, at a level $r_b > -1$ – a case, however, which is of no economic importance since (F) could never, then, provide a solution of (E), where $r_b > -1$ implies $P_{b0} > 0$, and is therefore incompatible with an inequality sign in the second of relations (5.3e).

6 Some implications of endogenous contract enforcement for general equilibrium theory*

Herbert Gintis†

6.1 INTRODUCTION

This chapter is a critique of Walrasian general equilibrium (GE) theory. Since the only good critique is a better alternative, I will be spending lots of time discussing alternatives. I deeply regret the fact that Walrasian general equilibrium theory is irremediably flawed, because it would be a really wonderful world if it were true. It is just that it would not be our world.[1]

I have two general criticisms of Walrasian general equilibrium theory. The first is substantive: the assumption of costless third-party contract enforcement is not an acceptable starting point for a model of the market economy. I will develop this critique using examples from labor, capital, managerial, and consumer goods markets.

The second is methodological: most research in the Walrasian tradition model is based on the notion that the economy can be understood without being grounded in specific, empirically-based, natural regularities. For the Walrasian general equilibrium theorist, empirical regularities are unimportant, because the world is always an "imperfect" realization of the theory (imperfect information, imperfect competition, externalities, bounded rationality, missing markets). I do not fault Walras and a century of his followers for hoping that economics could get a "free lunch" (theory not grounded in empirically-deduced regularities), but it is now time to move on to more fertile research areas.[2]

To illustrate this methodological critique, I will present some recent developments in experimental economics that suggest a radical revision of our conception of the human actor. I will stress three points. First, in many decision-making and strategic settings people do not behave like the self-interested "rational" actor depicted in neoclassical economics. Second, despite its increased complexity in comparison with traditional *Homo economicus*, human behavior can be modeled using game theory and optimization subject to constraints. Third, there are plausible models of

* Prepared for the International School of Economic Research, "General Equilibrium: Problems, Prospects and Alternatives," Siena, Italy, July 1999. This chapter is based partly on research undertaken with Samuel Bowles and partly on my book *Game Theory Evolving*, Princeton University Press (2000a).

† External Faculty, Santa Fe Institute. I would like to thank the MacArthur Foundation for financial support, and participants at the conference, and Professor Fabio Petri in particular, for thoughtful comments. Proofs of theorems are not included in this chapter, but are presented in Gintis (2000). The author can be reached at http://www-unix.oit.umass.edu/~gintis.

human cultural and genetic evolution that explain how we have gotten to be the way we are. Our analytical and evolutionary models, however, leave considerable room for improvement, and we are presently on the steep portion of the learning curve in developing analytical models of human behavior.

Laboratory experiments show that the *Homo economicus* of neoclassical economics is but one of several *persona* exhibited by human subjects engaging in strategic interaction. Because of limited space, I will discuss only one of several such *persona*, but one that I think is extremely illuminating in helping us to understand the nature of human cooperation and competition. I call him *Homo reciprocans. Homo reciprocans* exhibits what may be called *strong reciprocity*, by which I mean a propensity to cooperate and share with others similarly disposed, even at personal cost, and a willingness to punish those who violate cooperative and other social norms, even when punishing is personally costly, and even when there are no plausible future rewards or benefits from so behaving.

6.2 WALRASIAN GENERAL EQUILIBRIUM MODEL: THE PROBLEM OF ENDOGENOUS CONTRACT ENFORCEMENT

In the Walrasian general equilibrium model, firms and households make decisions based on commodity and factor prices, markets generate prices that equate supply-and-demand, and ownership rules assign the return to factor inputs to individuals. Under a suitable set of assumptions, the so-called *Fundamental Theorem of Welfare Economics* holds: Every competitive equilibrium is Pareto-optimal and every attainable distribution of welfare among agents can be achieved by a suitable initial distribution of property rights, followed by a Walrasian allocation.[3]

Its supporters often justify this model as an ideal type with "perfect information," and "complete contracting," of which real economies, with their "imperfect information" and "incomplete contracting," are distorted by confounding factors. An argument of this type is quite acceptable in physics, where the ideal type can be shown to hold in the absence of confounding factors, and where the confounding factors can themselves be modeled in terms of ideal type laws. But in economics the ideal type laws have no independent physical or social existence, and they do not provide a basis for modeling the confounding factors.

It might be argued that general equilibrium theory deals with the "broad outline" of the economy and other techniques (e.g. game theory) fill in the "fine detail." However, general equilibrium theory wrongly depicts the economy even in broad outline, since it cannot explain several key aspects of successful economies, including: (a) why market economies appear to require private ownership; (b) why central planning cannot substitute for private markets in allocating resources; (c) why there is generally an excess supply of labor and an excess demand for capital; (d) why most firms are quantity constrained, and increase profits by expanding sales in a market where the price exceeds marginal cost; and (e) why the wealthy control production and investment. Moreover, in each case game theoretic models that do not presume costless third-party contract enforcement better explain empirical regularities.

6.3 GENERAL EQUILIBRIUM, MARKETS, AND PRIVATE PROPERTY

The general equilibrium model and the Fundamental Theorem are often considered justifications for a private-ownership market economy. In fact, as Oskar Lange and others (Barone, 1935; Lange and Taylor, 1938) demonstrated in the famous "socialism debate" with Friedrich von Hayek and other supporters of *laissez faire* capitalism (Hayek, 1935) in the 1930s, these principles can just as easily be used to justify the social ownership of property and the control of the economy by the state. Indeed, the Fundamental Theorem asserts that *any* pattern of ownership is compatible with economic efficiency, so long as prices are chosen to equate supply-and-demand. Moreover, such prices need not be set by market interactions or any other particular mechanism – price-setting by a central planner is perfectly compatible with economic efficiency.

Lange pointed out that markets and private property play a purely metaphorical role in general equilibrium theory: they are *alluded to* to account for profit maximization and market clearing, but they play no formal role in the model and many other institutional forms can just as easily be alluded to account for the same events. There is in fact no competition in the common sense of the term, since agents never meet other agents and agents do not care what other agents are doing. The only factors determining individual behavior are prices.

Nor do markets do anything in the general equilibrium model. In Léon Walras' original description of the general equilibrium model (Walras, 1954 [1874]), market clearing was not effected by markets at all, but rather by an "auctioneer" who calls out prices, measures the degree of excess supply and demand in all markets, adjusts prices accordingly, and repeats the process until equilibrium prices are determined. The auctioneer then freezes these equilibrium prices, and agents are allowed to trade freely at these prices. How ironic! Not the buzzing confusion of market competition, but the cool hand of the centralized state apparatus brings about "market" equilibrium.

Walras fiction of the auctioneer was a reasonable first cut, but no one considered it ultimately acceptable. Most economists thought the auctioneer would eventually be replaced by a plausible decentralized, market-oriented equilibration mechanism. But such has not been the case. No one has succeeded in producing a plausible dynamic model of market interaction in which prices move towards their market-clearing levels. Only under implausible assumptions can even the "auctioneer" dynamic be shown to be stable (Fisher, 1983). Moreover, for any possible (continuous) behavior of excess demand as a function of price – even the most chaotic and complex, stable or unstable – there is some set of preferences and an initial endowment that gives rise to this behavior, and the preferences need not be "exotic" at all to generate this behavior (Saari, 1995).

The socialists won the academic debate, virtually everyone agrees. Joseph Schumpeter's classic *Capitalism, Socialism and Democracy* (1942), in which this staunch supporter of capitalism predicts its imminent demise, is perhaps the greatest tribute to the socialist victory in this debate. Hayek himself apparently concluded that it had been a mistake to conduct the debate in terms of the neoclassical theory, and in the late 1930s and early 1940s developed the analytical foundations of an alternative to the neoclassical model (Hayek, 1945). But the socialists lost the real world conflict of economic systems. Private property and markets apparently *are* central to a well-functioning advanced economic system. Therefore, there must be something fundamentally wrong with the Walrasian model, which says they are not. What might that be?

The general equilibrium model is inspired by models of physical systems, especially field-theoretic formulations of Newtonian mechanics, in which particles create force fields (electromagnetic, gravitational, and the like), and each particle interacts with the field. In general equilibrium theory, we replace particles by people and the field becomes the price system. But in fact society involves, centrally and probably irreducibly, the strategic interaction of agents.[4] This is all absent from general equilibrium theory.

6.4 ENDOGENOUS CONTRACT ENFORCEMENT

The Walrasian general equilibrium model assumes contracts are costlessly enforceable by third parties. This may be reasonable for standardized goods (for instance, raw materials, basic chemicals, and easily graded agricultural goods) but it is implausible when applied more generally.

In the absence of exogenous contract enforcement, and if producers have information concerning production that is not available to the Planning Board in the socialist state (or equivalently to Walras' auctioneer), there may be no way to elicit the knowledge that leads to high productivity without strategic interaction in the form of head-to-head market competition.

In effect, under the proper circumstances, competitive interactions subject firms to a "prisoner's dilemma" in which it is in the interest of each producer to reveal private information and supply high effort, even in cases where consumers and the planner/auctioneer cannot observe or contract for information/effort itself. In the words of Holmström (1982), "Competition among agents…has merit solely as a device to extract information optimally."

If Holmström is right – and both game theoretic modeling and practical experience suggest that he is – the defense of competitive markets in neoclassical economics must be one of the all-time great intellectual ironies. Since Adam Smith supporters of the market system have defended markets on the grounds that they allocate goods and services efficiently. Much to the consternation of those who take empirical facts seriously, the estimation of the "Harberger triangles" that represent the losses from misallocation, monopoly, tariffs, quotas and the like have little effect on per capita income or the growth rate (Browning, 1997). The real benefits of competition, by contrast, have only come to light with the development of game theoretic models of competitive interactions based on endogenous contract enforcement.

6.5 CONTINGENT RENEWAL CONTRACTS

In many exchanges, including those between (a) employer and employee; (b) lender and borrower; and (c) firm and customer, the agent on one side of the exchange gives money (employer, lender, customer), while the agent on the other side of the exchange gives a promise (employee, borrower, firm). The employee promises to work hard, the borrower promises to repay the loan, and the firm promises to provide high-quality products. Rarely, however, is this promise subject to a contract that can be enforced at reasonably low cost.

Let us call the player who gives money the *principal*, and the player who gives promises the *agent*. In the absence of an enforceable contract, why do agents keep their promises? Perhaps the threat of suing in a court of law is sufficient to secure agent compliance. But generally such threats are not credible. The idea of taking an employee to court for not working hard enough is ludicrous. A lender can sue a borrower for non-payment, but if the borrower was imprudent, winning the suit is a Pyrrhic victory – there is not much to collect! A customer can sue a firm for faulty goods, of course, but very few of us have ever done such a thing, and it is not reasonable to suppose that, except in special cases (for instance products that cause personal injury), firms satisfy customers' needs because they are afraid of being taken to court. So why, then, do agents generally keep their promises?

The answer is, of course, that if agents do not keep their promises, principals dump them: employers fire workers who shirk, lenders refuse future loans to borrowers who have defaulted, and customers switch to new suppliers when dissatisfied. All the three actions represent *trigger strategies in a repeated game*: the exchange between principal and agent is renewed indefinitely (perhaps with some exogenous probability of dissolution), the principal using the threat of non-renewal to secure compliance. We call these *contingent renewal* exchanges.

6.5.1 Contingent renewal markets do not clear in equilibrium

A *contingent renewal market* is a market in which exchanges between buyers and sellers are regulated by contingent renewal relationships. Since the principal (employer, lender, consumer) in such markets uses a trigger strategy (the threat of non-renewal) to elicit performance from the agent (worker, borrower, firm), the loss of the relationship must be costly to the agent. But if price is set in such markets to equate supply-and-demand, the cost to an agent of being cut off by the principal is zero, since the agent will secure another position in the next period at the prevailing price. Hence if the principal uses a trigger strategy, there must be a positive probability that there is an excess supply of agents. It follows that in a Nash equilibrium of a contingent renewal market, there is an excess supply of agents. There are many models in the literature exhibiting this phenomenon, including Gintis (1976), Stiglitz and Weiss (1983), Shapiro and Stiglitz (1984), Bowles and Gintis (1993a) and Bowles and Gintis (1998c).

Notice how nicely this conclusion explains some of the most pervasive facts about market economies.

1 *Labor markets*: In the neoclassical model, the wage rate adjusts to equate the supply of and the demand for labor. The general condition of labor markets, however, is *excess supply*. Often this takes the form of explicit unemployment, which neoclassical economists develop complex models to explain, using search costs, friction, adaptive expectations, exotic intertemporal elasticities, and the

like. Using Occam's razor a contingent renewal labor market does the job. There simply cannot be full employment in such models (Gintis, 1976; Shapiro and Stiglitz, 1984; Bowles and Gintis, 1993b). Excess supply in labor markets takes the form not only of unemployment, but also of "underemployment": workers hold one position but are capable and willing to fill a "better" position, even at the going wage or a bit below, but cannot secure such a position.

2 *Credit markets*: In the neoclassical model, the interest rate adjusts to equate the supply of and the demand for loans. The general condition of credit markets, however, is excess demand. Why does the interest rate not rise to cut off this excess demand? There are two basic reasons (Stiglitz and Weiss, 1981; Stiglitz, 1987). First, an increase in the interest rate will drive borrowers who have low-risk, low-expected-return projects out of the market, and increase the expected riskiness of the remaining pool of borrowers. Second, an interest rate increase will induce borrowers to increase the riskiness of their investment projects, thus lowering the lender's expected return.

Since risk-sharing – requiring the borrower to put up a fraction of the equity in a project – is the most widely used and effective means of endogenous contract enforcement in credit markets, it follows that lending is directed predominantly toward wealthy agents. This basic fact of life, which seems so perverse from the neoclassical standpoint (loans should be from the wealthy to the non-wealthy), is perfectly comprehensible from the standpoint of models in which contract enforcement is endogenous, even without contingent renewal. Contingent renewal (making available a line of credit, contingent on performance) adds the dimension that a certain subset of non-wealthy borrowers with good projects can get loans, facing the threat of falling into the pool of unemployed "credit-seekers" should their credit line be terminated.

3 *Consumer goods markets*: In the Walrasian model, the price adjusts until supply and demand are equal. This implies that firms can sell as much as they want, subject to the market price, and choose how much to produce according to cost considerations. Everyday observation tells a different story: firms try to *Sell More Stuff*, and except in their wildest dreams, they can produce with ease however much stuff they manage to sell. Only under the threat of a failing grade can we convince our students that the common-sense view is wrong. Of course there are sophisticated neoclassical models in which firms have "differentiated products" and "downward-sloping demand curves," but this is not really what is happening. What's really happening is that firms do not have their own, private demand curves – they want to Sell More Stuff at the prevailing price, in head-to-head competition with other firms trying to do the same thing. For instance, automobile manufacturers all serve the same array of markets, and buyers frequently shift among firms. They also sell at about the same price, for a given quality product, and can produce as much as they can sell at constant marginal cost. Downward-sloping demand curves do not capture this reality.

If the common-sense view is correct, so sellers want to Sell More Stuff in equilibrium, price must exceed marginal cost in equilibrium. How can this be? The simplest explanation is that where product quality cannot be ensured by explicit contract, goods are in excess supply. Consumers typically pay a price *in* *excess* of marginal cost, the implicit threat to switch suppliers if dissatisfied inducing firms to supply high quality products (Klein and Leffler, 1981; Gintis, 1989). Since price exceeds marginal cost, each firm wishes to expand its sales at the current price, but cannot. Firms are thus necessarily quantity constrained, and the market fails to clear at the equilibrium price. We provide an analytical model of such an equilibrium in Section 6.7.

6.5.2 Money confers short-side power

We say a principal *P has power over* an agent *A* if *P* can impose, or credibly threaten to impose, sanctions on *A*, but *A* has no such capacity vis-à-vis *P* (Bowles and Gintis, 1992). This definition is doubtless incomplete and unnuanced, but conforms to standard notions in analytical political theory (Simon, 1953; Dahl, 1957; Harsanyi, 1962). In Walrasian general equilibrium there is no power, because all markets clear and contracts are costlessly enforced. In contingent renewal markets, however, principals have power over agents because they can impose costs on agents by terminating them. Since agents are in excess supply, absent collusion, agents can exercise no parallel threat over their principals. It follows that employers have power over employees, lenders have power over borrowers, and consumers have power over the firms from whom they buy. We may call this *short-side power* because it always lies with the transactor on the short side of the market – i.e. the side for which the quantity of desired transactions is the lesser.

So contingent renewal markets do not clear, and in equilibrium they allocate power to agents located on the short side of the market.

6.5.3 When money talks, people listen

If we review the cast of characters in our various contingent renewal markets, we find a strong regularity: the principal gives money to the agent, and the principal is on the short side of the market. For instance the employer, the lender, and the consumer hand over money to the worker, the borrower, and the supplying firm and the latter are all short-siders. The reason for this is clear: the money-side of contracts is relatively easy to enforce.

The application of the notion that "money talks" is particularly dramatic in the case of consumer goods markets. In Walrasian theory, consumer sovereignty means that free markets (under the appropriate conditions) lead to efficient allocations. What the term *really* means in people's lives is that since firms are on the long side of the market (they are quantity constrained) consumers can tell producers how to behave – people are truly sovereign. Probably nowhere in the daily lives of ordinary people do they feel more power, and gain more respect, than when acting as consumers, constantly pandered to by obsequious suppliers interested in keeping in their good graces – and benefiting from the difference between price and marginal cost.

6.5.4 The economy is controlled by the wealthy

The wealthy control the economy. This is not a criticism, and it's not profound. It's just true.

Economists basing their intuitions on the Walrasian general equilibrium model, of course, disagree. The wealthy have great *purchasing power*, they opine, but this does not translate into power in any *political* sense. As Paul Samuelson (1957: 894) has noted, "in a perfectly competitive market it really doesn't matter who hires whom; so let labor hire capital." The result, expressed long ago by Joseph Schumpeter (1934) is a decentralization of power to consumers: "The people who direct business firms only execute what is prescribed for them by wants." These views taken together imply the touchingly apolitical conception of the competitive economy expressed by Abba Lerner (1972: 259), who said "An economic transaction is a solved political problem. Economics has gained the title of Queen of the Social Sciences by choosing solved political problems as its domain." Unfortunately, it is not always plausible, as we illustrate in the next three examples, one from labor markets, one from consumer goods markets, and the last from the market for managers.[5]

6.6 CONTINGENT RENEWAL LABOR MARKETS

In this section, we develop a repeated game between employer and employee, in which the employer pays the employee a wage higher than (the expected value of) his next best alternative, using the threat of termination (a trigger strategy) to induce a high level of effort, in a situation where it is infeasible to write and enforce a contract for labor effort. When all employers behave in this manner, we have a non-clearing market in equilibrium.

Suppose an employer's income per period is $q(e)$, an increasing, concave function of the effort e of an employee. The employee's payoff per period $u = u(w, e)$ is an increasing function of the wage w and a decreasing function of effort e. Effort is known to the employee but it is only imperfectly observable by the employer. In each period the employer pays the employee w, the employee chooses effort e, and the employer observes a signal that registers the employee as "shirking" with probability $f(e)$, where $f'(e) < 0$. If the employee is caught shirking, he is dismissed, and receives a fallback with present value z. Presumably z depends on the value of leisure, the extent of unemployment insurance, the cost of job search, the startup costs in another job, and the present value of the new job. The employer chooses w to maximize profits. The tradeoff the employer faces is that a higher wage costs more, but it increases the cost of dismissal to the employee. The profit-maximizing wage equates the marginal cost to the marginal benefit.

The employee chooses $e = e(w)$ to maximize the discounted present value v of having the job, where the flow of utility per period is $u(w, e)$. Given discount rate ρ and fallback z, the employee's payoff from the repeated game is

$$v = \frac{u(w,e) + [1 - f(e)]v + f(e)z}{1+\rho},$$

where the first term in the numerator is the current period utility, assumed for convenience to accrue at the end of the period, and the others measure the expected present value obtainable at the end of the period, the weights being the probability of retaining or losing the position. Simplifying, we get

$$v = \frac{u(w, e) - \rho z}{\rho + f(e)} + z.$$

The term ρz in the numerator is the forgone flow of utility from the fallback, so the numerator is the net flow of utility from the relationship, while $f(e)$ in the denominator is added to the discount rate ρ, reflecting the fact that future returns must be discounted by the probability of their accrual as well as by the rate of time preference.

The employee varies e to maximize v, giving the first order condition

$$\frac{\partial u}{\partial e} - \frac{\partial f}{\partial e}(v - z) = 0, \tag{6.1}$$

which says that the employee increases effort to the point where the marginal disutility of effort is equal to the marginal reduction in the expected loss occasioned by dismissal. Solving equation (6.1) for e gives us the employee's best response $e(w)$ to the employer's wage offer w.

We assume that the employer can hire any real number n of workers, all of whom have the effort function $e(w)$, so the employer solves

$$\max_{w,n} \pi = q(ne(w)) - wn.$$

The first order conditions on n and w give $q'e = w$, and $q'ne' = n$, which together imply

$$\frac{\partial e}{\partial w} = \frac{e}{w}. \tag{6.2}$$

This is the famous *Solow condition* (Solow, 1979).

The best response function and (part of) the employer's choice of an optimal enforcement strategy (w^*) are shown in Figure 6.1, which plots effort against the salary. The iso-v function v^* is one of a family of loci of effort levels and salaries that

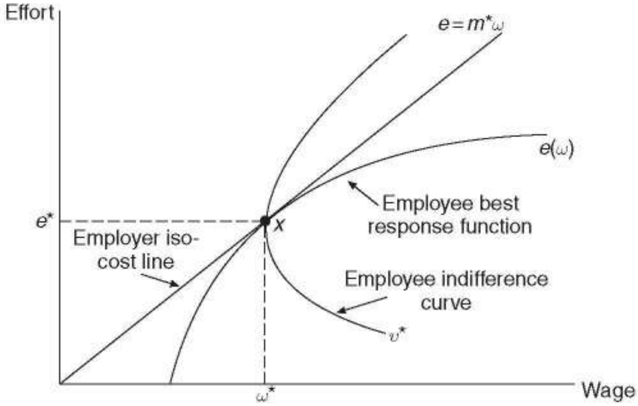

Figure 6.1 The employee's best response function.

yield identical present values to the employee. Their slope, $-(\partial v/\partial w)/(\partial v/\partial e)$, is the marginal rate of substitution between wage and effort in the employee's objective function. Preferred iso-v loci lie to the right.

By the employee's first order conditions (1), the iso-v loci are vertical where they intersect the best response function (because $\partial v/\partial e = 0$). The negative slope of the iso-v functions below $e(w)$ results from the fact that in this region the contribution of an increase in effort (via $(\partial f/\partial e)(v - z)$) to the probability of keeping the job outweigh the effort-disutility effects. Above $e(w)$, the effort-disutility effects predominate. Because v rises along $e(w)$ the employee is unambiguously better off at a higher wage. One of the employer's iso-cost loci is labeled $e = m^*w$, where m^* is the profit-maximizing effort per dollar. The employer's first order condition identifies the equilibrium wage w^* as the tangency between the employer's iso-cost function, $e = m^*w$, and the employee's effort function, with slope e', or point x in the figure.

It should be clear that the contingent renewal equilibrium at x is not the first-best, since if the parties could write a contract for effort, any point in the lens-shaped region below the employee's indifference curve v^* and above the employer's iso-cost line $e = m^*w$ makes both parties strictly better off than at x. Notice that if we populated the whole economy with firms like this, we would in general have $v > z$ in market equilibrium, since if $v = z$, equation (6.1) shows that $\partial u/\partial e = 0$, which is impossible so long as effort is a disutility. This is one instance of the general principle enunciated above, that contingent renewal markets do not clear in (Nash) equilibrium, and the agent whose promise is contractible (usually the agent paying money) is on the long side of the market.

Perhaps an example would help visualize this situation. Suppose the utility function is given by

$$u(w, e) = w - \frac{1}{1 - e}$$

and the shirking signal is given by

$$f(e) = 1 - e.$$

You can check that $e(w)$ is then given by

$$e(w) = 1 - a - \sqrt{a^2 + \rho a},$$

where $a = 1/(w - \rho z)$. This function is increasing and concave. It is zero when $w = 2 + \rho(1 + z)$, and approaches unity with increasing w. The solution for the employer's optimum w, given by the Solow condition in equation (6.2), is very complicated, so I will approximate the solution. Suppose $\rho = 0.05$ and the employment rate is $q \in [0, 1]$. An employee dismissed at the end of the current period therefore has a probability q, of finding a job right away (we assume all firms are alike), and so regains the present value v. With probability $1 - q$, however, the ex-employee remains unemployed for one period, and tries again afterward. Therefore we have

$$z = qv + (1 - q)z/(1 + \rho),$$

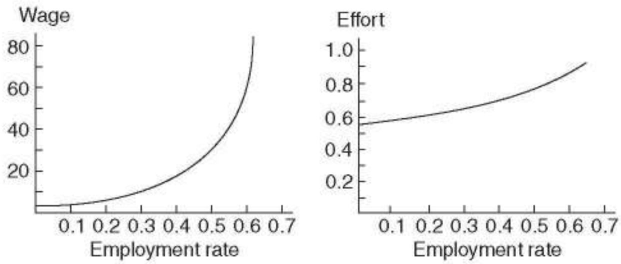

Figure 6.2 Wage and effort as functions of the employment rate in a contingent renewal labor market.

assuming the flow of utility from being unemployed (in particular, there is no unemployment insurance) is zero. Solving, we have

$$z = \frac{(1+\rho)q}{q+\rho}v.$$

For a given unemployment rate q, we can now find the equilibrium values of w, e, v, and z, and hence the employer's unit labor cost e/w. Running this through Mathematica, the equilibrium values of w and e as the employment rate q goes from zero to 0.67 is depicted in Figure 6.2.

Notice that while effort increases only moderately as the unemployment rate drops from 100 per cent to 33 per cent, the wage rate increases exponentially as the unemployment rate approaches 33 per cent. I could not find a solution for $q > 0.67$. The actual unemployment rate can be fixed by specifying the firm's production function and imposing a zero profit condition. However this is accomplished, there will be positive unemployment in equilibrium.

6.7 CONTINGENT RENEWAL PRODUCT MARKETS

In this section, we develop a repeated game between firm and consumer, in which product quality cannot be contracted for and can be verified only by consuming the good. The consumer pays a price greater than marginal cost using the threat of brand-switching (a trigger strategy) to induce a high level of quality on the part of the firm. The result is a non-clearing product market, with firms enjoying price greater than marginal cost, and hence are quantity constrained in equilibrium (i.e. they want to Sell More Stuff).

Every Monday families in Pleasant Valley wash clothes. To ensure brightness, they all use bleach. Low quality bleach can, with low but positive probability, ruin clothes, destroy the washing machine's Bleach Delivery Gizmo, and irritate the skin. High quality bleach is therefore deeply pleasing to Pleasant Valley families. However, high quality bleach is also costly to produce. Why should firms supply high quality?

Since people have different clothes, washing machines, and susceptibility to skin irritation, buyers cannot depend on a supplier's reputation to ascertain quality.

Moreover, a firm could fiendishly build up its reputation for delivering high quality bleach, and then when it has a large customer base, supply low quality for one period and then close up shop (this is called "milking your reputation"). Aggrieved families could of course sue the company if they have been hurt by low quality bleach, but such suits are hard to win and very costly to pursue. So no one does this.

If the quality q of bleach supplied by any particular company can only be ascertained after having purchased the product, and if there is no way to be compensated for being harmed by low quality bleach, how can high quality be assured?

Suppose the cost to a firm of producing a gallon of the bleach of quality q is $b(q)$, where $b(0) > 0$ and $b'(q) > 0$ for $q \geq 0$. Each consumer is a customer of a particular supplier, and purchases exactly one gallon of bleach each Friday at price p from this supplier. If dissatisfied, the customer switches to another supplier at zero cost. Suppose the probability of being dissatisfied, and hence of switching, is given by the decreasing function $f(q)$. We assume an infinite time horizon with a fixed discount rate ρ. We have

THEOREM 1 *(a) Considering both costs $b(q)$ and revenue q as accruing at the end of the period, the value $v(q)$ to a firm from having a customer is*

$$v(q) = \frac{p - b(q)}{f(q) + \rho}.$$

(b) If the price p is set by market competition, and so is exogenous to the firm, the firm chooses quality q so that

$$p = b(q) + b'(q)g(q), \tag{6.3}$$

where $g(q) = -[f(q) + \rho]/f'(q)$, provided $q > 0$.

(c) Quality is an increasing function of price.

Notice that firms are quantity constrained, since price is greater than marginal cost in market (Nash) equilibrium, and that consumers are on the long side of the market.

This model raises an interesting question. What determines firm size? In the standard perfect competition model, firm size is determined by the condition that average costs are at a minimum. This is of course just silly, since a firm can always produce at any multiple of the "optimal firm size" simply by working the production process, whatever it might be, in parallel.[6] The monopolistic competition model, in which a firm has a downward-sloping demand curve, is better, but it does not apply to a case like ours, where firms are price takers, as in the perfect competition model, and firm size is determined by the dynamic process of movement of customers among firms. Here is one plausible model of such a process.

Suppose there are n firms in the bleach industry, all selling at the same price p, which consumers choose to maximize their utility, given the price–quality relationship (6.3). Suppose firm j has market share m_j^t in period t. Suppose for $j = 1, \ldots, n$, a fraction f_j of firm j's customers leave the firm in each period, and a fraction a_j of customers who have left firms are attracted to firm j. We say the bleach industry is *in equilibrium* if the market share of each firm is constant over time. We have

THEOREM 2 *There is a unique asymptotically stable equilibrium in the bleach industry.*

6.8 MARKETS AS DISCIPLINING DEVICES: THE ALLIED WIDGETS MODEL

The following models show how market competition, in the form of strategic interaction, can induce high levels of managerial performance even when contracts for managerial effort cannot be written.[7]

Allied Widgets has two possible constant returns to scale production techniques: fission and fusion. For each technique, Nature decides in each period whether marginal cost is 1 or 2. With probability $\theta \in (0, 1)$ marginal cost is 1. Thus if fission is high-cost in a given production period, the manager can use fusion, which will be low-cost with probability θ. However it is costly for the manager to inspect the state of Nature, and if he fails to inspect, he will miss the opportunity to try fusion if the cost of fission is high.

Allied's owner cannot tell whether the manager had inspected or not, but he does know the resulting marginal cost, and can use this to give an incentive wage to the manager. Figure 6.3 shows the manager's decision tree, assuming the manager is paid a wage w_1 when marginal costs are low and w_2 when marginal costs are high, the cost of inspecting is α, and the manager has a logarithmic utility function over income: $u(w) = \ln w$.

To induce the manager to inspect the fission process, the owner decides to pay the manager a wage w_1 if marginal cost is low, and $w_2 < w_1$ if marginal cost is high. But how should the owner choose w_1 and w_2 to maximize profits? Suppose the manager's payoff is $\ln w$ if he does not inspect, $\ln w - \alpha$ if he inspects, and $\ln w_0$ if he does not take the job at all – in this case w_0 is called the manager's *reservation wage* or *fallback position*.

The expression that must be satisfied for a wage pair (w_1, w_2) to induce the manager to inspect the fission process is called the *incentive compatibility constraint*. To find this expression, note that the probability of using a low-cost technique if the manager does not inspect is θ, so the payoff to the manager from not inspecting (by the Expected Utility Theorem) is

$$\theta \ln w_1 + (1 - \theta) \ln w_2.$$

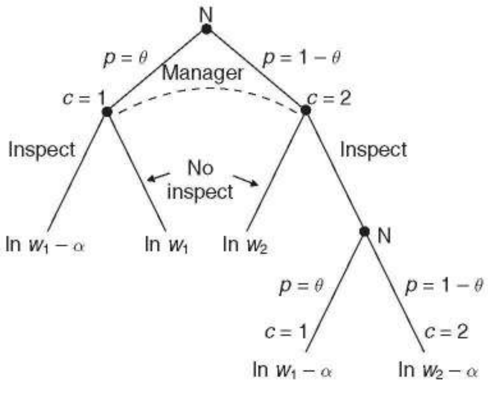

Figure 6.3 The allied widgets problem.

If the manager inspects, both techniques will turn out to be high cost with probability $(1-\theta)^2$, so the probability that at least one of the techniques is low-cost is $1-(1-\theta)^2$. Thus the payoff to the manager from inspecting (again by the Expected Utility Theorem) is

$$[1-(1-\theta)^2]\ln w_1 + (1-\theta)^2 \ln w_2 - \alpha.$$

The incentive compatibility constraint is then

$$\theta \ln w_1 + (1-\theta)\ln w_2 \leq [1-(1-\theta)^2]\ln w_1 + (1-\theta)^2 \ln w_2 - \alpha.$$

Since there is no reason to pay the manager more than absolutely necessary to get him to inspect, we can assume this is an equality,[8] in which case the constraint reduces to $\theta(1-\theta)\ln[w_1/w_2] = \alpha$, or

$$w_1 = w_2 e^{\frac{\alpha}{\theta(1-\theta)}}.$$

For instance, suppose $\alpha = 0.4$ and $\theta = 0.8$. Then $w_1 = 12.18 w_2$ – the manager must be paid more than 12 times as much in the good state as in the bad!

But the owner must also pay the manager enough so that it is worthwhile taking the job, rather than taking the fallback w_0. The expression that must be satisfied for a wage pair (w_1, w_2) to induce the manager to take the job is called the *participation constraint*. In our case, the participation constraint is:

$$[1-(1-\theta)^2]\ln w_1 + (1-\theta)^2 \ln w_2 - \alpha \geq \ln w_0.$$

Assuming this is an equality, and using the Incentive Compatibility Constraint, we find $w_0 = w_2 e^{\alpha/(1-\theta)}$, so

$$w_2 = w_0 e^{-\frac{\alpha}{(1-\theta)}}, \qquad w_1 = w_0 e^{\frac{\alpha}{\theta}}.$$

Using the above illustrative numbers, and assuming $w_0 = 1$, this gives

$$w_2 = 0.14, \qquad w_1 = 1.65.$$

The expected cost of the managerial incentives to the owner is

$$[1-(1-\theta)^2]w_1 + (1-\theta)^2 w_2 = w_0\left[\theta(2-\theta)e^{\frac{\alpha}{\theta}} + (1-\theta)^2 e^{-\frac{\alpha}{(1-\theta)}}\right].$$

Again, using our illustrative numbers, this gives the expected cost

$$0.96(1.65) + 0.04(0.14) = 1.59.$$

So where does competition come in? Suppose Allied has a competitor, Axis Widgets, subject to the same conditions of production. In particular, whatever marginal cost structure Nature imposes on Allied, Nature also imposes on Axis. Suppose also that the managers in the two firms cannot collude. We can show that Allied's owner can write a Pareto-efficient contract for the manager using Axis' marginal cost as a signal, satisfying both the participation and incentive compatibility constraints, and thereby increasing profits. They can do this by providing incentives that subject the managers to a prisoner's dilemma, in which the dominant strategy is to defect, which in this case means to inspect fission in search of a low-cost production process.

To see this, consider the following payment scheme, used by both Axis and Allied owners, where $\phi = 1 - \theta + \theta^2$, and the parameters β and γ are defined arbitrarily, but such that $\gamma < -\alpha(1-\theta+\theta^2)/\theta(1-\theta)$ and $\beta > \alpha(2-\phi)/(1-\phi)$. This gives rise to the following payoffs to the manager:

Allied cost	Axis cost	Allied wage	Numerical example
$c = 1$	$c = 1$	$w^* = w_0 e^{\alpha}$	$w^* = 1.49$
$c = 2$	$c = 2$	$w^* = w_0 e^{\alpha}$	$w^* = 1.49$
$c = 1$	$c = 2$	$w^+ = w_0 e^{\beta}$	$w^+ = 13.0$
$c = 2$	$c = 1$	$w^- = w_0 e^{\gamma}$	$w^- = 0.10$

We see that the manager will always inspect, and the owner's expected wage payment is $\phi w^* + (1-\phi)w^-$, which is less than the 1.59 in the previous case. The normal form for the game between the two managers is:

	Inspect	Shirk
Inspect	$\ln w^* - \alpha$ $\ln w^* - \alpha$	$\phi \ln w^* + (1-\phi)\ln w^+ - \alpha$ $\phi \ln w^* + (1-\phi)\ln w^-$
Shirk	$\phi \ln w^* + (1-\phi)\ln w^-$ $\phi \ln w^* + (1-\phi)\ln w^+ - \alpha$	$\ln w^*$ $\ln w^*$

Why is this so? The Inspect, Inspect box is obvious. The Inspect, Don't Inspect box is the case where the Axis manager checks, but the Allied manager does not. Then we have the following situation (writing c_a for allied costs and c_x for Axis costs):

Event	Probability	Outcome	Allied wage
$c = 1$	θ	$c_a = c_x = 1$	$w_0 e^{\alpha}$
$c = 2, c = 1$	$\theta(1-\theta)$	$c_a = 2, c_x = 1$	$w_0 e^{\gamma}$
$c = 2, c = 2$	$(1-\theta)^2$	$c_a = c_x = 2$	$w_0 e^{\alpha}$.

Thus the expected utility of Allied manager who does not inspect is

$$[\theta + (1-\theta)^2]\ln w^* + (1-\theta)\theta \ln w^-,$$

and the expected utility of Axis manager who does inspect, is

$$[\theta + (1-\theta)^2]\ln w^* + (1-\theta)\theta \ln w^+ - \alpha.$$

The Don't Inspect, Don't Inspect box is easier. Assuming that the managers choose their technologies in the same order, they both have either low or high costs. Hence their payoff is $\ln w^*$.

To show that this is a prisoner's dilemma, we need only show that

$$\ln w^* - \alpha > \phi \ln w^* + (1-\phi) \ln w^-$$

and

$$\phi \ln w^* + (1-\phi) \ln w^+ - \alpha > \phi \ln w^* + (1-\phi) \ln w^0.$$

The first of these becomes

$$\ln w_0 > \phi \ln w_0 + \phi\alpha + (1-\phi) \ln w_0 + (1-\phi)\gamma,$$

or $\gamma < -\phi\alpha/(1-\phi)$, which is true by assumption. The second becomes

$$\ln w^+ > \frac{\alpha}{1-\phi} + \ln w^*$$

or $\beta > \alpha \frac{2-\phi}{1-\phi}$, which is also true by assumption.

Note that in our numerical example the cost to the owner is $w^* = 1.49$, and the incentives for the managers are given by the normal form matrix

	Inspect	Shirk
Inspect	0 0	0.56 −1.06
Shirk	−1.06 0.55	0.224 0.224

This example shows that markets may be disciplining devices in the sense that they reduce the cost involved in providing the incentives for agents to act in the interests of their employers or clients, even where enforceable contracts cannot be written – in this case, there can be no enforceable contract for managerial inspecting. Note that in this example, even though managers are risk averse, imposing a structure of competition between the managers means each inspects and the cost of incentives is no greater than if a fully specified and enforceable contract for inspecting could be written.

Of course, if we weaken some of the assumptions, Pareto-optimality will no longer be attainable. For instance, suppose when a technique is low-cost for one firm, it is not necessarily low-cost for the other, but rather is low-cost with probability $q > 1/2$. Then competition between managers has an element of uncertainty, and optimal contracts will expose the managers to a positive level of risk, so their expected payoff must be greater than their fallback.

6.9 EXPERIMENTAL ECONOMICS: THE LABORATORY MEETS STRATEGIC INTERACTION

I will now turn from the issue of endogenous contract enforcement to that of modeling the human actor. The two topics are in fact intimately related. Because cooperation cannot be ensured by costless third-party contract enforcement, conditions that promote prosocial behavior allow for more cooperation with lower transactions cost. Classical game theory shows that with a sufficiently low discount rate and repeated interactions, such behavior will be exhibited by self-interested agents. However, low discount rates cannot be assumed, and prosocial behavior occurs in the laboratory even when the classical game theoretic requirements are absent. Thus the model of the individual in Walrasian general equilibrium theory does not accurately represent the human actor.

As a basis for interpreting a broad range of experiments, I will introduce one new *persona*, whom I call *Homo reciprocans*. *Homo reciprocans'* behavior in market situations, in which punishing and rewarding are impossible or excessively costly, is much like that of *Homo economicus*. But *Homo reciprocans* comes to strategic interactions with a propensity to cooperate, responds to cooperative behavior by maintaining or increasing his level of cooperation, and responds to non-cooperative behavior by retaliating against the "offenders," even at a cost to himself, and even when he could not reasonably expect future personal gains to flow from such retaliation. When other forms of punishment are not available, *Homo reciprocans* responds to defection with defection, leading to a downward spiral of non-cooperation. *Homo reciprocans* is thus neither the selfless altruist of utopian theory, nor the selfish hedonist of neoclassical economics. Rather, he is a conditional cooperator whose penchant for reciprocity can be elicited under circumstances in which personal self-interest would dictate otherwise.[9]

6.9.1 The ultimatum game

The *ultimatum game*, invented by the economists Werner Güth, Rolf Schmittberger and Berndt Schwarze (1982), is a showcase for costly retaliation in a one-shot situation. Under conditions of anonymity, one player, called the "proposer," is handed a sum of money, say $10, and is told to offer any number of dollars, from $1 to $10, to the second player, who is called the "responder." The responder, again under conditions of anonymity, can either accept the offer, or reject it. If the responder accepts the offer, the money is shared accordingly. If the responder rejects the offer, both players receive nothing.

There is only one responder strategy that is subgame perfect: accept anything you are offered. However, when actually played by people, the subgame perfect outcome is almost never attained or even approximated. In fact, as many replications of this experiment have documented, under varying conditions and with varying amounts of money, proposers routinely offer respondents very substantial amounts (50 per cent of the total being the modal offer), and respondents frequently reject low offers (e.g. offers below 30 per cent). These results are obtained in experiments with stakes as high as three months' earnings.[10]

In the United States and other complex societies, when asked why they offer more than the lowest possible amount, proposers commonly say that they are afraid that respondents will consider low offers unfair and reject them. When respondents reject offers, they give virtually the same reasons for their actions.[11]

6.9.2 The public goods game

Another important experimental setting in which strong reciprocity has been observed is that of the *public goods game*, designed to illuminate such problems as the voluntary payment of taxes and contribution to team and community goals. Public goods experiments have been run many times, under varying conditions, beginning with the pioneering work of the sociologist G. Marwell, the psychologist R. Dawes, the political scientist J. Orbell, and the economists R. Isaac and J. Walker in the late 1970s and early 1980s.[12] The following is a common variant of the game. Ten subjects are told that $1 will be deposited in each of their "private accounts" as a reward for participating in each round of the experiment. For every $1 a subject moves from his "private account" to the "public account," the experimenter will deposit $0.50 in the private accounts of each of the subjects at the end of the game. This process will be repeated ten times, and at the end, the subjects can take home whatever they have in their private accounts.

If all ten subjects are perfectly cooperative, each puts $1 in the public account at the end of each round, generating a public pool of $10; the experimenter then puts $5 in the private account of each subject. After ten rounds of this, each subject has $50. Suppose, by contrast, that one subject is perfectly selfish, while the others are cooperative. The selfish one keeps his $1-per-round in his private account, whereas the cooperative ones continue to put $1 in the public pool. In this case, the selfish subject who takes a free ride on the cooperative contributions of others ends up with $55 at the end of the game, while the other players end up with $45 each. But if all players opt for the selfish payoff, then no one contributes to the public pool, and each ends up with $10 at the end of the game. And if one player cooperates, while the others are all selfish, that player will end up with $5 at the end of the game, while the others will get $15. It is thus clear that this is indeed an "iterated prisoner's dilemma" – whatever other players do on a particular round a player's highest payoff comes from contributing nothing to the public account. If others cooperate, it is best to take a free ride; if others are selfish, it is best to join them. But if no one contributes, all receive less than they would had all cooperated.

Public goods experiments show that only a fraction of subjects conform to the *Homo economicus* model, contributing nothing to the public account. Rather, in a one-stage public goods game, people contribute on average about half of their private accounts. The results in the early stages of a repeated public goods game are similar. In the middle stages of the repeated game, however, contributions begin to decay, until at the end, they are close to the *Homo economicus* level – i.e. zero.

Could we not explain the decay of public contribution by *learning*: the participants really do not understand the game at first, but once they hit upon the free-riding strategy, they apply it? Not at all. One indication that learning does not account for the decay of cooperation is that increasing the number of rounds of play (when this is known to the players) leads to a decline in the rate of decay of cooperation (Isaac *et al.*, 1994). Similarly, Andreoni (1988) finds that when the whole process is repeated with the same subjects but with different group composition, the initial levels of cooperation are restored, but once again cooperation decays as the game progresses. Andreoni (1995) suggests a *Homo reciprocans* explanation for the decay of cooperation: public-spirited contributors want to retaliate against free-riders and the only way available to them in the game is by not contributing themselves.

6.9.3 The public goods game with retaliation

Could the decay of cooperation in the public goods game be due to cooperators retaliating against free-riders by free-riding themselves? Subjects often report this behavior retrospectively. More compelling, however, is the fact that when subjects are given a more constructive way of punishing defectors, they use it in a way that helps sustain cooperation (Dawes *et al.*, 1986; Sato, 1987; Yamagishi, 1988a,b, 1992).

For instance, in Ostrom *et al.* (1992) subjects interacted for about 25 periods in a public goods game, and by paying a "fee," subjects could impose costs on other subjects by "fining" them. Since fining costs the individual who uses it, but the benefits of increased compliance accrue to the group as a whole, the only subgame perfect Nash equilibrium in this game is for no player to pay the fee, so no player is ever punished for defecting, and all players defect by contributing nothing to the public account. However, the authors found a significant level of punishing behavior. The experiment was then repeated with subjects being allowed to communicate, without being able to make binding agreements. In the framework of the *Homo economicus* model, such communication is called *cheap talk*, and cannot lead to a distinct subgame perfect equilibrium. But in fact such communication led to almost perfect cooperation (93 per cent) with very little sanctioning (4 per cent).

The design of the Ostrom–Walker–Gardner study allowed individuals to engage in strategic behavior, since costly retaliation against defectors could increase cooperation in future periods, yielding a positive net return for the retaliator. It is true that backward induction rules out such a strategy, but we know that people do not backward induct very far anyway. What happens if we remove any possibility of retaliation being strategic? This is exactly what Fehr and Gächter (2000) studied. They set up a repeated public goods game with the possibility of costly retaliation, but they ensured that group composition changed *in every period* so subjects knew that costly retaliation could not confer any pecuniary benefit to those who punish. Nonetheless, punishment of free-riding was prevalent and gave rise to a large and sustainable increase in cooperation levels.

6.9.4 The common pool resource game

In 1968, Garrett Hardin wrote a famous article in the journal *Science* entitled "The Tragedy of the Commons" (Hardin, 1968). The term "commons" referred originally to the region of an English village that belonged to the villagers as a group, and on which villagers were permitted to graze their sheep or cows. The "tragedy" in the tragedy of the commons was that the commons tended to be overgrazed, since each villager would graze to the point where the *private* costs equals the benefits, whereas grazing imposed additional *social* costs on the rest of the community. Some involve social problems of the highest importance, including air and water pollution, overfishing, overuse of antibiotics, traffic congestion, excessive groundwater use, overpopulation, and the like.

The general implication of Hardin's analysis was that some centralized entity, such as a national government or international agency had to step in to prevent the tragedy by regulating the common. The historical experience in regulating the commons, however, has been a patchwork of successes and failures, and in 1990 Elinor Ostrom published an influential book, *Governing the Commons*, suggesting that the Hardin analysis did not apply generally, since local communities often had ways of self-organizing and self-governing to prevent overexploitation of the commons, and that government policy often exacerbated rather than amelioriating the problem, by undermining the social connections on which local regulation was based.

When formalized as a game, the common pool resource problem is simply an n-person repeated prisoner's dilemma, in which each player hopes the other players will cooperate (not take too much of the common resource), but will defect (take too much), no matter what the others do. But the public goods game is also an n-person repeated prisoner's dilemma, so it is not surprising that both in real world and experimental setting, under the appropriate conditions, we see much more cooperation than predicted by the *Homo economicus* model.

Ostrom *et al.* (1994) used both experimental and field data to test game-theoretic models of common pool resources. They found more spontaneous cooperation in the field studies than predicted, and when communication and sanctioning were permitted in the laboratory, the level of cooperation became quite high.

While common pool resource and public goods games are equivalent for *Homo economicus*, people treat them quite differently in practice. This is because the *status quo* in the public goods game is the individual keeping all the money in the private account, while the *status quo* in the common pool resource game is the resource not being used at all. This is a good example of a *framing effect*, since people measure movements from the *status quo*, and hence tend to undercontribute in the public goods game, and overcontribute (underexploit) in the common pool resource game, compared to the social optimum (Ostrom, 1998).

It is clear that in the real world, of course, communities often do *not* manage their common pool resources well. The point of Ostrom's work is to identify the sources of failure, not to romanticize small communities and informal organization. Among other reasons, the management of common pool resources fails when communities are so large that it pays to form a local coalition operating against the whole community and when resources are so unequally distributed that it pays the wealthy to defect on the nonwealthy and conversely (Hackett *et al.*, 1994; Bardhan *et al.*, 2000).

6.10 *HOMO RECIPROCANS*: MODELING STRONG RECIPROCITY

Consider a two-person extensive form game $\mathcal{G}$.[13] Let $\pi_i(p_1,p_2)$ be the payoff to player $i = 1,2$ when i uses behavioral strategy p_i, and let $\pi_i(p_1,p_2|\nu)$ be the payoff to i, conditional on being at information set ν. The *fairness* $f_j(p_1,p_2|\nu)$ of j if it is $i \neq j$'s move at ν is defined by

$$f_j(p_1,p_2|\nu) = \pi_i(p_1,p_2|\nu) - \pi_j(p_1,p_2|\nu).$$

Thus at ν, j has been relatively generous if $f_j > 0$, and relatively selfish if $f_j < 0$.

For every pure action a available to i at ν, let $p_i(a)$ be the behavioral strategy for i that is the same as p_i everywhere except at ν, where i takes action a. We then define *i kindness* from taking action a at ν to be

$$k_i(p_1,p_2,a|\nu) = \pi_j((p_i(a),p_j)|\nu) - \pi_j(p_1,p_2|\nu),$$

where $(p_i(a),p_j) = p_1(a),p_2$ if $i = 1$ and $(p_i(a),p_j) = p_1,p_2(a)$ if $i = 2$. In other words, given the pair of strategies (p_1,p_2), player i who moves at node ν is being "kind" when choosing move a if this gives j a greater payoff than that indicated by p_i.

The total payoff to i at a terminal node $t \in T$ of $\mathcal{G}$ is then

$$u_i(t) = \pi_i(t) + \rho_i \sum_{\nu \in N_i(t)} f_j(p_1,p_2|\nu)k_i(p_1,p_2,a_\nu|\nu), \tag{6.4}$$

where $N_i(t)$ is the set of information sets where i moves on the path to t, and a_ν is the action at ν on the path to t. Note that if $f_j > 0$ at a certain node, then *ceteris paribus* player i gains from exhibiting positive kindness, while if $f_j < 0$, the opposite is the case. Notice also that these payoffs are relative to a specific pair of behavioral strategies (p_1,p_2). This aspect of equation (6.4) reflects the fact that *Homo reciprocans* cares not only about payoffs, but also about the actions of the other player. We say that a pair of strategies (p_1^*,p_2^*) of $\mathcal{G}$ is a *reciprocity equilibrium* if (p_1^*,p_2^*) is a Nash equilibrium of (4) when (p_1,p_2) is replaced by (p_1^*,p_2^*) on the right hand side of (4).[14]

THEOREM 3 *Suppose both players in an ultimatum game have preferences given by (4), where $\rho_1,\rho_2 > 0$ are known by both players, and let s be the share the proposer offers the respondent. Let $p^*(s)$ be the respondent's best reply to the offers, and let $(p^*(s^*),s^*)$ be a reciprocity equilibrium. Then the respondent surely accepts (i.e. $p^*(s^*) = 1$), and the proposer chooses*

$$s^* = \max\left[\frac{1+3\rho_2-\sqrt{1+6\rho_2+\rho_2^2}}{4\rho_2},\frac{1}{2}\left(1-\frac{1}{\rho_1}\right)\right].$$

The theorem also holds when either or both of ρ_1,ρ_2 is zero, and if both are zero, we have the *Homo economicus* equilibrium. Notice that the second expression for the equilibrium offer s^* will hold when the proposer is highly motivated by fairness, while the first expression holds if the proposer is motivated to make an offer sufficiently high so as not to be rejected.

This theorem assumes the proposer knows the respondent's ρ_2, which accounts for the fact that offers are never refused. It is not difficult to see how to modify this by assuming the proposer knows only the probability distribution over respondent types.

As another example of a reciprocity equilibrium, let the game $\mathcal{G}$ be the Prisoner's Dilemma, with cooperative payoffs (b, b), mutual defect payoffs (c, c), and where a cooperator receives 0 against a defector, and a defector receives a against a cooperator. We assume $a > b > c > 0$. Suppose $\mathcal{G}$ is *sequential*, with One going first and choosing "cooperate" with probability p, then Two choosing "cooperate" with probability q if One cooperated, and choosing "cooperate" with probability r if One defected. We have

THEOREM 4 *Suppose players in the sequential Prisoner's Dilemma game $\mathcal{G}$ have utility functions given by equation (6.4), where $\rho_1, \rho_2 > 0$ are known by both players. Then there is a unique reciprocity equilibrium (p^*, q^*, r^*) with the following characteristics:*

(a)

$$r^* = 0;$$

(b)

$$q^* = 1 - \frac{a-b}{\rho_2 ab}$$

unless this quantity is negative, in which case $q^* = 0$;

(c)

$$p^* = \frac{q^*b - c}{\rho_1 a(1-q^*)(q^*b + (1-q^*)a - c)}, \tag{6.5}$$

provided this quantity is between 0 and 1. If the right hand side of equation (6.5) is negative, then $p^* = 0$, and if the right hand side of equation (6.5) is greater than 1, then $p^* = 1$.

Part (a) says that if One defects, Two defects as well. Part (b) says that if One cooperates and if the strength of Two's reciprocity motive ρ_2 is sufficiently strong, Two cooperates with positive probability. Also, this probability is increasing in the strength of Two's reciprocity motive, but it never reaches 100 per cent. Part (c) is a little more complicated. The numerator is the expected gain from cooperation q^*b over defection c. If this is positive, the denominator is as well, so a selfish One (low ρ_1) will cooperate with certainty, whereas a reciprocator (high ρ_1) may not, because he is averse to giving Two a high payoff from defecting. The denominator is necessarily positive, so if the numerator is negative, no proposer will cooperate.

6.11 THE EVOLUTION OF STRONG RECIPROCITY

Walrasian general equilibrium theory explains cooperation on the basis of costless third-party contract enforcement, whereas classical game theory explains cooperation on the basis of repeated interactions and reputation effects. In both cases, cooperation is completely compatible with self-interested actors. A critical of the classical game-theoretic approach is that when a social group is threatened with extinction or dispersal, say through war, pestilence, or famine, cooperation is most needed for survival. But the discount rate, which depends inversely on the probability of future interactions, increases sharply when the group is threatened. Thus precisely when society is most in need of prosocial behavior, cooperation based on repeated interactions will collapse, for the simple reason that the discount rate will rise to levels where cooperation is no longer a Nash equilibrium. This observation serves as the basis for the following model of the evolutionary emergence of strong reciprocity (Gintis, 2000b).

Consider an n-player public goods game in which each player has an amount c that may be kept or contributed to the "common pool." If the money is contributed, an amount $b > c$ is distributed equally among the members of the group. Thus if k players contribute, each contributing player receives kb/n and each non-contributing member receives $c + kb/n$. If $b/n < c$, the only Nash equilibrium is universal defection, in which each player keeps c. The Folk Theorem states that if this game is repeated indefinitely, full cooperation becomes a subgame perfect Nash equilibrium, provided the discount rate is sufficiently low.

We model early human society as a collection of small communities, each of which is engaged in this public goods game. Defecting is always detected and is common knowledge. When the discount factor is high enough to induce cooperation, defectors are excluded from participation in the community for a number of periods just sufficient to make defecting a suboptimal strategy, at zero cost to the community.

We suppose that in each "good" period the community will persist into the next period with probability δ^*, so δ^* is the discount factor. In each "bad" period there is a high probability $1 - \delta_*$ that the community will disband, so the discount factor is $\delta_* < \delta^*$. We suppose that the "bad" state occurs with small probability $p > 0$, and for simplicity, we suppose that the threat to the community does not affect the cost c or the return b.

Suppose at the beginning of each period, prior to agents deciding whether or not to cooperate, the state of the community for that period is revealed to the members. Let π^* be the present value (total fitness) of a member if all members cooperate forever, and the state of the community is "good," and let π_* be the present value of universal cooperation if the state is "bad." Then the present value before the state is revealed is $\pi = p\pi_* + (1-p)\pi^*$, and we have the following recursion equations:

$$\pi^* = b - c + \delta^*\pi,$$
$$\pi_* = b - c + \delta_*\pi,$$

which we can solve, giving

$$\pi^* = \frac{1 + p(\delta^* - \delta_*)}{1 - \delta^* + p(\delta^* - \delta_*)}(b - c), \tag{6.6}$$

$$\pi_* = \frac{1 - (1-p)(\delta^* - \delta_*)}{1 - \delta^* + p(\delta^* - \delta_*)}(b - c), \tag{6.7}$$

$$\pi = \frac{1}{1 - \delta^* + p(\delta^* - \delta_*)}(b - c). \tag{6.8}$$

Notice that $\pi^* - \pi_* = \pi(\delta^* - \delta_*)$, which is strictly positive, as expected. These equations assume the fitness of a member of a community that disbands is 0, which is thus the benchmark for all fitness values, and to which we must add an exogenous "baseline fitness" to account for the change in population of the set of communities.

When can cooperation be sustained? Clearly if it is worthwhile for an agent to cooperate in a bad period, it is worthwhile to cooperate in a good period, so we need to only check the bad period case. The current cost of cooperating is $c - b/n$, which we approximate by c for notational convenience (the approximation is good for a large community), so the condition for cooperation is $c < \delta_* \pi$. There is a Nash equilibrium in which members thus cooperate in the good state but not in the bad when the following inequalities hold:

$$\delta^* \pi > c > \delta_* \pi, \tag{6.9}$$

which will be the case if δ^* is near unity and δ_* is near zero. We assume that these inequalities hold.

Suppose community i has a fraction f_i of strong reciprocators, who cooperate and punish defectors independent of the state of the community. Suppose each cooperator inflicts a total amount of harm $l_r < 1$ on defectors, at a cost $c_r < 1$ to themselves. Because of equation (6.9), in a bad state, selfish agents always defect unless punished by strong reciprocators. If there are n_i community members, in a bad state $n_i(1 - f_i)$ defect, and the total harm inflicted on those caught is $n_i f_i l_r$, then the harm per defector imposed by strong reciprocators is $f_i l_r/(1 - f_i)$. The gain from defecting in equation (6.9) now becomes $c - f_i l_r/(1 - f_i)$. Thus if the fraction f_i of strong reciprocators is at least

$$f_* = \frac{c - \pi\delta_*}{c - \pi\delta_* + l_r}, \tag{6.10}$$

complete cooperation will hold. Note that f_* is strictly between 0 and 1, since the numerator, which is the gain from defecting prior to being punished by reciprocators, is positive. Also the larger is l_r, the smaller is the minimum fraction f_* of reciprocators needed to induce cooperation.

If $f_i < f_*$ there will be no cooperation in a bad period (we continue to assume the parameters of the model are such that there is always cooperation in the good period). In this situation the community disbands and each member takes the fallback fitness 0. The fitness π_s of members of such "selfish" communities then satisfies the recursion equation $\pi_s = (1 - p)(b - c + \delta^* \pi_s)$, which becomes

$$\pi_s = \frac{(1 - p)}{1 - (1 - p)\delta^*}(b - c). \tag{6.11}$$

Our assumption that there is always cooperation in the good state requires that $\delta^* \pi_s > c$, which becomes

$$\frac{\delta^*(1 - p)}{1 - (1 - p)\delta^*}(b - c) > c,$$

which we will assume holds. Notice that the relative fitness benefit from being in a cooperative community is

$$d\pi = \pi - \pi_s = p\pi \frac{1 - (1-p)(\delta^* - \delta_*)}{1 - (1-p)\delta^*} > 0. \tag{6.12}$$

We suppose that the fraction of strong reciprocators in a community is common knowledge, and strong reciprocators punish defectors only in communities where $f_i \geq f^*$, and in doing so they each incur the fixed fitness cost c_r. We shall interpret c_r as a surveillance cost, and since punishment is unnecessary except in "bad" periods, strong reciprocators will incur this cost only with probability p, so the expected fitness cost of being a strong reciprocator is pc_r.

Let q_f be the fraction of the population in cooperative communities, so

$$q_f = \sum_{f_i \geq f_*} q_i,$$

where q_i is the fraction of the population in community i. Let $f_s = \sum_{f_i < f^*} q_i f_i/(1 - q_f)$, which is the mean frequency of strong reciprocators in noncooperative communities, and let $f_c = \sum_{f_i \geq f^*} q_i f_i/q_f$, which is the mean fraction of strong reciprocators in cooperative communities. We have

THEOREM 5 *The condition for the increase in strong reciprocity is*

$$(1 - q_f)\left(1 - \frac{f_s}{f_c}\right) - c_r > 0, \tag{6.13}$$

and equilibrium occurs when the left hand side of the equation is zero.

We then have

THEOREM 6 *The fraction of strong reciprocators in the population lies strictly between zero and one in equilibrium. Moreover, a small number of strong reciprocators can invade a population of selfish types, provided f_s/f_c is sufficiently small; i.e. provided the strong reciprocators have a sufficiently strong tendency to associate with one another.*

Suppose communities are of size n and form randomly, the overall frequency of strong reciprocators being f. Then the expected frequency of strong reciprocators in each community will be f, with variance $f(1-f)/n$. Therefore if $f < f_*$ and if n is large (say 100), with high probability, no communities will have $f_i > f_*$, and even if some such communities exist, f_s/f_c will be very close to unity. Therefore

THEOREM 7 *Without a positive level of assortative interactions strong reciprocators cannot invade a population of selfish types.*[15]

So let us assume that there is some way that strong reciprocators can recognize each other. Without attempting to model community formation too closely, let us simply say that communities are of equal size, and that a fraction g is formed by assortative interactions, formed by a fraction r of strong reciprocators and a fraction $1 - r$ drawn randomly from the population. If the fraction of strong reciprocators in the population is f, then the assortative groups have a fraction $f_c = r + f(1 - r)$ of strong reciprocators. To determine f_s, note that the fraction of strong reciprocators in assortative groups is gf_c, so the fraction in randomly formed groups is $f - gf_c$, and since such groups form a fraction $1 - g$ of the total, the fraction of strong reciprocators in a randomly formed group is $f_s = (f - gf_c)/(1 - g)$. Then if assortative groups are cooperative while randomly mixed groups are not, we have $g = q_f$, and equation (6.13) becomes

$$\frac{r(1-f)}{r+f(1-r)} - c_r > 0. \tag{6.14}$$

This inequality holds for any value of $r > 0$ when f is very small, which is thus the condition for the invadability of strong reciprocators *however small is the level of assortative interaction*. The level r of assortative interaction does, however, determine the equilibrium frequency of strong reciprocators. Setting the left hand side of equation (6.14) to zero and solving for the equilibrium frequency $\hat{f}$ of strong reciprocators, we get

$$\hat{f} = \frac{r(1-c_r)}{r(1-c_r)+c_r}. \tag{6.15}$$

The fraction of strong reciprocators thus varies from zero when $r = 0$ to $1 - c_r$ when $r = 1$. We may summarize this argument by saying

THEOREM 8 *Suppose there is a degree $r > 0$ of assortative interaction among strong reciprocators. Then a small number of reciprocators can invade a population of selfish types, and the equilibrium fraction of reciprocators is given by $\hat{f}$ in equation (6.15).*

6.12 CONCLUSION

It is important to expand the search for a general equilibrium model of contemporary market economies, and of the world as an integrated economic system. But such a model will not likely resemble the Walrasian general equilibrium model. Indeed, it will probably look more like an ecological, biological, and/or thermodynamic system than the smoothly oiled mechanical system envisaged by Walras and his followers. A step towards this goal might be to model the interrelation among "communities," each of which is modeled in game theoretic terms, but some of whose parameters are exogenous to the community, and are given by the larger constellation of communities.

Trade among communities (depending on relative prices), migration in and out of communities (depending on relative costs and benefits of relocation), and the birth/death of communities then can be used to control the equilibrium of the model. Samuel Bowles and I have developed some relatively simple models of this type, but without prices (Bowles and Gintis, 1998a,b).

NOTES

1 Gandhi was once asked what he thought of Western Civilization. He replied that "it would be a good idea." I feel the same way about Walrasian general equilibrium theory.
2 Computable general equilibrium theory (Piggott and Whalley, 1991) may appear to be an exception to the generalization, but in fact it uses empirical data only to fill in the parameters of the general equilibrium model, and not to specify the axioms according to which the model is generated.
3 I prefer the term "Walrasian allocation" to the more common "competitive exchange" because there is really nothing "competitive" about such an allocation mechanism: there is no known, plausible, dynamic competitive mechanism for which a Walrasian allocation is a stable fixed point.
4 Even in physics, it is worth nothing, fields are just a convenient means of representing particle interactions, and must be gone beyond to arrive at the deeper principles of matter and energy.
5 For a capital market example, see Bardhan *et al.* (2000).
6 Why they teach the standard model to students these days is quite beyond me – it's totally bogus, since it is a model of *plant* size, not *firm* size. The important questions of vertical and horizontal integration, the real determinants of firm size, are virtually orthogonal to the question of plant size. Industrial economists have known this for a very long time – for a contemporary review of the literature on the subject, see Sutton (1997).
7 This problem is adapted from Tirole (1988).
8 Actually, this can easily be proven using the appropriate Kuhn–Tucker conditions. This remark applies also to our assumption that the participation constraint, defined below, is satisfied as an equality.
9 Another aspect of reciprocity is commonly known as "gift exchange," in which one agent behaves more kindly than required towards another, with the hope and expectation that the other will respond kindly as well (Akerlof, 1982). For instance, for a laboratory-simulated work situation in which "employers" can pay higher than market-clearing wages in hopes that "workers" will reciprocate by supplying high level of effort, see Fehr *et al.* (1998) and Fehr *et al.* (1997).
10 For a review of ultimatum game experiments, see Güth and Tietz (1990), Roth (1995) and Camerer and Thaler (1995).
11 In all of the above experiments a significant fraction of subjects (about a quarter, typically) conform to the self-interested preferences of *Homo economicus*, and it is often the self-serving behavior of this minority that, when it goes unpunished, unravels initial generosity and cooperation when the game is repeated.
12 For a summary of this research and an extensive bibliography, see Ledyard (1995).
13 This model of strong reciprocity follows Falk and Fischbacher (1998).
14 Following Rabin (1993), Falk and Fischbacher (1998) use the concept of a *psychological game* (Geanakoplos *et al.*, 1989) to formulate the notion of a reciprocity equilibrium. Our formulation accomplishes the same end without requiring a notion of "subjective beliefs."
15 It might be thought that a pattern of outmigration from cooperative groups might allow strong reciprocity to increase, but extensive analysis by population biologists fails to turn up any plausible models of this type. For an important contribution and review of the literature, see Rogers (1990).

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7 Macroeconomics and general equilibrium

Frank Hahn

7.1 WHY MACROECONOMICS?

If we want to understand the behaviour of macro-variables like GNP and employment, do we need a special theory? In physics, the behaviour of elementary particles gave rise to quantum mechanics while Newtonian theory sufficed for the behaviour of objects containing many particles. These two theories are different and not inconsistent. In economics we have a well developed theory of agents' decisions and of the interaction of these. I shall refer to this as "canonical micro-theory" (CM). The question is whether this theory suffices for macro-purposes.

There are economists – indeed very influential economists, who believe the answer to be yes. Indeed for these economists – let us call them Lucasians – macroeconomics is micro-economics with one type of representative agents: a representative household and a representative firm. I shall want to examine whether this is a coherent, theoretically soundly based, procedure. I anticipate now what I shall conclude: for a perfectly competitive complete market economy in a unique long-run equilibrium the approach can be justified as theory. So if we hold the view that these conditions are satisfied we can indeed be Lucasians with a good conscience. But it will be my contention that the result is an emasculated theory incapable of even asking some of the most important questions, leave alone answering them.

If I am right then the question with which I started: "Is there a need for a special macro-economic theory?" remains open. But the question can also be inverted: "Are there macroeconomic phenomena which have been illegitimately ignored by canonical microtheory?" In other words, is there more to macro-economics than more or less imperfect aggregation, and is there more to microeconomics than decisions based on prices, preferences, endowments and technology and the study of their consistency?

7.2 A PREVIEW

I shall now give a preliminary account of how I propose to proceed.

1 I shall start with a very brief recapitulation of Arrow–Debreu (A–D) general equilibrium (GE) theory and its recent extension to a sequence economy. It will be easy to demonstrate that in the absence of an account of the aggregation

procedure used, the natural interpretation of Lucasian theory is that it is a particular account of A–D general equilibrium theory.

2 I shall note that Lucasians are concerned with a monetary economy and that such an economy cannot be encompassed by A–D theory as first conceived – that is a non-sequential trading economy. But even in a sequential setting problems remain. I note them but do not discuss them here.

3 But primarily I shall want to draw attention to lacunae and difficulties with A–D theory itself. Here are some of these:

(a) A–D theory gives no account of the mechanism of transactions. In particular it has ignored the lack of information agents have of possible exchange partners. This lack, when remedied, leads to a class of new agents – mediators – and provides a first step in monetary theory.

(b) In a sequence economy agents need to form expectations of spot prices. This is not required in the original formulation of A–D theory. Agents are said to have rational price expectations if the latter are properly conditioned on all the information available to agents. Typically what comprises this information is not precisely stated – I shall want to persuade you that it is natural to include macro-variables in information. I have in mind GNP and price indices. If you are persuaded, then we now have a first link between micro- and macro-variables.

(c) We know (Aumann, 1964) that one rigorous justification of a perfectly competitive economy is the assumption that there is a continuum of agents. This may perhaps be a suitable postulate for consumers and for a pure exchange economy but seems plainly false for an economy with producers. But even in the case of Bertrand competition we must think of the competitors facing limited demand at given (possibly competitive) prices. This suggests that perfect competition may be an idealisation which obscures a number of important matters. Just as in any argument concerning expectations it is natural to suppose that demand at given prices depends on aggregate income (and perhaps on its distribution). This has two consequences: firms need macro-expectations to make decisions and consumers need them also in order to predict prices.

These observations suggest that the questions with which I started will have to await a change in micro-economic foundations before they can be answered.

4 There is one further important matter concerning A–D in its canonical and in its sequence form. Not only is it equilibrium theory but it is long-run equilibrium theory. By this I mean that a state of the economy is being described in which there is no more for agents to learn about the economy. In the canonical theory, all agents can distinguish between every state of nature (have the finest partition of the set of states of nature) and know their probabilities of occurrence, while in the sequence economy they additionally know spot prices conditional on event and date. These are extremely limiting assumptions. More importantly they are open to a number of objections. In any event, macro-economics based on these assumptions cannot engage in any debate with Keynesian macro-economics which is based on short-run equilibrium analysis. The "long-run" can only be regarded

as useful if some powerful arguments are developed to show that economies must be in long-run equilibrium for most of the time. No such powerful arguments are at present available.

5 The point just made leads me to attach considerable importance to the present lack of persuasive dynamics, in particular of learning. I shall give reasons for this lack, but there will be no miraculous revelations. I shall briefly discuss several proposals.

While the list I have just given does not exhaust – particularly does not exhaust in detail – what I shall be discussing, it does mark out the most important signposts. There is one exception. I shall finish in a speculative section of what an adequate macro-economics might be like. I emphasise that it will be speculative. I do this in the hope that one or two of you might be encouraged to think well beyond the macro-economic textbook.

7.3 MACRO-VARIABLES IN MICRO-RELATIONS

In A–D equilibrium analysis excess demands are maps from the space of A–D prices. This space considers goods at different dates and locations and in different states of nature and of different specification as different. Notice that no expected prices are involved. At the very least this means that there is a way in which agents at one moment of time can deal in all these goods – A–D prices are market prices or "virtual" market prices. Evidently in the world of current macro-theory, where agents have rational price expectations which here means that every one knows the market clearing price of every A–D good from the beginning and sequential trading is of no economic significance. Much has been assumed away, in particular many macro-problems can no longer be discussed.

But since the current macroeconomists learned GE theory, much progress has been made in weakening the requirements of macroeconomics equilibrium, and in particular there has been increased understanding of the situation where markets are not rich enough. This is explained either by asymmetric information or transaction costs or both. From our point of view it is interesting that these phenomena have also been appealed to in the "co-ordination failure" literature of macro-economics.

It is clear at the outset that if we want to allow for the possibility of economic inefficiency we shall have to study economies which are not always in A–D equilibrium. Since in the light of economic history we have many grounds for supposing that actual economies have at the very least inefficient phases, seems to settle the matter.

I now want to stress that A–D equilibrium, especially in its sequential form, must be taken to represent what used to be called "long-run equilibrium", that is, not only do all markets at a given moment clear but all agents are optimally adjusted at all times. Since learning involves noting the mistakes one has made, agents in this equilibrium have ceased to learn. (To be more exact they have learned everything they could.)

In the light of my earlier arguments it now follows that if we want to study a serious macro-economics, by which I mean one not committed to the ad hoc postulate of perpetual efficiency, we must avoid long-run equilibrium analysis. It may well be that on certain postulates, we can show that learning leads to long-run equilibrium, but one doubts that these postulates will not contradict the Schumpeterian foundations of a capitalist economy.

To some, these remarks will appear heretical: have we not got many journal pages on tâtonnement stability? Have we not got an equally large number of pages on convergence to steady state? If one considers these models then one will quickly see that learning is not part of the stories told. The same goes for endogenous growth theories which are not strong on learning.

But while there are powerful arguments to leave the long period to historians or at best as signposts, it is also true that short period analysis is fraught with sufficient difficulties as to make it almost foolhardy to attempt it. At least this is so when one is seeking a closed model. There are two cardinal difficulties: if the plans of agents are incompatible at some date, how is that incompatibility resolved? How are plans changed by observed incompatibility? To raise these difficulties is to become aware of the incompleteness of our knowledge.

Certainly I shall not attempt a complete model of transitions. But while it would be desirable to have one, for my purposes I can do without it. My purpose is to delineate some of the features which any eventually satisfactory model should have. This does not entail a commitment to any one form. Indeed given our ignorance of these matters it may be an advantage to leave matters open as much as possible.

I shall start with an example. Let $F(w, \hat{u})$ be a wage distribution. We do not inquire into its origin at the moment. Notice that we have written it as dependent on $\hat{u}$, the aggregate level of unemployment. If searching agents take the wage distribution as given, their optimum search, given initial $\hat{u}$, yields u, the resulting unemployment level. Firms when deciding on wage-employment offers take u as given, but we assume that offers are declining in u. So if $u < \hat{u}$ the distribution will move to the right. The macro-variable u here acts as an externality. Let $U(F(w, u))$ give total unemployment when the initial unemployment is u. Then an obvious candidate for equilibrium is u^* such that

$$U(F(w, u^*)) = u^*$$

If one considers this example it plainly would need more in the way of detail. But even at this stage I want to emphasise the special feature: that a macro-variable enters the micro-behavioural specifications of the model. The fact that it has the role of an externality strikes me as particularly interesting. However, the idea is by no means new, for instance no one would be surprised by the argument that a higher rate of inflation may lead agents to behave in a manner which increases the rate further. This is familiar to practical economists but has not been seriously considered by General Equilibrium theorists.

Plainly we are implicitly invoking a theory, or an introduction to a theory, of expectation formations. If one starts off by postulating perfect foresight then actual u always turns out to be what it was expected to be for a particular state of nature. It is one of the limitations to that postulate that it leaves no room for the macro–micro loop I have been discussing, and so leaves us unprepared when it is important. Yet the "Lucas Critique" (1972) makes a not unrelated point: a change in macro-policy which is exogenous to agents will affect their behaviour. But exogeneity is not of the essence. Agents may treat the given rate of inflation as exogenous but will be affected in their behaviour if the rate is different.

I have taken highly simplified examples. For instance, wage offers by firms will clearly depend on the prices they themselves can charge without selling less than they planned or facing a demand greater than that. But that really does not affect the essence of the hypothesis. For instance, we may regard the macro-variable as a signal which conditions price or demand expectations.

For instance, we are familiar with expected prices as conditioned by some signal. Let p_i be the price of good i and write P for the observed price index then $E(p_i, P)$ is the expected price of the good, given the level of prices. If planned supply depends in the usual way on expected price and everywhere markets clear, we have, once again, the required macro–micro loop.

I am here being more specific than seems usual about signals which affect expectations. I do not just write "omega". This means that we propose a hypothesis of the theory of the world held by agents. Thus it seems reasonable to have the theory that a higher price level will also entail a higher price for the good sold by oneself, or at least not a lower price. Of course, there may be exceptions even to this but there seems no good reason to worry about these.

Indeed it seems obvious that most economists are prepared for macro-data to affect micro-behaviour. Consider a higher stock of money. The general view is that it is likely to raise prices. However, the stock of money is a macro-variable which agents take as given. If a change in money stock is to influence the private agent's decision, it must be the privately held stock, that is why a correct money-neutrality proposition has all privately held money stocks change in the same proportion. This is obscured by the use of the representative agent. However, if an increase in the aggregate stock is taken to increase the probability of higher prices this is no longer the case. I do not know whether this is what Lucas had in mind (1972). In any case it establishes that a link from macro to micro has long been part of the literature.

It is the precise nature of these links which strikes me as being an important area of future research, research which cannot yield interesting insights until one turns seriously to the study of expectation formation. An element in the story will, I believe, be the following: (a) agents have macrotheories of their own; and (b) they also have theories of how macro-events affect the micro-variables which are pay-off relevant to them. But it will not always be a simple relation. When the aggregate stock of money is known to be higher, the theory may be that all prices will be higher in the same proportion; but it need not be. For instance, there may be an expectation of money wage stickiness. That is, the theory may postulate a lag of money wages behind the price level with consequential differential effect on the composition of demand and so on different prices. Of course, the theory itself will affect what actually happens. This in a way is well rehearsed economics but it has only here or there affected how economists theorise. We seem too much committed to the view that in the end there is a reality which is only briefly obscured by beliefs.

7.4 A LITTLE MORE FORMALITY AND GENERALITY

We consider the economy at date t and state s. Let $h(t)$ stand for the relevant economic history up to t which determines not only endowments but beliefs at t. Let $p(t)$ be the vector of money prices of the goods traded in the economy. Let m be the vector of relevant macro-variables which govern expectations of the future. Assume that the components of m are observed with a one period lag which does not seem unreasonable. Then we write the vector of excess demands at t, $X(t)$, as $X(p(t), m(t-1))$ and define $E(m(t-1)) = \{p/X(p; m(t-1)) = 0\}$. Then for all p in $E(m(t-1))$, $m(t)$ is also determined since zero excess demands will together with p give all the information that is needed for the calculation of components of m at t. Of course this means that there is, for instance, no search for unemployment by definition. If E is a singleton then this set up induces a difference equation. But even so there is far too little structure to make this into an acceptable model. For instance, I have justified the macro-variable in the excess demand function by the argument that it is an important signal for expectations formation. However, I have not specified how the functional form of X must be altered from its usual appearance if X is to include the complicated force of expectation formation. For instance, how does a higher price index affect the demand for various goods?

But I did not set out to develop a closed model of a sequence of short period equilibria, but rather to show in general terms that it makes sense to include macro-variables among those that determine the development of an economy. One does this in order to take account of the theory of the economy held by agents so that new macro-information is reflected since that is the economy wide information there is. I hope to produce a more articulated model in the future; but it is bound to be highly speculative. When it seems silly to postulate perfect foresight one must put up with this disadvantage until a well-documented theory of expectation formation becomes available.

I return to the earlier model. At t, agents form expectations of the value of $m(t)$ to be revealed at $t+1$. That expectation is conditioned by $m(t-1)$, which is the last macro-report available. To clarify then notation write $\hat{E}(m(t))$ as the expected value of $m(t)$. Then I am assuming that $\hat{E}(m(t)) = F(m(t-1))$ and as before that $m(t) = G(p^*(t)m(t-1))$. So if I had any idea of what these relations actually were we could have a closed theory of the path taken by the economy. Since I only have banal general ideas on this matter I must leave it there. After all, the behaviour of the model will be pretty sensitive to its lag structure of which we know practically nothing. (p^* denotes the equilibrium price vector given $m(t-1)$.)

Nonetheless certain lessons seem to be available. We now have different possible equilibrium concepts. One evidently can call an equilibrium a situation where all excess demands are zero but the values of the macro-variables are not as expected. That makes it likely that agents have taken decisions which had they known the true m, they would not have taken. It is of course these errors which generate a dynamic. Nonetheless we may call this situation a short-period equilibrium – the consequences of the error are postponed by one period. If now in addition all macro-variables revealed at any date are those expected for that date we may say that we are in a long-run equilibrium. But of course it is not hard to describe equilibria for varying periods. But even then we have not exhausted the typology of equilibria. To simplify I have taken a perfectly competitive economy with markets clearing at every date by prices. If it took time for prices to be changed, say the auctioneer needs time to do so, then of course one needs to consider various rationing schemes. The rationing equilibrium of Belgian and French economists does just that but by and large they have not gone beyond t. Once again one will need to keep track how the out-turn of m compares with the expectations held of it.

7.5 SHORT-PERIOD EQUILIBRIUM (SPE)

I am thinking of an economy going from one short-period equilibrium to another. That is a method of analysis which finesses the difficulties of resolving inconsistent plans at some time t. Consider a pure exchange economy with perfect competition. Evidently plans at t will depend on the expected path of prices. It is my contention in this essay that agents need to take short cuts in forming these expectations and that they are aware that they cannot readily compare expected prices with actual ones in the case of all goods. The short cut proposed is that as a proxy for all the other variables which might be relevant to the choice at t, they observe and form expectations regarding macro-variables. These they take to be independent of their own actions. One may thus treat them like states of nature (for simplicity I assume that the usual state is known to be constant). By SPE, I simply mean that at t, prices clear all markets. The subset of these observed by an agent will be compared by him to what was expected conditional on the macro-state. The latter will depend on prices and production at t. Hence it, too, may not correspond to expectations. In whatever way we define errors of expectations the characteristic of SPE is that they play no part in its definition. It is only when one considers the sequence of short period equilibria that one must know something of the learning rule. Of course one needs to ensure that these definitions are not vacuous, i.e. an SPE exists. I clearly will not attempt that here, in particular, since I have left the structure of the economy very vague. However, I want to remind the reader that too large a difference in expectations may lead to unbounded trade and so put the existence of equilibrium at risk. All of this is spelled out at length in a well-known essay by Grandmont (1982).

As I have already argued the demand at t depends in principle on all current prices, the expectations of future prices as well on the expectation of the stream of wealth if that is exogenous to the agent. As an example of this I have taken the probability that the agent's supply of labour is demanded at or above the wages which it expects. If one assumes possibilities like that then one is committed to clearing markets, that is the micro-relations only hold in that case. Which means that models of that sort are not usable in the study of unemployment. I have argued that it is inconceivable that agents always act in a full knowledge of not only future variables but of all contemporary ones, at least that is so outside a stationary state. Hence like governments they have recourse to macro-variables, both as information signals and as summaries of current states. They form expectations regarding these and their plans are conditional on these expectations.

With this in mind, I now look for a formal definition of short-period equilibrium. I shall take the case of a perfectly competitive economy. Of course there is a certain artificiality in this concept since the length of the short period is not given naturally. But that seems to be the case with almost all of our models, for instance, models in continuous time face the objection that to take decision time intervals as infinitesimal, one is doing violence to reality.

I shall suppose that demand of households adjusts faster than the supply decisions of firms. So in "period" t demand depends on current prices, and the "effective" wage which I define as the market wage at t multiplied by the fraction of the work force employed plus unemployment pay multiplied by the probability of being unemployed; I write it as $\hat{w}$. So at t there are two kinds of households: the employed and the unemployed each with its budget constraint at t which depends on their employment status. Let w be the wage of the employed at t and e the fraction of the work force which earns it. Then $\hat{w} = ew + (1 - e)u$, where u is unemployment pay. The employed and the unemployed are assumed to have the same tastes so that we add e times the budget constraints of the employed and $(1 - e)$ times the budget constraints of the unemployed.

Notice that total demand will depend on e which is a macro-variable. In general equilibrium models in which there is search, unemployment is consistent with equilibrium. This is defined by a wage distribution such that given the optimum search for jobs, the employment ratio is e, which is together with prices just such that firms make job offers consistent with the wage distribution and the optimum search strategy of households.

Evidently this is not satisfactory since a wage distribution for homogeneous labour does not seem consistent with a long-run equilibrium. At best we have described a possible state on a transition path, that is a short-run equilibrium. As firms become aware of the wage distribution the latter will have a tendency to collapse. As it does so optimum search strategies will change and so therefore will e. There is here the making of a complicated dynamics which is due to the assumption of homogeneous labour. However, matching models run into similar distinctions between long and short-run and hence lead to a macro-variable driven dynamics.

Apart from difficulties in modelling a perfectly competitive economy with unequal wages for homogeneous labour there is also the objection that the model is mis-specified and that for two reasons: (1) it defines a job by the instantaneous wage rate and ignores the likely duration of the job (for economists devoted to the infinitely lived agent that is peculiar). Solow and I have offered an alternative which includes the duration of a job in the latter's description (1998) and since we have done that I shall ignore the matter here; and (2) a wage distribution is not the same thing as a distribution of employment offers at different wages. It seems more reasonable to distinguish the wage an employer is willing to pay to potential employees from the employment offered at that wage. In fact it seems more reasonable to assume that the worker knows the wage paid by different firms but is searching for an offer of employment at a wage above or equal to a critical one. If one assumes that the disutility of work is the same everywhere we could simply do our Bellman argument for a given distribution of potential income. If one were to take account of different skills and types generally these distributions would differ over types, and the analysis will become more complex. There is now however a significant difference: the unemployment need not be search unemployment in which workers wait for a better wage but instead wait to be employed. They are literally searching for employment, not at any wage but at one not smaller than a critical one. If they are willing to accept employment at $w*$ but are not offered it, then they are involuntarily unemployed. I now want to stress that all of this is part of a short-period analysis and one would not expect matters to remain unchanged. But in this chapter I am only concerned with setting the scene, dynamics is on the agenda for the future.

7.6 REMARKS ON THE CASE WHERE COMPETITION IS NOT PERFECT

There are many ways in which macro-variables could impinge on micro-markets. I discuss only one: the case of imperfect competition. Various authors e.g. Hart and Blanchard have asked themselves whether this move would make much difference to macro-theory. The conclusion was: not much, although some relatively unimportant differences were noted.

This conclusion is not surprising in models of perfect foresight which also pay no attention to uncertainty. The perfect foresight in this case consists of knowledge of the demand conditions of the firm – the "objective demand curve". This is to be distinguished from the case where demand functions are conjectured. But there is no reason why we should not consider expected demand curves as well as conjectured ones.[1]

Since I will not, indeed am unable to, give a satisfactory account of GE under imperfect competition I shall only consider a simple example leaving to a later occasion, a thorough account. I therefore take it that the true demand curve facing firm i is given by

$$x_i = H_i(Y)g_i(P, p_i),$$

where Y is a measure of GDP and P is an index of prices and the firm takes both as stochastic maps from the state of nature. It then, given its theory, formulates an expected demand function and once that has been done decides on its production plan, in the light of the expected prices of inputs. If one could now assume that these plans are not only carried out but that they yield an output and price which leads to the good being demanded in the quantity in which it is produced. There would then be an easy way to generate the following period's Y and P. The assumption which I have made could be regarded as being one of perfect foresight of demand conditions. This would not necessarily imply that Y and P are perfectly foreseen but the assumption has all the characteristic of long-run equilibrium. So I have no business making it at this stage.

I have therefore to allow for firms to make mistakes in foresight. To keep matters as simple as possible, suppose that the functional form of demand is as expected. However, P and Y are not as expected which in the first instance will be revealed by demand at the set price not being as expected. There is now the difficulty of reconciling *ex ante* and *ex post* to which reference has already been made. An easy option is to suppose that market prices are such that all predetermined supplies are taken off the market. It is not very attractive but in principle allows the analysis to proceed. Firms receive new macro-signals and adjust their expectations may be à la Bayes. One of the questions in a study of possible convergence will be whether indeed firms knew their demand functions so that errors can only be ascribed to errors in the expectations of P and Y or whether other errors were made, say in the response of demand to a price either of the good itself or of others.

Once again I propose to give up but in my view for the very good reason that neither I nor others know what would be appropriate assumptions. It shows the power of rational expectations but there seems no more reason to believe this postulate to be true than any other. The fact that it makes for an easy life seems insufficient and given that rationality only implies that one will not stick to beliefs systematically falsified by facts, it has no more axiomatic foundation than anything else. It is unlikely that this will be widely agreed until a new generation of economists arises, but they will have also plenty of things to disagree with in what I have said earlier.

So where does all this leave us? It seems to me that the effect of macro-data on micro-behaviour is clear and it is a pity that we do not seem to have any macro-foundations of microeconomics. No doubt there are common sense things to say such as that the discovery of Y greater than anticipated will not in general lead firms to reduce output or prices. But there is no reason why this should be always true. However, there are also important matters which I have neglected in the above, in particular strategic considerations which may have great influence on the behaviour of firms, especially large firms. But large firms have relatively large effects on the macroeconomy. Yet game theory is not a great help since it yields results (apart from insights) in very special cases only. It makes life more difficult than it is already since it is rather hard except in special cases to use it in the construction in the sort of dynamics which I have in mind here.

NOTE

1 A conjecture arises when the form of the demand curve is uncertain given the state of nature; an expected demand curve is made up of known demand functions but differing over states of nature. (Hahn, 1978)

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8 'Classical' vs. 'neoclassical' theories of value and distribution and the long-period method*

Heinz D. Kurz and Neri Salvadori

8.1 INTRODUCTION

In this chapter we ask whether classifying economic theories in distinct analytical approaches to certain economic problems and even in different schools of economic thought is a futile enterprise. More particularly, we shall argue that there is a thing that may, for good reasons, be called 'classical' economics, which is distinct from other kinds of economics, in particular 'neoclassical' economics. We shall focus attention on the theory of value and distribution. What we have in mind is a particular rational reconstruction of 'classical' economics, which, in our view, is both useful for an understanding of certain important arguments found in several classical authors and for an understanding of important present-day problems.

The chapter consists essentially of two parts. The first part (Sections 8.2–8.5), responding to a request of one of the organisers of the conference, Prof. Fabio Petri, is historical and provides a bird's eye view of the developments in the theory of value and distribution that took place since the inception of systematic economic analysis at the time of the classical economists. The second part (Section 8.6) instead is designed to demonstrate the power of the long-period method elaborated by the classical economists in terms of two examples. More specifically, we begin, in Section 8.2, with a brief account of alternative answers to the question raised by this chapter. Next, in Section 8.3, we turn to a discussion of the complexity of most economic problems and of economic theory as an attempt to come to grips with that complexity. This leads us to the identification of a first characteristic feature of classical economics: its *long-period* method. As we shall see in Section 8.4, a version of this method was also shared by all major neoclassical authors until the late 1920s. However, the similarity of the methods adopted by two theories must not be mistaken for a similarity of the content of the theories. This aspect becomes clear when we turn, in Sections 8.3 and 8.4, to the scope and content of traditional classical and traditional neoclassical economics. The emphasis is on the sets of data, or independent variables, on the basis of which these theories attempt to explain the respective unknowns, or dependent variables, under consideration. It will be seen that in this regard classical economics differs

* We should like to express our gratitude to Fabio Petri for several valuable comments and suggestions. It goes without saying that the responsibility for the content of the chapter is entirely ours.

markedly from neoclassical economics, the main difference being the way in which income distribution is determined. Sections 8.3 and 8.4 also raise the question whether the sets of data contemplated by these theories are compatible with the long-period method or whether there exist tensions and contradictions between the method and content of a theory. It is argued that whilst traditional classical theory can be formulated in a consistent way, traditional neoclassical theory faces insurmountable difficulties in this regard. The latter come to the fore in the shape of inconsistencies that undermine the logical foundation of the approach to the problem of income distribution in terms of the demand for and the supply of the factors of production collaborating in the generation of the social product, when there are produced means of production, i.e. 'capital', among these factors. Section 8.5 turns to the attempts of neoclassical authors from the late 1920s onwards to remedy this defect and at the same time render the theory more 'realistic', and indeed 'dynamic', in terms of models of temporary and intertemporal equilibria. It can be argued, however, that these alternatives are beset by a number of methodological difficulties. Moreover, it is close at hand to ask the following question within the neoclassical framework: Are there problems that can fairly easily be grasped using the long-period method, whereas these problems are very difficult to deal with in an intertemporal analysis? Section 8.6 exemplifies the power and fecundity of the long-period method in terms of two special problems: the first problem concerns a multisector variant of the AK growth model, the steady-state properties of which can be analysed without invoking some bold assumptions needed in an intertemporal model; the second concerns the determination of one of the distributive variables and relative prices in the empirically important case in which fixed capital items are jointly used.

8.2 A BRIEF ACCOUNT OF ALTERNATIVE VIEWS OF 'CLASSICAL' ECONOMICS

The view that the economics of the classical authors from Adam Smith to David Ricardo is essentially just an early variant of neoclassical theory, that is, an explanation of quantities and relative prices, including the prices of factor services, in terms of demand and supply functions, was advocated by Alfred Marshall. According to him classical analysis is characterised by a fairly well developed supply side, whereas the demand side is still in its infancy. Marshall's interpretation has been widely accepted in the history of economic thought and was shown to be untenable only relatively recently by Piero Sraffa with his reconstruction of the development of Ricardo's theory of value and distribution in his introduction to Volume I of *The Collected Works and Correspondence of David Ricardo* (Sraffa, 1951) and Sraffa's reformulation of the classical approach to the theory of value and distribution in his book *Production of Commodities by Means of Commodities* (Sraffa, 1960). For a while it looked as if Sraffa's alternative interpretation was about to be generally accepted. Yet from the mid 1970s, things began to change somewhat. It was particularly due to contributions by John R. Hicks and Samuel Hollander that a version of Marshall's integrationist perspective on the history of economic thought – also known as the 'continuity thesis' – gained momentum again. Amongst historians of economic thought the main spokesman of this point of view is Samuel Hollander according to whom all major economic theorists, including Smith, Ricardo and Marx, were demand and supply theorists of sorts. Hence from the point of view under consideration there is no such thing as a distinct classical approach to the theory of value and distribution.

This point of view has been taken to receive some support with regard to Sraffa's (1960) reformulation of the classical theory of value and distribution by attempts to show that Sraffa's analysis is but a 'special case' of intertemporal general equilibrium (GE) theory. This view was put forward, for example, by Christopher Bliss (1975), but it was particularly Frank Hahn (1982) who gave prominence to it. Many economists today appear to share some version of the view that there is essentially a continuity of ideas from the very inception of systematic economic thought beginning with the time of the classical economists. According to this view the history of economic theory can essentially be conceived of as a one-way avenue leading from primitive conceptualisations of the demand and supply approach to all sorts of economic phenomena to ever more sophisticated ones, merely leaving behind errors of reasoning and unnecessarily restrictive assumptions.

However, there are exceptions to the rule even within the group of contemporary authors who are major proponents of one version or another of neoclassical theory. One such exception appears to be Kenneth Arrow who in a paper on Ricardo's theory of distribution (Arrow, 1991) and also in a paper co-authored with Starret (Arrow and Starret, 1973) stated clearly that Ricardo's theory defies of being subsumed under neoclassical theory, because of a different analytical structure, in which demand functions play no role in the determination of the general rate of profits and the real wage rate. The latter is rather taken as given, when determining the profit rate and relative prices. (On Arrow's view see especially Garegnani, 2000.)

More than a century ago, around the time of the so-called 'marginalist revolution',[1] the perception that the theories advanced by Jevons, Menger and Walras constituted a fundamental break with the classical tradition was even more pronounced. Jevons stressed that he once and for all wanted to do away with the 'mazy and preposterous assumptions' of the classical economists (Jevons [1871], 1965: xiii); in addition, he accused them of having put forward a theory that is indeterminate. He wrote:

> Another part of the current doctrines of Economics determines the rate of profit of capitalists in a very simple manner. The whole produce of industry must be divided into the portions paid as rent, . . . profits and wages . . . Rent also may be eliminated, for it is essentially variable, and is reduced to zero in the case of the poorest land cultivated. We thus arrive at the simple equation –
>
> Produce = profit + wages.
>
> A plain result also is drawn from the formula; for we are told that if wages rise profits must fall, and *vice versâ*. But such a doctrine is radically fallacious; *it involves the attempt to determine two unknown quantities from one equation*. I grant that if the produce be a fixed amount, then if wages rise profits must fall, and *vice versâ*. Something might perhaps be made of this doctrine if Ricardo's

> theory of a natural rate of wages, that which is just sufficient to support the labourer, held true. But I altogether question the existence of any such rate.
>
> (Jevons [1871], 1911: 268–269; emphasis in the original)

The accusation of having provided an indeterminate system recurred, *inter alia*, in the writings of Walras ([1874], 1954, § 368) and Wicksteed (1894). In contradistinction, Wicksell with his typical acuteness stressed that 'the way in which Ricardo develops his argument . . . is a model of strictly logical reasoning about a subject which seems, at first glance, to admit of so little precision'; and 'Ricardo's theory of value is, one finds, developed with a high degree of consistency and strictness' (Wicksell [1893], 1954: 34 and 40). He added: 'Since, according to Ricardo, wages represent a magnitude fixed from the beginning, and since – as he later shows – the level of rent is also determined by independent causes, the cause of capital profit is already settled. It is neither possible nor necessary to explain capital profit in other ways, if the other assumptions are sound' ([1893], 1954: pp. 36–37). Therefore, in Wicksell's view Ricardo's system was not underdetermined. This did not mean, of course, that Wicksell agreed with the content of Ricardo's theory; it only meant that he was willing to admit that there was a classical alternative to the then already conventional demand and supply approach, which he, Wicksell, was engaged in further elaborating by way of integrating the Austrian, essentially Böhm-Bawerkian, temporal view of production and consumption into a Walrasian general equilibrium framework.

Although Wicksell and some other commentators were prepared to concede that there was a distinct classical or Ricardian theory of value and distribution, the analytical structure of this theory was far from clear to most economists. In fact, one of the reasons for the abandonment of the classical approach to the theory of value and distribution was that it was considered to be unable to accomplish the task it had set itself, that is, to determine the rate of profits and relative prices in a logically consistent manner. Another reason for the attacks on and eventually abandonment of that theory was, of course, the use to which it, and especially the labour value-based reasoning, had been put by the so-called Ricardian socialists. The classical approach was close to falling into oblivion and a proper understanding of it vanished. It was only after a long period of time that the classical approach was rediscovered and its analytical structure gradually laid bare from under thick layers of interpretation (see below).

So what are the characteristic features of the classical approach to the theory of value and distribution?

8.3 THE CLASSICAL APPROACH TO THE THEORY OF VALUE AND DISTRIBUTION

The concern of the classical economists from Adam Smith to David Ricardo was the laws governing the emerging capitalist economy, characterised by wage labour, an increasingly sophisticated division of labour, the coordination of economic activities via a system of interdependent markets in which transactions are mediated through money, and continuing technical, organisational and institutional change. In short, they were concerned with an economic system in motion. The attention focused on the factors affecting the pace with which capital accumulates and the economy expands and how the growing social product is shared out between the different classes of society: workers, capitalists and landowners.

8.3.1 Long-period method

How to analyse such a highly complex system? The ingenious device of the classical authors to see through these complexities and intricacies consisted of distinguishing between *market* or *actual* values of the relevant variables, in particular the prices of commodities and the rates of remuneration of primary inputs (labour and land), on the one hand, and *natural* or *normal* values, on the other. The former were taken to reflect all kinds of influences, many of an accidental and temporary nature, whereas the latter were conceived of as expressing the persistent, non-accidental and non-temporary forces governing the economic system. The classical authors did not consider the 'normal' values of the variables as purely ideal or theoretical; they saw them rather as 'centres of gravitation' of actual or market values (cf. Smith, *WN*, I. vii). This assumed gravitation of market values around their natural levels was seen to be the result of the self-seeking behaviour of agents and especially of the profit-seeking actions of producers. In conditions of *free competition*, that is, the absence of significant and lasting barriers to entry in and exit from all markets – the case with which the classical authors were primarily concerned – this involved *cost minimization*. This was well understood by the authors under consideration, and hence their attention focused on what may be called *cost-minimizing systems of production* (see also Kurz and Salvadori, 1995, esp. Chapters 1 and 13).

The method of analysis adopted by the classical economists is known as the *long-period method* or method of *long-period positions* of the economy. Any such position is nothing but the constellation towards which the system is taken to gravitate, given the fundamental forces at work in the particular situation under consideration. In conditions of free competition the resulting long-period position is characterised by a *uniform rate of profits* (subject perhaps to persistent inter-industry differentials), *uniform rates of remuneration* for each particular kind of primary input in the production process (such as different kinds of labour and natural resources), and prices that are assumed not to change between the beginning of the uniform period of production and its end, that is, *static prices*. Such a constellation is to be understood as reflecting the salient features of a competitive capitalist economy in an ideal way: it expresses the pure logic of the relationship between relative prices and income distribution in such an economic system. The prices are taken to allow producers to cover just the costs of production at the normal levels of the distributive variables, including profits at the ordinary rate. These prices have aptly been called also *prices of production* (Torrens, Ricardo).

Ever since the advent of systematic economic analysis in the seventeenth and eighteenth centuries, economists have aspired to elaborate a proper dynamical theory and many ingenious and hard-working people have made great efforts in this regard. However, given the complexity of the object of their analyses – a socioeconomic system incessantly in travail – they understood that the long-period method was the best available to them. The latter proved indeed quickly to be a powerful tool for studying certain properties of complex interdependent systems; that is, systems which it would be extremely difficult to model and analyse in a dynamic framework, even with the advanced tools of modern mathematics at one's disposal. Moreover, the classicals themselves occasionally ventured probing steps in the direction of such a dynamical analysis. Think, for example, of David Ricardo's discussion of the introduction and diffusion of improved machinery in the newly added chapter 'On Machinery' in the third edition of his *Principles*, published in 1821 (cf. *Works*, I, Chapter 31). However, a general dynamic analysis of the highly complex system under consideration, paying due attention to all relevant interdependencies, was not considered possible in principle, and it is doubtful that had they considered such an analysis possible, they would have considered it to be of much use. The long-period method was envisaged as the best available in order to come to grips with the basic driving forces, and their interplay, of an ever-changing world characterised by continuing technical progress, the depletion of natural resources and a changing distribution of income. Long-period analysis was precisely devised to overcome the impasse in which the social scientist found himself, confronted with a reality which, at first sight, looked impenetrable, made up of a myriad of relationships between people amongst themselves and people and natural objects. The long-period method gave some transparency to the complex object of study and allowed the theorist to derive a large number of interesting insights into the functioning (and the sources of malfunctioning) of the economic system. Because of its fecundity, the long-period method was almost generally adopted in political economy until the 1930s and still plays an important role in contemporary economics.

This does not mean that there was no interest in short-period problems amongst the classical economists; there was, of course. However, the important point to be made is that, in the majority of authors dealing with such problems, the short-period analyses elaborated by them had – as their backbones, so to speak – fully specified long-period theories. In other words, the long-period theory was considered the core of economic analysis from which they derived several short-period analyses designed to tackle special problems of a short-run nature, such as the implications of a capital stock that is not fully adjusted to the other data of the system or a sudden increase of the quantity of money in circulation.

8.3.2 Characteristic features of the classical approach

It is a first characteristic feature of the classical economists' approach to the problem of value and distribution that the 'data' or rather independent variables contemplated all refer to magnitudes that can, in principle, be observed, measured or calculated. This point of view, which may be called 'objectivist', is clearly expressed, for example, in William Petty's *Political Arithmetick* and in the physiocrats, in particular in François Quesnay's *Tableau Économique*. It is also present in the writings of Adam Smith, David Ricardo and Karl Marx: these authors refrained from having recourse to any magnitudes that are non-observable, non-measurable or non-calculable in determining the general rate of profits and relative prices.

Second, the many differences between different authors notwithstanding, the contributions to the theory of value and distribution of 'classical' derivation typically start from the same set of data. In general, these concern the following:

(1) The set of technical alternatives from which cost-minimising producers can choose.
(2) The size and composition of the social product, reflecting the needs and wants of the members of the different classes of society and the requirements of reproduction and capital accumulation.
(3) The ruling real wage rate(s) (or, alternatively, the rate of profits).
(4) The quantities of different qualities of land available and the known stocks of depletable resources, such as mineral deposits.

The treatment of wages (or, alternatively, the rate of profits) as an independent variable and of the other distributive variables, the rate of profits (the wage rate) in particular, as dependent residuals exhibits a fundamental *asymmetry* in the classical approach to the theory of value and distribution. In correspondence with the underlying long-period competitive position of the economy, the capital stock is assumed to be fully adjusted to these data, especially to the given levels of output. Hence the 'normal' desired pattern of utilisation of plant and equipment would be realised and a uniform rate of return on its supply price obtained. Prices of production are considered the medium of distributing the *social surplus* in the form of profits between different sectors of the economy and thus different employments of capital and, with scarce natural resources, in the form of differential rents of lands and mines.

It deserves to be emphasised that these data are sufficient to determine the unknowns, that is, the rate of profits (the wage rate), the rent rates, and the set of relative prices supporting the cost-minimising system of producing the given levels of output. No other data, such as, for example, demand functions for commodities and factors of production, are needed. The classical approach allows a consistent determination of the variables under consideration: it accomplishes the task put to itself. It does so by separating the determination of income distribution and prices from that of quantities, taken as given in (2) above. The latter were considered as determined in another part of the theory, that is, the analysis of capital accumulation, structural change and socioeconomic development. More precisely, the magnitudes referred to in the set of 'data' (1)–(4) are only treated as known or given in one part of the classical theory: the determination of the shares of income other than wages, and relative prices, in given conditions of the economy. In other parts of the theory they are themselves treated as dependent variables or unknowns. Hence, variables (1)–(4), while magnitudes external to the classical approach to the theory of value and distribution in particular, are magnitudes internal to the classical theory as a whole. This draws attention to the fact that the classical authors distinguished between different spheres of economic analysis necessitating the employment of different methods. While one sphere is suited to the application of deductive reasoning – this relates to the investigation of the relations between the distributive variables and relative prices, given the system of production – the other sphere requires more inductive lines of reasoning and research – this relates to an investigation of the sources and consequences of economic change, in particular technological progress, economic growth, changing consumption patterns, the exhaustion of natural resources etc.

8.3.3 Modern classical economics

It hardly needs to be stressed that with the above specification of 'classical' political economy, this school of thought did not vanish with the death of Ricardo or some other early classical economist. Constituting a fertile research programme, the classical approach managed to survive during the two centuries since its inception, albeit with several ups and downs. The danger of extinction was repeatedly warded off by scholars who, after decades, during which the classical approach had been 'submerged and forgotten' (Sraffa), managed to lay bare again its genuine significance and clarify its characteristic features.

Early formalisations of the classical approach to the problem of value and distribution were provided, among others, by Vladimir K. Dmitriev and Ladislaus von Bortkiewicz. John von Neumann contributed to the classical approach in terms of his famous model of economic growth. However, there is one author in particular whose work is uniquely important for the revival of classical political economy: Piero Sraffa (1951, 1960).[2] He deserves credit both for his work in the history of economic thought, tracing the classical approach back to David Ricardo and before him the physiocrats, and his coherent reformulation of that approach. Sraffa's work entailed a renewed interest in the writings of the old classical authors and induced analytical contributions, leading to a host of new findings concerning complex economic systems characterised by the production of commodities by means of commodities.

Independently from Sraffa's work there were contributions, mainly in the tradition of the von Neumann model, with a strong classical flavour. These concerned, for example, the so called 'non-substitution' and 'turnpike theorems'. The non-substitution theorem states that, under certain specified conditions (one primary factor, no joint production and constant returns to scale), and taking the rate of profits (rate of interest) as given from outside the system, relative prices are independent of the pattern of final demand. The theorem was received with some astonishment by authors working in the neoclassical tradition since it seemed flatly to contradict the importance attached to consumer preferences for the determination of relative prices. As Samuelson wrote: 'From technology and the interest rate alone, and completely without regard to the demand considerations ... [,] price relations can be accurately predicted as constants' (1966: 530; emphasis in original). In the usual Marshallian price–quantity diagram in order for demand to exert an influence on the price of a good, the supply function must not be horizontal. It is the hypothesis that the rate of profits (or, alternatively, the wage rate) is given and independent of the level and composition of output which accounts for the theorem. This hypothesis is completely extraneous to the neoclassical approach in competitive conditions and in fact assumes away the role played by one set of data from which that analysis commonly begins: given initial endowments (at most one could have a given amount of labour which, however, would not matter for the issue under consideration because returns to scale are constant: it would determine the size of unemployment because employment is determined by the other data). The assumption of a given rate of profits radically transforms the substance of that theory. With the endowment side chopped off, the concept of 'scarcity' of factors of production loses the significance usually attributed to it in neoclassical explanations of relative prices (there is no relative scarcity of primary and non-primary factors). Hence the demand for goods, and thus preferences, can no longer exert an influence on prices via the derived demand for factor services which are available in given supply: prices of goods are independent of demand because income distribution is *assumed* to be independent of demand. It goes without saying that, in the framework of classical analysis with its different approach to the theory of value and distribution, a characteristic feature of which is the non-symmetric treatment of the distributive variables, there is nothing unusual or exceptional about the non-substitution theorem. A similar argument could be developed with respect to turnpike theorems.

8.4 TRADITIONAL NEOCLASSICAL THEORY

8.4.1 Long-period equilibrium

The appeal exerted by the long-period method can be inferred from the fact that all early major marginalist authors, including William Stanley Jevons, Léon Walras, Eugen von Böhm-Bawerk, Alfred Marshall, Knut Wicksell and John Bates Clark, fundamentally adopted it. Like the classical economists and Marx they were concerned with explaining the normal rate of profits and normal prices: the concept of long-period 'equilibrium' is the neoclassical adaptation of the classical concept of normal positions (see Garegnani, 1976). For example, in Marshall's *Principles of Economics* it is stated:

> The actual value at any time, the market value as it is often called, is often more influenced by passing events, and by causes whose action is fitful and short lived, than by those which work persistently. But in long periods these fitful and irregular causes in large measure efface one another's influence so that in the long run persistent causes dominate value completely.
>
> (Marshall [1890], 1977: 291)

And Böhm-Bawerk suggested that the investigation of the permanent effects of changes in what are considered the dominant forces shaping the economy should be carried out by means of comparisons between long-period equilibria. These comparisons are taken to express the 'principal movement' entailed by a variation in the basic data of the economic system (cf. Böhm-Bawerk [1889], 1959, vol. 2: 380). This view was shared by Ludwig von Mises, one of the most radical subjectivists of the Austrian school of economic thought, who advocated the long-period method, or, as he preferred to call it, the 'static method', in the following terms:

> One must not commit the error of believing that the static method can only be used to explain the stationary state of an economy, which, by the way, does not and never can exist in real life; and that the moving and changing economy can

> only be dealt with in terms of a dynamic theory. *The static method is a method which is aimed at studying changes*; it is designed to investigate the consequences of a change in *one* datum in an otherwise unchanged system. This is a procedure which we cannot dispense with.
>
> (Mises, 1933: 117; emphasis added)

However, the adoption of the long-period method was not, by itself, prejudicial as to the *content* of the theory. In order to see this we have to turn to the forces which the neoclassical approach, in contradistinction to the classical one, conceptualised in order to determine normal income distribution and the corresponding system of relative prices.

8.4.2 The neoclassical set of data

Since the new theory was to be an alternative to the classical theory, it had to be an alternative theory about the same thing, in particular the normal rate of profits and normal prices. However, the set of data in terms of which the neoclassical approach attempted to determine these variables exhibits some striking differences with respect to the classical approach. First, it introduced independent variables, that is, explanatory factors, that were not directly observable, such as marginal utilities and agents' preferences. Second, it took as given not only the amounts of natural resources available, but also the 'initial endowment' of 'capital'. The data from which neoclassical theory typically begins its reasoning are:

(1) The set of technical alternatives from which cost-minimising producers can choose.
(2) The preferences of consumers.
(3) The initial endowments of the economy with all 'factors of production', including 'capital', and the distribution of property rights among individual agents.

The basic novelty of the new theory consisted of the following. While the received classical approach conceived the real wage as determined prior to profits and rents, in the neoclassical approach all kinds of income were explained simultaneously and *symmetrically* in terms of the forces of demand and supply with regard to the services of the respective factors of production: labour, 'capital' and land. This was obtained by invoking substitution mechanisms acting symmetrically on labour, land and capital from technical and consumption choices. The neoclassical theory was able to elaborate *functional relationships* between the quantity demanded, or supplied, of a service (or good) on one side and the price of that service (or good), and eventually other prices, on the other. It was the seemingly coherent foundation of such functional relationships that greatly contributed to the rapid success of neoclassical theory in economics.

Historically long-period neoclassical theory derives from a generalisation of the theory of rent in terms of land of uniform quality and 'intensive' margins to all factors of production, including 'capital' (see Bharadwaj, 1978). This generalisation presupposes that there is an analogy between land, labour and 'capital'. On this premise the principle of scarcity rent, which the classical economists had limited to natural resources in given supply, was thought to be applicable also in explaining the incomes of labour and 'capital', that is, wages and profits. However, in order to be able to conceive of the rate of profits as some kind of index expressing the relative scarcity of a factor called 'capital', that factor had to be assumed to be available in a given 'quantity'. The degree of (relative) scarcity of the given 'quantity of capital', which was taken to be reflected in the level of the rate of profits, was then envisaged to be the result of the interplay of data (1)–(3). The smaller the overall amount of capital at the disposal of producers, other things being equal, the greater in general the relative scarcity of that factor and the higher the rate of profits, and *vice versa*.

As regards the conceptualisation of the 'capital' endowment of the economy, the advocates of long-period neoclassical theory, with the exception of Walras (at least until the fourth edition of the *Eléments*), were aware of the following fact. Whereas different kinds of labour and land can be measured in terms of their own physical units, 'capital', conceived of as a bundle of heterogeneous produced means of production, had to be expressed in terms of a *single magnitude*, related in a known way to the *value* of capital goods, allowing 'capital' to assume the physical composition or 'form' best suited to the other data of the system. For, if the capital endowment were to be given in kind, only a short-period equilibrium, characterised by differential rates of return on the supply prices of the various capital goods, could be established by the forces constituting demand and supply. Such an equilibrium could not, however, be considered a 'full equilibrium' (Hicks, 1932: 20). Whereas differential wage and rent rates for different qualities of labour and land are perfectly compatible with a long-period competitive equilibrium, differential profit rates are not: competition would enforce a tendency towards a uniform rate of profits. The discovery of *reverse capital deepening* and of the *reswitching of techniques* revealed the deficiency of the conventional neoclassical view: a central element of the explanation of distribution in terms of demand and supply – the principle of factor substitution as envisaged by the theory – cannot generally be sustained and, in particular, it cannot be applied to 'capital'.[3]

8.5 TEMPORARY AND INTERTEMPORAL EQUILIBRIUM THEORY

As early as the late 1920s some major protagonists of the demand and supply approach, especially Erik Lindahl, began to glimpse the deficiency of the conventional concept of 'quantity of capital' in an explanation of normal income distribution. However, confronted with the alternative of abandoning the demand and supply approach or the long-period method, in terms of which the former had been conceptualised, authors such as Lindahl, Friedrich August von Hayek and John Richard Hicks opted for the second alternative. The result of these attempts to overcome the impasse in which neoclassical theory then found itself was the development of the concepts of *intertemporal* and *temporary equilibrium*. In this way the demand and supply approach was meant to be rendered not only consistent but also more 'realistic' (cf. Lindahl [1929], 1939: 271; Hicks [1939], 1946: 116). Indeed, as the protagonists of the new developments kept stressing, economic theory had to be liberated from the straitjacket of 'static' analysis and turned into a proper 'dynamic' analysis.

8.5.1 Erik Lindahl, Friedrich August von Hayek and John Richard Hicks

This quest for greater 'realism' of economic theory could only have been strengthened by the observation that traditional demand and supply theory was not merely turning a blind eye to the complications posed by time, but was also logically inconsistent. Amongst the three Lindahl was perhaps best aware of this inconsistency. In a footnote added to the English text of his 1929 paper, he pointed out that the received versions of 'modern' capital theory 'have the disadvantage that the measure of capital is made dependent on the prices of the services invested and on the rate of interest – which belong to the unknown factors of the problem' (Lindahl [1929], 1939: 317).

Following Wicksell, Lindahl was concerned with incorporating the insights of the Austrian theory of capital into the time structure of production in a Walrasian theory of general equilibrium. He proceeded in terms of a sequence of models designed to exhibit rising degrees of 'realism'. This sequence was eventually to be crowned by a model capable of portraying, in abstract terms, a 'real' economy moving through time. It goes without saying that Lindahl did not achieve this bold aim and openly admitted this.[4] He was, however, convinced that the 'dynamic' approach to economic problems advocated by him liberated economic theory from a static dead end and shunted it on to the right track.

Lindahl himself did not use the term 'intertemporal equilibrium'; yet what he had developed in some parts of his analysis were clearly intertemporal equilibrium models with a finite time horizon. Since these were explicitly based on the assumption of perfect foresight they could represent no more than preliminary steps on the way to a 'general dynamic theory' that truly deserved this name.[5] Hence, while it is true, as Debreu (1959: 35, n. 2) observed, that Lindahl provided 'the first general mathematical study' of this sort, its author can most certainly not be accused of having attributed too much importance to it.

In this conceptualisation, in accordance with Walras's analysis, the capital stock available at a particular point in time, which is the beginning of the first period of the economy contemplated by Lindahl's theory, is given in kind. Hence: 'Produced capital goods have the same significance for price formation as true *original* sources of similar kinds' (ibid.: 320–321; emphasis added). The importance of the initial conditions for the dynamic behaviour of the economy is particularly stressed by Lindahl in his later paper: 'The first step in this analysis is to explain a certain development as a result of certain given conditions prevailing at the beginning of the period studied. These given conditions must be stated in such a way that they contain *in nuce* the whole subsequent development' (Lindahl, 1939: 21).

According to Lindahl the main feature distinguishing an intertemporal from a long-period equilibrium, which he identified with a 'stationary state', concerns the 'original' factors, including the capital goods in given supply at the beginning of the first period: 'The real difference from the stationary case lies in the circumstance that the primary factors, there regarded as given, are assumed to undergo change from one period to another. In this way a movement arises in the system' (Lindahl [1929], 1939: 330). This movement concerns prices and quantities. As regards prices, Lindahl characterised the new 'dynamic' view of the economic system as opposed to the old 'static' one as follows: 'while in the stationary case the prices in succeeding periods are equal to the prices in the present period and thus do not introduce any new unknowns into the problem, in the dynamic case they will differ more or less from the prices in the first period' (ibid.: 319). Correspondingly, the notion of a uniform rate of interest turns out to be generally devoid of any 'clear and precise content' (ibid.: 245).[6] Nevertheless, Lindahl did not think that 'static theory' was entirely useless. He maintained rather that the system, if not disrupted by exogenous shocks, would gradually converge to a long-period equilibrium: 'If this tendency were alone operative, the community would in time reach stationary conditions' (ibid.: 331), characterised by a uniform rate of interest throughout the economy.[7]

Similarly to Lindahl, Hicks, who was strongly influenced by Lindahl, emphasised that 'static' theory neglected important features of the 'real world', such as uncertainty and expectations, and thus was 'quite incompetent to deal properly with capital and interest' (Hicks [1939], 1946: 116). 'Static theory', Hicks argued, would be applicable, 'if we could say that the system of prices existing at any moment depends upon the preferences and resources existing at that moment and upon nothing else'. Yet this is not the case: 'supplies (and ultimately demands too) are governed by expected prices quite as much as by current prices' (ibid.: 115–116). In his view the economic system had to be conceived of, 'not merely as a network of interdependent markets, but as a process in time' (ibid.: 116). This process, he contended, was best represented as a sequence of temporary equilibria, each temporary equilibrium being dependent on individuals' expectations of the future. Just as in Lindahl, the productive equipment of the economy is assumed to be given in kind. While 'the economic problem' was traditionally conceived of, in an atemporal way, as consisting in the allocation of given resources to alternative ends, it now had to be specified explicitly as involving 'the allotment of these resources, inherited from the past, among the satisfaction of present wants and future wants' (ibid.: 130). In such a framework prices are bound to change. While own rates of interest can be defined, they are said to be 'of little direct importance for us' (ibid.: 142).

Hicks's break with traditional neoclassical theory was even more radical than Lindahl's. The concept of the 'stationary state' as a position towards which the system is taken to gravitate if not perturbated by a series of exogenous factors of a more or less short-lived nature is rejected on the grounds that 'the stationary state is, in the end, nothing but an evasion' (ibid.: 117).[8]

For quite some time Hayek's part in the development of the notion of intertemporal equilibrium had not received the attention it deserves.[9] One year prior to Lindahl Hayek had published a paper, in German, which for the first time bore the notion 'intertemporal equilibrium' (*intertemporales Gleichgewicht*) in its title. Hayek argued that contrary to the received opinion the existence of such 'equilibria' is not merely 'incompatible with the idea that constant prices are a prerequisite to an undisturbed economic process, but is in the strictest opposition to it' (Hayek, 1928: 37). Implicit in an intertemporal price system is a multiplicity of commodity-own-rates of interest (ibid.: 43).

8.5.2 Gérard Debreu

A total break with the traditional method of analysis and its concern with long-period positions of the economic system characterised by a uniform rate of interest (profit) was finally effectuated in the so-called Arrow–Debreu model, developed by Kenneth Arrow and Gerard Debreu in the 1950s (Arrow and Debreu, 1954). Here we focus attention on Debreu's *Theory of Value*, published in 1959.

Debreu set himself two tasks: '(1) the explanation of the prices of commodities resulting from the interaction of the agents in a private ownership economy through markets; and (2) the explanation of the role of prices in an optimal state of the economy' (Debreu, 1959: ix). Our concern will be exclusively with task (1). An 'economy' is defined in terms of three sets of data: (1) a given number of consumers, characterised by their consumption sets and their preferences; (2) a given number of producers, characterised by their production sets; and (3) total resources (cf. ibid.: 74). As regards the latter, Debreu specified: 'They include the capital of the economy at the present instant, i.e. all the land, buildings, mineral deposits, equipment, inventories of goods, ... now existing and available to the agents of the economy. All these are a legacy of the past; they are *a priori* given' (ibid.: 75). The property rights as to these resources are also taken as given; all resources are owned by consumers.

The abandonment of any concern with the long period in Debreu's analysis is also clearly involved in the assumptions of a given and constant number of producers and given shares of the profit of the various producers received by consumers (Ibid.: 39 and 78).[10] As is well known, in Marshall, the assumption of a given number of firms was entertained only in the short-run, but not in the long-run. Consequently, in the short-run the supply function for any commodity for the economy as a whole equals the horizontally summed-up marginal cost functions, whereas in the long run for each quantity supplied by the respective industry the supply price equals the minimum of the average cost function of the single firm.

Commodities are not only specified in terms of their physical characteristics, but also in terms of their date and location of availability. By means of this generalisation of the concept of commodity, Debreu sought to accommodate time and space into the model and thus construct a *general* theory: 'By focusing attention on changes of dates one obtains, *as a particular case* of the general theory of commodities ... a theory of saving, investment, capital, and interest. Similarly by focusing attention on changes of locations one obtains, *as another particular case* of the same general theory, a theory of location, transportation, international trade and exchange' (ibid.: 32). Debreu assumed that there is only a finite number of distinguishable commodities (ibid.), which implies that the time-horizon of the model is finite. In a note appended to Chapter 2 he admitted that 'there are, however, conceptual difficulties in postulating a predetermined instant beyond which all economic activity either ceases or is outside the scope of the analysis' (ibid.: 35–36; see also Malinvaud, 1953). In addition, it is assumed that 'the interval of time over which economic activity takes place is divided into a finite number of compact *elementary intervals* of equal length' (ibid.: 29).

Debreu assumed that there exist current markets for *all* commodities, whatever their physical, temporal (within the given time horizon), or spatial specification.[11] Hence, in the 'economy' contemplated all trade for the entire time-horizon takes place at the beginning of the first period. If markets were reopened at later dates, then no additional trade would take place. As Arrow and Hahn stressed, the hypothesis that there exists a complete set of markets for current goods '"telescopes" the future into the present' (Arrow and Hahn, 1971: 33). Given a set of prices, each agent chooses a plan for all the elementary periods. An equilibrium for a 'private ownership economy' requires that all individual plans are, from the initial date onwards, mutually consistent for all future dates and compatible with 'the capital of the economy at the present instant', that is, initial endowments. Since Debreu assumes free disposal (1959: 42), in equilibrium some prices may be zero (in some periods): this concerns goods for which there is a negative excess demand.

Debreu's model exhibits several features that are disquieting. Here we cannot enter into a detailed discussion of these (see, however, Geanakoplos, 1987; Malinvaud, 1987; McKenzie, 1987 and Currie; Steedman, 1990, Chapter 7). Some critical remarks must suffice. A major difficulty concerns the treatment of time. 'The principal objection to the restriction to a finite number of goods is that it requires a finite horizon and there is no natural way to choose the final period. Moreover, since there will be terminal stocks in the final period there is no natural way to value them without contemplating future periods in which they will be used' (McKenzie, 1987: 507). What Debreu in fact assumed in his formal model is that all economic activity stops at the arbitrarily given terminal instant, that is, resources existing at the end of the time-horizon have *zero* value. Because of the recursive structure of the model, all economic activities decided in the initial instant are derived with regard to the final period.

As regards the instant from which the economy is analysed, that is, the 'present instant', the question arises whether there has been no economic activity prior to that date. Debreu's answer was in the negative: the economy is not created 'now'; it is rather assumed that, for the purpose of analysing the economy's future development, the legacy of the past is exclusively and completely reflected in the amounts of resources inherited and the distribution of private ownership of these resources. In particular, it is assumed that there are no commitments carrying over from the past that constrain agents' present decisions. This implies of course that the logic of the model does not extend to the past, because otherwise Debreu would have to admit that at some dates in the past agents entered into contracts referring to dates that are still in the future. In addition there is the following conceptual problem pointed out by Joan Robinson and others. If in equilibrium some of the capital stocks turn out to be in excess supply these stocks assume zero prices. This possibility appears to indicate that the expectations entrepreneurs held in the past when deciding to build up the present capital stocks are not realised. Hence, strictly speaking we are faced with a disequilibrium situation because otherwise the wrong stocks could not have been accumulated. Therefore, the problem arises how the past or, more exactly, possible discrepancies between expectations and facts influence the future.

As we have seen, earlier neoclassical authors, most notably Walras, were concerned with the long- and short-run equilibrium relationships between the prices of durable capital goods and the prices of their services, that is, the rates of return on different kinds of capital goods, and whether the short-run relationships gravitate towards some long-run relationship characterised by a uniform rate of return throughout the economy. To this effect Walras proposed an explicit *tâtonnement* procedure which he conjectured converged to long-period equilibrium. These concerns are not present in the Debreu model. It is not even asked how the economy is supposed ever to get into equilibrium. The notion of equilibrium is simply one of simultaneous clearing of all markets; there is no discussion of any *adjustment process* when defining equilibrium. Hence, in the Debreu analysis, as opposed to that presented by Walras with its long-period orientation, general equilibrium cannot be thought of as a 'centre of gravitation'.

8.5.3 Infinite time horizon

Until a few decades ago the time horizon in intertemporal general equilibrium theory was assumed to be finite and, therefore, arbitrary.[12] The introduction of an *infinite* horizon turned out to be critical (see also Burgstaller, 1994: 43–48). It pushed the analysis towards steady-state analysis (It ought to be stressed that the latter is a special case of long-period analysis and must not be identified with it.). This was clearly spelled out, for instance, by Robert Lucas in a contribution to the theories of endogenous growth. Lucas observed that 'for *any* initial capital $K(0) > 0$, the optimal capital-consumption path $(K(t), c(t))$ will converge to the balanced path asymptotically. That is, the balanced path will be a good approximation to any actual path "most" of the time' and that 'this is exactly the reason why the balanced path is interesting to us' (Lucas, 1988: 11). Lucas thus advocated a *(re-)switching* from an intertemporal analysis to a steady-state one. Since the balanced path of the intertemporal model is the only path analysed by Lucas, the intertemporal model may be regarded simply as a step to obtain a rigorous long-period (steady-state) setting (Paraphrasing a dictum put forward by Paul Samuelson in a different context, we may say that intertemporal analysis is a *detour* with regard to steady-state analysis.).

Moreover, Lucas abandoned one of the characteristic features of all neoclassical theories, that is, income distribution is determined by demand and supply of factors of production: if we concentrate on the 'balanced path', capital in the initial period *cannot* be taken as given along with other 'initial endowments'. Since distribution cannot be determined by demand and supply of capital and labour, in Lucas's model it is determined in the following way. Labour is just the vehicle of 'human capital', that is, a producible factor, hence all factors are producible and the rate of profit is determined as in Chapter II of Sraffa's Book. This is not surprising since the assumption of a given real wage rate is formally equivalent to the assumption that there is a technology producing 'labour'. The 'human capital' story could be seen as just a retorical artifice to render the idea of a given real wage more palatable to modern scholars. As regards its basic analytical structure (as opposed to its building blocks), some of the so-called 'new' growth theories belong within the realm of what we have called 'classical' economics. In particular, in the free competition versions of the theory, the 'technology' to produce 'human capital' (or, alternatively, 'knowledge' in some approaches) plays the same role as the assumption of a given real wage rate in 'classical' economics.

This leads to two questions. First, in the case in which we are exclusively, or at least mainly, interested in the steady state of an intertemporal model, and this steady state can be directly analysed using the long-period method, is it convenient to embark on a full-fledged intertemporal analysis? Or is it more convenient to study directly the long-period position? Note that if the answer to the last question is 'yes', we cannot but use the classical idea of an asymmetrical determination of distribution.

Second, are there issues that can easily be treated using the long-period method, while it would be difficult to treat them in an intertemporal analysis? With regard to those cases where this question has to be answered in the positive, there are a number of economic phenomena which can be grasped by using the long-period method, whereas they would remain obscure (at least temporarily) if this method were not used. This idea has been expressed in the literature. For instance, Burmeister (1996: 1346) has argued:

> It is natural to try to answer the easiest questions first, and it is much easier to study economies in a "long-period equilibrium" than ones in which the rate of profit is not uniform and is changing over time.

As is well known, the proof of the pudding is in the eating. Hence two simple examples will suffice to answer the above questions with regard to the special case of steady-states.

8.6 THE USEFULNESS OF THE LONG-PERIOD METHOD: TWO EXAMPLES

We begin with an example which deals with a multisector version of the AK model known from the literature on endogenous growth. In line with Lucas's argument referred to at the end of Section 8.5 we may ask: Why do we have to make the bold assumptions required to study an intertemporal model if, in the end, we are only interested in the steady state of that model? Why not use the long-period method directly? This does not mean, of course, that the problem whether and how an economic system gravitates towards a long-period position or, in the present case, a steady-state growth path is uninteresting. Not at all. However, this problem is very difficult and will be set aside here. It suffices to remark that in order to study it we would need a truly dynamic analysis in which agents can make errors etc. It hardly needs to be stressed that the intertemporal equilibrium model does not constitute such a dynamic analysis.

8.6.1 Endogenous growth: a multisector version of the AK model

Let us consider an economy with the conventional representative agent who is faced with the following problem ($t \in \mathbf{R}$):

$$\max \int_0^\infty \mathrm{e}^{-\rho t} \frac{C_t^{1-\sigma} - 1}{1-\sigma}\, \mathrm{d}t \tag{8.1a}$$

$$\text{s. to } \mathbf{x}_t^T(\mathbf{I} - \delta\mathbf{A}) \geqq C_t \mathbf{e}_1^T + \dot{\mathbf{x}}_t^T \mathbf{A}, \tag{8.1b}$$

$$\mathbf{x}_t \geqq \mathbf{0}, \quad \mathbf{x}_0^T \mathbf{A} \leqq \bar{\mathbf{x}}, \quad C_t \geq 0, \tag{8.1c}$$

where $\rho > 0$ is rate of time preference, $1/\sigma$ is the elasticity of substitution between present and future consumption ($1 \neq \sigma > 0$), C_t is the consumption of commodity 1, the only consumption good, at time t, $\mathbf{A}$ is an $n \times n$ instantaneous capital goods matrix, the corresponding $n \times n$ instantaneous output matrix being $\mathbf{I} + (1 - \delta)\mathbf{A}$, where $\mathbf{I}$ is the $n \times n$ identity matrix and δ is the uniform rate of depreciation of capital goods, $0 \leq \delta < +\infty$ (that is, no primary factor is used in production and there is no choice of technique[13]), $\mathbf{x}_t$ is the vector of intensities of operation of processes defined by matrix $\mathbf{A}$ and depreciation rate δ, $\dot{\mathbf{x}}$ is the derivative of $\mathbf{x}$ with respect to time, $\bar{\mathbf{x}}$ is the given positive vector of initial stocks of commodities, and $\mathbf{e}_1$ is the first unit vector. Matrix $\mathbf{A}$ is assumed to be non-negative and indecomposable. The ρ is assumed to satisfy the inequalities

$$(\lambda^{-1} - \delta)(1 - \sigma) < \rho \leq (\lambda^{-1} - \delta), \tag{8.2}$$

where λ is the eigenvalue of maximum modulus (also known as the Frobenius eigenvalue, cf. Takayama, 1974, Chapter 4; or Kurz and Salvadori, 1995: 509–519) of matrix $\mathbf{A}$. It will also be assumed for simplicity that $\mathbf{A}$ is invertible and has n distinct eigenvalues. By using optimal control theory it is possible to show

Proposition 1 *There are scalars $g > 0$ and $C_0 \geq 0$ such that $\dot{\mathbf{x}} = \mathbf{x}_0 e^{gt}$ and $\dot{C} = C_0 e^{gt}$ are solutions to problem* (8.1) *if and only if*

$$g = \frac{1 - \delta\lambda - \rho\lambda}{\lambda\sigma} \tag{8.3}$$

and there is a scalar $\theta > 0$ such that

$$\bar{\mathbf{x}} = \theta \mathbf{e}_1^T [\mathbf{I} - (\delta + g)\mathbf{A}]^{-1}\mathbf{A}.$$

For a proof, see Salvadori (1998).[14]

Let us now consider another economy with the same technology. As in the previous exercise it is assumed that only commodity 1 is consumed and that the saving–investment mechanism determines the following relationship between the real profit rate r and the growth rate g

$$g = \frac{r - \rho}{\sigma}, \tag{8.4}$$

whatever the meaning of ρ and σ. Note that these assumptions are much less strong than those of the previous exercise. If some agent owns the commodities $\mathbf{e}_j^T\mathbf{A}$ at time 0 and uses them to produce continuously commodity j from time 0 to time t, so that (1) at time t he owns the commodities $e^{-\delta t}\mathbf{e}_j^T\mathbf{A}$; and (2) at each time τ, $0 \leq \tau < t$, he has a flow of product of $e^{-\delta\tau}$ units of commodity j which is invested in another business; and if all investments earn a nominal rate of profit i (to be determined); then

$$\int_0^t e^{(t-\tau)i} e^{-\delta\tau} \mathbf{e}_j^T \mathbf{p}_\tau d\tau + e^{-\delta t}\mathbf{e}_j^T \mathbf{A}\mathbf{p}_t = e^{it}\mathbf{e}_j^T\mathbf{A}\mathbf{p}_0. \tag{8.5}$$

In a long-period position relative prices are constant and the rate of inflation is also constant, so that for each t

$$\mathbf{p}_t = e^{\pi t}\mathbf{p},$$

where $\mathbf{p}$ is a vector to be determined and π is the rate of inflation (or deflation). Hence, if long-period conditions are assumed to hold (and if $i \neq \pi - \delta$),[15] from equation (8.5) we obtain

$$[e^{it} - e^{(\pi-\delta)t}]\left[\frac{1}{i-\pi+\delta}\mathbf{e}_j^T\mathbf{p} - \mathbf{e}_j^T\mathbf{A}\mathbf{p}\right] = 0,$$

which can be written as

$$[e^{(r+\pi)t} - e^{(\pi-\delta)t}]\left[\frac{1}{r+\delta}\mathbf{e}_j^T\mathbf{p} - \mathbf{e}_j^T\mathbf{A}\mathbf{p}\right] = 0,$$

where r is the real rate of profit ($r = i - \pi$). Since this equation must hold for each j we have

$$\left[\frac{1}{r+\delta}\mathbf{p} - \mathbf{A}\mathbf{p}\right] = \mathbf{0}.$$

Since $\mathbf{p}$ must be semipositive, then we have from the Perron-Frobenius Theorem that $\mathbf{p} > 0$ is the right eigenvector of matrix $\mathbf{A}$ corresponding to the eigenvalue of maximum modulus $\lambda > \mathbf{0}$, and

$$r = \frac{1-\delta\lambda}{\lambda} > 0,$$

the inequality being a consequence of the second of inequalities (8.2). Moreover, because of equation (8.4), equation (8.3) holds. Note that inequalities (8.2) imply

$$0 \leq g < r.$$

Finally, since only commodity 1 is consumed and the economy grows at rate g, the consumption C_t and the intensity vector $\mathbf{x}_t$ must satisfy the following equations

$$C_t = C_0 e^{gt},$$
$$\mathbf{x}_t^T = C_0\mathbf{e}_1^T[\mathbf{I} - (\delta+g)\mathbf{A}]^{-1}e^{gt},$$

provided that $\delta + g < \lambda^{-1}$, which is certainly the case, since the first of inequalities (8.2) holds.

This completes the demonstration that one can analyse the steady-state properties of a multisector variant of the AK-model without being compelled to invoke the bold assumptions necessary in an intertemporal model.

We turn now to our second example. This concerns the case of jointly utilised fixed capital items. (For a more detailed discussion, see Kurz and Salvadori, 1995, Chapter 9.)

8.6.2 Jointly utilised fixed capital items

It is assumed that there are m processes and n commodities. Each process of production i ($i = 1, 2, \dots, m$) is defined by the triplet $(\mathbf{a}_i, \mathbf{b}_i, \mathbf{l}_i)$, where $\mathbf{a}_i = (a_{i1}, a_{i2}, \dots, a_{in})^T$ is the non-negative material input vector, $\mathbf{b}_i = (\mathbf{b}_{i1}, \mathbf{b}_{i2}, \dots, \mathbf{b}_{in})^T$ is the non-negative output vector, and $\mathbf{l}_i$, a scalar, is the non-negative labour input. Thus, the whole technology is defined by the triplet $(\mathbf{A}, \mathbf{B}, \mathbf{l})$, where

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{a}_m^T \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{b}_1^T \\ \mathbf{b}_2^T \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{b}_m^T \end{bmatrix}, \quad \mathbf{l} = \begin{bmatrix} \mathbf{l}_1 \\ \mathbf{l}_2 \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{l}_m \end{bmatrix}$$

It is assumed that

Assumption 1 It is not possible to produce something without using some material inputs, i.e.

$$\mathbf{e}_j^T \mathbf{A} \geq \mathbf{0} \quad j = 1, 2, \dots, m$$

Assumption 2 All commodities are producible, i.e.

$$\mathbf{B}\mathbf{e}_j \geq \mathbf{0} \quad j = 1, 2, \dots, n$$

Assumption 3 Labour is indispensable for the reproduction of commodities, i.e.

$$(\mathbf{x} \geq 0,\ \mathbf{x}^T(\mathbf{B} - \mathbf{A}) \geqq \mathbf{0}) \Rightarrow \mathbf{x}^T \mathbf{l} > 0.$$

The example we are going to analyse concerns jointly utilised fixed capital items that are subject to the assumption that they cannot be transfered among sectors, that is, an oven once utilised to produce bread cannot be used during its lifetime to produce biscuits.[16] On this assumption a number of 'desirable' properties will be shown to hold. Among these properties there is the fact that consumption patterns do not matter at all in determining prices or operated processes, whereas the growth rate can play a role in determining the cost minimising technique and, therefore, prices. The assumption we are going to investigate is the following.

Assumption 4 There are two subsets, S and **T**, of the set of commodities N such that
(A.4.1) $\mathrm{S} \cap \mathrm{T} = \phi$, $\mathrm{S} \cup \mathrm{T} = \mathrm{N}$;
(A.4.2) commodities in T are never consumed;

(A.4.3) each process produces one and only one commodity in S;

(A.4.4) if commodity $i \in \mathrm{T}$ is produced by process j producing commodity $h \in \mathrm{S}$, then there is no process producing a commodity $k \in \mathrm{S}$, $k \neq h$, such that it either produces i or utilises i as an input;

(A.4.5) for each process producing commodity $j \in \mathrm{T}$ there is a process with the same inputs and the same outputs except that commodity j is not produced.

It is immediately recognised that if and only if single production holds, Assumption 4 is satisfied with $\mathrm{S} = \mathrm{N}$ and $\mathrm{T} = \phi$. On the contrary, if Assumption 4 holds with $\mathrm{S} \neq \mathrm{N}$ and $\mathrm{T} \neq \phi$, then, for the sake of simplicity, we can refer to the commodities in T as 'old machines' and to the commodities in S as 'finished goods'. The rationale for the above axioms can now be stated as follows. Axiom (A.4.1) implies that a commodity is either an old machine or a finished good, but never both. Axiom (A.4.2) implies that old machines are never consumed. Axiom (A.4.3) rules out joint production proper. Axiom (A.4.4) states that old machines cannot be transferred among sectors. Axiom (A.4.5) implies that old machines can be disposed of at no cost (leaving no scrap behind), that is there is free disposal of old machines, but not necessarily of finished goods.

If Assumption 4 holds, we may reorder commodities and processes in the following way: the first s commodities are in S, the next t_1 commodities are in T and are produced jointly with commodity 1, the next t_2 commodities are in T and are produced jointly with commodity $2, \ldots$, the next t_s commodities are in T and are produced jointly with commodity s, some t_i may be equal to zero, $t_1 + t_2 + \cdots + t_s = t$, $s + t = n$; the first m_1 processes produce commodity 1, the next m_2 processes produce commodity $2, \ldots$, the next m_s processes produce commodity s, $m_1 + m_2 + \cdots + m_s = m$. In order to simplify the notation, let us also introduce $t_0 = 0$ and $m_0 = 0$.

Therefore Axiom A.4.4, i.e. old machines cannot be transferred among sectors, is equivalent to the following properties of matrices $\mathbf{A}$ and $\mathbf{B}$:

1 if $k \in \mathrm{S}$, $b_{hk} > 0$ if and only if $\sum_{j=0}^{k-1} m_j < h \leq \sum_{j=0}^{k} m_j$;

2 if $k \in \mathrm{S}$ and $s + \sum_{j=0}^{k-1} t_j < i \leq s + \sum_{j=0}^{k} t_j$, then $b_{hi} > 0$ or $a_{hi} > 0$ only if $\sum_{j=0}^{k-1} m_j < h \leq \sum_{j=0}^{k} m_j$.

In Figures 8.1 and 8.2 matrices $\mathbf{A}$ and $\mathbf{B}$, respectively, are represented on the assumption that $s = 5$ and that all commodities except commodity 3 are produced by using old machines: grey areas represent non-negative elements, white areas represent zero elements, and black areas represent positive elements.

We say that there is a long-period position corresponding to rate of profit r and demand function $\mathbf{d}(r, \mathbf{p}, \mathbf{x})$ if there is a solution to the following set of equations and inequalities.

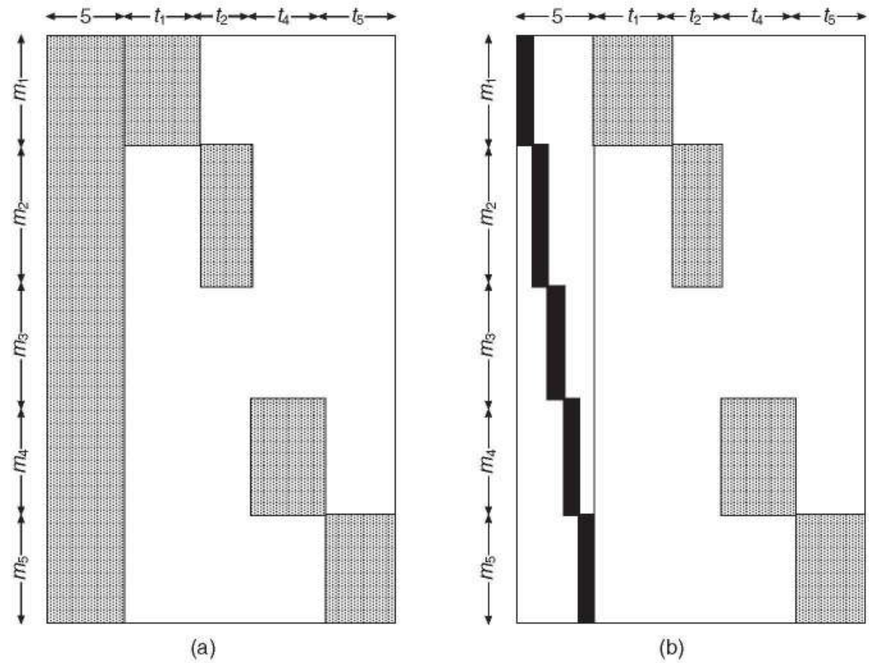

Figure 8.1

$$[\mathbf{B}-(1+r)\mathbf{A}]\mathbf{p} \leqq \mathbf{l} \tag{8.6a}$$

$$\mathrm{x}^T[\mathbf{B}-\mathbf{A}] \geqq \mathbf{d}(r, \mathbf{p}, \mathbf{x})^T \tag{8.6b}$$

$$\mathbf{x}^T[\mathbf{B}-(1+r)\mathbf{A}]\mathbf{p} = \mathbf{x}^T\mathbf{l} \tag{8.6c}$$

$$\mathbf{x}^T[\mathbf{B}-\mathbf{A}]\mathbf{p} = \mathbf{d}(r, \mathbf{p}, \mathbf{x})^T\mathbf{p} \tag{8.6d}$$

$$\mathbf{x} \geqq 0, \quad \mathbf{p} \geqq 0. \tag{8.6e}$$

Since we want to prove that prices are independent from $\mathbf{d}(r, \mathbf{p}, \mathbf{x})$ provided the economy is growing at a uniform rate which will be called g and that Assumption 4 holds, it will be assumed that $\mathbf{d}(r, \mathbf{p}, \mathbf{x}) = g\mathbf{x}^T\mathbf{A} + \mathbf{c}(r, \mathbf{p}, \mathbf{x})^T$, where the last t elements of vector $\mathbf{c}(r, \mathbf{p}, \mathbf{x})$ are identically nought. Hence system equation (8.6) is better stated as

$$[\mathbf{B}-(1+r)\mathbf{A}]\mathbf{p} \leqq \mathbf{l} \tag{8.7a}$$

$$\mathbf{x}^T[\mathbf{B}-(1+g)\mathbf{A}] \geqq \mathbf{c}(r, \mathbf{p}, \mathbf{x})^T \tag{8.7b}$$

$$\mathbf{x}^T[\mathbf{B}-(1+r)\mathbf{A}]\mathbf{p} = \mathbf{x}^T\mathbf{l} \tag{8.7c}$$

$$\mathbf{x}^T[\mathbf{B}-(1+g)\mathbf{A}]\mathbf{p} = \mathbf{c}(r, \mathbf{p}, \mathbf{x})^T\mathbf{p} \tag{8.7d}$$

$$\mathbf{x} \geqq 0, \quad \mathbf{p} \geqq 0. \tag{8.7e}$$

Furthermore function $\mathbf{c}(r, \mathbf{p}, \mathbf{x})$ satisfies the following obvious assumptions.

Assumption 5 Function $\mathbf{c}(r, \mathbf{p}, \mathbf{x})$

(A.5.1) is continuous in $\mathbf{x}$;

(A.5.2) is homogeneous of degree 1 in $\mathbf{x}$, that is, it satisfies the equality

$$\alpha\mathbf{c}(r, \mathbf{p}, \mathbf{x}) = \mathbf{c}(r, \mathbf{p}, \alpha\mathbf{x})$$

for each $\alpha \geq 0$, $r \geq 0$, $\mathbf{p} \geq 0$, $\mathbf{x} \geq 0$;

(A.5.3) is non-negative everywhere it is defined, that is,

$$\mathbf{c}(r, \mathbf{p}, \mathbf{x}) \geqq 0$$

for each $r \geq 0$, $\mathbf{p} \geq 0$, $\mathbf{x} \geq 0$;

(A.5.4) satisfies Walras's law, that is

$$\mathbf{c}(r, \mathbf{p}, \mathbf{x})^T\mathbf{p} = \mathbf{x}^T\mathbf{l} + (r - g)\mathbf{x}^T\mathbf{A}\mathbf{p}$$

for each $r \geq 0$, $\mathbf{p} \geq 0$, $\mathbf{x} \geq 0$.

Axioms (A.5.1), (A.5.2), and (A.5.4) are needed in order to allow the existence of a uniform growth rate which is constant over time, whereas Axiom (A.5.3) just asserts that a negative amount of a commodity cannot be consumed. Before stating conditions for the existence and uniqueness of a cost-minimising technique some Lemmata will be introduced.

LEMMA 1 *If the following Assumption 6 holds, and if $g \leq r$, then the following system of equations and inequalities has a solution*:

$$[\mathbf{B} - (1 + r)\mathbf{A}]\mathbf{y} \leqq \mathbf{l} \tag{8.8a}$$

$$\mathbf{q}^T[\mathbf{B} - (1 + g)\mathbf{A}] \geqq \mathbf{a}^T \tag{8.8b}$$

$$\mathbf{q}^T[\mathbf{B} - (1 + r)\mathbf{A}]\mathbf{y} = \mathbf{q}^T\mathbf{l} \tag{8.8c}$$

$$\mathbf{q}^T[\mathbf{B} - (1 + g)\mathbf{A}]\mathbf{y} = \mathbf{a}^T\mathbf{y} \tag{8.8d}$$

$$\mathbf{q} \geqq \mathbf{0}, \quad \mathbf{y} \geqq \mathbf{0}, \tag{8.8e}$$

where $\mathbf{a}$ *is a given semipositive vector.*

Proof See Lippi (1979) or Salvadori (1980).

Assumption 6 There is a vector $\mathbf{z}$ such that

$$\mathbf{z} \geqq 0, \quad \mathbf{z}^T[\mathbf{B} - (1 + r)\mathbf{A}] \geqq \mathbf{a}^T.$$

The following two Lemmata introduce the procedure which will be followed to derive the consequences of Assumption 4.

LEMMA 2 *Let Assumptions 4 and 6 hold, let $g \leq r$, and let $(\mathbf{q}^*, \mathbf{y}^*)$ be a solution to system equation (8.8) for $\mathbf{a} = \hat{\mathbf{a}}$, where $\hat{\mathbf{a}}$ is a vector with each of the first s elements being equal to 1, and each of the others being equal to 0. Then there is a vector $\mathbf{q}^{**}$ such that $(\mathbf{q}^{**}, \mathbf{y}^*)$ is a solution to system (8) for $\mathbf{a} = (\mathbf{c}_s^T, \mathbf{0}^\mathbf{T})$, where $\mathbf{c}_s$ is any semipositive vector in $\mathbf{R}^s$.*

Proof Let $\mathbf{Q} = [q_{hk}]$ be an $s \times m$ matrix such that

$$q_{hk} = \begin{cases} \mathbf{e}_h^T \mathbf{q}^* & \text{if } \sum_{i=0}^{h-1} m_i < k \leq \sum_{i=0}^{h} m_i \\ 0 & \text{elsewhere} \end{cases}$$

Let us define the matrices $\mathbf{D}_0$, $\mathbf{C}_0$, $\mathbf{D}_1$, $\mathbf{C}_1$, obtained by the following partition of matrices $\mathbf{QA}$ and $\mathbf{QB}$: $(\mathbf{D}_0, \mathbf{D}_1) = \mathbf{QB}, (\mathbf{C}_0, \mathbf{C}_1) = \mathbf{QA}$, $\mathbf{C}_0$ and $\mathbf{D}_0$ being square. Since Assumption 4 holds and since the first s entries of $\hat{\mathbf{a}}$ are equal to 1, all the others being equal to zero,

$\mathbf{D}_0, \mathbf{C}_0, \mathbf{D}_1, \mathbf{C}_1$ are non-negative, $\mathbf{D}_0$ is diagonal, diag $\mathbf{D}_0 > \mathbf{0}$;

$$[\mathbf{D}_0 - (1+r)\mathbf{C}_0]\mathbf{y}_s^* + [\mathbf{D}_1 - (1+r)\mathbf{C}_1]\mathbf{y}_t^* = \mathbf{Ql}; \tag{8.9}$$

$$[\mathbf{D}_1 - (1+g)\mathbf{C}_1] \geqq \mathbf{0},\ [\mathbf{D}_1 - (1+g)\mathbf{C}_1]\mathbf{y}_t^* = \mathbf{0}; \tag{8.10}$$

$$\mathbf{e}^T[\mathbf{D}_0 - (1+g)\mathbf{C}_0] \geqq \mathbf{e}^T; \tag{8.11}$$

where $(\mathbf{y}_s^{*T}, \mathbf{y}_t^{*T}) = \mathbf{y}^{*T}$. It is immediately recognised, because of equation (8.10), that if there is a non-negative solution $\mathbf{v}^*$ to the equation

$$\mathbf{v}^T[\mathbf{D}_0 - (1+g)\mathbf{C}_0] = \mathbf{e}_s^T,$$

then vector $\mathbf{q}^{**} = \mathbf{Q}^T\mathbf{v}^*$ satisfies the Lemma. To prove that $\mathbf{v}^*$ exists, it is enough to remark that matrix $\mathbf{C}_0\mathbf{D}^{-1}$ is non-negative and that inequality

$$\mathbf{e}^T[\mathbf{1} - (1+g)\mathbf{C}_0\mathbf{D}_0^{-1}] > \mathbf{0}^T$$

holds, because of equation (8.11); then we obtain from a well known theorem that matrix $[\mathbf{D}_0 - (1+g)\mathbf{C}_0]$ is invertible and

$$[\mathbf{D}_0 - (1+g)\mathbf{C}_0]^{-1} \geq \mathbf{0}. \tag{8.12}$$

Q.E.D.

LEMMA 3 *Let Assumptions 4 and 6 hold and let $\mathbf{y}^* = (\mathbf{y}_s^{*T}, \mathbf{y}_t^{*T})^T$ be defined as in Lemma 2, then $\mathbf{y}_s^{*T} > 0$ and the weak inequality equation (8.11) is satisfied as an equation.*

Proof Let $(\mathbf{q}_i^{**}, \mathbf{y}^*)$ be a solution to system equation (8.8) for $\mathbf{a} = \mathbf{e}_i$, $i \in \{1, 2, \ldots, s\}$. Thus,

$$\mathbf{q}_i^{**T}[\mathbf{B} - (1+g)\mathbf{A}]\mathbf{y}^* = \mathbf{e}_i^T\mathbf{y}^*$$
$$\mathbf{q}_i^{**T}[\mathbf{B} - (1+r)\mathbf{A}]\mathbf{y}^* = \mathbf{q}_i^{**T}\mathbf{l}$$

i.e.

$$\mathbf{e}_i^T\mathbf{y}^* = \mathbf{q}_i^{**T}\mathbf{l} + (r-g)\mathbf{q}_i^{**T}\mathbf{A}\mathbf{y}^*.$$

Then, the first part of the Lemma is obtained since $(r-g) \geq 0$ and $\mathbf{q}_i^{**T}\mathbf{l} > 0$ because of Assumption 3. The second part of the Lemma is an immediate consequence of the first part. Q.E.D.

Now the main theorem concerning the existence and irrelevance of the form of function $\mathbf{c}(r, \mathbf{p}, \mathbf{x})$, apart from the elements mentioned in Assumption 5 and in Axiom (A.4.2), can be proved.

THEOREM 1 *If Assumptions 4 and 5 hold and $g \leq r$, then Assumption 6 is sufficient for the existence of a long-period position corresponding to rate of profit r and demand function $g\mathbf{x}^T\mathbf{A}\mathbf{p} + \mathbf{c}(r, \mathbf{p}, \mathbf{x})$. The operated processes and the price vector $\mathbf{p}$ in the long-period position are independent of function $\mathbf{c}(r, \mathbf{p}, \mathbf{x})$.*

Proof Axiom (A.4.2) implies that

$$\mathbf{c}(r, \mathbf{p}, \mathbf{x}) = \begin{bmatrix} \mathbf{c}_s(r, \mathbf{p}, \mathbf{x}) \\ \mathbf{0} \end{bmatrix}$$

where sub-vector $\mathbf{c}_s(r, \mathbf{p}, \mathbf{x})$ has size s and is a function of r, $\mathbf{p}$, $\mathbf{x}$, which is continuous and homogeneous of degree 1 in $\mathbf{x}$ (because of Axioms (A.5.1) and (A.5.2)), non-negative for non-negative values of r, $\mathbf{p}$, $\mathbf{x}$ (because of Axiom (A.5.3)), and such that

$$\mathbf{c}_s^T(r, \mathbf{p}, \mathbf{x})\mathbf{p}_s = \mathbf{x}^T\mathbf{l} + (r-g)\mathbf{x}^T\mathbf{A}\mathbf{p} \tag{8.13}$$

where $\mathbf{p}_s$ is the vector consisting of the first s elements of vector $\mathbf{p}$ (because of Axiom (A.5.4)). Then, because of Lemma 2 it is enough to prove that there is a semipositive vector $\mathbf{v}$ such that

$$\mathbf{v}^T[\mathbf{D}_0 - (1+g)\mathbf{C}_0] = \mathbf{c}_s^T(r, \mathbf{y}^*, \mathbf{Q}^T\mathbf{v}),$$

that is that function

$$\mathbf{v}(\mathbf{u})^T := \mathbf{c}_s^T(r, \mathbf{y}^*, \mathbf{Q}^T\mathbf{u})[\mathbf{D}_0 - (1+g)\mathbf{C}_0]^{-1}$$

has a fixed point. In order to prove this, let us consider the set

$$\mathrm{S} = \{\mathbf{u} \in \mathrm{R}^s | \mathbf{u} \geqq 0, \mathbf{u}^T[\mathbf{D}_0 - (1+g)\mathbf{C}_0]\mathbf{y}_s^* = 1\}.$$

It is immediately checked that if $\mathbf{u}$ is in S, then $\mathbf{v}(\mathbf{u})$ is also in S, and since it is continuous, it has in S a fixed point. In fact, if $\mathbf{u}$ is in S, then $\mathbf{v}(\mathbf{u}) \geqq \mathbf{0}$ since inequality (12) and Axiom (A.5.3) hold and

$$\begin{aligned}\mathbf{v}(\mathbf{u})^T[\mathbf{D}_0 - (1+g)\mathbf{C}_0]\mathbf{y}_s^* &= \mathbf{c}_s^{\mathrm{T}}(r, \mathbf{y}^*, \mathbf{Q}^T\mathbf{u})\mathbf{y}_s^* = \mathbf{u}^T\mathbf{Q}\mathbf{l} + (r-g)\mathbf{u}^T\mathbf{Q}\mathbf{A}\mathbf{y}^* \\ &= \mathbf{u}^T[\mathbf{D}_0 - (1+g)\mathbf{C}_0]\mathbf{y}_s^* = 1\end{aligned}$$

since equations (8.13) (second equality) and (8.9) and (8.10) (third equality) hold. Q.E.D.

This shows that given the rate of profits and the uniform rate of growth, we can determine which technique will be chosen by cost-minimising producers and the system of relative prices associated with that technique even in the presence of sophisticated (and realistic) assumptions about fixed capital. In the conditions specified, consumption patterns have no impact on the processes operated and prices.

8.7 CONCLUSION

The paper has two parts. In the first part (Sections 8.2–8.5), we provide a short summary account of the developments in the theory of value and distribution that took place since the beginnings of systematic economic analysis at the time of the classical economists. We distinguish between a 'classical' approach to the theory of value and distribution and a 'neoclassical' one. The emphasis is on the distinguishing features of the two approaches. It is argued that the classical authors from Smith to Ricardo developed what is known as the method of 'long-period positions' of the economic system, focusing attention on the persistent and systematically operating factors shaping the economy at any given moment of time and over time. This method was adopted by basically all neoclassical authors up until the late 1920s and has made a reappearance in more recent times especially with the so-called 'new' growth theories. It had been abandoned essentially because of internal problems of the theory originating from its concept of a 'quantity of capital' as a magnitude that could be given independently of relative prices and income distribution. In terms of the methods of intertemporal equilibrium and temporary equilibrium, championed by Friedrich August von Hayek, Erik Lindahl and John Richard Hicks, it was sought to overcome the capital theoretic difficulties and yet preserve the demand and supply approach to the problem of income distribution. It is argued that these alternatives are beset by a number of disquieting features.

The second part (Section 8.6) is designed to show the power of the traditional long-period method in terms of two examples. The first example concerns a multi-sector variant of the 'linear' or 'AK growth model'; it is shown that its steady-state properties can be analysed in relatively simple terms, whereas in an intertemporal setting some very bold assumptions are required to do so. The second case refers to the case of fixed capital within a classical framework of the analysis, in which several fixed capital items are jointly utilised.

We hope to have made clear: (1) that the long-period method is an extremely powerful tool of analysis, if handled correctly; and (2) that a correct long-period analysis cannot take the endowment of 'capital' as given. However, our tribute to long-period analysis of 'classical' derivation must not be mistaken to imply an opposition on our part to the development of a proper dynamical analysis. We are rather convinced that a correct long-period analysis provides the best ground for starting to elaborate a dynamical analysis. As Edwin Burmeister recently stressed: 'Very little is known about the properties of such more realistic economies . . . , and even the little that is known usually is only about special and quite unrealistic cases (such as the one-good case). Almost nothing is known about the dynamic behaviour of the more complex models' (Burmeister, 1996: 1346) which can be studied within a long-period classical framework.

NOTES

1 As we shall see, the theories that filtered into the profession in the final third of last century constituted no revolution at all because all main building blocks of these theories had already been forged in the five decades between Ricardo's death (1823) and the publication of the works of Jevons (1871), Menger (1871) and Walras (1874).
2 For a recent critical assessment of Sraffa's contributions to economics, see Kurz (2000) and Cozzi and Marchionatti (2000).
3 For a summary statement of the different versions of the theory and the debates around them, see Kurz (1987), Garegnani (1990a) and Kurz and Salvadori (1995, Chapter 14).
4 Towards the end of his paper Lindahl (Ibid.: 348) wrote that his investigation has been brought to a point 'at which a further approximation to reality is associated with ... considerable difficulties' which he, at the time, felt unable to tackle in a theoretically satisfactory way. These difficulties derived largely from the need to accomodate imperfect foresight and uncertainty in the model. His disenchantment with the achievements of his 1929 paper were also the main impetus for him to write the 1939 paper on 'The Dynamic Approach to Economic Theory' (Lindahl, 1939, Part I).
5 Repeatedly, Lindahl expressed his uneasiness with the assumption of perfect foresight; see Lindahl (Ibid.: 285 and 339–340).
6 See also the following statement by Koopmans: 'The irrelevance-in-principle of the concept of interest to the problem of efficient allocations over time is clearly implied, if not explicitly stated, in Lindahl's penetrating exposition of capital theory' (Koopmans, 1957: 114).
7 It is interesting to note the parallel between the view expressed by Lindahl and that of Duménil and Lévy (1985) in their response to Hahn (1982).
8 It is interesting to notice that Hicks later in his career became increasingly sceptical as to the usefulness of the temporary equilibrium method; see, for example, Hicks (1965: 66). On Hicks's recantation of the temporary equilibrium method, see Petri (1991).
9 It was only recently that his contribution to this field was given proper credit (cf. Milgate, 1979; Huth, 1989).
10 Fabio Petri remarked to us that the given number of firms in a General Equilibrium analysis starts with Hicks *Value and Capital*, where the temporary equilibrium framework made it quite reasonable. But in an intertemporal model there is no reason why the entry of firms in periods subsequent to the first one could not be accomodated, since to form a firm takes time.
11 In the final chapter of his book, Debreu tried to cope also with the problem of uncertainty by generalising the notion of commodity still further: a contract for the transfer of a commodity now includes in addition the specification of 'an event on the occurrence of which the transfer is conditional'. Debreu added: 'This new definition of a commodity allows one to obtain a theory of uncertainty free from any probability concept and formally identical with the theory of certainty developed in the preceding chapters' (Ibid.: 98). The theory makes use of Arrow's concept of 'choices of Nature'. In what follows we shall set aside this aspect.
12 The study of intertemporal models with an infinite time horizon was begun by Bewley (1972).
13 In a framework treating time as a discrete variable, it would be simple to assume that all capital is consumed in a finite period of time. In a continuous time framework, in order to allow for such a situation we would need to introduce an infinite number of commodities for each type of capital good, each of this infinite number of commodities representing the capital good at the appropriate (continuous) vintage. With continuous time, then, the idea that a capital good depreciates in the sense that a part of it evaporates is not only the simplest one available to capture the idea of capital, but also the only one which, as far as we know, avoids the need to have recourse to an infinite number of capital goods.
14 Freni *et al.* (2001) have investigated more deeply a variant of this model in order to analyse the dynamics outside the steady-state solution. They found also that if the second of the inequalities (8.2) is not satisfied, some other long-period position can be found with a negative growth rate: the only process which is operated in the interest of

consumption is process 1 and the growth rate is equal to or lower than $-\delta$. We refrain here from showing how these steady-state solutions are found using the long-period method.

15 If $i = \pi - \delta$, then from equation (8.5) we would obtain

$$te^{it}\mathbf{e}_j^T\mathbf{p} = 0,$$

which can hold for each t only if $\mathbf{e}_j^T\mathbf{p} = 0$. And since this equation should hold for each j, $\mathbf{p}$ could not be semipositive.

16 A case in which machines are transferable among sectors, but are not used jointly, has been recently analysed by Salvadori (1999).

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9 Endogenous uncertainty and rational belief equilibrium

A unified theory of market volatility

*Mordecai Kurz**

The theory of rational belief equilibria (RBE) offers a unified paradigm for explaining market volatility by the effect of "Endogenous Uncertainty" on financial markets. This uncertainty is propagated within the economy (hence "endogenous") by the beliefs of asset traders. The theory of RBE was developed in a sequence of papers assembled in a recently published book (Kurz, 1997) and the present paper provides an exposition of both the main ideas of the theory of RBE as well as a summary of the main results of the book regarding market volatility.
Section 9.1 starts by reviewing the standard assumptions underlying models of (rational expectations equilibria) and their implications to market volatility. The paper then reviews four basic problems which have constituted puzzles or anomalies in relation to the assumptions of REE: (1) Why are asset prices much more volatile than their underlying fundamentals? (2) The equity premium puzzle: why under REE is the predicted riskless rate so high and the equity risk premium so low? (3) Why do asset prices and returns exhibit the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) behavior without exogenous fundamental variables to explain it? (4) The "Forward Discount Bias" in foreign exchange: why are interest rate differentials poor predictors of future changes in the exchange rates? Section 9.2 outlines the basic assumptions of the theory of RBE and the main propositions which it implies for market volatility. Section 9.3 develops the simulation models which are used to study the four problems above and explains that the domestic economy is calibrated, as in Mehra and Prescott (1985), to the US economy. Then for each of the four problems the relevant simulation results of the RBE are presented and compared to the results predicted by a corresponding REE and to the actual empirical observations in the United States.
The paper concludes that the main cause of market volatility is the dynamics of beliefs of agents. The theory of RBE shows that if agents disagree then the state of belief of each agent, represented by his conditional probability, *must fluctuate over time*. Hence the distribution of the individual states of belief in the market is the root cause of all phenomena of market volatility. The GARCH phenomenon of time varying variance of asset prices is explained in the simulation model by the presence of both *persistence* in the states of beliefs of agents as well as *correlation* among these states. Correlation makes beliefs either narrowly distributed (i.e. "consensus") or widely distributed (i.e. "non-consensus"). In a belief regime of consensus (due to persistence it remains in place for a while) agents seek to buy or sell the same portfolio leading to high volatility. In a belief regime of non-consensus there is a widespread disagreement which causes a balance between sellers and buyers leading to low market volatility. In short, the GARCH phenomenon is the result of shifts

* This work has been supported by Fondazione ENI Enrico Mattei, Milano, Italy. I thank Horace W. Brock, William A. Brock, Michael Magill and Martine Quinzii for extremely valuable comments on an earlier draft of this chapter. I thank Stanley Black and Maurizio Motolese for their dedicated assistance.

in the distribution of beliefs in the market induced by the dynamics of the individual states of belief.

Turning to the equity risk premium, the key question is what are the distributions of beliefs which ensure that the average riskless rate is low and the average equity risk premium is high. It turns out that *the only circumstances* when the mean riskless rate falls to around 1 per cent and the mean equity premium rises to around 7 per cent arise when, on the average, the majority of agents are relatively optimistic about the prospects of capital gains in the subsequent period. In such a circumstance the rationality of belief conditions imply that the pessimists (who are in the minority) must have a higher intensity of pessimism than the intensity of the optimists. In a large economy with this property the state of belief of any one agent may fluctuate but on the average there will be a minority of intensely pessimistic agents. This asymmetry between optimists and pessimists flows directly from the rationality conditions of beliefs and implies that at most dates the pessimists have a stronger impact on the bill market. At those dates the pessimists protect their wealth by increasing their purchases of the riskless bill. This bids up the price of the bill, lowers the riskless rate and results in a higher equity risk premium. In sum, the theory of RBE offers a very simple explanation to the observed riskless rate and equity premium. It says that the riskless rate is, on average, low and the premium high because at most dates there is a minority of pessimists who, by the rationality of belief conditions, have the higher intensity level of belief about high stock prices in the future. These agents drive the riskless rate lower and the equity premium higher.

The "Forward Discount Bias" in foreign exchange markets is the result of the fact that in an RBE, agents often make the wrong forecasts although they are right on the average. Hence, in an RBE the exchange rate fluctuates excessively due to the errors of the agents and hence at almost no date is the interest differential between two countries an unbiased estimate of the rate of depreciation of the exchange rate one period later. The bias is positive since agents who invest in foreign currency demand a risk premium on endogenous uncertainty which is above and beyond the risk which exists in an REE. The size of the bias is equal to the added risk premium due to endogenous uncertainty.

9.1 THE BASIC ISSUE

This chapter surveys the unified view of market volatility that flows from the insight that volatility has two components. One is generated by "fundamental" forces which are outside the economy and I refer to them as *exogenous*. The second is propagated within the economic system and I refer to it as the *endogenous* component. Since the nature of exogenous shocks is well known, explaining the endogenous component is the main task of this chapter. My exposition style in this survey is mostly non-technical but several sections are mathematically formal in order to ensure that a precise statement of the main concepts is provided.

Before explaining my theory, I briefly outline the perspective of the Market Efficiency Theory, or rational expectations equilibrium (REE) on market volatility. My aim is to use REE as a reference point for the evaluation of the problems which market volatility generates. My account is brief since the theory of REE is well known.

The standard formulation of an equilibrium of an economy starts with the dynamic portfolio and consumption choices of households and the production, investment and dividend decisions of firms. The theory is closed with market-clearing conditions but given the random nature of the underlying economy it follows that equilibrium quantities are all stochastic processes with an underlying probability law. I call this probability the "true" probability law. Most of what is done in modern research depends upon the utilization of this probability for computing objects like forecasts, theoretical covariances or security prices. Thus, the idea that equilibrium is represented by a *true* stochastic process is fundamental to modern thinking in economics.

The REE theory is based on several assumptions, but three of them are fundamental to my discussion here. These are:

A.1 The true probability law of the economy is stationary. In a *stationary* economy the joint probabilities of economic variables remain the same over time.

A.2 Economic agents know the true probability law underlying the equilibrium variables of the economy. This is the first component of "structural knowledge" which the agents are *assumed* to possess.

A.3 Agents know the demand and supply functions of all other agents. They can compute equilibrium prices of commodities and assets in the present and in the future given any possible exogenous fundamental information in the future. This is the second component of structural knowledge which they possess.

When formulating uncertainty, the standard theory specifies an *exogenous* "state space" which describes all that the agents are uncertain about with respect to the external environment. Examples of exogenous variables include: weather conditions, earthquakes, technological changes, fire destruction etc. All equilibrium magnitudes depend upon the realization of the exogenous state but according to (A.3) all agents know the map between equilibrium magnitudes (e.g. production decisions of firms, prices, etc.) and the state. Consequently, all economic magnitudes vary only with the variability of the exogenous state over time. Moreover, it is then an assumption that given any observed information, all agents agree on the *interpretation* of such information.

The implication of these assumptions is that *all* financial risks and observed volatility arise from causes which are external to the economy. I call such uncertainty "Exogenous Uncertainty". Under the above theory, no risk can be propagated from within the economic system via human beliefs or actions and the volatility of equilibrium variables is exactly equal to the level justified by the variability of exogenous conditions.

The above discussion enables me to offer a simple summary of the conclusions of the theory of REE with respect to the nature of market volatility:

1 For each state of the exogenous fundamentals there is a *correct* equilibrium price of all securities in the market.

2 If you possess all exogenous fundamental information you are able to compute the correct prices of securities and hence all uncertainty about prices is resolved. By implication, hedging against the risks of all exogenous fundamentals is possible, in principle, and can control all risk associated with market volatility.

3 All market volatility is caused by exogenous forces.

These conclusions of REE have been at the foundation of contemporary research into the structure of market volatility. Unfortunately, they are in conflict with many empirical observations and with common experience of market participants. Indeed, the implications of this theory have been rejected in broad areas of economics. In order to discuss specific issues I note that there are several outstanding problems or paradoxes (sometimes called "anomalies") related to the functioning of financial markets which the REE theory failed to resolve, and current academic research has aimed to develop special theories to explain each one of these paradoxes. Since I will offer a *unified* view of market volatility, such a *single* theory would be more convincing if it could solve simultaneously many of these problems. Here I focus on four such central problems:

1. *Problem A*: Why are asset prices and foreign exchange rates much more volatile than their underlying fundamentals?
2. *Problem B*: Why do models based on REE predict an equity risk premium over the riskless rate of around 0.5 per cent and a rate of return on riskless short term debt of around 5.5 per cent while over the last hundred years the average equity risk premium in the United States has been around 7 per cent and the riskless rate has been around 1 per cent?
3. *Problem C*: Why do asset prices and returns exhibit the "GARCH" behavior of time varying variances when there are no fundamental factors to explain it?
4. *Problem D*: Why have interest rate differentials (between two countries) been such poor predictors of future changes in foreign exchange differentials in contrast with rational expectations, giving rise to the "Forward Discount Bias"?

Those who rejected the theory of rational expectations have tended to drift in diverse directions. Some have concluded that financial markets are dominated by investors who perceive probabilities incorrectly or are vulnerable to the impact of fads and mass psychology. Others have concluded that for some unexplained reason the market can be irrational *sometimes* and each failed prediction of the theory has been ascribed to a corresponding incident of such irrationality. As a result, it is common to find in the investment community the argument that each instant of such *presumed* irrationality offers an opportunity for excess returns (i.e. when an investment opportunity is viewed as "excellent" and inexpensive). These perspectives are in conflict both with principles of rationality as well as with the hope of finding *one* explanation for all these phenomena.[1] This is my motivation for seeking a *unified* theory for market volatility.

I proceed by reviewing in Section 9.2 the basic premises of my theory of Rational Beliefs and the allied concept of "Endogenous Uncertainty" which are the cornerstones of my approach. Section 9.3, which is the main section of this chapter, is devoted to showing via simulation results how the theory which I propose resolves the four Problems outlined. Most of the material presented here is based on papers published in a volume by Kurz (1997)[2] and on Kurz and Motolese (2001).

9.2 ENDOGENOUS UNCERTAINTY AND RATIONAL BELIEFS

9.2.1 Rational beliefs

My theory of RBE developed in Kurz (1994a,b) is based on the following alternative assumptions:

AA.1 Despite the fact that the economy may undergo structural changes yielding non-stationarity, the economic universe is *stable* in the sense that statistical and quantitative analysis can be successfully carried out in it. In such a system the concept of "normal" patterns makes empirical sense and provides useful knowledge. It is represented by the long-term averages of economic variables. Thus, although our economy experiences technological and economic changes, the price/earning ratios of major indices have well known "normal" ranges and long-term (i.e. asymptotic) means, variances and covariances. Interest rates, growth rates, capital/output ratios etc. all have well known long-run average behavior which reveals some important dimensions of the true structure of the economy.

AA.2 Economic agents do not know the true probability law underlying equilibrium magnitudes. This is the first component of structural knowledge which agents are assumed to lack.

AA.3 Agents do not know the map from exogenous variables to equilibrium quantities in general and prices in particular. They have, however, access to the very large volume of all *past* data on the performance of the economy. This data they can use to statistically test any theory which they may develop about the functioning of the economy and of the financial markets. In this sense agents may learn something about structural relationships in the economy.

In formal terms, let $x_t \in X \subseteq \mathbb{R}^N$ be a vector of all observables at date t and assume it to be N, finite. The sequence $\{x_t, t = 0, 1, \ldots\}$ is a stochastic process with true probability Π. I use the notation $x = (x_0, x_1, \ldots)$ for members of $(X)^\infty$ and denote by $\mathscr{B}((X)^\infty)$ the Borel σ-field of $(X)^\infty$. The space $((X)^\infty, \mathscr{B}((X)^\infty), \Pi$ is *the true* probability space and the dynamical system $((X)^\infty, \mathscr{B}((X)^\infty), \Pi, T)$ represents the true economy. T is the shift transformation defined as follows: let $x^t = (x_t, x_{t+1}, x_{t+2}, \ldots)$ then $x^{t+1} = Tx^t$, $t = 0, 1, 2, 3, \ldots$ A belief of an agent is a probability Q; the agent is adopting the theory that the probability space is $((X)^\infty, \mathscr{B}((X)^\infty), Q)$.

An agent who observes the data does not know Π and using past data he tries to learn this probability. I assume that date 0 has occurred "a long time ago" and at date t, when agents form their beliefs about the future beyond t, they have an *ample* supply of past data. Denote by $x = (x_0, x_1, x_2, x_3, \ldots)$ the vector of observations generated by the economy. In studying joint distributions among observables, one considers *blocks* rather than individual observations. For example, to study the distribution of $(x_{\text{today}}, x_{\text{today}+1})$ one uses the blocks $(x_0, x_1), (x_1, x_2), (x_2, x_3), \ldots$ Hence, for any $B \in \mathscr{B}((X)^\infty)$ let the set $T^{-n}B$, which is the pre-image of B under T^n, be defined by

$$T^{-k}B = \{x \in X^\infty : T^k x \in B\}.$$

$T^{-k}B$ is the event B occurring k dates later. A dynamical system $((X)^\infty), \mathscr{B}((X)^\infty), \Pi, \mathrm{T})$ is *stationary* if $\Pi(B) = \Pi(T^{-1}B)$ for all $B \in \mathscr{B}((X)^\infty)$. A set $S \in \mathscr{B}((X)^\infty)$ is *invariant* if $S = T^{-1}S$. A dynamical system is *ergodic* if $\Pi(S) = 1$ or $\Pi(S) = 0$ for any invariant set S. For simplicity I assume that $((X)^\infty, \mathscr{B}((X)^\infty), \Pi, T)$ is ergodic although this is not needed (see Kurz (1994a) where this assumption is not made). In order to learn probabilities agents study the *frequencies* of all economic events. For example, consider the event B

$$B = \left\{ \begin{array}{l} \text{price of commodity 1 today} \leq \$1, \text{price of commodity 6 tomorrow} \geq \$3, \\ 2 \leq \text{quantity of commodity 14 consumed two months later} \leq 5 \end{array} \right\}.$$

Now using past data agents can compute for any finite dimensional set B the expression

$$m_n(B)(x) = \frac{1}{n}\sum_{k=0}^{n-1} 1_B(T^k x) = \left\{ \begin{array}{l} \text{The relative frequency at which } B \text{ occurred} \\ \textit{among } n \text{ observations since date } 0 \end{array} \right\},$$

where

$$1_B(y) = \begin{cases} 1 & \textit{if} \quad y \in B \\ 0 & \textit{if} \quad y \notin B. \end{cases}$$

This leads to a definition of a basic property which $((X)^\infty, \mathscr{B}((X)^\infty), \Pi, \mathrm{T})$ has:

Definition 1 A dynamical system is called *stable* if for any finite dimensional set (i.e. cylinder) B

$$\lim_{n\to\infty} m_n(B)(x) = \hat{m} \text{ exists} \quad \Pi \text{ a.e.}$$

The assumption of ergodicity ensures that the limit in Definition 1 is independent of x. In Kurz (1994a) it is shown that the set function $\hat{m}$ can be uniquely extended to a probability m on $((X)^\infty, \mathscr{B}((X)^\infty))$. Moreover, relative to m the dynamical system $((X)^\infty, \mathscr{B}((X)^\infty), m, T)$ is stationary. There are two observations to be made.

(a) Given the property of stability, in trying to learn Π all agents end up learning m which is a *stationary* probability. In general $m \neq \Pi$: the true dynamical system $((X)^\infty, \mathscr{B}((X)^\infty)), \Pi, T)$ may not be stationary. Π *cannot be learned.*

(b) Agents know that m may not be Π but with the data at hand m is the only thing that they can learn and agree upon.

These conclusions mean that although agents have no structural knowledge they do have a common empirical knowledge. I have noted that a *stationary* economy is one in which all the joint probabilities of economic variables remain the same over time. Stationary systems are stable but stable systems are not necessarily stationary. A system which experiences new technologies and new social organizations is not likely to be stationary but may be stable. This distinction is the central motive for the above assumptions and for this reason requires some explanation.

Our economy is driven by a process of technological and organizational change which dominates every aspect of life in human history. This process is very complex but has a distinct character: once a new technology or organizational structure is established, it remains in place for some time until a new one is developed to replace it. While a technology or social organization is in place, the economy appears to have a fixed structure (i.e. it is stationary) until the next change. For simplicity I use the term "regime" to refer to such episodes in which the structure of the economy and the market are relatively fixed. Note that a regime in which steam ships dominate the technological frontier is very long and will have within it many, much shorter, sub-regimes. Moreover, the term may be used for the description of short periods in which a market may be dominated by a fixed configuration of factors, some fundamental and others involving the beliefs and perceptions of investors. In Figure 9.1(a) I give an example of such a sequence of regimes and the data which they generate. The horizontal bars represent the mean value functions which are constructed as constant within each regime. Figure 9.1(b) shows how we see the data without the knowledge of either the start and end dates of each regime or the mean value function prevailing within it.

The important feature of a market characterized as a sequence of regimes is that in real time no one knows exactly the parameters of the prevailing regime or its starting and ending dates. Assumptions AA.1–AA.3 aim to capture this reality. They do not deny the fact that if a regime lasts long enough investors will figure out the character of the regime. Unfortunately, the fact that we can find out *in retrospect* the nature of the last regime does not mean that we learn the probability law of future observations or that we can correctly predict the next regime. This explains conclusion (a) above:

> R(1) The true probability underlying the system cannot be learned and even if an agent discovers it, he cannot be sure that it is the true probability. Equally so, economic agents cannot learn the equilibrium map between market prices and those variables which determine prices. Such a map may change over time.

Assumptions AA.1–AA.3 also specify what the agents do know and this fact is the basis for the next development. Specifically, assumption AA.3 means that all agents know the empirical distribution of past data from which they all deduce the same stationary probability m specified in conclusion (b). Observe that m summarizes the entire

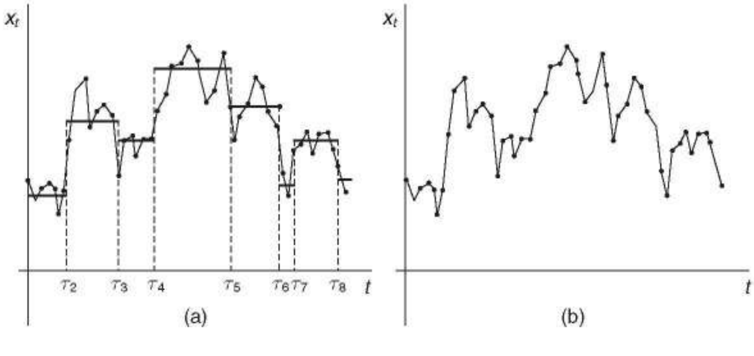

Figure 9.1 (a) (i) τ_j are dates of regime change; (ii) horizonal bars are mean value functions; (iii) data seen with parameters of structural change.
(b) data seen without any information about structural change.

collection of *asymptotic restrictions* imposed by $((X)^\infty, \mathscr{B}((X)^\infty)), \Pi, T)$ on the empirical distribution of all variables. This common empirical knowledge provides the basis for a new definition of the *rationality* of beliefs. I now proceed formally.

It is shown in Kurz (1994a) that for each stable dynamical system with probability Π there is a set $B(\Pi)$ of stable probabilities Q with dynamical systems which generate the same stationary probability m and hence impose the same asymptotic restrictions on the data as the true system with Π. The question is how to determine *analytically* if any proposed belief $((X)^\infty, \mathscr{B}((X)^\infty)), Q, T)$ generates m as a stationary measure. To examine this question consider, for any cylinder B the set function

$$m_n^\Pi(B) = \frac{1}{n}\sum_{k=0}^{n-1}\Pi(T^{-k}B).$$

I note that $m_n^\Pi(B)$ has nothing to do with data: it is an analytical expression derived from $((X)^\infty, \mathscr{B}((X)^\infty)), \Pi, T)$.

Definition 2 A dynamical system $((X)^\infty, \mathscr{B}((X)^\infty)), \Pi, T)$ is said to be weak asymptotically mean stationary (WAMS) if for all cylinders $S \in \mathscr{B}((X)^\infty))$ the limit

$$\mathring{m}^\Pi(S) = \lim_{n\to\infty}\frac{1}{n}\sum_{k=0}^{n-1}\Pi(T^{-k}S) \text{ exists.}$$

It is shown in Kurz (1994a) that $\mathring{m}^\Pi$ can be uniquely extended to a probability measure m^Π on $((X)^\infty, \mathscr{B}((X)^\infty))$ and $((X)^\infty, \mathscr{B}((X)^\infty)), m^\Pi, T)$ is stationary. I then have a theorem which is the main tool in Kurz (1994a):

Theorem 1 *$((X)^\infty, \mathscr{B}((X)^\infty)), \Pi, T)$ is stable if and only if it is WAMS. If m is the stationary measure calculated from the data, then $m(S) = m^\Pi(S)$ for all $S \in \mathscr{B}((X)^\infty))$.*

The implication of Theorem 1 is that every stable system $((X)^\infty, \mathscr{B}((X)^\infty)), Q, T)$ generates a *unique* stationary probability m^Q which is *calculated analytically* from Q. This last fact is the foundation of the following:

Definition 3 A selection of belief Q cannot be contradicted by the data m if

1 the system $((X)^\infty, \mathscr{B}((X)^\infty)), Q, T)$ is stable;
2 the system $((X)^\infty, \mathscr{B}((X)^\infty)), Q, T)$ generates m and hence $m^Q = m$.

Rationality axiom A belief Q by an agent is a *Rational Belief* if it satisfies

1 *Compatibility with the Data*: Q cannot be contradicted by the data.
2 *Non-Degeneracy*: if $m(S) > 0$, then $Q(S) > 0$.

To express a belief in the non-stationarity of the process, an agent selects a probability $Q^\perp$. This probability is said to be *orthogonal with m* if there are events S and S^c such that

1 $S \cup S^c = (X)^\infty, \quad S \cap S^c = \emptyset$;
2 $m(S) = 1, \quad m(S^c) = 0$;
3 $Q^\perp(S) = 0, \quad Q^\perp(S^c) = 1$.

I want to characterize the set $B(\Pi)$ of Rational Beliefs when the empirical distribution implies a stationary measure m induced by $((X)^\infty, \mathscr{B}((X)^\infty)), \Pi, T)$

THEOREM 2 *(Kurz, 1994a) Every Rational Belief must satisfy* $Q = \lambda Q_a + (1-\lambda)Q^\perp$ *where* $0 < \lambda \leq 1$, Q_a *and m are probabilities which are mutually absolutely continuous (i.e. they are equivalent) and* $Q^\perp$ *is orthogonal with m such that*

1 $((X)^\infty, \mathscr{B}((X)^\infty)), Q_a, T)$ and $((X)^\infty, \mathscr{B}((X)^\infty)), Q^\perp, T)$ *are both stable;*
2 $m^{Q_a} = m^{Q^\perp} = m$.

Moreover, any Q such that λ, Q_a *and* $Q^\perp$ *satisfy the above is a Rational Belief.*

A rational belief must then have the property that if one simulates the model, over time it will generate the same empirical distribution as the one that was generated by the historical record of the market. Thus the concept of a rational belief isolates that subset of all possible theories or models that cannot be contradicted by the available data.

In my approach, the rationality of beliefs rests on the premise that the economic universe (or some transformation of it, in case of a growing economy) is stable so that two rational agents holding two different theories cannot disagree about the long-run statistics which both of their individual theories are required to "reproduce". If any model generates long term statistics which differ from the empirical evidence, it is judged wrong and the underlying belief judged irrational. I will now explain other important interpretations and implications of Theorem 2.

9.2.2 Diversity of beliefs and mistakes in a rational belief equilibrium

A dynamically changing but stable economy is one in which economic variables may be transformed (e.g. into logs or into growth rates rather than absolute values if needed) so that although structural changes take place, all long term frequencies and averages converge. These frequencies and averages are learned by all agents and represent the "normal" probabilities of events. Investors often consult such information when they describe how frequently a certain pattern of events happened over the last two hundred years! An agent who believes that the world is stationary would adopt these normal frequencies as his belief. This result can be summed up by:

> R(2) The theory holds that an agent who adopts the normal frequencies as his belief is rational since his belief is compatible with the empirical distributions.

Note, however, that such a person must also believe that the joint probability distributions of economic and financial variables in the 1990s are the same as the joint distributions in the 1890s and both are equal, according to him, to the joint distributions computed as averages over many past years. That is, he believes that no structural changes ever take place or that technological or structural changes in the real economy have a *neutral* effect on financial markets and thus have no effect on the structure of market performance.

If the economic system is stationary and if all the agents *knew for sure* that it is stationary, then they will all learn the true probability law of motion and will know that this true law of motion is the one calculated from the empirical distributions of past events. Under such circumstances there will be no disagreement in that economy.

In contrast, I have already expressed my view that the process of structural change (i.e. non-stationarity) in our society is *the* central building block of its complexity and the root cause of the diversity of beliefs about it. In such a system the past is not an entirely satisfactory basis for assessment of risks in the future and at every date many agents hold the view that the market may be *similar* to the past but yet very different. Hence, an agent who forms a forecast which is different from the historical statistical average is adopting a sharper view of the future than can be deduced from the statistics of the past. Such a theory may not be contradicted by past data but past data is not required to support it either. That is, an agent who holds a theory of the market which insists that the situation today is *different* from the past does not support his theory by the statistics of the past. He may offer some statistical evidence of *recent* developments to bolster his model but such evidence would lack high statistical reliability and thus may not be acceptable to other agents. His theory may sometimes be right and sometimes wrong.

What are the patterns of disagreement among these rational agents? Motivated by the observations above, Theorem 2 shows that:

> R(3) The main source of disagreement among agents derives from the fact that they can hold different theories both about the *nature* and *intensity* of changes in the economy as well as their *timing*. As a result, given commonly observed news at any date, agents can have very different opinions regarding the significance of the news to future market performance. For example, some may be optimistic while others are pessimistic about it.

The mere fact that agents disagree has an immediate and very important implication.

> R(4) A group of economic agents who hold rational beliefs and disagree *pairwise* forever (at all times and in the limit rather than have a one-time disagreement) must also experience variations in the probabilities with which they forecast future economic events at different dates. *This means that in a world of disagreement the states of belief of these agents must fluctuate over time.*

I stress that conclusion R(4) is a *consequence* of the theory of rational beliefs together with the observations that agents disagree. To understand why this conclusion holds note that if a group of agents disagree pairwise forever then at least all but one of them must not believe that the economy is stationary and hence they do not permanently adopt the normal frequencies as their beliefs. However, their beliefs must be compatible with the normal frequencies in the exact sense that deviations of their one period probability beliefs from the normal frequencies must average to zero. That is, if you are optimistic relative to the normal frequencies in some dates you must be pessimistic relative to those frequencies in other dates so that on average you expect your deviations from the normal frequencies to average to zero. But then it follows that all permanent disagreements imply variability in probability beliefs around the normal frequencies.

Let me examine the implication of R(4). It says that if we observe a market in which there is always some disagreement among agents who hold rational beliefs, then their disagreements are not fixed. If we study those disagreements we shall find that they are the result of on-going reassessment and the states of beliefs of the disagreeing individuals are changing over time. Note that this does not mean that the distribution of beliefs in the market as a whole will be changing over time as well. I return to this important subject when I discuss in section 9.4(iv) the results regarding the equity risk premium.

The dual requirement of stability and of compatibility with the empirical distributions impose restrictions on the models of the economy which a rational agent can adopt as his belief. Nevertheless, the theory allows sufficient heterogeneity of beliefs to persist over time so that the subjective models used by the agents may imply forecast functions which can be different for different agents *at all dates*. In short, my theory permits two intelligent investors who observe the same vast information about the past to have different opinions and hence to make different forecasts of the future.

If there is a *true and unknown* equilibrium probabilistic law of motion underlying the dynamics of the market, and if there are substantial differences in probability beliefs among the agents about the future, then, although all the agents are rational, most may be holding wrong beliefs. This leads them to make *forecasting mistakes*. To clarify this point recall Figures 9.1(a), (b) which reveal the problem of an agent who forms a belief about the market. Suppose that the price/earnings ratio of an index of his interest is the highest in 40 years. If he follows the statistics of the long past he will compute the fact that, say, only in 7.8 per cent of past cases the price/earnings ratio went higher than the observed level and hence the probability of capital gains is 7.8 per cent. With such probability the investor decides that the index is too high and his portfolio decision is to sell. Another investor, observing the identical information about prices and earnings, formulates a model about the future productivity of the firms in the index on the basis of which he concludes that the statistical record of the past is not entirely applicable. Based on his model, he believes that the probability of higher prices is 60 per cent on the basis of which his portfolio decision is to buy.

I suggest that one or both of the two investors hold wrong beliefs and are thus making a mistake. More formally, the *mistake* of an agent at date t is defined as the difference between the collection of his forecasts at date t conditional upon the information at that date and the forecasts that would be made with the correct model, were it known. Since an agent selects his decisions based on his beliefs, these mistakes in beliefs get translated into mistaken *actions*. In equilibrium, quantities and prices will reflect those mistakes. Thus, the economic variables which we observe at each date contain the mistakes of the agents and this fact will be the foundation of the concept of Endogenous Uncertainty.

I caution against a simplistic interpretation of the term "mistake". In its daily use this term usually refers to acts or thoughts which are wrong but which could have been avoided. Here a "mistake" is a rule by which a rational agent utilizes information efficiently but fails to make the correct forecast. In fact, it is essential that there is no statistical way through which an agent can be assured of avoiding making a "mistake" in my sense. Thus, in the context of this theory *rational agents make mistakes*. The theory does not say that agents who form an opinion which deviates from the statistical norm be "certain" or sure of the truth of their model. What the agents do know is that without committing to an investment program that will take advantage of the changing conditions of the market, they cannot make excess returns.

My approach implies, therefore, that the nature of "risk assessment" by the agents is quite different from the usual analysis of the covariance structure among asset returns. For these agents the market is an arena for the competition among theories that seek to capture future excess returns. In such a market the risky nature of a decision is tied to a commitment to a theory of the market without having *statistically* reliable evidence in support of such a theory. "Assessment" of such risks has something to do with the way we *interpret* existing information rather than with a utilization of past covariances. This is particularly true in an environment of changing regimes where advanced signals about the coming regime may be available, but agents have insufficient evidence to be able to interpret such information with a high level of statistical reliability.

An economic equilibrium in which all agents hold rational beliefs is called an RBE. In such an equilibrium the investment, consumption and portfolio decisions are, in part, determined by the mistakes of the agents and these effects can be substantial. Hence the mistakes of agents have an effect on equilibrium prices and on the real allocations in the economy. Alternatively, in an RBE the beliefs of agents have *real effects* on the performance of the economy; they influence the volatility of economic variables such as output, investment and prices. This leads to the fifth result:

> R(5) If individual agents can make mistakes in the assessment of market values, then the market as a whole can also evaluate assets "incorrectly". This conclusion should be understood in the sense that such pricing can be different from that pricing that would be justified by the true market forecast. Equilibrium market prices may overshoot above "fundamental values" when asset prices rise and may overshoot downward, below "fundamental values" when asset prices decline.

This conclusion shows that an important component of the volatility of economic variables is generated by the mistakes of agents. To see why this could be important, suppose for example that some investors develop a theory according to which a particular imminent development may adversely affect the profits of some firm. The actions of these investors will induce a fall in the price of the shares of the firm with no exogenous event to "justify" it. Moreover, if the theory of these agents is wrong, prices will ultimately return to their original position and the entire move would have been induced only by the forecasting mistakes of the agents. Similar arguments apply to other variables such as an investment by a firm or a purchase of foreign currency by a trader: beliefs have real effects on the fluctuations of economic variables. That component of volatility beyond the level that is justified by the exogenous variables is therefore said to be *internally propagated.* I call this type of uncertainty *Endogenous Uncertainty.*[3]

9.2.3 Components of endogenous uncertainty

Anticipating the developments in Section 9.3, I briefly evaluate the specific factors which contribute to this component of market volatility. Think of a market in which, at any date or over a period, an agent holds a probability belief about future economic events which deviates from the normal pattern. For example, the agent may sometimes be relatively optimistic and sometimes relatively pessimistic about future increases of price/earnings ratios relative to the probability m. It turns out that in order to assess how these levels of *relative* optimism and pessimism contribute to market volatility over time we need to focus on the fluctuations in the distribution of beliefs. For example, compare a distribution in which 5 per cent of the agents are optimistic, 5 per cent are pessimistic and 90 per cent are neutral with a distribution in which 50 per cent are optimists and 50 per cent are pessimists. Although both distributions are "balanced," it is a fact that the latter *contributes to market volatility much more than the former*. It is important to understand the two components of endogenous volatility:

1 *Amplification of exogenous shocks* (Overshooting) In an REE, Markov exogenous shocks, which alter the profit stream of a firm have an effect on price volatility. I define the price fluctuations generated by such exogenous shocks as those fluctuations which are "justified by dividends." The impact of endogenous amplification is simply to increase the effect of these exogenous shocks so that the price fluctuations could be much higher than those justified by the dividends. This is what is commonly known as the "overshooting" phenomenon in stock prices. In the models that will be discussed later in Section 9.3, the degree of overshooting is very large.
2 *Pure endogenous volatility* The second component of endogenous volatility is pure volatility. In models that have a finite number of possible equilibrium prices this component simply generates new price states. That is, there are more possible prices in an RBE than in an REE. Indeed, in the typical model that will be discussed later, there are two exogenous dividend shocks leading to two prices under REE. Under RBE there are eight possible prices generated by the exogenous shocks *and by the states of beliefs*. Hence, the pure effect is represented by the additional prices. In REE with an infinite number of prices, this distinction is more complicated and can be defined by regressing prices on the exogenous shocks: the component of price variability over the REE level which is explained by the exogenous shocks is defined as *amplification* and the component that cannot be explained by exogenous shocks is defined as *pure endogenous volatility*.

9.2.4 Rational belief equilibrium as a general equilibrium concept

In the spirit of this conference, I review my RBE in relation to an Arrow–Debreu equilibrium with contingent claims or to an Arrow (1953) or a Radner (1972, 1979) equilibrium with securities. The distinguishing characteristic of the RBE theory is the role played by the diverse beliefs of the agents and the emergence of Endogenous Uncertainty as the central concept of uncertainty. I thus start with the Arrow–Debreu view of uncertainty.

The full generality of the Arrow–Debreu formulation enables the incorporation of uncertainty merely by a reinterpretation of the symbols employed. In the original Arrow–Debreu (1954) paper terms like "risk" or "uncertainty" are not even mentioned. In his explicit treatment of uncertainty Arrow (1953) defines the exogenous "state space" and explicitly introduces markets for state contingent claims on commodity bundles and the utility of such uncertain commodities. He notes that the treatment of the uncertain case is analogous to the case of certainty except for the enlarged dimension of the commodity space (which equals the number of physical commodities multiplied by the number of exogenous "states"). Apart from the formal interpretation of the concept of a "commodity" the uncertainty interpretation raises only one issue of substance with respect to the assumption of convexity of preferences. Since in the case of uncertainty,[4] convexity implies risk aversion, both the existence of competitive equilibrium in the Arrow–Debreu theory and the optimality theorem in Arrow (1953) are proved under the assumption of universal risk aversion.

The crucial step taken in the Arrow–Debreu formulation of uncertainty within general equilibrium (GE) theory is the introduction of the concept of "the state" into the theory. This concept, however, is the cornerstone of the theory of *individual* decision theory and subjective probability. In Savage's (1954) treatment, the concept of "the state of the world" is nothing more than a formal description of what a decision maker is uncertain about. Savage (1954) defined the "world" to be "the object about which the person is concerned" whereas "a state" (of the world) is defined as "...a description of the world, leaving no relevant aspect undescribed."[5]

Arrow adopted Savage's approach to uncertainty and in early papers he did not even provide a definition of the concept of the "state." He took it to be both known as well as naturally applicable to the economic problem at hand. In later papers (e.g. Arrow (1971)) he provided a precise definition of the "state of the world" as "...a description of the world so complete that, if true and known, the consequences of every action would be known."[6]

In the context of decision theory the "state" is no more than a tool for the formulation of an individual decision problem. As such, it is entirely satisfactory. In fact, it is hard to visualize how to formulate a stochastic dynamic decision problem without a concept like a "state." However, the formulation of any decision problem neither requires the "state" to be observable nor needs its description be communicable to or be understood by other decision makers.

The generality of the decision theoretic framework naturally led Arrow and Debreu to adopt this framework for the formulation of the problem of choice under uncertainty of every economic agent in a competitive economy. The important theoretical step which they took was to endow all agents with the same exogenous state space and to provide them with the market opportunity of trading the uncertainty defined by the "state." Thus, the concept of "the state" became a major tool of general equilibrium analysis. In contrast with the context of the individual decision problem where the "state of the world" is merely an expression of individual uncertainty, in the general equilibrium framework "the state of the world" is a description of commodities, it identifies markets and becomes a basis for specifying contracts and property rights. In such a framework the concept must satisfy the same marketability criteria as "navel oranges available in Palo Alto, California, on 29 November 2000": it must be precisely defined, commonly observable and unequivocally comprehended by all economic agents. These requirements raise some difficult practical problems of description.[7] However, the theoretical structure of the exogenous state space enabled Arrow and Debreu to achieve a complete integration of the theory of value.

Notwithstanding the importance of the integrated vision of the Arrow–Debreu theory, the construct of markets for claims which are contingent on the exogenous states is a cumbersome and unsatisfactory solution to the problem of allocating risk in a market economy. Arrow (1953) himself observed that outside the insurance framework, markets for commodity claims which are contingent upon the exogenous states do not exist. Moreover, even insurance markets do not function as visualized in the theory (see Malinvaud, 1972, 1973). The central observation at hand is even more fundamental: exogenous shocks account for only a small proportion of all observed social risks as reflected in the causes of economic fluctuations. It is evident that the list of *observed* variables which are truly exogenous to the economic universe is very short and the range of their variability and impact are much too small to account for the observed variability of economic variables. Thus, one must conclude that if the exogenous shocks are all that agents can insure against, then large and relevant components of the "state" are not observable and cannot provide a basis for contingent contracts.

The non-existence of markets for contingent claims posed a problem for equilibrium theory. Arrow's (1953) celebrated solution has become the foundation of modern general equilibrium theory of finance. He recognized that without markets for contingent claims one must think of an economy as a sequence of spot markets linked together by a market for securities which enable the reallocation of incomes across the different state–date combinations. In Arrow's (1953) formulation and in the extension by Radner (1972, 1979) an equilibrium consists of a set of market-clearing spot price functions $p_t(s)$ of commodities associated with each of the finite number of the state–date pairs (s, t), and a set of market-clearing prices of securities which pay different dividends in different pairs (s, t). Since the equilibrium is established at date $t = 0$ which we can think of as "the present," such an equilibrium requires the agents to know at $t = 0$ all prices $p_t(s)$ that would prevail at all future dates and all states s. This is the assumption of "Rational Expectations." This assumption is also the basis for all work which seeks to show that Pareto-optimality is obtained whenever the market is complete and the set of securities "spans" the set of exogenous states.

The rational expectations equilibrium concept of Arrow (1953) and its extension by Radner (1972) elevates the exogenously specified "state" substantially above Arrow's own definition (e.g. Arrow (1971: 20). It is no longer just such a complete description that the consequences of all *individual actions* are known; now the requirement is that the knowledge of the exogenously specified state enables every agent to know the consequences of all collective actions as well and, in particular, to know all future *prices* in the economy. These ideas extend further to the treatment of general equilibrium with private information (e.g. see Radner (1979, 1982)). The agent's knowledge of the *price maps* $p_t(s)$ plays a crucial role in the public revelation of private information.

The assumption of rational expectations in the Arrow–Radner equilibrium is viewed, almost universally, as placing excessive and unreasonable demands on the agents: since the map $p_t(s)$ is not observable, how could the agents know it at date 0? The term "rational" in connection to the knowledge of this map appears to mean that agents know the structure of the economy so completely (including technology and resources as well as preferences and endowments of other agents) that for each exogenous state s the agents can carry out all general equilibrium calculations needed to deduce the map $p_t(s)$[8] for all future dates. It is then natural to ask what if the agents do not know the map since they do not have the needed "structural knowledge".[9] A Rational Belief Equilibrium is then an extension of the Arrow–Radner theory to the case where agents know neither the equilibrium map nor the true probability distribution of prices and exogenous shocks. The theory of RBE leads, in a natural way, to the emergence of endogenous uncertainty. I will now explore this connection in some detail.

In an Arrow–Radner economy, without contingent claims, an equilibrium is a sequence of market-clearing spot prices of the reopened markets at the different dates. But if agents do not know the equilibrium map, then at $t = 0$ agents are uncertain about future exogenous states and about future spot prices at $t = 1, 2, \ldots$. According to Savage's (1954) dictate, future exogenous states and future spot prices are a part of the "world" about which all agents are uncertain. This means that the state, which is a description of the world, should include future spot prices. If prices are part of "the state of the world" then agents cannot view prices as *a known* equilibrium map like $p_t(s)$. Moreover, from the point of view of each agent the state space is the product of exogenous states and price states. With this enlargement of the "state space" we lower the concept of "the state of the world" back to where it is merely a terminology for the description of what agents are uncertain about. However, this change of the state space has far reaching implications for the way we should think about uncertainty in a general equilibrium context.

Once agents view prices as random variables they must form probability beliefs about future prices in the same way they form beliefs about exogenous variables. Since Savage (1954), Arrow (1953), and Radner (1972) allow agents to have different probability beliefs about the objects of their uncertainty, it follows that if an equilibrium concept is to permit agents to be uncertain about future prices, then equilibrium prices at each date must depend upon what agents expect future equilibrium prices to be!! Formally, suppose that in an economy with K agents we denote by $y_t = (y_t^1, \ldots, y_t^K)$ the date t vector of conditional probabilities of the K agents about all equilibrium events after date t conditional *upon the entire past*. y_t is the "state of belief" in the economy and y_t^k is the state of belief of agent k. The decision functions of the agent at each date take the general form

$$x_t^k = F^k(p_{(t)}, s_{(t)}, y_t^k), \tag{9.1}$$

where $z_{(t)} = (z_0, z_1, \ldots, z_t)$ denotes the entire history of a variable z. Market-clearing conditions establish equilibrium prices p_t at each date t as

$$p_t = \Phi(y_t, s_{(t)}) \tag{9.2}$$

and in the special and useful case of finite memory equilibria, equation (9.2) takes the simpler form

$$p_t = \Phi(y_t, s_t). \tag{9.2'}$$

The map equation (9.2) which is unknown to the agents in an RBE corresponds to the Arrow–Radner price map $p_t(s)$ which is assumed to be known to the agents. The crucial difference is the emergence of the state of belief which becomes part of the enlarged state space for the economy. In either case of equation (9.2) or equation (9.2′)

the riskiness of prices is then determined by the true state (y_t, s_t) consisting of the state of belief and the state of the exogenous shocks.

In a dynamic economy consisting of a sequence of markets, agents' perception of risk is directly linked to the fluctuations of the economy over time and against such variability they wish to insure themselves. Endogenous uncertainty is then that component of economic fluctuations which is caused by the agent's states of beliefs. Since agents do not know the true equilibrium map between states (s_t, y_t) and prices and endogenous variables, and since they do not observe states of beliefs, they can learn something from an examination of the data generated by the economy. One of the main conclusions of Kurz (1994a) is that there is no basis to expect that agents will learn the true structure of the maps in equations (9.2) or (9.2′) and the true probability distribution of prices and other endogenous variables. I can then sum up the nature of an RBE as follows. An RBE is an Arrow–Radner equilibrium in which

1. agents hold diverse Rational Beliefs about future exogenous states and endogenous variables;
2. agents do not know the equilibrium map and act on their beliefs about future prices;
3. the state space for the economy is enlarged to consist of the exogenous states and the states of belief, and equilibrium quantities are functions of these joint states.

The emergence of endogenous uncertainty in economies where agents do not have structural knowledge points to the observation that in such economies "expectations matter" and have real effects on equilibrium allocations.

9.3 EXPLAINING THE PARADOXES: SIMULATION ANALYSIS

I have suggested to the reader that my theory offers a unified paradigm to solve the four problems formulated in Section 9.1. Here I review these solutions in the form of simulation results of models with endogenous uncertainty. Since the questions span issues related not only to the domestic but also to the international economy, I present the results of two slightly different models: one of the domestic economy and a second of the international economy.[10] The two models have the same basic structure which I shall review first. After this review I shall present the results and interpret them.

9.3.1 The basic OLG models

I will review the domestic component of the OLG model and then comment on the multi country version.

9.3.1.1 The economy

I employ a two-agent, OLG, economy with a homogenous consumption good. Each agent lives two periods, the first when he is "young" and the second when he is "old." A young agent is a replica of the old agent who preceded him, where the term "replica" refers to *utilities* and *beliefs*, and hence this is a model of two infinitely lived "dynasties" denoted by $k = 1, 2$. Only young agents receive an endowment $\Omega_t^k, t = 1, 2, \ldots$ of the consumption good. This endowment is viewed as the labor income of the agents; the stochastic processes $\{\Omega_t^k, t = 1, 2, \ldots\}$ for $k = 1, 2$ will be specified below. Additional net output is supplied by a firm which produces exogenously, as in Lucas (1978), the strictly positive dividend process $\{D_t, t = 1, 2, \ldots\}$ with no input. D_t is paid out as dividends at date t to the owners of the shares of the firm. The shares are traded on a public stock market and their aggregate supply is 1.

The economy has (i) a market for the single consumption good; (ii) a stock market as specified above; and (iii) a market for a zero net supply, short term debt instrument which we call a "bill." Since $\{d_t, t = 1, 2, \ldots\}$ is a Markov process with two states, the economy has a complete financial structure: the number of assets equals the number of states. To ensure intergenerational efficiency, the financial sector is initiated at date 1 by distributing the supply of ownership shares among the old of that date. My notation is as follows: for $k = 1, 2$

C_t^{1k}	consumption of k when young at t;
C_{t+1}^{2k}	consumption of k when old at $t+1$ (implying that the agent was born at t);
D_t	total amount of profits or dividends produced exogenously at t;
θ_t^k	amount of stock purchases by young agent k at t;
B_t^k	amount of one period bill purchased by young agent k at t;
Ω_t^k	endowment of young agent k at t;
P_t	the price of the common stock at t;
q_t	the price of a one period bill at t. This is a discount price;
I_t	history of all observables up to t;
$d_{t+1} = \frac{D_{t+1}}{D_t}$	the random growth rate of profits or dividends;
$p_t = \frac{P_t}{D_t}$	the price/dividend ratio of the common stock at t.

Consumption is used as a numéraire and hence the optimization problem of agent k at date t has the following structure where Q^k is a probability belief of agent k:

$$\operatorname{Max}_{(C_t^{1k}, \theta_t^k, B_t^k, C_{t+1}^{2k})} E_{Q_t^k} \{u^k(C_t^{1k}, C_{t+1}^{2k}) | I_t\} \tag{9.3a}$$

subject to

$$C_t^{1k} + P_t \theta_t^k + q_t B_t^k = \Omega_t^k, \tag{9.3b}$$

$$C_{t+1}^{2k} = \theta_t^k (P_{t+1} + D_{t+1}) + B_t^k. \tag{9.3c}$$

To enable the computation of equilibria, I take the utility function agent k to be

$$u^k(C_t^{1k}, C_{t+1}^{2k}) = \frac{1}{1-\gamma_k}(C_t^{1k})^{1-\gamma_k} + \frac{\beta_k}{1-\gamma_k}(C_{t+1}^{2k})^{1-\gamma_k}, \quad \gamma_k > 0, \quad 0 < \beta_k < 1. \tag{9.4}$$

With this specification the Euler equations for agent **k** are

$$-P_t(C_t^{1k})^{-\gamma_k} + \beta_k E_{Q_t^k}((C_{t+1}^{2k})^{-\gamma_k}(P_{t+1} + D_{t+1}) | I_t) = 0 \tag{9.5a}$$

$$-q_t(C_t^{1k})^{-\gamma_k} + \beta_k E_{Q_t^k}((C_{t+1}^{2k})^{-\gamma_k} | I_t) = 0 \tag{9.5b}$$

The dividend process is as specified in Mehra and Prescott (1985). It takes the form

$$D_{t+1} = D_t d_{t+1}, \tag{9.6}$$

where $\{d_t, t = 1, 2, \ldots\}$ is a stationary and ergodic Markov process. The state space of the process is $J_D = \{d^H, d^L\}$ with $d^H = 1.054$ and $d^L = 0.982$ and a transition matrix

$$\begin{bmatrix} \phi & 1 & -\phi \\ 1 & -\phi & \phi \end{bmatrix} \tag{9.7}$$

with $\phi = 0.43$. Hence, over time agents experience a secular rise of total dividends and it is therefore convenient to focus on growth rates. To do that let

$\omega_t^k = \dfrac{\Omega_t^k}{D_t}$ is the endowment/dividend ratio of agent k at date t;

$b_t^k = \dfrac{B_t^k}{D_t}$ is the bill/dividend ratio of agent k at date t;

$c_t^{1k} = \dfrac{C_t^{1k}}{D_t}$ is the ratio of consumption when young to aggregate capital income;

$c_{t+1}^{2k} = \dfrac{C_{t+1}^{2k}}{D_{t+1}}$ is the ratio of consumption when old to aggregate capital income.

In the domestic model $\omega_t^k = \omega^k$ for $k = 1, 2$ are constant. Now divide equation (9.3b) by D_t, equation (9.3c) by D_{t+1}, equation (9.5a) by $D_t^{1-\gamma_k}$ and equation (9.5b) by $D_t^{-\gamma_k}$ to obtain for $k = 1, 2$

$$c_t^{1k} = -p_t\theta_t^k - q_t b_t^k + \omega^k, \tag{9.8a}$$

$$c_{t+1}^{2k} = \theta_t^k(p_{t+1} + 1) + \frac{b_t^k}{d_{t+1}}, \tag{9.8b}$$

$$-p_t(c_t^{1k})^{-\gamma_k} + \beta_k E_{Q_t^k}((c_{t+1}^{2k} d_{t+1})^{-\gamma_k}(p_{t+1} + 1)d_{t+1} | I_t) = 0, \tag{9.8c}$$

$$-q_t(c_t^{1k})^{-\gamma_k} + \beta_k E_{Q_t^k}((c_{t+1}^{2k} d_{t+1})^{-\gamma_k} | I_t) = 0. \tag{9.8d}$$

The optimum conditions (9.8a)–(9.8d) imply the following demand functions for $k = 1, 2$

$$b_t^k = b_t^k(p_t, q_t, d_t, I_t), \tag{9.9a}$$

$$\theta_t^k = \theta_t^k(p_t, q_t, d_t, I_t). \tag{9.9b}$$

Equilibrium requires the market-clearing conditions

$$\theta_t^1 + \theta_t^2 = 1, \tag{9.9c}$$

$$b_t^1 + b_t^2 = 0. \tag{9.9d}$$

The equilibrium implied by equations (9.9a)–(9.9d) depends upon the beliefs of the agents. I study the Markov equilibria with a finite number of prices. An equilibrium is characterized either by one Markov matrix or by a sequence of such matrices which describe the transition from a price state to another. However, the stationary measure m will be described by a *single* transition matrix from prices at t to prices at $t+1$ which I call Γ. The two agents hold rational beliefs Q^k which are stable Markov probabilities with stationary measures defined by Γ. It is clear that the rationality of belief conditions can be very complicated. The technique of "assessment variables" is the main technical development in Nielsen (1996) and Kurz and Schneider (1996). It enables a simple description of a large family of rational beliefs.

9.3.1.2 Rational belief equilibrium

Assessment variables of agents are sequences of random variables $\{y_t^k, t = 1, 2, \ldots\}$ for $k = 1, 2$ and here I assume that $y_t^k \in Y = \{0, 1\}$. The belief of agent k is a probability Q^k over the *joint* process $\{(p_t, q_t, d_t, y_t^k), t = 1, 2, \ldots\}$ which is a Markov process. The decision functions in equations (9.8a)–(9.8b) are selected based on the *conditional probability* of Q^k given the value of y_t^k.

As a matter of economic interpretation, assessment variables are parameters indicating how an agent perceives the state of the economy and are thus tools for the description of stable and non-stationary processes. In the model at hand they are the method of describing if an agent is optimistic or pessimistic at date t about capital gains at date $t+1$. I thus need to clarify how assessment variables enter the decision mechanism of agent k. Note that in equations (9.8c)–(9.8d) agent k specifies the probability of $(p_{t+1}, q_{t+1}, d_{t+1}, y_{t+1}^k)$ *conditional* on (p_t, q_t, d_t, y_t^k) – the value of his assessment variable jointly with the observed data. It then follows from the Markov assumptions that the demand functions of agent **k** for stocks and bills are functions of the form

$$b_t^k = b^k(p_t, q_t, d_t, y_t^k), \tag{9.10a}$$

$$\theta_t^k = \theta^k(p_t, q_t, d_t, y_t^k). \tag{9.10b}$$

Consequently the market-clearing conditions are

$$\theta^1(p_t, q_t, d_t, y_t^1) + \theta^2(p_t, q_t, d_t, y_t^2) = 1, \tag{9.10c}$$

$$b^1(p_t, q_t, d_t, y_t^1) + b^2(p_t, q_t, d_t, y_t^2) = 0. \tag{9.10d}$$

The system (9.10c)–(9.10d) implies that the equilibrium map of this economy takes the form

$$\begin{bmatrix} p_t \\ q_t \end{bmatrix} = \Phi^*(d_t, y_t^1, y_t^2). \tag{9.11}$$

The equilibrium map (9.11) shows that prices are determined by the exogenous shock d_t and by the endogenous "state of belief" (y_t^1, y_t^2). Here $y_t^k \in \{0, 1\}$ for $k = 1, 2$ and $y_t^k = 1$ means that k is in a state of optimism while $y_t^k = 0$ means that k is in a state of pessimism. The appearence of the endogenous vector (y_t^1, y_t^2) in the equilibrium map is the precise way in which *Endogenous Uncertainty* is present in the equilibrium.

Condition (9.11) implies that there are at most eight *distinct* price vectors (p_t, q_t) that may be observed corresponding to the eight combinations of (d_t, y_t^1, y_t^2). Moreover, the *true* equilibrium transition probability from the eight prices (p_t, q_t) to the eight prices (p_{t+1}, q_{t+1}) is determined by the transition from (d_t, y_t^1, y_t^2) to $(d_{t+1}, y_{t+1}^1, y_{t+1}^2)$. I select the joint process $\{(d_t, y_t^1, y_t^2), t = 1, 2, \ldots\}$ to be a stationary Markov process with a transition matrix Γ. This choice[11] implies that the true equilibrium process of prices $\{(p_t, q_t), t = 1, 2, \ldots\}$, has Γ as the *fixed* stationary transition probability from (p_t, q_t) to (p_{t+1}, q_{t+1}). Hence, the agents who compute the empirical distribution of the process will discover Γ. Although this matrix characterizes the empirical distribution of the equilibrium dynamics, the agents do not know this fact and form rational beliefs relative to Γ. Indeed, the fact that they form rational beliefs in accordance with their assessment variables is the reason why Γ is the equilibrium probability of the implied RBE.

An assessment variable y_t^k determines completely the transition matrix from (p_t, q_t) to (p_{t+1}, q_{t+1}) perceived by agent k at date t. Moreover, $y_t^k \in \{0, 1\}$ implies that the agent has at most two 8×8 Markov matrices and at each date the value taken by his assessment variable determines which of the two the agent uses. This means that over time the empirical frequencies determine Γ while each agent uses two Markov matrices (F_1, F_2) for agent 1 and (G_1, G_2) for agent 2. What are then the rationality of belief conditions? This technical question is fully answered by the fundamental Conditional Stability Theorem of Kurz and Schneider (1996) (see p. 494). It turns out that the rationality conditions depend upon the marginal distributions of y_t^k for $k = 1, 2$. For simplicity I assume that these distributions are i.i.d. with $Q^k\{y_t^k = 1\} = \alpha_k$ for $k = 1, 2$. The Conditional Stability Theorem then implies that the beliefs Q^1 and Q^2 of the two agents are described by the following rule:

$$
\begin{array}{llll}
Q^1 \text{ for agent 1:} & \text{use } F_1 \text{ if } y_t^1 = 1 & Q^2 \text{ for agent 2:} & \text{use } G_1 \text{ if } y_t^2 = 1 \\
& \text{use } F_2 \text{ if } y_t^1 = 0 & & \text{use } G_2 \text{ if } y_t^2 = 0
\end{array}
\tag{9.12a}
$$

The rationality of belief conditions then require that

$$
\alpha_1 F_1 + (1 - \alpha_1) F_2 = \Gamma, \qquad \alpha_2 G_1 + (1 - \alpha_2) G_2 = \Gamma. \tag{9.12b}
$$

The rational agents believe that the price-dividend process is not stationary and their beliefs are parametrized by their private assessment variables (y_t^1, y_t^2). Equation (9.12b) requires that the sequence of matrices which they adopt generates the same empirical distribution as the Markov process with transition matrix Γ. α_1 is the frequency at which agent 1 uses Matrix F_1 and α_2 is the frequency at which agent 2 uses matrix G_1.

I now specify the matrixes (F_1, F_2) and (G_1, G_2). One needs to specify only F_1 and G_1 since equation (9.12b) determines F_2 and G_2. I have already noted that the two agents can be optimistic or pessimistic about capital gains in the future. Since high prices are associated with d^H they are the first four prices in the matrix Γ. It follows that optimism in F_1 and G_1 is expressed by a proportion $\lambda > 1$ by which entries in the first four rows of F_1 and G_1 are increased relative to the corresponding entries in Γ.

Thus, optimism or pessimism is always defined relative to the long-run conditional frequencies defined by Γ. Think of λ as the *intensity of optimism*. The rationality conditions (9.12b) are linear; they specify that for any period of optimism there must be a corresponding period of pessimism. But pessimism is represented by lower conditional probabilities at t of capital gains at $t+1$ relative to Γ. Hence, as the intensity of optimism λ rises, the pessimistic probabilites approach 0. In the limit there is a finite intensity of optimism which brings the pessimist at t to the 0 probability of high prices in $t+1$. I mention these facts here since they play a crucial role in the analysis below.

I note that an REE is an RBE with $Q^1 = Q^2 = \Pi$ and, deduced from equation (9.7), the probabilities of $(p_{t+1}, q_{t+1}, d_{t+1})$ in equations (9.6c)–(9.6d) are conditioned only on the realized value of d_t. Since the growth rate of dividends takes two values, a Markov REE is, in fact, a stationary equilibrium with two prices and two optimal portfolios.

The international model includes money and allows for monetary policies of the two economies. Since it is not my aim to study different monetary policies, I fix the policies in the two countries. They are set so that each country responds to its own exogenous shock: the domestic central bank adjusts the money supply in response to the random changes in the growth rate of earnings and the foreign central bank adjusts the money supply to changes in the growth rate of wages. In either country the objective of the bank is to maintain price stability. The foreign economy is purely hypothetical.

9.3.2 On the method of simulations

What is the logic of a simulation model and why should we consider this method of analysis valid? To answer this question I note first that the parameters of the real economy are selected so as to conform to well known parameters of econometric models that were estimated for the United States economy. These include the long-term growth rates of wages and earnings and the coefficients of risk aversion and discount rates of the agents. As a result, the real part of the economy is required to act in conformity with what we know about the long-run tendencies of the United States economy. Hence, the fundamentals of the economy are exactly the same as we know from the statistics of the real economy. The parameters which I, as a model builder, will select are those that relate to the beliefs of the agents and their distributions. The simulation models then ask what would be the implications of alternative belief structures for price volatility, holding the fundamentals fixed. Since rational expectations are among the beliefs which can be examined in the model, the results below will provide a comparison between the implications of rational expectations and rational beliefs for price volatility, keeping the real economy the same.

It has been well documented that if one imposes on the real fundamentals of the simulation models the assumption of rational expectations by the agents, all the problems and paradoxes specified earlier will appear and I shall demonstrate that this remains true in the models at hand. However, if I can show that under the assumption that the agents hold rational beliefs the financial markets will not exhibit any of the paradoxes, then it follows that the belief structure of the agents does provide a unified paradigm to resolve the specified problems. It would then be useful to have an intuitive understanding of the structure of beliefs that generate the various conclusions and I will attempt to provide some interpretation in a later section.

The foreign economy is a purely hypothetical economy; it is not calibrated to any particular economy. The two economies will have a common ("world") stock market and the foreign economy will have an exogenous endowment shock.

9.3.3 Simulation demonstration of the solutions to the four problems

In the Tables below I present comparisons between the simulation results under rational expectations and under rational beliefs. The aim is first to exhibit what are the problems which arise under REE and then to show that these problems are significantly resolved under the unified paradigm of the theory of rational beliefs. The sequence of tables below correspond to the questions posed at the start.

9.3.3.1 Problem A: asset price volatility in the domestic economy

Table 9.1 reports two measures of price volatility. The first is the interval in which the price/earnings ratio fluctuates 95 per cent of the time. The long-term mean of this variable is fixed at 13.9 which is the actual long-term average of the price/earnings ratio of the S&P 500 index. This average has no significance in the table; it is used only as a reference for measuring the interval of fluctuations under each of the model assumptions. The second row reports the long-term standard deviation of the real rate of return on equity.

The table exhibits the problem which arises under rational expectations: if stock prices vary strictly in accordance with fundamentals they would not change very much! The variance of the price/earning ratio is bigger *by an order of magnitude* under rational beliefs than under rational expectations. The table shows that under rational beliefs the index would have spent 95 per cent of the time between 9.4 and 18.4 which is of the same order of magnitude as the historical record. This interval is somewhat smaller than the actual interval reported in the last column, a fact that may be explained by the generally agreed upon observation that the fluctuations of the reported price/earnings ratio are sensitive to tax and accounting practices. These tend to overstate the volatility of recorded earnings relative to the true economic earnings of the companies in the index. The actual long-term standard deviation of the return on the S&P 500 index is 18.4 per cent and the simulations under rational beliefs lead to a standard deviation of 17.5 per cent. These two measures of volatility are very close.

Table 9.1 Long-run volatility of the price/earnings ratio and the return on equity

	Under rational expectations	*Under rational beliefs*	*Actual*
Interval in which the price/earnings ratio fluctuates 95 per cent of the time	(13.8, 14.0)	(9.4, 18.4)	(5.5, 22.3)
σ_r – the long-term standard deviation of the return on equity	4.1%	17.5%	18.4%

Table 9.2 The long-run average riskless rate and the equity premium over the riskless rate

	Under rational expectations%	*Under rational beliefs%*	*Actual (%) (approx)*
r^F – the long-term average of the riskless rate	5.72	0.71	1.00
ρ – the long-term average risk premium of equity	0.49	7.97	7.00

9.3.3.2 Problem B: the equity premium and the riskless rate in the domestic economy

In Table 9.2, I record the long-term averages of the riskless real rate of return on short-term debt and of the equity risk premium. The table exhibits the problem which arises under rational expectations: the historical record over the last hundred years shows a riskless short-term interest rate in the order of magnitude of 1 per cent and an average risk premium of around 7 per cent. Under rational expectations the model fails to come close to these facts. Under rational beliefs the average equity premium is 7.97 per cent, the average riskless rate is 0.71 per cent and these figures correspond to the historical record.

9.3.3.3 Problem C: the GARCH property of stock prices in the domestic economy

It has been observed both by experienced market traders as well as by academic researchers that over time, the variance of stock prices and returns change *without* a corresponding change in fundamentals to account for it. This is known as "the GARCH property of stock prices" and this represents a problem for rational expectations since under such expectations prices change only in response to changes in fundamentals. In Figure 9.2, I exhibit a plot of the time series of 300 prices that were simulated in the domestic model. The growth rate of dividends is assumed to take two values in these calculations and since these are also random, I plot them at the bottom. It is clear that over time the model exhibits drastic changes in price volatility but there are only two volatility regimes: one is a *high volatility regime* and the second is a *low volatility regime*. Both regimes exhibit substantial persistence. Variations in the growth rate of dividends has a slight effect on these regimes so that within the high and low volatility regimes there are sub-regimes whose volatility depends to a small degree upon dividends.

9.3.3.4 Problem D: volatility of the foreign exchange rate and the forward discount bias

Table 9.3 reports selected results of the international model which I now draw upon for the first time. Before discussing those let me define exactly the concept of "forward discount bias" which was mentioned in Problem D above. Suppose you estimate a regression of the form

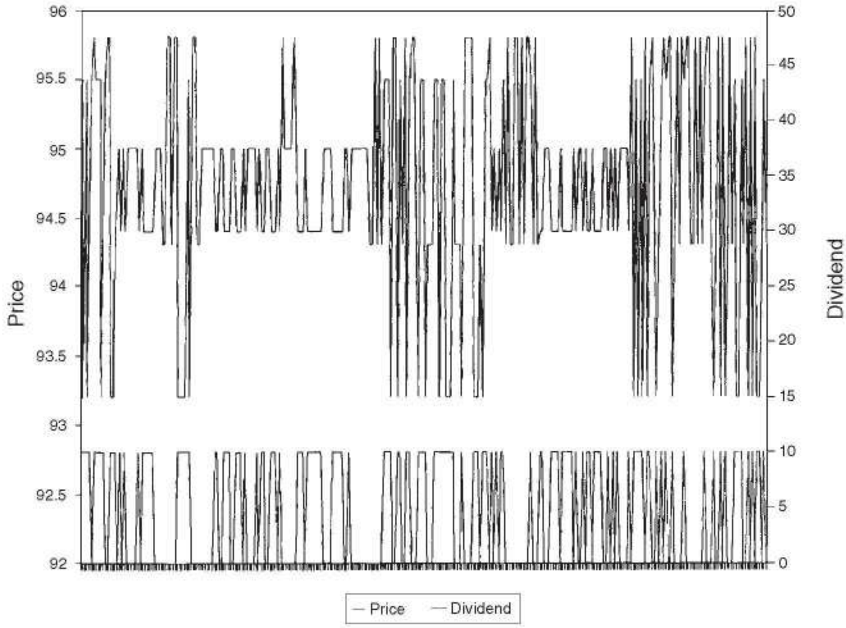

Figure 9.2 Time series of 300 prices simulated in the domestic model.

Table 9.3 The volatility of the exchange rate and the forward discount bias

	Under rational expectations	*Under rational beliefs*	*Actual Yen/Dollar*
Interval in which the exchange rate fluctuates 95 per cent of the time	[115, 125]	[67, 173]	[84, 156]
β – the forward discount bias parameter	0.957	0.152	Diverse <1

$$\frac{\mathrm{ex}_{t+1} - \mathrm{ex}_t}{\mathrm{ex}_t} = c + \beta(r_t^D - r_t^F) + \epsilon_{t+1},$$

where $(\mathrm{ex}_{t+1} - \mathrm{ex}_t)$ is the change of the exchange rate between date t and date $t+1$ while $(r_t^D - r_t^F)$ is the difference between the short-term nominal interest rates in the domestic and the foreign economies. Under rational expectations the differential of the interest rates between the two countries at date t should provide a correct predictor of the *actual* depreciation of the currency that will occur between date t and $t+1$. This means that apart from a technical correction for risk aversion, the parameter β should be close to 1. In 75 empirical studies in which equations like the above were estimated, the estimates of the parameter β are significantly less than 1. Indeed, Froot (1990) estimates that the average for all these studies is -0.88! The failure of this parameter to exhibit estimated values close to 1 has come to be known as the "forward discount bias" (see Engel (1996) for an extensive recent survey and Froot and Thaler (1990) for a simple exposition of the problem).

Table 9.3 reports (1) an interval in which the exchange rate fluctuates 95 per cent of the time and where the mean exchange rate has been *arbitrarily* calibrated to be 120; (2) the value of the parameter β which the simulation models predict. The selection of 120 as the mean of the exchange rate has no significance to the volatility measures reported. It is only meant to establish a comparable frame of reference. The actual Yen/Dollar rate of exchange in the last column of Table 9.3 has fluctuated in part due to different inflation rates in the United States and Japan and I have thus computed the variance of the exchange rate based *on logarithmic detrending of the data*. The "actual" variability in the table is then that part of the variability of the Yen/Dollar exchange rate around the average geometric trend. The table exhibits the problems which arise under rational expectations: the variance of the foreign exchange rate is negligible and the parameter β has a value close to 1. Under rational beliefs the results are drastically different: the variance of the foreign exchange is of the order of magnitude of observed fluctuations in the market. Finally, the forward discount bias parameter in the RBE reported in the table is 0.152 which is significantly less than 1. Within the class of models used here a negative parameter could not be predicted.

9.4 SIMPLE EXPLANATIONS OF HOW THE THEORY RESOLVES EACH OF THE FOUR PROBLEMS

In Section 9.3, I demonstrated that the unified paradigm offered by the theory of RBE goes a long way towards solving the four problems that could not be solved within the prevailing rational expectations paradigm. In this section I will offer a simple but systematic explanation of the results presented in Section 9.3. In doing so I will also demonstrate the workings of the model of RBE.

(i) *Volatility of prices and exchange rates* The explanation of why the volatility of prices and exchange rates in an RBE exceed the level determined by the exogenous fundamentals of the economy is simple. Each agent forms his own theory of what the future will bring and the distribution of the private models in the economy constitutes the "social state of belief." Variability in the state of belief in the market is then an important factor, together with the exogenous shocks, in explaining price volatility. Since the social state of belief is not observable we need to seek proxies for it. Incomplete proxies can be seen in the distribution of price forecasts announced by different forecasters in the market. Interesting distributions of short-term and long-term interest rate forecasts by professional economists are also revealing since all use the same data. Thus the "state of belief" in the market is simply a "distribution of beliefs."

Endogenous uncertainty is then that component of price volatility which is caused by the distribution of beliefs of the agents. Therefore, equilibrium price volatility can be represented as a sum of the form

$$\text{Market Uncertainty} = \text{Exogenous Uncertainty} + \text{Endogenous Uncertainty}$$

Since exogenous uncertainty is that component of market volatility which is determined by the volatility of the exogenous fundamental conditions in the market, it is then clear why total market volatility exceeds the level justified by fundamentals.

Without introducing technical details I stress that endogenous uncertainty has a dual effect on market volatility. One component of endogenous uncertainty is the *amplification* of the effect of fluctuations of exogenous fundamentals on price volatility. This is the effect whereby the distribution of beliefs in the market can cause changes in the fundamental exogenous variables to have a larger effect on price volatility than would be true in a corresponding rational expectations equilibrium. The second component of endogenous uncertainty arises from the fact that variations in the distribution of beliefs cause additional *pure price volatility* which is uncorrelated with any fundamental information. This component of endogenous uncertainty may have dramatic effects on the volatility of prices in an RBE since this component turns out to be affected by correlation and commonality of beliefs among traders. When a large number of agents become optimistic about capital gains, prices may rise. Conversely, when a large number of agents become pessimistic prices decline.

The amplification component of Endogenous Uncertainty provides a natural explanation of the phenomenon which is recognized as "market overshooting." This is usually a reference to the fact that when prices are high they often proceed to go higher than can be justified by fundamentals and when they go low, they go lower than can be justified by the exogenous variables. Naturally, excess volatility and overshooting is part of the historical record and is incorporated in the empirical distribution of any market. Hence it becomes part of the belief structure of agents: they expect the market to overshoot.

(ii) *The forward discount bias in foreign exchange rates* To see why this bias arises naturally in an RBE recall the rational expectations argument in favor of $\beta = 1$ (apart from the correction for risk aversion which I ignore here). Hence, in such an equilibrium it is a *theoretical* conclusion that the difference between the one period nominal rates in the two countries at date t is exactly equal to the expected percentage depreciation of the exchange rate between the two currencies between dates t and $t + 1$. This *expectational arbitrage* argument implies that in the real economy the differential between the one period nominal rates in the two countries will be an unbiased *statistical* forecast of the one period depreciation of the exchange rate in the next period. Under this proposition one would expect to have a regression coefficient of 1 between the percentage differential of the nominal rates at date t and the *actual* percentage change of the exchange rate between dates t and $t + 1$.

The theory of RBE predicts that agents holding rational beliefs may make significant forecasting mistakes. This would result in a true, equilibrium, process of the exchange rate which would fluctuate excessively in part due to these mistaken forecasts. Hence, at almost no date would the nominal interest differential between the two countries be an unbiased estimate of the rate of depreciation of the exchange rate one period later and under such circumstances one should not expect the regression coefficient to be close to one. Agents who want to take advantage of such a regression, basing their investment strategy on a nominal rate differential which appears to offer an investment opportunity, will find that this is not arbitrage in the standard riskless sense of the term. From the perspective of date t the exchange rate at date $t + 1$ *is a* *random variable*. In an RBE any trade on the spread between the nominal interest rates of two currencies requires agents to take a foreign exchange risk which is valued differently by agents holding diverse beliefs.

Should we expect that under rational beliefs the parameter β satisfies $\beta < 1$? The answer is yes for the following reason. Consider first a rational expectations equilibrium in which the difference between the domestic and foreign nominal interest rates is z per cent. In that equilibrium you do not need to form expectations on the currency depreciation itself. It is sufficient for you to believe that other investors or currency arbitrageurs know the true probability of currency depreciation and they have already induced the interest differential to be equal to the average rate of currency depreciation which will be z per cent. Now consider an RBE. All agents know that no one knows the true probability distribution of the exchange rate and therefore the exchange rate is subject to endogenous uncertainty. Being risk averse, agents who invest in foreign currency would demand a risk premium on endogenous uncertainty and over the long-run the difference $(1 - \beta)$ is the premium received by currency speculators for being willing to carry foreign currency positions. For a positive premium it follows that $\beta < 1$.

(iii) *The GARCH property of asset prices*[12] As indicated earlier (Section 9.3), the states of belief of different individual investors may be highly correlated and this is a consequence of the many modes of communication in our society. Investors talk to each other and this interaction causes them to influence each other; they all read the same newspapers, the same reports of the Wall Street analysts and watch the same television programs which feature expert views on the economic conditions in the future. Analysts and experts know each other, they talk to each other and attend the same conferences thus tend to correlate their views either in agreement or disagreement. The consequence of this correlation among the beliefs of agents is that the distribution of beliefs tends to switch across different "cognitive" centers of gravity. Indeed, each such center of gravity is a "belief regime." The important examples of such regimes of belief are regimes of "consensus" and "non-consensus." The persistence of the states of belief is an important element in the emergence of the GARCH property. In the models studied here the state of belief is a Markov process with degrees of persistence which depend upon the parametrization of each model.

The emergence of the GARCH property is a consequence of two different effects which the distribution of beliefs has on the market. These two effects are directly related to the relative strength of the two components of Endogenous Uncertainty discussed above: *amplification* and *pure endogenous volatility*. I will start with pure endogenous volatility since this effect is simpler. It turns out that what really matters for the effect of this component of endogenous volatility on the emergence of the GARCH phenomenon is the *persistence* of the regime of consensus vs. the regime of market non-consensus. A regime of *market consensus* is formed when the models of the majority of traders generate similar predictions and if the regime persists, then over time, if the real economy remains in the same state ("high" for expansion and "low" for recession) the traders move *together* between states of optimism and states of pessimism. Such fluctuations between optimistic and pessimistic outlooks on market prices may occur on many different frequencies. *Non-consensus* is a belief regime in which the distribution of models used by the agents is relatively spread out and consequently their predictions vary widely across the different possible outcomes in the future. If the regime of non-consensus persists then, at a given state of the real economy, the diverse forecasts tend to cancel each other out over time.

I now observe that when the regime of consensus is formed the pure volatility component of security prices will be high. This is so since all agents are either optimistic or pessimistic *at the same time*; when optimistic they want to buy the same portfolio and when pessimistic they want to sell the same portfolio leading to price volatility. Conversely, when a non-consensus regime occurs the opposite is true: the distribution of beliefs is one in which the excess demand of the optimists matches the excess supply of the pessimists leading to low volatility. This component of endogenous volatility is not correlated with real exogenous shocks.

I turn now to the effect of "amplification" which is drastically different from the first effect. To understand this second effect consider two different states of belief each of which has a high degree of persistence. For example, let the first state be one of "universal optimism" and the second state be one of "non-consensus" or "disagreement". Given the state of optimism, prices will vary with the state of the dividend growth. However, assuming a strong endogenous amplification, prices will overreact to changes in the fundamental information but the degree of amplification is not the same in the state of total optimism as in the state of non-consensus. If the economy is in a state of optimism (and remains there) the variations in asset prices due to cyclical output fluctuations is usually relatively small so that the variance of prices in that state is relatively small. If the economy is in the state of non-consensus (and remains there) the price response to cyclical output fluctuations is very different depending upon the intensity of optimism and pessimism. As we shall see later in this section, in all models presented here the pessimists are more intense than the optimists. Consequently, if in a state of non-consensus the net output state is "low" (i.e. recession) then the pessimists will dominate by making the price crash even further but if the state is "high" the pessimistic outlook has less force and prices will rebound sharply. As a result, the variance of prices in that state is very high. In short, as the economy moves among different states of belief the level of asset prices will change, but more importantly, the variance of prices will vary, giving rise to the GARCH property.

To generalize these conclusions beyond the simulation models, the theory of RBE shows that the variance of stock prices depends upon the *distribution of beliefs* in the market and since this distribution changes over time, so does the variance of stock prices. Also, in an RBE agents can utilize exogenous shocks and realized prices to determine their state of belief about the future. Consequently, the distribution of beliefs and hence the variance of prices may depend upon both the correlation among the beliefs of the agents as well as the exogenous shocks and realized prices. Both of these may change abruptly, and so can the induced regime of beliefs.

The models used in the simulations relate to events which occur over long stretches of time and hence the simulation results apply to low frequencies (i.e. months and years). These do not address the structure of volatility at high frequencies investigated by some papers of the GARCH literature (see for example Bollerslev *et al.* (1992) and Brock and LeBaron (1996)). This limitation of the results here should not obscure the main conclusion to which the theory of RBE leads: the GARCH phenomenon is caused both by the persistence as well as by the abrupt shifts in the distribution of beliefs. These forces hold over low or high frequencies.

(iv) *The equity premium and the riskless rate*[13] Explaining the factors which determine the equity risk premium (i.e. "the" premium) in an RBE is ultimately simple but demands the review of the technical conditions which formulate the rationality of beliefs of the agents. A *direct* and simple explanation flows naturally from the resolution of Problem A. It proposes that in an RBE, endogenous uncertainty causes the total level of uncertainty to exceed the level that would prevail under rational expectations. Risk averse investors would then demand a higher risk premium for holding equity which is more risky in an RBE than in a rational expectations equilibrium and for that reason the premium would be higher in an RBE. This explanation has a grain of truth but needs to be qualified by two additional considerations.

The first consideration suggests that due to the diversity of beliefs the equity premium arises in a world where optimists and pessimists reside together. The risk premium demanded by optimists is likely to be different from the premium demanded by pessimists and hence, the market premium must depend upon the *distribution* of beliefs. Indeed, there are proportions of optimists and pessimists which do not generate a higher equity risk premium than is generated under rational expectations. Second, an important component of the equity premium puzzle has been the question of why the riskless rate predicted by rational expectations models has been so much higher than the mean riskless rate realized over the last century and this question must be cleared as well. The *direct* explanation given above does not address the question of why the riskless rate is so much lower in the simulated RBE relative to rational expectations equilibria.

To gain intuition into the two issues above I must bring you into some of the more technical aspects of the theory and to do that I examine a very simple model (based on Kurz (1998)). Consider an economy with two types (α and β) of agents who are different only in their models of market price behavior (i.e. their beliefs). As part of their models, each of the type α agents has an assessment and when the assessment takes the value 1 the agent uses probability distribution F_1 of future prices and when it takes the value 2 the agent uses probability distribution F_2. These assessment variables are different for the two types. For this reason I denote the probabilities used by type β agents by G_1 when the assessment of a type β agent takes the value 1 and by G_2 when the assessment of a type β agent takes the value 2. However, I also assume that there is a very large number of agents of each type and each of them has his own separate assessment. Now, the assessments of the large number of agents of each of the types are, statistically speaking, the *same* random variables since these agents are of the *same* type but now comes the deeper question: are these assessments independent random variables? To address this question I must take an indirect route.

Some who object to models with heterogenous beliefs have suggested that in a large economy consisting of many agents with *independent* beliefs the law of large numbers would operate to average out the diversity of beliefs. Such averaging should render the model of diverse beliefs irrelevant, leading the model of a large economy to function like a model of the representative household with a *single*, rational expectations belief. This intuitive argument is misleading and the reasons are the key to understanding why a large equity premium and a low riskless rate can be generated in an RBE.

Let me then return to my simple model and make the strong assumption that all the assessments within each type are i.i.d. with the probability of assessment taking the value of 1 being, say, 0.60. The consequence of this assumption is that although the probability used by any one agent depends upon his assessment, the distribution of beliefs in the economy is fixed at ((0.60, 0.40), (0.60, 0.40)). That is, *at all times* 60 per cent of type α agents use probability distribution F_1 and 40 per cent of them use F_2. A similar situation is assumed with respect to type β agents. If I now interpret F_1 and G_1 to mean "optimistic beliefs about higher returns next period" and F_2 and G_2 to mean "pessimistic beliefs about higher returns next period" then I have an economy where the law of large numbers holds as required. At *all times* the distribution of beliefs is constant with 60 per cent of each type optimistic and 40 per cent pessimistic. The terms "optimism" and "pessimism" are exactly the same proportions defined before and hence I will call these proportions (which are fixed for each type but may be different across the two types) the "intensities of optimism" or the "intensities of pessimism." I use the term "intensities" rather than "intensity" since these intensities of optimism or pessimism may vary depending upon current prices.

In this economy 60 per cent of the agents are always optimistic, using F_1 or G_1, and hence each agent fluctuates between optimistic and pessimistic outlooks with a frequency of 0.60 in the optimistic mode and a frequency of 0.40 in the pessimistic mode. This would make sense only when I consider the rationality of belief conditions which the agents satisfy. These stipulate that the beliefs may fluctuate over time but must average to Γ. The RBE is then established if type α agents satisfy the condition $0.60F_1 + 0.40F_2 = \Gamma$ and type β agents satisfy condition $0.60G_1 + 0.40G_2 = \Gamma$. But now I need to compare two equilibria: an REE in which all the agents hold the belief Γ and the RBE in which 60 per cent are optimists and 40 per cent are pessimists relative to Γ. Those claiming that independent beliefs do not matter would propose that these two equilibria are the same in the sense that prices and allocations are the same. I will show that these two are very different equilibria with drastically different equity premia and volatility characteristics.

To convince you of that fact suppose that the initial percentage of pessimists in the economy is $x = 0.40$ and in equilibrium the rationality condition $(1 - x)F_1 + xF_2 = \Gamma$ is satisfied. Now I lower the percentage $x = 0.40$ to x'. Would the rationality of belief condition $(1 - x')F_1 + x'F_2 = \Gamma$ be satisfied with x'? The answer is no since my decrease of the percentage of pessimists from $x = 0.40$ to x' without changing the matrices (F_1, F_2, G_1, G_2) means that I reduced the weight assigned to the pessimistic matrix F_2 and increased the weight assigned to the optimistic matrix F_1 leading to the result that $(1 - x')F_1 + x'F_2 \neq \Gamma$. Hence, as the number of pessimists in the market declines, I must adjust the intensity parameters in F_2 and in G_2 so that the intensity of their pessimism *increases*. Indeed, a point will be reached at which I could not lower the fraction of pessimists any further since the intensity of their pessimism has reached a point where, given some price, they are virtually certain that they will lose money between date t and $t + 1$. I will then have an economy with a reduced proportion of pessimists but who are so intensely pessimistic that they are willing to pay a very high price for the bill to secure their wealth for the next period. What will happen to the interest rate and to the risky returns in the model under these circumstances? The price of the bill will rise, lowering the riskless rate, and the price of the stock will fall causing the equity risk premium to rise. In the section below I will provide an example that would apply to the situation under discussion. This concludes my demonstration that the RBE under discussion is very different from the REE with a representative agent.

The central observation is that the rationality of belief conditions are linear conditions of the form $(1-x)F_1 + xF_2 = \Gamma$ but variations in the percentage/intensity combinations of optimists and pessimists have a non-linear impact on the demand functions for securities. Hence, as these combinations vary over the feasible parameter space of the model, the riskless rate and the equity premium change. The reader may note that since the rationality condition $(1-x)F_1 + xF_2 = \Gamma$ is linear, any increase in probability in F_1 must be compensated by a reduction in F_2. Moreover, if $x = 0.5$ then the compensation in F_2 must be exactly the same as the increase in F_1 and this is entirely symmetric. So, if such changes have non-linear effect then the model at hand must imply some asymmetry. To understand it let me present a simple example which will clarify the issue.

Example Consider the case of a 3×3 matrix in which optimism is defined with respect to states 1 and 2. Assume $\alpha = 0.50$ and that $\lambda = 2.0$ is feasible. Both the matrix Γ and the optimistic matrix F_1 are entirely regular. The rationality conditions imply that the pessimistic matrix is

The Matrix Γ

	$s=1$	$s=2$	$s=3$
$s=1$	0.25	0.20	0.55
$s=2$	0.20	0.20	0.60
$s=3$	0.30	0.10	0.60

The Matrix F_1 Of Optimists

	$s=1$	$s=2$	$s=3$
$s=1$	0.50	0.40	0.10
$s=2$	0.40	0.40	0.20
$s=3$	0.60	0.20	0.20

$F_2 = \frac{1}{0.50}(\Gamma - 0.50F_1)$. It can be checked that the matrix F_2 is the one shown. I think

Matrix F_2 of Pessimists

	$s=1$	$s=2$	$s=3$
$s=1$	0	0	1
$s=2$	0	0	1
$s=3$	0	0	1

it is reasonable to think of the pessimists using matrix F_2 as being "more intense" in their pessimism than the intensity of the optimists which I quantified to be 2. I hasten to add that in the basic model discussed in this chapter $\lambda = 2.0$ is not feasible and the matrix Γ needs to be compatible with (9.7) and for this reason does not have the simple structure as in the example. Indeed, in the model above $\lambda = 1.7542$ and $\alpha = 0.57$.

Given the basic observation that at any date the risk premium is determined by the exogenous variables and by *the distribution of beliefs* in the market, I re-examine the assumptions made earlier. Recall that I have assumed that the assessments are i.i.d. in order to refute the criticism that heterogeneity of beliefs is irrelevant in a large economy with independent beliefs. Extensive research conducted in recent years has shown that it takes very little local interaction among agents in the market in order to remove the effect of the law of large numbers on equilibrium variables such as prices. More specifically, under small local interactions, equilibrium aggregate variables of a large economy *act as random variables* rather than as constants.[14] Given the natural interaction among the agents in financial markets there is ample theoretical justification for assuming that the beliefs of agents in the market are correlated and hence their assessments are not jointly i.i.d. On the empirical side, there is little data on the distribution of beliefs in the market. However, the litle evidence which is available (such as the forecasts of analysts on Wall Street) suggests that individual beliefs are highly correlated. Hence, both theoretical as well as empirical considerations imply that we should study models where the distribution of beliefs is a random variable, jointly distributed with prices and other equilibrium variables.

The argument developed earlier (for an economy with i.i.d. assessments) regarding the belief intensity of the pessimists remains valid in an economy with correlation among the assessments of the agents. The only difference is that now the distribution of beliefs changes over time and the riskless rate and equity premium vary with the states of the economy. Hence, the RBE model's prediction of the long-term averages of the riskless rate and of the equity premium depends now also upon the frequency at which the system visits those distributions of beliefs which generate low riskless rate and high premium. As we consider patterns of correlation among the beliefs of agents we may also expand the range of empirical evidence that needs to be explained. In this chapter I considered only four variables which needed to be explained. In a complete analysis of the equity premium one may ask for the model prediction to match other empirical regularities. These would include:

1 the first and second moments of the price/earning or price/dividend ratio;
2 the first and second moments of the riskless rate on short term debt;
3 the first and second moments of the risky return of equities;
4 the equity premium.

The remarkable fact is that the basic model presented in this chapter can explain all these regularities *simultaneously*. In order to do that, the parametrization needs to be specified as follows:

1 the optimists need to constitute a majority of about 57 per cent;
2 the intensity of the optimists needs to be set at 1.7542 which is approximately the maximal rate feasible;
3 Condition (2) ensures that the pessimists are more intense in that they act as if they are virtually certain at each date t that at date $t + 1$ the economy will slide into a recession;
4 the dynamics of the state of belief has to be such that the market prices cannot rise directly from the crash-recession levels to the highest prices of the bull market but it can crash from the high prices to the crash prices.

In sum, the RBE theory presented here offers a very simple explanation for the observed low average riskless rate of around 1 per cent and a high equity premium

Table 9.4 Real rates of return on debt instruments (Segal, 1994)

	On short term government (%)	*On long term government (%)*
1802–1870	5.1	4.8
1871–1925	3.2	3.7
1926–1997	0.6	2.0

of about 7 per cent. The theory proposes that such a pattern arises as a consequence of the diversity of beliefs in our financial markets when the majority of traders are optimistic but where there is always a minority of intense pessimists. The identity of these agents changes at all times since no rational agent is always optimistic or always pessimistic. This distribution of beliefs has two drastic consequences. First, it causes our financial markets to "overshoot" in the sense of experiencing much larger fluctuations of prices than could be explained by exogenous, fundamental, factors. Second, and this is the main conclusion of this section, the high intensity of the pessimists is the decisive factor which, in the long-run, dominates the market for short-term debt instruments. These are the agents who push the riskless rate down and the equity premium up. This ability of the theory of Rational Beliefs to provide this explanation of the empirical evidence is a central dimension of the unified paradigm proposed in this chapter. That is, our explanation of the empirical evidence flows directly from the conditions of rationality of the agents since the crucial asymmetry of the argument which grants the pessimists the greater intensity is a direct consequence of the rationality conditions.

A final observation regarding the historical record is of interest. There is some evidence that the riskless rate has exhibited a rather irregular pattern over the last 200 years. Table 9.4, drawn from data provided in Siegel (1994), shows that the very low average rate of return of less than 1 per cent on riskless debt instruments is a phenomenon which occurred mostly after the great depression. Indeed, Siegel (1994) shows that the large spread between rates of return on long-and short-term government debt instruments opened up exactly around 1930 and remained high until 1997. I might caution the reader that historical data prior to World War I are subject to large errors and could be interpreted in many different ways. Suppose, however, that Siegel (1994) is correct in identifying the data on the riskless rate. In that case, it appears that the 1930 depression has something to do with the low riskless rate. But such a fact provides further support for the theory offered in this chapter since this may establish the fact that the pessimists in my RBE model based their pessimism on the experience of the 1930s. This does not mean that the probability which the pessimists attached to capital losses are exactly the empirical frequencies of the great depressions. These empirical frequencies are part of the average historical record in the matrix Γ. Rationality of belief permits the pessimists to hold a probability F_2 or G_2 which do not correspond to any specific empirical frequency. However, it is the great depression that may have been responsible for the *nature* of the RBE which we have been discussing all along.

NOTES

1 Models of "Noisy" rational expectation equilibria have also attempted to address this problem within the rational expectations paradigm. In these models the noise in prices is assumed to be generated by the erratic trades of "noise traders" who are uninformed and irrational traders constituting a significant proportion of all traders in the market. I do not review this work in the present paper since it stands in sharp contrast to the basic rationality postulates of that paradigm. That is, since all the conclusions of a model of noisy rational expectations are driven by the arbitrary market actions of irrational traders, such a model should be viewed as a theory of irrational behavior with which one can prove anything. Also, from the empirical perspective it is hard to see who these noise traders are and since on average they lose money it is not clear what makes such traders survive.

2 Kurz (1997) *Endogenous Economic Fluctuations: Studies in the Theory of Rational Beliefs*. Studies in Economic Theory No. 6, Berlin and New York: Springer-Verlag. The introductory Chapter 1 (Kurz, 1997a) and the "Applications" Part B consisting of Chapters 9, 10, 11 and 12 contain the details which explain the ideas and support the results reported in the present paper.

3 This component of market uncertainty is called *Endogenous Uncertainty* in Kurz (1974).

4 And assuming expected utility maximization with preferences which are state independent.

5 See Savage (1954: 9).

6 See Arrow (1971: 20).

7 For a description of the exogenous state see Debreu (1959: 98).

8 For this reason the assumption is sometimes called "conditional perfect foresight."

9 I have introduced this term (see Kurz (1994a)) in order to distinguish knowledge about the *state of the economy* which is "information" and knowledge about *the functioning of the economy* which I call "structural knowledge".

10 All numerical results for the domestic economy are developed in Kurz and Schneider (1996) in Kurz and Beltratti (1997) and Kurz and Motolese (2001) who utilize similar models. The results for the international economy are in Kurz (1997b) and Black (1997).

11 The choice of the equilibrium dynamics being generated by a fixed, stationary matrix is a matter of convenience and simplicity. The process $\{(d_t, y_t^1, y_t^2), t = 1, 2, \ldots\}$ could have been selected to be any stable process with a Markov stationary measure induced by the empirical distribution. In such a case the fixed transition matrix Γ would characterize only the *stationary measure* of the equilibrium dynamics rather than be the matrix of the *true* probability of the equilibrium dynamics of prices.

12 For more details about the nature of GARCH and related processes see Bollerslev *et al.* (1992, 1994).

13 The debate regarding "The Equity Premium Puzzle" was initiated by Mehra and Prescott (1985). A sample of other papers on the subject include Mankiw (1986), Reitz (1988), Weil (1989), Epstein and Zin (1990), Constantinides (1990), Campbell and Cochrane (1999), Brennan and Xia (1998) and Abel (1999).

14 See, for example, the papers by Brock (1993, 1996), Durlauf (1993, 1994) and Föllmer (1974).

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10 Incentives and the stock market in general equilibrium

Michael Magill and Martine Quinzii

10.1 RECENT TRENDS IN GENERAL EQUILIBRIUM THEORY

The objective of general equilibrium theory is to understand how the complex structure of contractual markets, characteristic of a modern economy, provides a mechanism for agents (consumers, firms and government) to coordinate their decisions, share their risks and create appropriate incentives, in an evolving intertemporal setting with uncertainty. The basic skeleton on which the theory is constructed is the classical theory, first envisioned as the felicitous "invisible hand" of Adam Smith (1776), enriched by the theoretical fabric contributed by Walras (1874) and Pareto (1909), and elegantly transformed into a mathematical framework some two hundred years later in the Arrow–Debreu theory (Debreu, 1959). This theory provides a highly idealized, abstract model of markets working at their best: the nature of the markets and the underlying contracts envisioned is austere and idealized in the extreme. For the Arrow–Debreu theory conceives a fictitious initial moment of time where all agents that are to live for the indefinite future assemble together to exchange contractual commitments, fully aware of all possible future scenarios, and fully confident that all the contractual commitments will be delivered in the future. The agents look up into the future – expressed as an immense date-event tree of possible scenarios, spelled out with meticulous detail and agreed upon by all agents – and in truly Olympian fashion, trade a complete set of Arrow–Debreu contracts, that rich family of promissory notes, each committing to deliver a good of carefully defined quality and characteristics at some future date-event: a truly grandiose thought experiment of uniquely ambitious proportions in the Social Sciences. The model maps all goods at all future date-events into the present, and assembles all future and present generations of agents onto a grand theatrical stage: it is clear that the model cannot be taken literally as a description of reality – indeed some would argue that the whole problem with the model is that it is pure theater – this in essence is the argument of Gintis in this volume.

This is unfortunate for a fundamental insight of the Arrow–Debreu theory that the co-ordination of productive activities and the exchange of goods and services between agents in an economy is effected through *contracts*, i.e. promises made by one party to another to deliver a specified stream of goods and services from some initial date (the date of issue) until some specified future date (the date of maturity). Precisely because everything about the economy is so perfect – all agents are very knowledgeable and symmetrically informed, no agent ever acts opportunistically or defaults on his promises, being monitored by a perfect, costless legal system – society's problem of resource allocation can be solved by a very simple system of contracts all issued and signed at the initial date. The central insight of the model is that the resulting equilibrium outcome is "best" in a precise sense – being Pareto optimal – and every Pareto optimal allocation can always be achieved by such a complete system of Arrow–Debreu contracts.

The development of general equilibrium theory over the last 30 years can be viewed as an attempt to introduce various "imperfections" into the Arrow–Debreu description of an economy, for example, missing markets and asymmetry of information, retaining the idea that exchange and production is effected through contracts – but this time contracts that conform more closely with what we observe in the real world, including spot contracts and financial contracts such as bonds, equity and insurance. Building a theory which is consistent from top to bottom is a very difficult task. For example, we have the intuition that when agents have limited knowledge about the actions and characteristics of other agents, and when the possibility arises that agents will behave opportunistically, then the Arrow–Debreu system essentially becomes unworkable. Agents would flee to the relative safety of spot markets where delivery is assured, making only limited forward trades through financial markets to smooth their income and diversify their risks. This explains why the subsequent generation of general equilibrium models has focused on sequential models, where agents trade at each date on spot markets and make limited forward commitments through financial markets.

One of the interesting properties of models with incomplete markets and asymmetry of information, is that they can often be viewed as constrained versions of an Arrow–Debreu model – and for analytical purposes studying them in this form usually leads to the most tractable mathematical formulation. As a result, even though we do not use the Arrow–Debreu model directly as a descriptive model for the reasons just indicated, its canonical mathematical structure, and the tools and methods which were developed for analyzing it, reach far beyond the confines of the original model. This perhaps explains why there is still much active research on developing techniques for versions of the Arrow–Debrue model in relatively abstract settings – for example, with infinite dimensional commodity spaces (Mas-Colell and Zame, 1991; Shannon, 1999) or in finance models without non-negativity constraints on consumption (Werner, 1986; Page and Wooders, 1996), and Chichilnisky (1997) whose research is presented in this volume.

10.1.1 Sequential models

A significant part of the research on general equilibrium models in the last 30 years has been devoted to studying *sequential* models – the Arrow–Debreu assumption that all contracts are traded at an initial date being replaced by the more realistic assumption that there is trade at each date-event. Once the sequential nature of trade is admitted, two rather different ways present themselves for closing the model to obtain an equilibrium concept. To trade on markets today, agents must form expectations regarding prices and outcomes tomorrow. In a *temporary equilibrium* every agent is assumed to have an exogenously given, essentially arbitrary, expectation function regarding prices and outcomes tomorrow, and prices are sought which satisfy the minimal property of clearing the markets of today. When tomorrow arrives, the prices which clear the markets will typically prove that agents' expectations in the previous period regarding future prices were false (see Grandmont, 1977, 1982). The economy thus stumbles, as it were, through a sequence of false expectations. At the other end of the spectrum agents are assumed to find themselves in a market environment where they understand a great deal about what is going on: in a *correct (or rational) expectations equilibrium* agents correctly anticipate outcomes and prices tomorrow: not only do current prices clear the markets of today, but also the anticipated prices will clear the markets tomorrow in every possible contingency (see Radner, 1972, 1982).

The temporary equilibrium model was studied extensively in the 1970s and was viewed as a promising candidate for integrating Keynesian macroeconomic ideas with general equilibrium theory. The analytical and conceptual difficulties encountered in obtaining a more satisfactory theory of expectation formation, and the rather arbitrary nature of the results obtained when agents have exogenously given expectations, led the temporary equilibrium model to be essentially abandoned. This is unfortunate since the spirit of the equilibrium concept, namely that agents can hold different beliefs regarding future prices and payoffs, and that such differences of opinion provide an important motive for trade on financial markets, is surely correct as a stylized fact.[1] In the language of Wall Street, a trader is at any moment either a bull or a bear, and the course of prices is importantly influenced by the proportion of bulls to bears. In an important recent contribution, Kurz (1994a,b, 1997) has introduced a concept of equilibrium with rational beliefs which permits agents to trade with such differences in beliefs: this equilibrium concept lies half way between a temporary and a rational expectations equilibrium – for rational beliefs are not required to be correct but they are not arbitrary either, since they must be consistent with the realized frequencies of events as revealed in past data (see Kurz's contribution in this volume).

It is the concept of rational expectations equilibrium, however, which has become the predominant equilibrium concept for sequential models since the mid 1970s. The idea that rational agents trading on forward-looking (speculative) markets will use available information to predict the future, and that they are not likely to be fooled into making systematic mistakes in their predictions, is compelling – and it is perhaps no accident that the areas of economics in which the concept of rational expectations has been felt to provide a good first approximation are precisely those in which there is most money at stake for those willing to make intelligent forecasts: the *efficient markets hypothesis* dominates all the modern theory of equilibrium on financial markets, and the idea that agents make use of their information regarding government monetary policy in forming their expectations of inflation has become a basic tenet of the rational expectations school of macroeconomics. We take no stand here as to the correctness or incorrectness of such models: what is certain, is that models with rational expectations have provided important new insights into the anticipatory behavior of security prices, and on the possible ineffectiveness of monetary policy in settings where agents can anticipate the monetary authority's future actions.

The hypothesis of rational expectations is an equilibrium concept which depends both on individual behavior (how agents form their expectations and make their decisions) and on market clearing (the determination of prices). It presupposes a setting where agents make informed predictions of future prices – correct predictions based on a correct model of the economy. The subsequent market clearing prices that arise from the decisions based on these expectations confirm their predictions. A rational expectations equilibrium is essentially the only equilibrium concept which is consistent in the sense that agents' expectations are self-fulfilling.

The basic building block for trade in a sequential general equilibrium model with uncertainty is the system of *spot markets* at the nodes of the underlying event-tree. As time and uncertainty unfold, the economy walks, as it were, along a path through the event-tree and, at any given node, an agent sells his endowment at this node and purchases a vector of goods at the current spot prices. Since agents buy and sell goods on current spot markets (at each node of the event-tree) each agent faces a sequence of budget constraints, and this sequence of budget constraints is the key distinguishing feature of the sequential model. Since agents' endowments and the outputs of firms (whose ownership is distributed among the agents) are subject to shocks, the agents will typically want to redistribute their income across the nodes of the event-tree, "borrowing and lending" to smooth their income over time, and buying or selling "rights to income streams" to diversify their risks across the nodes: this of course is the canonical role of the *financial markets*, for example, the bond and equity markets, the commodity futures markets, derivative securities (puts and calls) and insurance markets. The richer the structure of these financial markets, the more readily agents can redistribute income across the event-tree: and when – at a cost – an agent can obtain any desired profile of income across the event-tree by the appropriate choice of a portfolio of the currently available securities at each node, the financial markets are said to be *complete*: roughly speaking what is required is that at each node there should be as many tradeable securities as there are immediately succeeding contingencies (the so-called *branching number* of the event-tree at that node). A sequential model with correct anticipations in which the financial markets are complete has the same equilibrium allocations as the Arrow–Debreu model (see Magill and Shafer, 1990).

The bulk of research on the sequential GE model in the 1980s and 1990s has been directed to exploring the consequences of a lack of complete markets for equilibrium allocations, a research agenda known as GEI, or *general equilibrium with incomplete markets*. In the discrete-time, discrete-state-space model which is commonly used, discontinuities in the demand for securities can occur when the security payoff matrix has a change of rank. This can happen when the payoffs depend on the spot prices (for example for futures contracts) or if the securities are long-lived so that the per-period payoff involves capital value terms (for example for equity) – and the discontinuities in demand can create non-existence of equilibrium. In essence every sequential GE model that involves either more than one good or more than two periods has a potential problem with non-existence of equilibrium. This problem, first uncovered by Hart (1975), discouraged work in the area for almost 10 years: the problem was solved in the mid-1980s when it was proved that an equilibrium exists "generically," that is, for almost all endowments and security structures (Duffie and Shafer, 1985; Hirsch *et al.*, 1990; Magill and Shafer, 1991). This result was a high point of mathematical economics since it involved beautiful mathematical arguments drawing on the powerful techniques of differential topology.[2] It established that the problem of non-existence would typically only arise for "exceptional" parameter values, so that research could justifiably focus on establishing qualitative properties of the equilibria.

10.1.2 Nominal assets and money

One of the striking attributes of the Arrow–Debreu model is that it describes the economic world entirely in real terms: there is no role and hence no room for money to affect the equilibrium outcome. As Hahn (1971, 1973a,b) has convincingly argued, any attempt to introduce money into a general equilibrium model must be placed in the setting of a sequence economy. Once we enter a world of sequential markets, new possibilities arise for exploring the way money enters into the determination of an equilibrium. Modeling money in a satisfactory way in a general equilibrium model is a notoriously difficult task, and we are far from having even the elements of a monetary general equilibrium theory. Money is an asset which can be held like the other financial securities in a GEI model: however, a distinct function must be assigned to money relative to the other financial assets if agents are to be induced to hold it, since typically its rate of return is dominated by those on other assets. The characteristic role of money, which distinguishes it from other assets, is that it serves as a medium of exchange. Magill and Quinzii (1992) have shown how, by introducing an appropriate transactions technology which formalizes Clower's idea that "money buys goods and goods buy money, but goods cannot buy goods," a real GEI equilibrium can be transformed into a monetary GEI equilibrium, in which a monetary authority decides at each date-event how much money to inject into the economy for use for transactions purposes. In essence a new equation is added at each node of the event-tree – over and above the equations equating demand and supply for each of the goods and each of the securities – which expresses equality between the demand for money for transactions purposes and the supply made available by the monetary authority. These quantity-theory equations determine the price-level at each node of the event-tree.

Once such a transactions technology has been introduced, it becomes useful to distinguish between two types of financial securities. We say that a security is *real* if its payoff at any node is a linear function of the spot prices at this node: this is true for a commodity futures contract or an equity contract. A security is said to be *nominal* if its payoff at a node is an amount of money which is independent of the spot prices – most bonds are nominal assets. An important property of a real security is that it is inflation-proof – if the price-level doubles at some node then its payoff at that node doubles – while for a nominal security, when the price-level doubles, its purchasing power is cut in half. Monetary policy is said to be *neutral* if changing the monetary policy leaves the real equilibrium allocation unchanged. Building on results on the indeterminacy of an equilibrium allocation when nominal securities are introduced into an otherwise real GEI model (Balasko and Cass, 1989; Geanakoplos and Mas-Colell, 1989), the following properties of a monetary equilibrium can be established (see Magill and Quinzii, 1992, 1996): if all securities are real, then regardless of the degree of incompleteness of the markets, monetary policy is neutral; if there are nominal securities and the financial markets are *complete*, then perfectly anticipated monetary policy is also neutral – in essence when agents anticipate a different monetary policy they rearrange their portfolios so as to offset the changed purchasing power of the nominal securities across the date-events; however, if the financial markets are *incomplete* and there are nominal securities, then monetary policy, even when perfectly anticipated, is non-neutral. In essence, when monetary policy is changed agents do not have enough instruments at their disposal to compensate for the change in the purchasing power of the nominal securities across the date-events. This latter result can in turn be used to establish some simple stylized properties of an "optimal" monetary policy: for example, it can be shown that, in an economy without fundamental uncertainty, monetary policy should not introduce "monetary shocks," i.e. be a new source of uncertainty for the agents in the economy.

Of course, it might be asked: why would agents ever want to hold nominal securities since it exposes them to fluctuations in the purchasing power of money? This is an old puzzle in monetary theory which has long seemed difficult to explain.[3] The GEI model of a monetary equilibrium can be used to throw light on this question. For once it is recognized that real shocks create fluctuations in the relative prices of goods across date-events, then we can understand why "indexing," while it eliminates the risks arising from fluctuations in the price level, necessarily introduces a new risk, namely that arising from the fluctuations in the *relative prices* of goods. Magill and Quinzii (1997) show that in an economy exposed to fluctuations in relative prices arising from normal technological and supply or demand shocks, then there is a critical level of the variability of purchasing power of money such that agents will prefer to hold a nominal rather than an indexed bond if the fluctuations in the purchasing power of money are less than this critical level – basically agents prefer a small exposure to price level fluctuations to the greater risk arising from changes in the relative prices of the goods in the index. Of course, when as in a number of Latin American countries, such as the Argentine or Brazil, fluctuations in price levels exceed the critical level, then agents prefer the indexed bond. These results show in a clear way some of the useful insights that can be obtained from the monetary version of the GEI exchange model.

The sequential model provides a natural setting for studying contracts – that is, promises to deliver a stream of goods or services from the date the contract is signed to its date of maturity. The financial contracts mentioned above represent a subset of these contracts, but the model could be greatly enriched by including a much broader array of contractual agreements between firms to deliver or accept delivery of goods, as well as labor contracts promising to render labor services. To the extent that such contracts embody nominal elements, fluctuations in monetary policy will have even more pervasive real effects in a production economy.

10.1.3 Understanding incompleteness of markets

A property of the GEI model which has received much attention in the recent literature is the inefficiency property of equilibrium with incomplete markets. More precisely, as soon as we move out of the setting of a two-period one-good model (the basic model of a finance economy) into a world with more than two periods or more than two goods, if the markets are incomplete, not only is an equilibrium not typically Pareto optimal – which is to be expected if some markets are missing – but more surprisingly it is not even *constrained Pareto optimal* (Stiglitz, 1982; Geanakoplos and Polemarchakis, 1986; Geanakoplos *et al.*, 1990). This means that a planner, even if forced to respect the limited availability of instruments for transferring income across the date-events imposed by the incompleteness of the markets, can find better consumption, portfolio and production decisions than those which agents are induced to undertake through the sequential system of markets.

The intuition underlying this constrained inefficiency is best understood in the simplest case of a two-period exchange economy with many goods: if the planner changes agents' portfolios, then the agents will change their planned trades on the spot markets at the next date, causing the spot prices to change. If the financial markets are complete, then, since agents' rates of substitution are equalized at equilibrium, the increased utility of the winners from the price change will exactly compensate the decreased utility of losers; however, when markets are incomplete, rates of substitution typically differ and a change of portfolios can be found such that, under the Hicks–Kaldor criterion, the winners can compensate the losers, and there is social gain to the planner's intervention. In a multiperiod setting, or in a model with production, the argument is a bit more involved, but the intuition is the same: a change in decisions taken at date t influences the income distribution and the supply of goods at date $t + 1$, and thus the prices of the goods and securities at date $t + 1$. As a result, when rates of substitution differ because of missing markets, the planner can find a small reallocation such that the winners can compensate the losers. In a market economy, there are spillover effects across markets which by definition (of competitive equilibrium) agents do not take into account: with incomplete markets, this leads to coordination failure in the overall system of markets.

This result poses rather serious problems of interpretation for the GEI model: should it be interpreted to mean that government intervention is called for as soon as there are incomplete markets? This seems unlikely. For to be successful, such intervention would require the planner to have access to enormous amounts of information on agents' and firms' characteristics – information that may be difficult to come by, in view of the well-known difficulty of inducing truthful revelation of preferences by consumers and technology by firms, not to speak of the massive calculation problem involved. Would it not be better for the government to seek to "complete" the markets rather than intervening in an incomplete markets environment? To answer questions of this kind, the structure of the model and the possible sources of the incompleteness of markets need to be more explicitly specified. For example, the private sector does not offer insurance against unemployment – fundamentally because of the problem of moral hazard and adverse selection involved in a labor contract. The government steps in, but offers only limited insurance since it also has to cope with the underlying incentive problem. To enrich the model and suggest appropriate normative analysis, we need to model more directly the factors which cause markets to be incomplete.

Contracts theorists often object to general equilibrium models on the grounds that they do not take into account the problems posed by asymmetry of information and by strategic (especially opportunistic) behavior in the contracting process that supports the exchange and production activities (see Gintis's critique of general equilibrium). Much of contract theory is essentially bilateral and strategic, and hence outside the normal purview of markets – or is at most set in highly simplified partial equilibrium models. Perhaps the line of division between the general equilibrium and contract theory camps has been drawn too severely and naively: the two approaches are complementary and a more productive research agenda would be to seek ways of reconciling the two approaches, retaining the competitive assumption of standard price-taking behavior with perfect information whenever it is a reasonable approximation, while invoking strategic behavior whenever asymmetries of information or differences in bargaining strengths of the parties make the standard competitive assumption inappropriate.

10.1.4 Adverse selection and moral hazard in GEI

Some progress has recently been made in modeling the functioning of financial markets in a general equilibrium setting when agents act opportunistically. As soon as the contracts which are traded promise delivery of goods or income in the future, there is a possibility that agents may renege on their promises. One of the important functions of the legal system is to ensure that contracts are respected – and the standard GEI model assumes that agents only make promises which they can and do fulfill. This however is clearly an idealization, for there are limits to what a legal system can achieve at reasonable cost in enforcing the repayment of debts. Moreover, even if it were possible to monitor agents' actions, it would not always be socially desirable to enforce complete repayment of debts, since a limited tolerance for default encourages innovation and risk taking. In this volume, Geanakoplos presents recent research with his co-authors on the modeling of strategic default in a GEI model, under various assumptions on the enforcement mechanism – for example, utility penalties imposed by the legal system and/or a collateral requirement. The analysis is based on the assumption that there is a large number of anonymous buyers and sellers on each contract, and that all buyers of "promises" (lenders) can foresee correctly the average repayment rate of the sellers (borrowers), given the enforcement mechanism.

Despite the presence of a moral hazard created by the possibility of default, the contracts which are traded are *anonymous* and the risks are implicitly pooled by intermediaries. In a related paper, Bisin–Gottardi (1999) study the functioning of markets for anonymous insurance contracts in the presence of moral hazard or adverse selection. In these models, intermediaries pool risks for investors, so that the equilibrium price of each contract only depends on the average (aggregate) behavior (default rate, accident rate) which results from the strategic behavior of many individuals. Since the underlying equilibrium concept only exploits a minimal amount of information concerning the actions or characteristics of the agents, the resulting allocation can only hope to have limited efficiency properties.

Our analysis, which we outline in the remainder of the chapter, has focused on the other extreme of *named contracts* where the issuer of the contract for which there is a problem of a moral hazard is a single legal entity, namely a corporation. In this setting, the buyers of the contract have access to much more detailed information regarding the actions of the corporation which influence the payoff of its security, and as a result we show that the market acts as a disciplining device, attenuating the problem of moral hazard: the equilibrium price, instead of reflecting the average behavior of a pool of agents, becomes a complex equilibrium pricing function which reveals to the corporation the consequences for the price of its security of a whole family of out-of-equilibrium actions which it could have taken.

Introducing the moral hazard problem of corporate management into a general equilibrium framework permits two opposing views on the merits of the stock market to be studied in a common framework. In one view, the merit of the stock market is that it permits the substantial production risks of society to be diversified among many investors: this view underlies the capital asset pricing model (CAPM) which forms the basis for much of the modern theory of finance. On the other hand, the traditional view of classical economists (Adam Smith, 1776; Mill, 1848; Marshall, 1890), revived in modern times by Berle and Means (1932), Jensen and Meckling (1976) and the ensuing agency-cost literature, has emphasized the negative effect on incentives of the separation of ownership and control implied by the corporate form of organization. The objective of the analysis which follows is to show how these two approaches can be captured in a general equilibrium model, and to discover the circumstances under which capital markets can provide an optimal trade-off between the beneficial effect of risk diversification and the distortive effect on incentives.

10.2 INCENTIVES VS. RISK-SHARING IN CAPITAL MARKETS

10.2.1 The basic model

To capture the dual role of capital markets in affecting risk sharing and incentives, we consider a simple general equilibrium model of a finance economy with production. In the spirit of Knight (1921), we model the firm as an entity arising from the organizational ability, foresight and initiative of an *entrepreneur*. The activity of a firm consists in combining entrepreneurial effort and physical input (the value of capital and non-managerial labor) at an initial date: this gives rise to a random profit stream at the next date. In addition to entrepreneurs there is another class of agents which we call *investors*: they have initial wealth at date 0 but no productive opportunities.

More formally consider a two-period one-good economy with production in which investment of both capital and effort at date 0 is required to generate output at date 1, the output at date 1 being uncertain. There are two types of agents in the economy, *entrepreneurs* and *investors*: $\mathcal{I}_1 \neq \emptyset$ is the set of entrepreneurs, $\mathcal{I}_2 \neq \emptyset$ the set of investors and $\mathcal{I} = \mathcal{I}_1 \cup \mathcal{I}_2$ is the set of all agents, which is assumed to be finite.[4] Every agent $i \in \mathcal{I}$ has an initial wealth $\omega_0^i > 0$ at date 0. If agent i is an entrepreneur, then by investing capital $\kappa^i \in \mathbb{R}_+$ (an amount of the good (income)) and effort $e^i \in \mathbb{R}_+$ at date 0 he can obtain the uncertain stream of income

$$\boldsymbol{F}^i(\kappa^i, e^i) = (F_1^i(\kappa^i, e^i), \ldots, F_S^i(\kappa^i, e^i))$$

at date 1, where $\mathcal{S} = \{1, \ldots, S\}$ is the set of states of nature describing the uncertainty and $\boldsymbol{F}^i: \mathbb{R}_+^2 \longrightarrow \mathbb{R}_+^S$ is a concave, differentiable, non-decreasing function, with $\boldsymbol{F}^i(\kappa^i, 0) = \boldsymbol{F}^i(0, e^i) = 0$. Investors are agents who do not undertake productive ventures, so if $i \in \mathcal{I}_2$, then $\boldsymbol{F}^i(\kappa^i, e^i) \equiv 0$.

Each agent has a utility function $U^i\colon \mathbb{R}_+^{S+1} \times \mathbb{R}_+ \longrightarrow \mathbb{R}$, where $U^i(\boldsymbol{x}^i, e^i)$ is the utility associated with the consumption stream $\boldsymbol{x}^i = (x_0^i, x_1^i, \ldots, x_S^i)$ and the effort level e^i. The utility function is assumed to be separable

$$U^i(\boldsymbol{x}^i, e^i) = u_0^i(x_0^i) + u_1^i(x_1^i, \ldots, x_S^i) - c^i(e^i),$$

where the functions u_0^i, u_1^i are differentiable, strictly concave, increasing, and c^i is differentiable, convex, increasing, with $c^i(0) = 0$.

To ensure that the technology of each entrepreneur i is sufficiently productive to make it worthwhile to operate, we assume that, as soon as entrepreneur i invests some positive effort in his firm, the marginal productivity of capital at zero is infinite. More precisely we assume that for all $i \in \mathcal{I}_1$ there is a smooth path $e^i\colon [0,1] \longrightarrow \mathbb{R}_+$ with $e^i(0) = 0$ and $e^{i\prime}(t) > 0$ such that

$$\lim_{t\to 0} c^{i\prime}(e^i(t))e^{i\prime}(t) < \infty \quad \text{and} \quad \lim_{t\to 0} \frac{\partial F_s^i}{\partial z^i}\left(t, e^i(t)\right) = \infty, \quad \text{for some } s \in \mathcal{S}.$$

It is easy to see that this assumption implies that for all $\boldsymbol{x}^i = (x_0^i, \boldsymbol{x}_1^i) \in \mathbb{R}_+^{S+1}$ with $x_0^i > 0$, there exists $(\kappa^i, e^i) \gg 0$ such that

$$u_0^i(x_0^i - \kappa^i) + u_1^i(\boldsymbol{x}_1^i + \boldsymbol{F}^i(\kappa^i, e^i)) - c^i(e^i) > u_0^i(x_0^i) + u_1^i(\boldsymbol{x}_1^i),$$

so that even if there were no market to finance the capital investment, entrepreneur i would choose to produce. To bound the economy we assume that, for any positive level of capital input, the marginal cost of effort eventually exceeds its marginal product

$$\frac{\partial \boldsymbol{F}^i(\kappa^i, e^i)}{\partial e^i} \to 0, \quad \text{and} \quad c^{i\prime}(e^i) \to \infty \quad \text{when } e^i \to \infty, \quad i \in \mathcal{I}_1.$$

This implies that, for a given level of capital, the effort chosen by entrepreneur i will always remain bounded.

10.2.2 An ideal world: sole proprietorship with Arrow securities

In this model, there is a moral hazard problem when two imperfections are present simultaneously. The first is that *effort is not observable*, so that contracts cannot be written contingent on the effort invested by entrepreneurs in their firms. This would not create a problem without the constraint that *states of nature are not verifiable*, so that the enforcement of contracts contingent on states is not feasible.

To see this, let us imagine an ideal world in which financial contracts could be written contingent on the states of nature, so that the securities have payoffs which are independent of the agents' actions, and suppose in addition that such contracts are complete – in short that there is a complete set of Arrow securities. Then there would be a simple way of obtaining a Pareto optimal(PO) allocation, despite the non-observability of effort. It would suffice to let each entrepreneur be the *sole proprietor* of his firm so that he has both the full marginal benefit and cost of his effort and there is no distortion of incentives: to adjust the risk profile of his income stream, he trades Arrow securities whose payoffs are independent of his actions. To make these statements precise, let us introduce the concept of a sole-proprietorship equilibrium with Arrow securities. Letting the price of income at date 0 be normalized to 1 and letting π_s denote the price (at date 0) of the Arrow security for state s (which delivers one unit of income in state $s \in \mathcal{S}$), the budget set of agent i with Arrow securities and sole proprietorship is given by

$$B(\boldsymbol{\pi}, \omega_0^i, \mathbf{F}^i) = \left\{ (\boldsymbol{x}^i, e^i) \in \mathbb{R}_+^{S+2} \middle| \begin{array}{l} x_0^i = \omega_0^i - \boldsymbol{\pi}\boldsymbol{\zeta}^i - \kappa^i \\ \boldsymbol{x}_1^i = \boldsymbol{\zeta}^i + \boldsymbol{F}^i(\kappa^i, e^i) \\ (\kappa^i, \boldsymbol{\zeta}^i) \in \mathbb{R}_+ \times \mathbb{R}^S \end{array} \right\},$$

where $\boldsymbol{\pi} = (\pi_1, \ldots, \pi_S)$ and $\boldsymbol{\zeta}^i = (\zeta_1^i, \ldots, \zeta_S^i)$ is agent i' portfolio of the Arrow securities. As usual, the $S + 1$ budget constraints with Arrow securities can be reduced to a single budget constraint, i.e. the set $B(\boldsymbol{\pi}, \omega_0^i, \boldsymbol{F}^i)$ can be written as

$$B(\boldsymbol{\pi}, \omega_0^i, \boldsymbol{F}^i) = \left\{(\boldsymbol{x}^i, e^i) \in \mathbb{R}_+^{S+2} | x_0^i + \boldsymbol{\pi}\boldsymbol{x}_1^i = \omega_0^i + \boldsymbol{\pi}\boldsymbol{F}^i(\kappa^i, e^i) - \kappa^i\right\}. \tag{10.1}$$

This is the budget set of an agent with contingent markets for income and sole proprietorship, expressing equality of the present value of the agents' lifetime expenditure and income.

Definition 1 $(\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{e}}; \bar{\boldsymbol{\pi}})$ is a sole-proprietorship equilibrium with Arrow securities (SPA) if

(i) $(\bar{\boldsymbol{x}}^i, \bar{e}^i, \bar{\kappa}^i) \in \arg\max \left\{U^i(\boldsymbol{x}^i, e^i) \,\middle|(\boldsymbol{x}^i, e^i) \in B(\bar{\boldsymbol{\pi}}, \omega_0^i, \boldsymbol{F}^i)\right\}$ and $\bar{\boldsymbol{y}}^i = \boldsymbol{F}^i(\bar{\kappa}^i, \bar{e}^i)$, $i \in \mathcal{I}$.

(ii) $\sum_{i\in\mathcal{I}} x_0^i = \sum_{i\in\mathcal{I}} \omega_0^i - \sum_{i\in\mathcal{I}} \kappa^i$, $\sum_{i\in\mathcal{I}} \boldsymbol{x}_1^i = \sum_{i\in\mathcal{I}} \boldsymbol{F}^i(\kappa^i, e^i)$.

An SPA is not precisely an Arrow–Debreu equilibrium, since there are $S + 1 + I_1$ "goods" in the economy – the $S + 1$ incomes at dates 0 and 1, and the I_1 effort levels of the entrepreneurs – but there are only $S + 1$ markets. Despite the absence of the I_1 markets for the effort levels of entrepreneurs, the first and second welfare theorems – as well as the existence of equilibrium – are satisfied by SPA equilibria. This is due to the following two properties of "Robinson Crusoe" economies:

(i) An agent who is both a producer and a consumer in a convex economy can be split into two "personalities": an entrepreneur who maximizes profit and a consumer who takes the profit as given and maximizes utility over his budget set (see Magill and Quinzii (1996) for an account of this property in a general framework).
(ii) Agent i as the entrepreneur running firm i buys the input "effort for firm i" from only one agent, himself as a consumer. The market for effort e^i can thus be "internalized" in the joint consumer–producer maximum problem of agent i in a SPA. Any other ownership structure of the firm would fail to lead to Pareto optimality in the absence of a market for effort.

PROPOSITION A (Properties of SPA equilibrium).

(i) *For any $\omega_0 \in \mathbb{R}^I_+, \omega_0 \neq 0$, there exists an SPA equilibrium.*
(ii) *If $(\bar{x}, \bar{y}, \bar{\kappa}, \bar{e}; \bar{\pi})$ is an SPA equilibrium, then the allocation $(\bar{x}, \bar{y}, \bar{\kappa}, \bar{e})$ is Pareto optimal.*
(iii) *For any Pareto optimal allocation $(\bar{x}, \bar{y}, \bar{\kappa}, \bar{e})$ there exist incomes $\omega_0 \in \mathbb{R}^I$ and state prices $\bar{\pi} \in \mathbb{R}^S_{++}$ such that $(\bar{x}, \bar{y}, \bar{\kappa}, \bar{e}; \bar{\pi})$ is an SPA equilibrium.*

The existence proof is standard: to prove the equivalence between PO allocations and SPA equilibria in the differentiable case it suffices to note that the first-order conditions (FOC) for Pareto optimality are the same as the FOC for the maximum problems of the agents in an SPA equilibrium:

$$\frac{\partial u_1^i(\bar{x}_1^i)}{\partial x_s^i} \Big/ u_0^{i\prime}(\bar{x}_0^i) = \bar{\pi}_s, \quad c^{i\prime}(e^i) = \sum_{s \in \mathcal{S}} \bar{\pi}_s \frac{\partial F_s^i}{\partial e^i}(\bar{\kappa}^i, \bar{e}^i), \quad 1 = \sum_{s \in \mathcal{S}} \bar{\pi}_s \frac{\partial F_s^i}{\partial \kappa^i}(\bar{\kappa}^i, \bar{e}^i), \quad i \in \mathcal{I}.$$

In both cases the problems are convex so that the FOC are necessary and sufficient.

Proposition A asserts that in an ideal world where states of nature are verifiable, society's problem of resource allocation – even when faced with the problem of non-observability of effort of the entrepreneurs who run the economy's firms – can be solved by letting entrepreneurs be sole proprietors of their firms and permitting agents to transfer income by trading Arrow securities. This structure of ownership and markets solves the twin problem of incentives and risk sharing by keeping them separate: sole proprietorship creates incentives and Arrow securities induce optimal risk sharing.

10.2.3 The real world: moral hazard with output-contingent contracts

In practice few insurance contracts based on "primitive causes" exist for sharing the risks of business. For most of the important risks which influence a firm's profits – shocks of varying magnitude to demand, to technology, to the competitive environment, and to input availability, as described in approximate terms in corporate quarterly and annual reports to shareholders – *no insurance contracts are available*. To an experienced businessman this is obvious – for it is the essence of business that such risks cannot be insured.[5] Given the difficulty of describing precisely *ex ante* and verifying accurately ex-post the precise nature of the primitive shocks, most financial contracts that are used for financing firms and sharing productive risks are either non-contingent (bonds) or based directly on the realized *outputs* of firms (equity and derivative securities, like options on equity). To reflect these restrictions imposed by the real world on the nature of the contracts that can be used for facilitating exchange and production, let us assume that only financial contracts based on the realized outputs of firms are feasible. To make the model coherent, we must however assume that investors and entrepreneurs *understand* the basic nature of the uncertainty (the shocks) to which firms are subjected.

Trading contracts contingent on realized output when there is no separate market for "effort" is liable however to lead to inefficiencies. For typically entrepreneurs need funds to finance their projects. If they only have access to borrowing (debt), they will be exposed to rather risky leveraged positions, and if there are penalties for default, then they will have to restrict the amount they borrow for fear of bad contingencies. One traditional remedy lies in introducing the possibility of financing by issuing equity: this additional source of funds permits entrepreneurs to share the risks involved in their productive activity with investors, and opens up the possibility for all agents to diversify their risks. But selling ownership shares of his firm has a negative incentive effect on the entrepreneur, since any increment to effort is no longer rewarded by the full value of its marginal product.

Debt and equity contracts however constituted only the first stage in the development of financial markets to meet the financing needs of firms: at a later stage financial markets became more sophisticated, permitting the introduction and systematic use of derivative securities. Such securities serve two roles: they increase the risk-sharing possibilities of agents (the span of the markets) and provide instruments for creating incentives for managers of firms. Holding an option which is worthless unless the profit of the firm exceeds the striking price of the option provides a strong incentive for a manager to exert the extra effort needed to assure that the profit stream is likely to surpass this level. The extraordinary increase in the use of options as incentive devices in the last 10 years in the US comes from the recognition of their great power as instruments for motivating CEO's. The use of options can potentially restore some, or, as we shall see in Section 10.5, even all of the incentives of entrepreneurs lost in reducing their equity shares to finance their firms.

10.2.3.1 The financial markets

The financial contracts which the agents in the economy can trade are thus taken to be: first, the default-free bond with (date 0) price q_0 and payoff stream $\mathbf{1} = (1, \ldots, 1)$ at date 1; second for each firm $i \in \mathcal{I}_1$, its equity contract with price q_y^i and date 1 payoff stream $\boldsymbol{y}^i = (y_1^i, \ldots, y_S^i)$; finally for each firm a family of derivative securities on its equity, consisting of call options with different striking prices. Let $\mathcal{J}^i \subset \mathbb{N}$ denote the index set for the call options on the equity of firm i, and let $\boldsymbol{\tau}^i = (\tau_j^i)_{j \in \mathcal{J}^i}$ denote the vector of associated striking prices, with $\boldsymbol{\tau}^i \in \mathbb{R}_+^{J^i}$. The call option (i, j) – the jth option of firm i – has the price q_j^i at date 0 and the payoff stream $\boldsymbol{R}_j^i = (R_{j,1}^i, \ldots, R_{j,S}^i)$ across the states at date 1 given by

$$R_{j,s}^i = \max\{y_s^i - \tau_j^i, 0\}, \quad s \in \mathcal{S},$$

where $y_s^i = F_s^i(\kappa^i, e^i)$ denotes the output of firm i in state s. When it is important to stress that the choice of (κ^i, e^i) influences the payoff of the equity and thus of the option, we use the notation

$$R_{j,s}^i(\kappa^i, e^i) = \max\{F_s^i(\kappa^i, e^i) - \tau_j^i, 0\}, \quad s \in \mathcal{S}.$$

Let $\boldsymbol{R}^i$ (or $\boldsymbol{R}^i(\kappa^i, e^i)$) denote the $S \times J^i$ matrix of payoffs of the J^i options of firm i, $\boldsymbol{R}_s^i$ the row vector of payoffs of the J^i options on firm i in state s and $\boldsymbol{R}_j^i$ the column vector of payoffs of option j across the states. Let $\mathcal{J} = \cup_{i \in \mathcal{I}_1} \mathcal{J}^i$ denote the set of all options and let $\boldsymbol{\tau} = (\boldsymbol{\tau}^i)_{i \in \mathcal{I}_1}$ denote the associated striking prices. The economy with characteristics

$\boldsymbol{U} = (U^i)_{i\in\mathcal{I}}$, $\boldsymbol{\omega}_0 = (\omega_0^i)_{i\in\mathcal{I}}$, $\boldsymbol{F} = (\boldsymbol{F}^i)_{i\in\mathcal{I}_1}$ for the agents, and with a market structure composed of the riskless bond, the equity contracts of the I_1 firms, and the set of options $\mathcal{J}$ with striking prices $\boldsymbol{\tau}$, will be denoted $\mathcal{E}(\boldsymbol{U},\boldsymbol{\omega}_0,\boldsymbol{F},\boldsymbol{\tau})$. In such an economy, we let $\boldsymbol{q}_y = (q_y^i)_{i\in\mathcal{I}_1}$ denote the vector of equity prices, $\boldsymbol{q}_c^i$ the vector of prices of the J^i call options of firm i, $\boldsymbol{q}^i = (q_y^i, \boldsymbol{q}_c^i)$ the vector of prices which are influenced by the actions of entrepreneur i, and $\boldsymbol{q} = (q_0, (\boldsymbol{q}^i)_{i\in\mathcal{I}})$ the vector of all security prices.

10.2.3.2 The agents' accounts

To simplify the analysis we assume that the penalty for default for individual agents and for bankruptcy by firms is infinite, so that the personal debt of an entrepreneur and the debt incurred by his firm are both default-free debt. With no default and no bankruptcy there is no loss of generality in assuming that the entrepreneur is personally responsible for the debt of his firm: thus the accounts of the firm and the entrepreneur can be lumped together, leading to a considerable simplification of the model.

At date 0 entrepreneur i decides on the amount of capital κ^i to invest in his firm, on the amount to borrow ξ_0^i (if $\xi_0^i > 0$, lend if $\xi_0^i < 0$), and on the share $(1-\theta_i^i)$ of his firm to sell. He also purchases shares θ_k^i of other firms $k \neq i$, as well as amounts $\xi_{k,j}^i$ of the options of these firms ($j \in \mathcal{J}^k, k \neq i$) to diversify his risks. The purchase of a portfolio of options $(\xi_{i,j}^i)_{j\in\mathcal{J}^i}$ on his own equity contract serves as an incentive device to "bond" his personal interest to those of the outside shareholders of his firm. Let $\boldsymbol{\theta}^i = (\theta_k^i)_{k\in\mathcal{I}_1}$, denote the equity portfolio, $\boldsymbol{\xi}_k^i = (\xi_{k,j}^i)_{j\in\mathcal{J}^k}$ the portfolio of options of firm k and $\boldsymbol{\xi}^i = (\xi_0^i, (\boldsymbol{\xi}_k^i)_{k\in\mathcal{I}_1})$ the portfolio of all securities in zero net supply (bond and options) held by agent i. If entrepreneur i anticipates the production ($\boldsymbol{y}^k$) of other entrepreneurs, then a choice of the financial variables $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$, in conjunction with a choice of effort e^i, leads to a vector of consumption $\boldsymbol{x}^i = (x_0^i, x_1^i, \ldots, x_S^i)$ given by

$$x_0^i = \omega_0^i - q_0\xi_0^i - \sum_{k\neq i} q_y^k\theta_k^i - \sum_{k\neq i} \boldsymbol{q}_c^k\boldsymbol{\xi}_k^i + q_y^i(1-\theta_i^i) - \boldsymbol{q}_c^i\boldsymbol{\xi}_i^i - \kappa^i. \tag{10.2}$$

$$x_s^i = \xi_0^i + \sum_{k\neq i} \theta_k^i y_s^k + \sum_{k\neq i} \boldsymbol{R}_s^k\boldsymbol{\xi}_k^i + F_s^i(\kappa^i, e^i)\theta_i^i + \boldsymbol{R}_s^i(\kappa^i, e^i)\boldsymbol{\xi}_i^i, \quad s \in \mathcal{S}. \tag{10.3}$$

If agent i is an investor, then the budget equations are the same with $\kappa^i = 0, e^i = 0, \boldsymbol{F}^i = \boldsymbol{0}, q_y^i = q_j^i = 0$, so that the terms related to his own "firm" are just dummy variables.[6]

It is clear from equations (10.3) that the date 1 reward of an entrepreneur for his effort depends on his choice of financial variables $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$. This captures the idea that the capital structure of a firm (in particular, the inside equity and options held by the manager, and the firm's debt) affects the performance of its management. Since financing arrangements must be in place before a firm can become operational, we assume that the choice of effort e^i by an entrepreneur is made after the financial decisions $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$ have been determined. To make this sequential structure of decision-making explicit, suppose that date 0 is divided into two subperiods, 0_1, 0_2. In subperiod 0_1 entrepreneurs use the financial markets to obtain the capital required to set up their firms and diversify their risks: in the second subperiod 0_2, after the investment and financing decisions have been made, firms become "operational" and entrepreneurs decide on the appropriate effort to invest in running their firms. At date 1 "nature" chooses a state of the world $s \in \mathcal{S}$: production takes place and profit is realized.

10.2.3.3 How an entrepreneur decides his optimal effort

After entrepreneur i has chosen his financial variables $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$ in subperiod 0_1 (in a way that we will study shortly), in subperiod 0_2 he chooses the effort level e^i which maximizes $u_1^i(\boldsymbol{x}_1^i) - c^i(e^i)$, where $\boldsymbol{x}_1^i = (x_1^i, \ldots, x_S^i)$ is the date 1 consumption stream given by equation (10.3). Entrepreneur i's financial variables are of two kinds: *inside* variables (those internal to the firm) which directly affect the payoff (reward) of the entrepreneur from his effort, and the *outside* variables (external to the firm) which determine the income that agent i gets independently of his effort. $(\kappa^i, \theta_i^i, \boldsymbol{\xi}_i^i)$ are the inside financial variables which determine his *inside income* (the last two terms in equation (10.3)), while $(\xi_0^i, (\theta_k^i, \boldsymbol{\xi}_k^i)_{k \neq i})$ are the outside variables which determine his *outside income* $\boldsymbol{m}^i = (m_1^i, \ldots, m_S^i)$ defined by

$$\boldsymbol{m}^i = \mathbf{1}\xi_0^i + \sum_{k \neq i} (\, \boldsymbol{y}^k \theta_k^i + \boldsymbol{R}^k \boldsymbol{\xi}_k^i), \tag{10.4}$$

namely the first three terms in equation (10.3). The *effort correspondence* of entrepreneur i is defined as

$$\tilde{e}^i(\boldsymbol{m}^i, \kappa^i, \theta_i^i, \boldsymbol{\xi}_i^i) = \arg\max_{e^i \geq 0} \{u_1^i(\boldsymbol{x}_1^i) - c^i(e^i) | \boldsymbol{x}_1^i = \boldsymbol{m}^i + \boldsymbol{F}^i(\kappa^i, e^i)\theta_i^i + \boldsymbol{R}^i(\kappa^i, e^i)\boldsymbol{\xi}_i^i\}. \tag{E}$$

Since we have assumed that the marginal productivity of effort tends to zero when effort tends to infinity while its marginal cost tends to infinity, the *effort choice problem* equation (E) has a maximum for some finite value of e^i and the correspondence $\tilde{e}^i$ is well-defined on the domain $\mathcal{D}^i \subset \mathbb{R}_+^S \times \mathbb{R}_+ \times \mathbb{R}^J \times \mathbb{R}^{\mathcal{J}^i}$ consisting of the variables $(\boldsymbol{m}^i, \kappa^i, \theta_i^i, (\xi_{i,j}^i)_{j \in \mathcal{J}^i})$ such that $\boldsymbol{m}^i + \theta_i^i \boldsymbol{F}^i(\kappa^i, e^i) + \sum_{j \in \mathcal{J}^i} \xi_{i,j}^i \boldsymbol{R}_j^i(\kappa^i, e^i) \in \mathbb{R}_{++}^S$ for some $e^i > 0$. In the special case where there are no options ($\mathcal{J}^i = \emptyset$), the assumptions of strict concavity of u_1^i, convexity of c^i and concavity of $\boldsymbol{F}^i$ ensure that the solution to equation (E) is unique, so that $\tilde{e}^i$ is a function on $\mathcal{D}^i$. When $\mathcal{J}^i \neq \emptyset$, the payoffs of the options introduce a non-convexity into the constraint set in E, and the solution of the maximum problem may not be unique: in this case $\tilde{e}^i$ is a correspondence defined on $\mathcal{D}^i$.

10.2.3.4 Equilibrium with rational competitive price perceptions

Consider an investor who is thinking of buying either the equity or options of firm i. To anticipate what the firm's profit will be, the investor needs to anticipate the entrepreneur's inputs (κ^i, e^i). In this model, we assume that the capital input κ^i is observable, while the effort e^i is not. However, as we have seen, e^i can be deduced if the entrepreneur's characteristics $(u_1^i, \boldsymbol{F}^i, c^i)$ and his financial variables, or more precisely his outside income $\boldsymbol{m}^i$ and the inside financial variables $(\kappa^i, \theta_i^i, \boldsymbol{\xi}_i^i)$, are known: in the analysis that follows we assume that investors do indeed have access to this information and hence can deduce the effort e^i that the entrepreneur will invest in his firm.

In practice there is an important distinction between accessibility of information regarding the inside financial variables $(\kappa^i, \theta^i_i, \xi^i_i)$ and information regarding the outside wealth $\boldsymbol{m}^i$ and characteristics $(u^i_1, \boldsymbol{F}^i, c^i)$ of a firm's manager. Disclosure rules of the Securities and Exchange Commission require that proxy statements of publicly traded firms contain information regarding capital projects of the firm, as well as the equity and options holdings of the top management. Thus the assumption that inside variables are known by investors conforms with the regulations of capital markets in the United States.

More detailed information regarding the characteristics of the firm and its manager is less directly accessible, and it is essentially the job of security analysts to gain access to this type of information. While this information may not be available with the precision required by the model, analysts will however in the course of scrutinizing the earnings prospects of the firms they follow, acquire a good knowledge of the characteristics of the firms and their top management. Analysts who have followed the careers of top executives are likely to have a good estimate of the magnitude of their personal wealth and hence can impute at least the orders of magnitude of their outside incomes. Past performance gives information on their ability – which in the model is included in the function $\boldsymbol{F}^i$ – and their motivation and ability to take risks – in the model, the functions u^i_1 and c^i. The information collected by analysts spreads to investors through advisory services and the recommendations given by large brokerage companies. The assumption that the characteristics and financial trades of the entrepreneurs are known by all agents is thus the theoretical limit of a situation in which both the rules of disclosure and the activity of professionals in financial services result in a large amount of information being available to investors in the market.

If entrepreneurs' financial trades are known to investors, if investors make optimal use of this information to anticipate the outputs of firms, and in this way come to decide on the prices they are prepared to pay for the equity and options of the firms, then it seems reasonable to suppose that entrepreneurs will come to understand this. Hence our second assumption regarding anticipations: entrepreneurs are aware that investors will use their financial decisions as "signals" of the effort that they will exert in their firms. The next step is to incorporate these two assumptions – namely that (1) *investors use the available information (the financial variables) to correctly anticipate the firms' outputs*; (2) *entrepreneurs understand this* – into a concept of equilibrium.

The description of an equilibrium thus consists of two parts. The first is the standard part which enumerates the *actions* of the agents and the *prices* of the securities; the second part describes the entrepreneurs' *perceptions* of the way their financial decisions affect the price that the "market" will pay for the securities – equity and options – based on the profit of their firm. Let $\tilde{\boldsymbol{Q}}^i = (\tilde{Q}^i_y, \tilde{\boldsymbol{Q}}^i_c) = (\tilde{Q}^i_y, (\tilde{Q}^i_j)_{j \in \mathcal{J}^i})$ denote the price perception of entrepreneur i where

$$\tilde{Q}^i_\beta : \mathbb{R}_+ \times \mathbb{R}^I \times \mathbb{R}^J \longrightarrow \mathbb{R}_+, \quad \beta = y \text{ or } j, \quad j \in \mathcal{J}^i$$

is the price that entrepreneur i expects for security β (his equity, or an option on his equity) if he chooses the financial variables $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$. Let $\tilde{\boldsymbol{Q}} = (\tilde{\boldsymbol{Q}}^1, \ldots, \tilde{\boldsymbol{Q}}^I)$ denote the price perceptions of all entrepreneurs.

It is useful to define the following date 1 payoff matrices associated with the different securities in the economy. Let $V^0 = (1,\ldots,1)^T$ be the payoff of the riskless bond and, for a vector $y = (y^i)_{i\in\mathcal{I}_1}$, let $V^i(y) = [y^i, R^i(y^i)]$ denote the $S \times (1 + J^i)$ matrix of payoffs of the securities of firm i. $V(y) = [V^0, V^1(y), \ldots, V^{I_1}(y)]$ denotes the $S \times [1 + (1 + J^1) + \cdots + (1 + J^{I_1})]$ payoff matrix of all the securities and $V_{-i}(y) = [V^0, \ldots, V^{i-1}(y), V^{i+1}(y) \ldots V^{I_1}(y)]$ is the payoff matrix of all securities other than those of firm i. The associated subspaces of $\mathbb{R}^S$ generated by the columns of the above matrices are denoted by $\mathcal{V}^0, \mathcal{V}^i(y)$, $\mathcal{V}(y)$ and $\mathcal{V}_{-i}(y)$ respectively: we call $\mathcal{V}^i(y)$ the *firm i-subspace* and $\mathcal{V}(y)$ the *marketed subspace* at y.

A vector of prices q which prices the basic securities in the model (the columns of the matrix $V(y)$) leads to a valuation of every income stream in the marketed subspace $v_q: \mathcal{V}(y) \to \mathbb{R}$ defined by

$$v_q(m) = q_0\xi_0 + q_y\theta + \sum_{i\in\mathcal{I}_1} q_c^i \xi_i,$$

where $z = (\xi_0, \theta, (\xi_i)_{i\in\mathcal{I}_1})$ is any portfolio such that $m = V(y)z$. The valuation v_q is well-defined if the vector of prices q does not offer any arbitrage opportunities – a property which is equivalent to the existence of a strictly positive vector $\pi \in \mathbb{R}^S$ such that $\pi V(y) = q$ (see Magill and Quinzii, 1996, Section 9). Since we have assumed that there are investors ($\mathcal{I}_2 \neq \emptyset$) who can take advantage of arbitrage opportunities, any vector of equilibrium prices for the securities must be arbitrage free, and thus admit an associated vector of state prices.

Definition 2 A financial market equilibrium with price perceptions $\tilde{Q}$ for the economy $\mathcal{E}(\mathbf{U}, \omega_0, F, \tau)$ is a triple

$$((\bar{x}, \bar{y}, \bar{e}, \bar{\kappa}, \bar{\theta}, \bar{\xi}), \bar{q}; \tilde{Q})$$

consisting of actions, prices and price perceptions such that

(i) for each agent $i \in \mathcal{I}$ the action $(\bar{x}^i, \bar{e}^i, \bar{\kappa}^i, \bar{\theta}^i, \bar{\xi}^i)$ maximizes $U^i(x^i, e^i)$ among consumption-effort streams such that

$$x_0^i = \omega_0^i - v_{\bar{q}}(m^i) + \tilde{Q}_y^i(\kappa^i, \theta^i, \xi^i)(1 - \theta_i^i) - \tilde{Q}_c^i(\kappa^i, \theta^i, \xi^i)\xi_i^i - \kappa^i,$$

$$x_1^i = m^i + F^i(\kappa^i, e^i)\theta_i^i + R^i(\kappa^i, e^i)\xi_i^i,$$

for $\kappa^i \in \mathbb{R}_+$ and $(\theta^i, \xi^i) \in \mathbb{R}^{I_1} \times \mathbb{R}^J$, with $m^i = \mathbf{1}\xi_0^i + \sum_{k\neq i}(\bar{y}^k\theta_k^i + \bar{R}^k\xi_k^i)$.

(ii) $\bar{q}^i = \tilde{Q}^i(\bar{\kappa}^i, \bar{\theta}^i, \bar{\xi}^i), i \in \mathcal{I}_1$.

(iii) $\sum_{i\in\mathcal{I}} \bar{\theta}_k^i = 1, k \in \mathcal{I}_1$.

(iv) $\sum_{i\in\mathcal{I}} \bar{\xi}_0^i = 0$.

(v) $\sum_{i\in\mathcal{I}} \bar{\xi}_k^i = 0, k \in \mathcal{I}_1$.

Note that this definition uses some obvious notations: $\bar{y}^k = F^k(\bar{\kappa}^k, \bar{e}^k)$ is the equilibrium output of firm k and $\bar{R}_j^k$ is the payoff of the jth option on firm k when its output is $\bar{y}^k$.

In an equilibrium with price perceptions, each entrepreneur takes the production plans and the prices of the securities of other entrepreneurs' firms as given, correctly anticipating the effort they will invest in their firms. He chooses his own actions, anticipating that those which are observable (his financial decisions) will influence the prices of his securities in the way indicated by the function $\tilde{\boldsymbol{Q}}^i(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$. By (ii), the price perceptions are consistent with the observed equilibrium prices $\bar{\boldsymbol{q}}^i$ for each firm, and by (iii)–(v) the security markets clear.

10.2.3.5 The price perception functions

Without more precise assumptions on the price perceptions $\tilde{\boldsymbol{Q}}^i$, the definition of equilibrium given so far only incorporates the first assumption that we discussed above – namely that investors have correct expectations – but it does not yet explicitly incorporate the second – namely that entrepreneurs are fully aware of this fact. To form his anticipations $\tilde{\boldsymbol{Q}}^i$, entrepreneur i needs to predict:

(a) the *output* of his firm that investors expect if they observe $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$;
(b) how the market will *price* this expected output and the associated options of his firm.

For part (a) we use the assumption that entrepreneur i knows that investors will deduce from the observation of $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$ what his likely effort $e^i \in \tilde{e}^i$ will be, and hence what the likely output $\boldsymbol{F}^i(\kappa^i, e^i)$ of his firm will be. For part (b) we assume that the entrepreneur is, like an investor, a price-taker in the market for risky income streams. This price-taking assumption for price perceptions can be formalized as follows. If $\boldsymbol{m} \in \mathbb{R}^S$ is a potential income stream in $\mathcal{V}(\bar{\boldsymbol{y}})$, then its anticipated value is $v_{\bar{q}}(\boldsymbol{m}) = \sum_{s \in \mathcal{S}} \pi_s m_s$, where $\boldsymbol{\pi} \in \mathbb{R}^S_{++}$ is any vector of state prices satisfying $\boldsymbol{\pi} V(\bar{\boldsymbol{y}}) = \bar{\boldsymbol{q}}$. As long as the entrepreneur envisions alternative production plans lying in the marketed subspace $\mathcal{V}(\bar{\boldsymbol{y}})$, he does not perceive the possibility of affecting the state prices implicit in the equilibrium prices $\bar{\boldsymbol{q}}$. While the price-taking assumption leads to a well-defined valuation of income streams in the marketed subspace, it does not extend in any natural way to income streams outside of the marketed subspace: for if $\boldsymbol{m} \notin \mathcal{V}(\bar{\boldsymbol{y}})$, the value $\sum_{s \in \mathcal{S}} \pi_s m_s$ can change when the vector of state prices satisfying $\boldsymbol{\pi} \boldsymbol{V}(\bar{\boldsymbol{y}}) = \bar{\boldsymbol{q}}$ is changed, so that the valuation of the stream $\boldsymbol{m}$ is no longer well-defined. To stay within a framework that permits the competitive assumption to be retained without raising conceptual difficulties, we introduce the assumption of partial spanning.

DEFINITION 3 We say that there is partial spanning (PS) at $\bar{\boldsymbol{y}}$ if for all $i \in \mathcal{I}_1$, for all $(\kappa^i, e^i) \in \mathbb{R}^2_+$ and $\boldsymbol{y}^i = \boldsymbol{F}^i(\kappa^i, e^i)$, the firm i-subspace $\mathcal{V}^i(\boldsymbol{y})$ is contained in the marketed subspace at $\bar{\boldsymbol{y}}$, i.e. $\mathcal{V}^i(\boldsymbol{y}) \subset \mathcal{V}(\bar{\boldsymbol{y}})$.

The partial spanning assumption is classical – and is often used in the finance literature: it means that a firm cannot create a "new security," i.e. an income stream which is not in the existing marketed subspace $\mathcal{V}(\bar{\boldsymbol{y}})$, by changing its production plan. With partial spanning the market prices of the securities are sufficient signals to value all possible alternative production plans of any firm and its associated options.

DEFINITION 4 A financial market equilibrium with rational competitive price perceptions (RCPP) is an equilibrium $((\bar{\boldsymbol{x}}, \bar{\boldsymbol{y}}, \bar{\boldsymbol{e}}, \bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{\theta}}, \bar{\boldsymbol{\xi}}), \bar{\boldsymbol{q}}; \tilde{\boldsymbol{Q}})$ with price perceptions such that:

(i) PS holds at $\bar{\boldsymbol{y}}$
(ii) for each $i \in \mathcal{I}_1$ the price perceptions are given by

$$\tilde{Q}^i_y(\kappa^i,\boldsymbol{\theta}^i,\boldsymbol{\xi}^i) = \bar{\boldsymbol{\pi}}\boldsymbol{F}^i(\kappa^i,\hat{e}^i), \quad \tilde{Q}^i_j(\kappa^i,\boldsymbol{\theta}^i,\boldsymbol{\xi}^i) = \bar{\boldsymbol{\pi}}\boldsymbol{R}^i_j(\kappa^i,\hat{e}^i), \quad j \in \mathcal{J}^i$$

for any $\bar{\boldsymbol{\pi}} \in \mathbb{R}^S_{++}$ such that $\bar{\boldsymbol{\pi}}V(\bar{\boldsymbol{y}}) = \bar{\boldsymbol{q}}$ and $\hat{e}^i \in \tilde{e}^i(\kappa^i,\boldsymbol{\theta}^i,\boldsymbol{\xi}^i)$ which maximizes

$$\bar{\boldsymbol{\pi}}\boldsymbol{F}^i(\kappa^i,e^i)(1-\theta^i_i) - \bar{\boldsymbol{\pi}}\boldsymbol{R}^i(\kappa^i,e^i)\boldsymbol{\xi}^i_i.$$

Note that we use the notation $\tilde{e}^i(\kappa^i,\boldsymbol{\theta}^i,\boldsymbol{\xi}^i)$ instead of $\tilde{e}^i(\boldsymbol{m}^i,\kappa^i,\theta^i_i,\boldsymbol{\xi}^i_i)$, since $\boldsymbol{m}^i$ is a function of the financial variables $(\theta^i_k,\boldsymbol{\xi}^i_k, k \neq i)$ given by equation (10.4). To check if his equilibrium financial decisions $(\bar{\kappa}^i,\bar{\boldsymbol{\theta}}^i,\bar{\boldsymbol{\xi}}^i)$ are optimal, entrepreneur i considers alternative decisions $(\kappa^i,\boldsymbol{\theta}^i,\boldsymbol{\xi}^i)$, recognizing that investors are rational and will deduce from $(\kappa^i,\boldsymbol{\theta}^i,\boldsymbol{\xi}^i)$ what his associated optimal effort will be – namely the solution of the optimal effort problem equation (E) if it is unique, or if it is multivalued, the solution which yields the highest date 0 income for entrepreneur i (recall that $u^i_1(\boldsymbol{x}^i_1) - c^i(e^i)$ has the same value for each of the solutions). This is the "rational" part of his anticipations. To evaluate the prices $\tilde{\boldsymbol{Q}}^i(\kappa^i,\boldsymbol{\theta}^i,\boldsymbol{\xi}^i)$ that he would get for the alternative output or that he would pay for the options on his firm, he uses any state price vector $\bar{\boldsymbol{\pi}}$ compatible with the equilibrium vector of security prices $\bar{\boldsymbol{q}}$. This is the "competitive" part of his expectations, which requires that PS hold at equilibrium.

PS is automatically satisfied if the financial markets are complete at equilibrium (rank $V(\bar{\boldsymbol{y}}) = S$), but it can also be satisfied when the markets are incomplete as shown by the following examples.

Example 1 The financial markets are simple: they consist solely of the bond and equity markets, so that $\mathcal{J}^i = \emptyset$ for all $i \in \mathcal{I}_1$. The production function of each firm has a factor structure: $\boldsymbol{F}^i(\kappa^i,e^i) = f^i(\kappa^i,e^i)\boldsymbol{\eta}^i$ where $f^i\colon \bar{\mathbb{R}}^2_+ \longrightarrow \mathbb{R}$ is a concave increasing function and $\boldsymbol{\eta}^i \in \mathbb{R}^S_+$ is a fixed vector, characterizing the risk structure of the firm. Then PS is satisfied if $f^i(\bar{\kappa}^i,\bar{e}^i) > 0$ for all $i \in \mathcal{I}_1$. This case is studied in detail in Magill and Quinzii (1999).

Example 2 The financial securities consist of the riskless bond, equity and options on each firm. Suppose the uncertainty (shocks) affecting the production in the economy is decomposed into a product of $\mathcal{I}_1$ spaces

$$\mathcal{S} = \mathcal{S}^1 \times \ldots \times \mathcal{S}^{I_1} = \{1,\ldots,S^1\} \times \ldots \times \{1,\ldots,S^{I_1}\}$$

so that a state of nature is an I_1-triple $s = (s^1,\ldots,s^{I_1})$ where s^i is the shock experienced by firm i. Then for any pair of states $s = (s^1,\ldots,s^{I_1}) \in \mathcal{S}$, $\hat{s} = (\hat{s}^1,\ldots,\hat{s}^{I_1}) \in \mathcal{S}$ with $s^i = \hat{s}^i$, $F^i_s(\kappa^i,e^i) = F^i_{\hat{s}}(\kappa^i,e^i)$ for all $(\kappa^i,e^i) \in \mathbb{R}^2_+$. If the vector $\boldsymbol{F}^i(\bar{\kappa}^i,\bar{e}^i)$ takes on S^i different values for the S^i individual states of firm i, and if there are options with striking prices in between the S^i different values taken by the output of firm i, for each firm $i \in \mathcal{I}_1$, then PS is satisfied.

10.3 CONSTRAINED OPTIMALITY OF RCPP

The concept of an RCPP equilibrium is a natural way of describing market behavior in a production economy with a moral hazard in which participants on the financial markets are well informed. To get a feel of how natural this concept is we turn to a study of its normative properties. As we mentioned in Section 10.2, at the first stage of development, when the contracts traded consist solely of the bond and the equity of firms, there is a clear trade-off between incentives and risk sharing. Entrepreneurs who want to finance their investment without incurring a large debt (which would put them in an inordinately risky situation) can choose to finance some of their investment by issuing equity, thus opening the way to risk sharing and diversification. But issuing equity means they no longer receive the full marginal benefit of their effort, so their incentives to exert effort are reduced. Do markets induce entrepreneurs to make the optimal trade-off between incentives and risk sharing in their choice of debt and equity?

We also argued that, at a more mature stage of development, in addition to the bond and equity markets, options on the firms' profits (equity) are introduced. Such contracts not only augment the opportunities for risk sharing, but also permit the introduction of non-linear reward schedules for entrepreneurs: non-linear schedules incorporate "high powered" incentives which can help to solve the moral-hazard problem induced by the reduced equity shares of entrepreneurs. If the entrepreneur receives a larger share of output when the firm's realized output is high than when it is low, then he will (typically) be induced to increase effort, to increase the likelihood of a high realization of output. Such an incentive scheme can be obtained by adding options to his share of equity: but would an entrepreneur choose to buy options to increase his incentives in this way, given that the income stream received from his firm will tend to be more risky? In short, do market-induced choices of bonds, equity and options by entrepreneurs and investors lead to the best possible use of these instruments?

To answer this question we consider another way of arriving at an allocation where a "planner" – rather than the agents – chooses the financial variables, and examine if the planner could obtain a better allocation (in the Pareto sense) than that achieved in a RCPP equilibrium. Such a comparison only makes sense if the planner faces the same problem of unobservability of effort of the entrepreneurs and is restricted to the same opportunities for risk sharing as those available to the agents with the system of financial markets. In particular, the planner cannot dictate effort levels to entrepreneurs – rather, these effort levels are chosen optimally by the entrepreneurs who take the reward structure given by the debt-equity-option choice of the planner and the effort levels of other agents (and hence their outputs) as given.

Definition 5 An allocation $(\boldsymbol{x}, \boldsymbol{e}) \in \mathbb{R}_+^{(S+1)I} \times \mathbb{R}_+^I$ is *constrained feasible* if there exist inputs and portfolios $(\boldsymbol{\kappa}, \boldsymbol{\theta}, \boldsymbol{\xi}) \in \mathbb{R}_+^I \times \mathbb{R}^{2I} \times \mathbb{R}^{IJ}$ such that

(i) $\sum_{i \in \mathcal{I}} \xi_0^i = \mathbf{0}$.

(ii) $\Sigma_{i \in \mathcal{I}} \theta_k^i = 1, \quad k \in \mathcal{I}$.

(iii) $\Sigma_{i \in \mathcal{I}} \boldsymbol{\xi}_k^i = \mathbf{0}, \quad k \in \mathcal{I}$.

(iv) $\Sigma_{i \in \mathcal{I}} x_0^i = \Sigma_{i \in \mathcal{I}} \omega_0^i - \Sigma_{i \in \mathcal{I}} \kappa^i$.

(v) $\boldsymbol{x}_1^i = \boldsymbol{m}^i + \boldsymbol{F}^i(\kappa^i, e^i)\theta_i^i + \boldsymbol{R}^i(\kappa^i, e^i)\boldsymbol{\xi}_i^i, \quad i \in \mathcal{I}$.

(vi) $\boldsymbol{m}^i = \mathbf{1}\xi_0^i + \Sigma_{k \neq i}(\boldsymbol{F}^k(\kappa^k, e^k)\theta_k^i + \boldsymbol{R}^k(\kappa^k, e^k)\boldsymbol{\xi}_k^i), \quad i \in \mathcal{I}$.

(vii) $e^i \in \tilde{e}^i(\boldsymbol{m}^i, \kappa^i, \theta_i^i, \boldsymbol{\xi}_i^i), \quad i \in \mathcal{I}$.

An allocation $(\boldsymbol{x},\boldsymbol{e})$ is *constrained Pareto optimal* (CPO), if it is constrained-feasible, and if there does not exist any alternative constrained feasible allocation $(\hat{\boldsymbol{x}},\hat{\boldsymbol{e}})$ such that $U^i(\hat{x}^i,\hat{e}^i) \geq U^i(x^i,e^i)$, $i \in \mathcal{I}$, with strict inequality for at least one i.

(i)–(iii) are the feasibility constraints for the planner's choice of financial variables $(\boldsymbol{\theta},\boldsymbol{\xi})$. Constraint (iv) indicates that the planner does not need to respect a system of prices for the securities and the associated date 0 budget constraint implied for each agent: it is in this sense that the "planner" replaces the "market". (v) and (vi) indicate that the planner's choice of date 1 consumption streams, and hence risk sharing, for the agents respects the existing structure of the financial securities. (vii) are the incentive constraints which reflect the fact that the choice of effort is made by entrepreneur i (and not the planner), and is the one that is optimal given the financial variables attributed to him, and given the effort levels of other agents (since agent i takes $\boldsymbol{m}^i$ as given).

Despite the fact that a planner chooses the financial variables $(\kappa,\boldsymbol{\theta},\boldsymbol{\xi})$ being fully aware of their consequences for the choices of effort by entrepreneurs and of the effect of each entrepreneur's effort on the consumption of other agents (the outside shareholders), he cannot improve on an RCPP equilibrium allocation arising from the self-interested choices of agents coordinated by the financial markets, provided we invoke the following strengthening of the partial spanning assumption.

Definition 6 We say that there is strong partial spanning (SPS) at $\bar{\boldsymbol{y}}$ if for all $(\kappa,e) \in \mathbb{R}^{2I_1}$ and $\boldsymbol{y} = (\boldsymbol{F}^k(\kappa^k,e^k)_{k\in\mathcal{I}_1})$, $\mathcal{V}_{-i}(\boldsymbol{y}) \subset \mathcal{V}_{-i}(\bar{\boldsymbol{y}})$ for all $i \in \mathcal{I}_1$.

SPS ensures that there is partial spanning for every subset of $I_1 - 1$ firms. Note that even if markets were complete, SPS would not automatically be satisfied. It holds if the securities based on the outputs of any subset of $I_1 - 1$ firms suffice to complete the markets, or if each firm spans its own subspace, as in Examples 1 and 2. SPS implies PS: if firm i cannot create an income stream which lies outside $\mathcal{V}^{-k}(\bar{\boldsymbol{y}})$ for $k \neq i$, it cannot create an income stream lying outside $\mathcal{V}(\bar{\boldsymbol{y}})$.

The need for the additional assumption SPS comes from the fact that in the RCPP equilibrium there are two potential sources of inefficiency. The first arises from the property that the equilibrium is a *Nash equilibrium*, in which the effort decision of each entrepreneur depends on the decisions of other entrepreneurs. The second arises from the *moral hazard problem*: the choice of effort of an entrepreneur affects the payoff of all the securities based on his firm and thus has an external effect on the investors who buy these securities. The assumption SPS cancels the possible inefficiency due to the Nash aspect of the concept. For the decisions of an entrepreneur depends on the decisions of the others only through the outside income possibilities offered by the securities based on their firms. Under SPS, a planner could not, by changing all the portfolios and capital choices of the entrepreneurs, create outside income possibilities which would induce a particular entrepreneur to a better effort decision, since the subspace of outside income has to be the same or a reduced version of the equilibrium subspace. In Magill and Quinzii (2000) we show that without the assumption SPS, an RCPP equilibrium can indeed be constrained inefficient, but that once SPS is invoked, an RCPP equilibrium is constrained efficient. The result of course depends on the fact that the model is a one-good, two-period finance model: without this assumption, i.e. if there were more than two goods or more than two periods, the constrained inefficiency mentioned in the introduction – due to the feedback between agents' decisions and the spot prices – would reappear when security markets are incomplete.

PROPOSITION B (RCPP is CPO) *If an RCPP equilibrium* $((\bar{x},\bar{y},\bar{e},\bar{\kappa},\bar{\theta},\bar{\xi}),\bar{q};\tilde{Q})$ *of the economy* $\mathcal{E}(U,\omega_0,F,\tau)$ *satisfies SPS at* $\bar{y}$, *then* $(\bar{x},\bar{e})$ *is constrained Pareto optimal.*

The choice of financial variables (θ,ξ) creates a reward structure for each entrepreneur, namely a *contract* linking his payoff to the performance of his firm and the rest of the economy

$$\phi^i(y) = \theta^i_i y^i + \sum_{j\in\mathcal{J}^i} \xi^i_{i,j} \max\{y^i - \tau^i_j, 0\} + m^i(y^{-i}), \quad i \in \mathcal{I}_1,$$

where $y = (y^i, y^{-i}), y^i$ being the random output of the firm and y^{-i} the random output of all other firms. $\mathcal{J} = \bigcup_{i\in\mathcal{I}_1} \mathcal{J}^i$ determines the richness of the incentive structure over and above the basic equity contracts. If $\mathcal{J} = \emptyset$, the market and the planner are restricted to linear contracts, while if $\mathcal{J} \neq \emptyset$ the admissible contracts are nonlinear (piecewise linear): the larger the sets $\mathcal{J}^i$, the larger the admissible class of piecewise linear functions.

The CPO problem, which amounts to choosing optimally the investment, risk and incentive structure for the economy, is a generalized *principal-agent problem*, in which the planner (the principal) chooses the investment in each firm and the (constrained) optimal contract for each entrepreneur and investor in the economy. When agents rationally anticipate in the way described by an RCPP equilibrium, then Proposition B asserts that *a system of markets is capable of solving society's principal-agent problem*. The basic driving force for this optimality property of an RCPP equilibrium is that the social effect of each entrepreneur's choice of capital and reward structure – in particular the effect on outside investors – is transmitted to the entrepreneur through the rational price perceptions.

10.4 FIRST-ORDER CONDITIONS FOR CPO AND RCPP

A way – indeed probably the best way – of understanding the forces which lead agents to optimally coordinate their actions is to study the first-order (i.e. the marginal) conditions that must be satisfied at a CPO and to show how these end up being achieved at an RCPP equilibrium through the disciplining effect of the perception functions $\tilde{Q}$.

10.4.1 First-order conditions for CPO

Let $(x,y,e,\kappa,\theta,\xi)$ be a CPO allocation such that: (i) the striking prices are strictly between the values $F^i_s(\kappa^i,e^i), s \in \mathcal{S}$, so that the payoffs $R^i_j(y^i)$ of the options are locally differentiable; (ii) each agent's consumption vector x^i is strictly positive; (iii) each entrepreneur i's effort level e^i is a locally differentiable selection of the effort correspondence $\tilde{e}^i$, which with a slight abuse of notation we denote by $\tilde{e}^i(\boldsymbol{m}^i, \kappa^i, \theta^i_i, \boldsymbol{\xi}^i_i)$. This CPO allocation is an extremum of the social welfare function

$$\sum_{i\in\mathcal{I}} \nu^i (u^i_0(x^i_0) + u^i_1(\boldsymbol{x}^i_1) - c^i(e^i)),$$

subject to the constraints (i)–(vii) in Definition 4, for some vector of relative weights $\boldsymbol{\nu} \in \mathbb{R}^I_{++}$. It must therefore satisfy the FOC for this constrained maximum problem. To express the cost of each constraint in units of date 0 consumption, we divide all the multipliers by the multiplier induced by the resource availability constraint (iv) at date 0. Let $(q_0, q^k_v, \boldsymbol{q}^k_c, 1, \boldsymbol{\pi}^i, \boldsymbol{\mu}^i, \epsilon^i)$ denote the resulting normalized multipliers associated with the constraints (i)–(vii). For each $s \in \mathcal{S}$ and $i \in \mathcal{I}$, let $\mathcal{J}^i_s \subset \mathcal{J}^i$ denote the subset of options which are "in the money" at the CPO in state s, i.e. $j \in \mathcal{J}^i_s$ implies $F^i_s(\kappa^i, e^i) > \tau^i_j$. The first-order conditions with respect to the variables $(\boldsymbol{x}^i, e^i, \boldsymbol{m}^i, \kappa^i, \boldsymbol{\xi}^i, \boldsymbol{\theta}^i)$ are:

$$\frac{\partial u^i_1/\partial x^i_s}{u^{i\prime}_0} = \pi^i_s, \quad s \in \mathcal{S}. \tag{10.5}$$

$$\frac{c^{i\prime}}{u^{i\prime}_0} = \left[\sum_{s\in\mathcal{S}} \pi^i_s \left(\theta^i_i + \sum_{j\in\mathcal{J}^i_s} \xi^i_{i,j}\right) + \sum_{k\neq i}\sum_{s\in\mathcal{S}} \mu^k_s \left(\theta^k_i + \sum_{j\in\mathcal{J}^i_s} \xi^k_{i,j}\right)\right] \frac{\partial F^i_s}{\partial e^i} - \epsilon^i. \tag{10.6}$$

$$\mu^i_s = \pi^i_s + \epsilon^i \frac{\partial \tilde{e}^i}{\partial m^i_s}, \quad s \in \mathcal{S}. \tag{10.7}$$

$$1 = \left[\sum_{s\in\mathcal{S}} \pi^i_s \left(\theta^i_i + \sum_{j\in\mathcal{J}^i_s} \xi^i_{i,j}\right) + \sum_{k\neq i}\sum_{s\in\mathcal{S}} \mu^k_s \left(\theta^k_i + \sum_{j\in\mathcal{J}^i_s} \xi^k_{i,j}\right)\right] \frac{\partial F^i_s}{\partial \kappa^i} + \epsilon^i \frac{\partial \tilde{e}^i}{\partial \kappa^i}. \tag{10.8}$$

$$q_0 = \sum_{s\in\mathcal{S}} \mu^i_s. \tag{10.9}$$

$$q^k_j = \sum_{s\in\mathcal{S}} \mu^i_s R^k_{j,s}. \tag{10.10}$$

$$q^k_y = \sum_{s\in\mathcal{S}} \mu^i_s F^k_s. \tag{10.11}$$

$$q^i_j = \sum_{s\in\mathcal{S}} \pi^i_s R^i_{j,s} + \epsilon^i \frac{\partial \tilde{e}^i}{\partial \xi^i_{i,j}}. \tag{10.12}$$

$$q^i_y = \sum_{s\in\mathcal{S}} \pi^i_s F^i_s + \epsilon^i \frac{\partial \tilde{e}^i}{\partial \theta^i_i}. \tag{10.13}$$

To these equations should be added the FOC for the choice of optimal effort by entrepreneur i

$$\frac{c^{i\prime}(e^i)}{u_0^{i\prime}} = \sum_{s\in\mathcal{S}} \pi_s^i \left(\theta_i^i + \sum_{j\in\mathcal{J}_s^i} \xi_{i,j}^i\right) \frac{\partial F_s^i}{\partial e^i}, \tag{10.14}$$

where we have divided both sides by $u_0^{i\prime}$. Equation (10.14) is just the marginal way of expressing the incentive constraint $e^i = \tilde{e}^i(\,\cdot\,)$ in (vii). Using equations (10.6) and (10.14) gives

$$\epsilon^i = \sum_{k\neq i}\sum_{s\in\mathcal{S}} \mu_s^k \left(\theta_i^k + \sum_{j\in\mathcal{J}_s^i} \xi_{i,j}^k\right) \frac{\partial F_s^i}{\partial e^i}. \tag{10.15}$$

Note that for an investor $\partial F_s^i/\partial e^i = 0$, $\partial F_s^i/\partial \kappa^i = 0$ which implies $\epsilon^i = 0$ and $\mu_s^i = \pi_s^i$.

10.4.2 Economic interpretation of FOC

Equation (10.5) defines the present-value vector $\boldsymbol{\pi}^i = (\pi_1^i, \dots, \pi_S^i)$ of agent i: for any date 1 income stream $\boldsymbol{v} = (v_1, \dots, v_S)$, $\boldsymbol{\pi}^i\,\boldsymbol{v}$ is the present value to agent i of the income stream v (i.e. what he is prepared to pay for it at date 0). The components of the vector $\boldsymbol{q} = (q_0, q_y^k, \boldsymbol{q}_c^k, k \in \mathcal{I})$ are the shadow prices of the securities i.e. the social gain from giving one (marginal) unit of the relevant security to any agent in the economy. μ_s^i is the social gain from giving one more unit of income to entrepreneur i in state s: in most models this social gain would coincide with the private gain π_s^i, but in this setting, giving more income to entrepreneur i influences his effort, and thus has a consequence on other agents (equity or option holders of firm i), which creates a discrepancy between social and private benefit. ϵ^i is the social value of an additional unit of effort by entrepreneur i; by equation (10.6) ϵ^i is the difference between the social marginal benefit – namely the (marginal) benefit to entrepreneur i *plus* the benefit to every "outside investor" holding either the equity or options of firm i – and the social marginal cost, which here coincides with the private cost $c^{i\prime}/u_0^{i\prime}$, since entrepreneur i is the only one to bear the cost of his effort. Since effort is chosen optimally by entrepreneur i, by the "envelope theorem," or more precisely by the FOC equation (10.14), the welfare effect on the entrepreneur of a marginal change in his effort is zero. This explains why equations (10.6) and (10.14) lead to equation (10.15), namely that the social value of an additional unit of effort by entrepreneur i is the value to agents other than himself of the additional output that his effort would create. Note that the benefit to these agents $k \neq i$ is evaluated using μ_s^k rather than π_s^k and thus when k is an entrepreneur it incorporates the incentive cost of giving him a marginal increment of income in state s. As soon as $\theta_i^k \neq 0$ or $\boldsymbol{\xi}_i^k \neq 0$ for some $k \neq i$, a marginal increment of effort by agent i has an *external effect* on agent k which is not taken into account when entrepreneur i makes his effort decision. ϵ^i, which is the cost of the incentive constraint (vii), is the sum of these external effects, and is in essence the cost of separating the ownership and control of firm i. This cost is explicitly taken into account by the planner when he chooses the financial variables $(\kappa, \boldsymbol{\theta}, \boldsymbol{\xi})$.

Equations (10.9)–(10.13), i.e. the first-order conditions with respect to the financial variables $(\boldsymbol{\xi}, \boldsymbol{\theta})$, express the limited sense in which there must be equalization of marginal rates of substitution to achieve a CPO allocation, full equalization (in the general case) being prevented by the fact that income can only be distributed indirectly using securities, and that the incentive constraints of the agents must be satisfied. Equations (10.9)–(10.13) require that the social marginal cost of each security equal its social marginal benefit, the latter being a sum of two terms, one direct, the other indirect: the direct effect is the private benefit to an agent of the security's income stream, and the indirect effect is the social cost of the reduced effort made by agent i as a result of this increment to his income stream. For the outside variables $(\xi_0, \xi^i_{k,j}, \theta^i_k)$ the indirect effect is taken into account by μ^i_s, for the inside variables $(\xi^i_{i,j}, \theta^i_i)$ it depends on the specific way in which the variable affects the entrepreneur's effort. The FOC equation (10.8) for the capital stock κ^i of firm i differs in that an increment to κ^i affects all agents holding one of the securities of firm i.

10.4.3 How the FOC for CPO are achieved at equilibrium

Since an RCPP equilibrium is constrained Pareto optimal, in such an equilibrium entrepreneurs must – just like the planner in a CPO problem – be induced to take into account the external effect of their effort on the welfare of others, namely the terms involving ϵ^i in equations (10.6)–(10.13). How is this effect transmitted to entrepreneurs?

The first point to note is that entrepreneur i raises money by selling a share $(1 - \theta^i_i)$ of his equity and is thus concerned with the valuation $q^i_y = \bar{\boldsymbol{\pi}}\, \boldsymbol{y}^i$ that investors will assign to his firm. The assumption of competition implies that he does not perceive any effect of his actions on the vector of state prices $\bar{\boldsymbol{\pi}}$; the assumption of rationality implies that he perceives that the output $\boldsymbol{y}^i$ that investors anticipate from his firm is influenced by his choice of financial variables. Actually since the entrepreneur can typically raise the value of his equity by holding options – to convince investors that it is in his interest to make a high effort – the net proceeds from selling equity is $q^i_y(1 - \theta^i_i) - \sum_{j \in \mathcal{J}^i} q^i_j \xi^i_{i,j}$, and it is this net value which is of concern to entrepreneur i. When he considers alternative financing decisions, he knows that investors will anticipate the output $\boldsymbol{y}^i = \boldsymbol{F}^i(\kappa^i, \tilde{e}^i(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i))$ and that this anticipation on their part will translate into the net proceeds for him $\bar{\boldsymbol{\pi}}\boldsymbol{y}^i(1 - \theta^i_i) - \bar{\boldsymbol{\pi}}\boldsymbol{R}^i(\boldsymbol{y}^i)\boldsymbol{\xi}^i_i$ at date 0. It is his concern for the value of the equity that he sells, net of the cost of options, which leads the entrepreneur to take into account the interests of outside investors when he chooses his financial variables $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$. This can be seen by studying the first-order conditions for an entrepreneur's maximum problem in an RCPP equilibrium and comparing them with the FOC for a CPO allocation. Consider the maximum problem of agent i in Definition 2(i). Let $\boldsymbol{\lambda}^i = (\lambda^i_0, \lambda^i_1, \ldots, \lambda^i_S) \in \mathbb{R}^{S+1}_+$ denote the vector of multipliers induced by the $S + 1$ budget constraints: the normalized vector

$$\bar{\boldsymbol{\pi}}^i = \frac{1}{\bar{\lambda}^i_0}(\bar{\lambda}^i_1, \ldots, \bar{\lambda}^i_S) = (\bar{\pi}^i_1, \ldots, \bar{\pi}^i_S)$$

is the present-value vector of agent i at the equilibrium. The FOC are

$$\frac{\partial u_1^i/\partial x_s^i}{u_0^{i\prime}} = \bar{\pi}_s^i, \quad s \in S, \tag{10.16}$$

$$\frac{c^{i\prime}}{u_0^{i\prime}} = \sum_{s \in S} \bar{\pi}_s^i \left(\bar{\theta}_i^i + \sum_{j \in \mathcal{J}_s^i} \bar{\xi}_{i,j}^i \right) \frac{\partial F_s^i}{\partial e^i}, \tag{10.17}$$

$$1 = \sum_{s \in S} \bar{\pi}_s^i \left(\bar{\theta}_i^i + \sum_{j \in \mathcal{J}_s^i} \bar{\xi}_{i,j}^i \right) \frac{\partial F_s^i}{\partial \kappa^i} + \frac{\partial \tilde{Q}_y^i}{\partial \kappa^i}(1 - \bar{\theta}_i^i) - \frac{\partial \tilde{\boldsymbol{Q}}_c^i}{\partial \kappa^i} \bar{\boldsymbol{\xi}}_i^i, \tag{10.18}$$

$$\bar{q}_\alpha = \sum_{s \in S} \bar{\pi}_s^i v_s^\alpha + \frac{\partial \tilde{Q}_y^i}{\partial z_\alpha^i}(1 - \bar{\theta}_i^i) \; - \; \frac{\partial \tilde{\boldsymbol{Q}}_c^i}{\partial z_\alpha^i} \bar{\boldsymbol{\xi}}_i^i, \tag{10.19}$$

where α is an index denoting any one of the traded securities [$\alpha = 0$ (bond) or $\alpha = k$ (equity of firm k) or $\alpha = (k, j)$ (option j of firm k)], $v^\alpha \in \mathbb{R}^S$ is its dividend stream, and z_α^i is the appropriate component of agent i's portfolio $z^i = (\boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$. By substituting the expression for $\tilde{Q}^i$ given in Definition 4, one can show that the FOC equations (10.16)–(10.19) are the same as the FOC equations (10.5)–(10.13) for constrained Pareto optimality. Since this requires a certain amount of computation, for the convenience of the reader we spell out these calculations in the Appendix.

The intuition for the result is nevertheless clear from equations (10.18) to (10.19). By paying attention to the way investors in the securities of firm i react to his financial decisions $(\kappa^i, \boldsymbol{\theta}^i, \boldsymbol{\xi}^i)$, through the partial derivatives $(\partial\tilde{Q}^i/\partial\kappa^i, \partial\tilde{Q}^i/\partial z_\alpha^i, \alpha = 0, \ldots)$ of his perception function $\tilde{Q}^i$, entrepreneur i is led to take their interests into account. Thus with sophisticated participants, the capital markets ensure that self-interested behavior leads to a constrained socially optimal outcome.

10.5 PARETO OPTIMALITY: CAN CAPITAL MARKETS MIMIC ARROW SECURITIES?

The standard GEI model is fundamentally a model of intertemporal risk sharing, and it can be used to study how limitations of the instruments available for transferring income across time or sharing intertemporal risks affect the equilibrium outcome. In Section 10.2, we have shown how incentives can be introduced into the model through the non-observability of the effort of the entrepreneurs who run (and are the original owners) of the firms. In Sections 10.3 and 10.4, we have shown how the presence of rational and informed investors can force entrepreneurs to take into account the interests of the shareholders: this was made precise in Proposition B which asserts that the capital markets lead to an optimal trade-off between risk sharing and incentives, relative to the possibly incomplete structure of the markets.

In Section 10.2, we have shown that if we lived in an ideal world where states of nature are verifiable, then sole proprietorship and a complete set of Arrow securities would solve the moral hazard problem. However, in the real world the complex array of business and technological shocks to which firms are subjected makes states of nature unverifiable, and hence a system based on Arrow securities unimplementable. To be enforceable in the courts of law, contracts must be based on events that are easily observable and verifiable by a third party: for a production economy of the kind considered in this chapter this essentially means that the contracts must be based on the *observed outputs* of firms – precisely the property satisfied by the standard capital market instruments consisting of debt, equity, and options on equity. What is intriguing is that there is a way of showing that these instruments, which constitute society's response to the problem of enforceability, are able to collectively mimic the ideal system of Arrow securities. More precisely, under appropriate conditions, an RCPP equilibrium with capital markets gives the same Pareto optimal outcome as the equilibrium with Arrow securities – the SPA equilibrium of Section 10.2.2.

The key property of an SPA equilibrium which leads to Pareto optimality is that each agent has a single budget constraint expressing equality of the present value his lifetime consumption expenditure and income: as a result each entrepreneur is led to maximize the present value of the profit of his firm, with a shadow price equal to his marginal cost for his effort. It can be shown (Magill and Quinzii, 2000) that the budget constraint of an RCPP equilibrium can be written in a form which mimics the budget constraint of an SPA equilibrium: each agent is subject to a budget constraint expressing the equality of the present value of his expenditure and of his income (including the full profit generated by his firm), but the incompleteness of the markets and the fact that the entrepreneur's effort must be credible, i.e. optimal given his choice of financing, lead to two additional constraints: the first is a *spanning constraint* and the second an *incentive constraint*. We show that there is an abstract condition on the security structure, which we call the *spanning-overlap condition*, under which these latter constraints are not binding for agents at an equilibrium, so that *the RCPP equilibrium outcome coincides with the SPA equilibrium allocation*. The "spanning part" is what is required for optimal risk-sharing, namely that markets be *complete* – the usual condition for optimality in a standard risk-sharing GEI equilibrium. However, complete markets is not enough to deal with incentives: some extra "control" is needed so that an entrepreneur can simultaneously choose his risk and retain appropriate incentives.

To formalize this controllability condition, note that for each firm i, the securities can be placed into two categories: those whose payoffs are independent of the effort of entrepreneur i (e.g. the equity and options of other firms, or the default-free bond) and those whose payoff is directly affected by his effort (e.g. his equity or options on his equity). The overlap condition requires that there is an intersection (of dimension at least one) between the subspace spanned by the i-dependent and i-independent securities, for each entrepreneur i in the economy. When the spanning-overlap condition is satisfied, by adjusting the component of an entrepreneur's future income on the i-dependent securities the magnitude of the incentive effect can be adjusted, while the component on the i-independent subspace permits the risk profile to be kept at any desired level. Thus risk sharing and incentives can be completely controlled.

Does this spanning-overlap condition have any chance of holding in the "real world"? In Magill and Quinzii (2000) we show that there are reasonable conditions on the structure of uncertainty that affects the firms, such that the condition can be satisfied by standard capital markets instruments. In order that output-contingent securities can do the work expected of state-contingent securities, the states must be distinguishable by the firms' outputs: two distinct shocks must always lead to different outcomes, for at least *some* firm in the economy. Then, if enough options are introduced on the equity of each firm (and recall that introducing such options is essentially costless), and if these options have "appropriate" striking prices, then a security structure consisting of debt, equity and options satisfies the spanning-overlap condition. Equity provides the basic output-contingent security for each firm: the options, contingent on the payoff of the equity, then provide a rich enough family of instruments so that not only can risks be shared, but also an appropriate non-linear incentive schedule can be created for each entrepreneur to induce him to make the appropriate effort. Thus the classical markets instruments consisting of debt, equity and options, when used by sophisticated and well-informed investors, can replace the ideal Arrow securities which are not observed in the real world. In this way, the capital market instruments characteristic of developed financial markets provide a class of indirect instruments which lead us back to Adam Smith's invisible hand, coordinating the self-interested actions of agents, even in the presence of moral hazard.

APPENDIX

Let us show that when the expression for rational, competitive price perceptions $\tilde{\boldsymbol{Q}}^i$ given by Definition 4 is substituted in equations (10.18, 10.19) the FOC of an RCPP equilibrium coincide with the FOC for constrained Pareto optimality. The partial derivatives of $\tilde{\boldsymbol{Q}}^i$ are

$$\frac{\partial \tilde{Q}^i_\beta}{\partial \kappa^i} = \sum_{x \in \mathcal{S}^i_\beta} \bar{\pi}_s \left(\frac{\partial F^i_s}{\partial \kappa^i} + \frac{\partial F^i_s}{\partial e^i} \frac{\partial \tilde{e}^i}{\partial \kappa^i} \right), \tag{10.20}$$

$$\frac{\partial \tilde{Q}^i_\beta}{\partial \theta^i_k} = \sum_{s \in \mathcal{S}^i_\beta} \bar{\pi}_s \frac{\partial F^i_s}{\partial e^i} \left(\sum_{s' \in \mathcal{S}} \frac{\partial \tilde{e}^i}{\partial m^i_{s'}} y^k_{s'} \right), \quad k \neq i, \tag{10.21}$$

$$\frac{\partial \tilde{Q}^i_\beta}{\partial \xi^i_{k,j}} = \sum_{s^i \mathcal{S}^i_\beta} \bar{\pi}_s \frac{\partial F^i_s}{\partial e^i} \left(\sum_{s' \in \mathcal{S}^k_j} \frac{\partial \tilde{e}^i}{\partial m^i_{s'}} (y^k_{s'} - \tau^k_j) \right), \quad k \neq i, \quad j \in \mathcal{J}^k, \tag{10.22}$$

$$\frac{\partial \tilde{Q}^i_\beta}{\partial \theta^i_i} = \sum_{s^i \mathcal{S}^i_\beta} \bar{\pi}_s \frac{\partial F^i_s}{\partial e^i} \frac{\partial \tilde{e}^i}{\partial \theta^i_i}, \tag{10.23}$$

$$\frac{\partial \tilde{Q}^i_\beta}{\partial \xi^i_{i,j}} = \sum_{s \in \mathcal{S}^i_\beta} \bar{\pi}_s \frac{\partial F^i_s}{\partial e^i} \frac{\partial \tilde{e}^i}{\partial \xi^i_{i,j}}, \tag{10.24}$$

where β is an index denoting one of the securities associated with firm i ($\beta = y$ or $\beta = j, j \in \mathcal{J}^i$), whose price is influenced by the action of entrepreneur i, and $\mathcal{S}^i_\beta$ is the subset of states in which security (i,β) has a positive payoff: thus $\mathcal{S}^i_\beta = \mathcal{S}$ if $\beta = y$ and $\mathcal{S}^i_\beta = \mathcal{S}^i_j = \{s \in \mathcal{S} | F^i_s(\bar{\kappa}^i, \bar{e}^i) > \tau^i_j\}$ if $\beta = j$, $j \in \mathcal{J}^i$.

For $i \in \mathcal{I}$, define

$$\epsilon^i = (1-\bar{\theta}^i_i)\sum_{s\in\mathcal{S}} \bar{\pi}_s \frac{\partial F^i_s}{\partial e^i} - \sum_{j\in\mathcal{J}^i} \bar{\xi}^i_{i,j} \sum_{s\in\mathcal{S}^i_j} \bar{\pi}_s \frac{\partial F^i_s}{\partial e^i}. \tag{10.25}$$

$$\bar{\mu}^i_{s'} = \bar{\pi}^i_{s'} + \epsilon^i \frac{\partial \bar{e}^i}{\partial m^i_{s'}}, \quad s' \in \mathcal{S}. \tag{10.26}$$

Substituting equations (10.21) and (10.22) into equation (10.19) for a security α whose payoff is not directly influenced by the effort of agent i ($\alpha = (k,\beta)$ with $k \neq i$, $\beta = y$ or $\beta = j, j \in \mathcal{J}^k$) and using the expressions in equations (10.25) and (10.26), we obtain

$$\bar{q}^k_\alpha = \sum_{s\in\mathcal{S}} \bar{\mu}^i_s v^\alpha_s,$$

so that each agent equalizes the price of a security which influences his outside income with its present value under the modified present-value vector equation (10.26). Thus for an agent $k \neq i$ and a security for firm i ($\alpha = (i,\beta)$)

$$\bar{q}_\alpha = \bar{q}^i_\beta = \sum_{s\in\mathcal{S}} \bar{\mu}^k_s v^\alpha_s = \sum_{s\in\mathcal{S}} \bar{\pi}_s v^\alpha_s, \tag{10.27}$$

where the second equality follows from the definition of $\bar{\boldsymbol{\pi}}$. Thus the valuations under the vectors $\bar{\boldsymbol{\pi}}$ and $\bar{\boldsymbol{\mu}}^k$ agree on the subspace $\mathcal{V}_{-k}(\bar{\boldsymbol{y}})$. A marginal change $\Delta\kappa^i$ in the input, or Δe^i in the effort of entrepreneur i, induces a change $\Delta y^i_s = (\partial F^i_s/\partial\kappa^i)\,\Delta\kappa^i$ or $(\partial F^i_s/\partial e^i)\,\Delta e^i$ in output in each state: this induces a change $\Delta \boldsymbol{y}^i$ in the payoff of equity and a change $\Delta \boldsymbol{R}^i_j$ in the payoff of option j where

$$\Delta R^i_{js} = \begin{cases} \dfrac{\partial F^i_s}{\partial \kappa^i}\Delta\kappa^i \quad \text{or} \quad \dfrac{\partial F^i_s}{\partial e^i}\Delta e^i & \text{if} \quad s \in \mathcal{S}^i_j, \\ 0 & \text{if} \quad s \notin \mathcal{S}^i_j. \end{cases}$$

By SPS the changes $\Delta\, \boldsymbol{y}^i$ and $\Delta\, \boldsymbol{R}^i_j$ must lie in $\mathcal{V}_{-k}(\,\bar{\boldsymbol{y}})$ for all $k \neq i$. In view of equation (10.27)

$$\sum_{s\in\mathcal{S}^i_\beta} \bar{\pi}_s \frac{\partial F^i_s}{\partial \kappa^i} = \sum_{s\in\mathcal{S}^i_\beta} \bar{\mu}^k_s \frac{\partial F^i_s}{\partial \kappa^i}, \quad \sum_{s\in\mathcal{S}^i_\beta} \bar{\pi}_s \frac{\partial F^i_s}{\partial e^i} = \sum_{s\in\mathcal{S}^i_\beta} \bar{\mu}^k_s \frac{\partial F^i_s}{\partial e^i}, \tag{10.28}$$

where we recall that $\mathcal{S}^i_\beta = \mathcal{S}$ when $\beta = y$. Since

$$1-\bar{\theta}^i_i = \sum_{k\neq i} \bar{\theta}^i_k, \quad \bar{\xi}^i_{i,j} = -\sum_{k\neq i} \bar{\xi}^k_{i,j} \tag{10.29}$$

using equations (10.28), ϵ^i in (10.25) can be written as

$$\epsilon^i = \sum_{k \neq i} \bar{\theta}_i^k \sum_{s \in \mathcal{S}} \bar{\mu}_s^k \frac{\partial F_s^i}{\partial e^i} + \sum_{k \neq i} \sum_{j \in \mathcal{J}^i} \xi_{i,j}^k \sum_{s \in \mathcal{S}_j^i} \bar{\mu}^{k_s} \frac{\partial F_s^i}{\partial e^i}$$

$$= \sum_{k \neq i} \sum_{s \in \mathcal{S}} \bar{\mu}_s^k \left(\bar{\theta}_i^k + \sum_{j \in \mathcal{J}_s^i} \bar{\xi}_{i,j}^k \right) \frac{\partial F_s^i}{\partial e^i}, \quad (10.30)$$

which is the same as equation (10.15). Substituting equations (10.20)–(10.24) into equations (10.17)–(10.19), using equation (10.28) and (10.30), gives the FOC equations (10.6)–(10.13) for a CPO.

It is interesting to note that when ϵ^i is defined by equation (10.25), and price perceptions satisfy equations (10.20)–(10.24), then for any change dz_α^i in the portfolio of entrepreneur i

$$\frac{\partial}{\partial z_\alpha^i} \left(\tilde{Q}_y^i (1 - \bar{\theta}_i^i) - \tilde{\boldsymbol{Q}}_c^i \bar{\boldsymbol{\xi}}_i^i \right) = \epsilon^i \frac{\partial \tilde{e}^i}{\partial z_\alpha^i}.$$

Thus in an RCPP equilibrium an entrepreneur acting purely in his own self interest is made aware of the value of his effort (ϵ^i) through the change in the date 0 income earned from the sale of his equity (net of options), arising from a change Δe^i in the effort that investors expect from him. The optimality property of an RCPP equilibrium is then explained by equality equation (10.30): market clearing and the common valuation of the traded securities imply that *the private value* ϵ^i *of his effort to entrepreneur i given by* equation (10.25) coincides with the social value of his effort to investors holding securities of firm i, given by the right side of equation (10.30).

NOTES

1 Keynes (1930, Chapter 15) was an articulate advocate of the idea that differences of opinion are an important explanatory factor for trade on financial markets.

2 For readers with a mathematical bent, it was shown that the underlying "fixed point" argument involves a more general theorem than the Brouwer fixed point theorem – essentially a "fixed point" theorem for maps from the Grassmannian of J-dimensional subspaces of $\mathbb{R}^S$ to $\mathbb{R}^{JS}$, for an economy with S states of nature and J securities (see Hirsch *et al.*, 1990).

3 As Shiller (1997) noted: "That the public should generally want to denominate contracts in currency units – despite all the evidence that it is not wise to do so ... should be regarded as one of the great economic puzzles of all time."

4 Sets are denoted by calligraphic letters: the same letter in roman denotes the cardinality of the set, e.g. $\mathcal{I} = 1, \ldots, I\}$. Vectors, matrices and vector-valued functions are written in boldface.

5 This is the central theme of Knight's (1921) classic treatise *Risk, Uncertainty and Profit*.

6 When we need unified notation for both types of agents, entrepreneurs and investors, we adopt the convention that for $k \in \mathcal{I}_2, \theta_k^i = 0$ if $i \neq k, \theta_k^k = 1$ and $\mathcal{J}^k = \emptyset$.

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11 Intermediation, the stock market and intergenerational transfers

Michael Magill and Martine Quinzii

Roughly speaking there are two classes of general equilibrium models which explicitly recognize that the future is open ended: the first, simplest and most idealized, views agents as being infinitely lived – they are all permanently on stage together. With a complete set of markets such an economy leads to an efficient outcome: in essence all contingencies can be traded out at an initial date and the future unfolds in a preordained way thereafter. The second, more realistic framework, recognizes that agents are finitely lived – agents in any generation are on stage for only a transient interval of time and are replaced by the agents of the next generation. In this latter model, markets face more serious problems for coordinating agents, decisions over time. In this chapter, we use the simplest overlapping generations (OLG) model with production to study how capital markets coordinate decisions of consumers and the investment decisions of firms, their joint decisions leading to a path of capital accumulation for the economy.

The distilled essence of our message is best understood by first looking at the simplest model of an OLG exchange economy, namely the canonical model of Allais (1947) and Samuelson (1958), as analyzed and interpreted in the elegant paper of Gale (1973). The first principle that we draw from this preliminary model is that economies can be classified into one of two types depending on the direction (sign) of the transfers between generations at the Golden Rule (positive if from young to old, negative if from old to young) and that the long-run dynamic behavior of an economy can be predicted once the sign of these transfers is known. The second principle is that financial markets lead to long-run efficiency for economies with negative transfers, but do not lead to long-run efficiency for economies with positive transfers, i.e. financial markets can successfully transfer income backwards, but not forwards.

We then consider the simplest model of an OLG production economy, namely the classic model of Diamond (1965), which has become one of the basic workhorses of modern macroeconomics. Many current policy debates – for example, on the nature and role of the social security system – draw heavily on the insights of this model, and it is here that our message comes with a warning. The Diamond model, and the host of literature that ensued (see for example, Blanchard and Fischer, 1989) assumes that borrowing and lending can only occur between agents of the same generation – in short, that the bond market is intragenerational. Once the smallest dose of intergenerational intermediation is introduced, the equilibrium of the model changes radically for all economies characterized by negative transfers at the efficient steady-state. This can be seen by analyzing a generalized version of the Diamond model in which infinitely-lived intermediaries permit intergenerational transfers to take place on the bond market. The resulting model generates equilibria whose qualitative behavior parallels that of the OLG exchange economies. The equilibria of economies with positive transfers at the Golden Rule (called economies with *overaccumulation*), converge to the Diamond steady-state, which is inefficient: at such a steady-state too much of the society's current output is devoted to investment rather than consumption. Thus the enlarged set of market opportunities offered by intergenerational intermediation does not lead to long-run efficiency for economies with overaccumulation. By contrast, for economies with negative transfers at the Golden Rule (called economies with *under-accumulation*), the equilibrium converges to the Golden Rule. In this case, the enlarged financial opportunities offered by the financial intermediaries lead to greater long-run efficiency than that obtained with the more restrictive lending structure of the Diamond model. If, as is often argued in the literature (e.g. Abel *et al.*, 1989), under-accumulation is the most realistic case, and if, as we would argue, there is a rich set of financial intermediaries which permit intergenerational transfers, then many of the important policy recommendations which are based on the Diamond model, can change in significant ways. For example, while a Diamond model predicts that a pay-as-you-go social security system lowers the long-run level of capital, in the model with intergenerational transfers, even though a pay-as-you-go system slows down the process of capital accumulation, it has no impact on its long-run behavior.

Finally, we argue that the bond market is not the only financial market through which intergenerational transfers can take place. Provided "firms" are modeled in a more realistic way, it can be shown that the canonical market for the transfer of ownership of firms, the *stock market*, provides a natural mechanism for the inter-generational transfer of funds. The greater realism consists in recognizing that capital once installed in a firm can no longer be unbolted and made equivalent to so many units of current output, in short that capital is a sunk cost. In such a setting, we show that the financial value of a firm is typically less than its replacement cost (i.e. it has a Tobin's q less than one) and that this discount on the value of the firm relative to its replacement cost in essence provides a mechanism for the transfer of funds from the old generation that owns the firm to the young generation which purchases it. Thus the framework of this chapter throws a new light on the role of the stock market as an instrument for transferring ownership of firms across generations.

11.1 OLG EXCHANGE ECONOMY

Consider an OLG exchange economy with one good, in which agents live for two periods and population grows at the rate n. Let N_t denote the number of young born at date t, then $N_t = (1+n)N_{t-1}$ where N_{t-1} is the number of old at date t. Each agent has a lifetime endowment $e = (e_0, e_1) \in \mathbb{R}^2_+$, representing income when young and old, and a preference ordering represented by a utility function $u(c_0, c_1)$, where (c_0, c_1) is the agent's lifetime consumption stream: thus all agents are identical, except for their date of birth. Let $\mathcal{E}(u, e, n)$ denote the associated exchange economy in which each agent has the utility function u, the endowment stream e, and the population grows at the rate n. In the analysis that follows we will restrict attention to a subset of preference orderings which leads to a simple structure for the dynamics of the equilibrium model. This subset is most conveniently defined by drawing on the standard framework of microeconomic theory which tells us how an agent's demand responds to changes in prices and income: the two "goods" are consumption when young and old (c_0, c_1), their prices are (p_0, p_1), and the agent's income (wealth) under these prices is denoted by $m = p_0 e_0 + p_1 e_1$.

Assumption U The utility function $u: \mathbb{R}^2_+ \to \mathbb{R}$ satisfies

(i) u is strictly quasi-concave, smooth, increasing, satisfies $u_0'(c) \to \infty$ if $c_0 \to 0^+$ and $u_1'(c) \to \infty$ if $c_1 \to 0^+$ (where $u_\ell' = \partial u / \partial c_\ell$, $\ell = 0, 1$)

(ii) the demand function $c(p, m) = \arg\max\{u(c) \mid pc = m\}$ with $c(p, m) = (c_0(p, m), c_1(p, m))$ satisfies

$$(\alpha)\ \frac{\partial c_0}{\partial m}(p, m) > 0, \quad \frac{\partial c_1}{\partial m}(p, m) > 0, \quad \text{for all} \quad p \gg 0, \quad m > 0$$

$$(\beta)\ \frac{\partial c_0}{\partial p_0}(p, pe) < 0, \quad \frac{\partial c_1}{\partial p_0}(p, pe) > 0 \quad \text{for all} \quad p \gg 0 \quad \text{and} \quad e \in \mathbb{R}^2_+ \text{ such that } pe > 0$$

(ii)(α) requires that consumption at each period of life be a normal good; (ii)(β) requires that when the income and substitution effects are of opposite signs, the substitution effect dominates. In the sequential model with borrowing and lending at the interest rate r, we will see that $(p_0, p_1) = ((1 + r), 1)$: thus an increase in p_0 is equivalent to an increase in the rate of interest r and (ii)(β) implies an increase in the rate of interest decreases date 0 consumption and increases date 1 consumption. Assumption U is satisfied by Cobb–Douglas utility functions and CES utility functions with elasticity of substitution greater than 1.

We assume that the story starts at an initial date, called date $t = 0$. Let $c_t = (c_{0,t}, c_{1,t+1})$ denote the (lifetime) consumption stream of a representative agent of the generation born at date t. An *allocation* $\boldsymbol{c} = (c_{1,0}, (c_t)_{t \geq 0})$ is a consumption stream $(c_t)_{t \geq 0}$ for all generations born from date $t = 0$ into the indefinite future, and consumption $c_{1,0}$ for the representative agent of the old generation at $t = 0$. Note that a consequence of imposing a starting date on the model ($t = 0$) is that the model cuts into the life story of agents of the very first generation in "midlife," specifying only what their consumption $c_{1,0}$ is in their old age and leaving completely unspecified what their consumption was when they were young. This asymmetric treatment of the first generation is an unavoidable consequence of the overlapping structure of the generations and the desire to start the model at some date in the finite past which we call date $t = 0$ – and not at $t = -\infty$ as the model would really like. Some of the tricky and unintuitive properties of the OLG model have their origin in this asymmetric treatment of the first generation. An allocation $\boldsymbol{c} = (c_{1,0}, (c_t)_{t \geq 0})$ is *feasible* if for all $t \geq 0$

$$N_t c_{0,t} + N_{t-1} c_{1,t} = N_t e_0 + N_{t-1} e_1 \iff (1 + n)(c_{0,t} - e_0) + (c_{1,t} - e_1) = 0, \quad (11.1)$$

namely if no more consumption is distributed to the young and old generations at date t than is available through their combined date t endowments. An allocation is *Pareto-optimal* if there is no other feasible allocation $\tilde{\boldsymbol{c}} = (\tilde{c}_{1,0}, (\tilde{c}_t)_{t \geq 0})$ such that $\tilde{c}_{1,0} \geq (c_{1,0})$ and $u(\tilde{c}_t) \geq u(c_t)$ for all $t \geq 0$, with at least one strict inequality.

The simplest kind of allocation is one in which all generations have the same lifetime consumption: more precisely, we say that the allocation $\boldsymbol{c} = (c_{1,0}, (c_t)_{t \geq 0})$ is a *steady-state* if there exists a lifetime consumption stream (c_0, c_1) such that $c_{1,0} = c_1,\ c_t = (c_0, c_1)$ for all $t \geq 0$. For a steady-state (c_0, c_1), the feasibility condition (1) becomes

$$(1+n)c_0 + c_1 = (1+n)e_0 + e_1. \tag{11.2}$$

A steady-state (c_0^*, c_1^*) is said to be *efficient* if it satisfies the equation (11.2) and there is no other feasible steady-state which is preferred by the representative agent of any generation. An efficient steady-state (c_0^*, c_1^*) is a solution to the problem

$$\max\{u(c) \,|\, (1+n)c_0 + c_1 = (1+n)e_0 + e_1,\ c \in \mathbb{R}^2_+\}$$

and is characterized by the first-order condition

$$\frac{u_0'(c_0^*, c_1^*)}{u_1'(c_0^*, c_1^*)} = 1 + n. \tag{11.3}$$

The geometric representation of the efficient steady-state is the same as the geometric representation of the optimal choice of consumption of an agent with initial endowment (e_0, e_1) maximizing utility under the budget constraint equation (11.2) (see Figure 11.1). Under assumption U there is a unique efficient steady-state (c_0^*, c_1^*) which is called the *Golden Rule (steady-state)*.

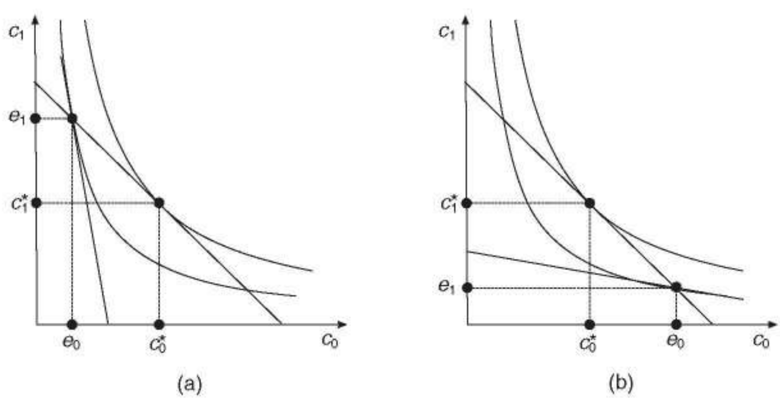

Figure 11.1 The Golden Rule (c_0^*, c_1^*): for the economy in (a) the endowment (e_0, e_1) is such that the transfers at (c_0^*, c_1^*) are negative; for the economy in (b), (e_0, e_1) is such that the transfers are positive.

It follows from Gale's (1973) analysis that there is an interesting way of classifying the economies $\mathcal{E}(u, e, n)$ depending on the behavior of the savings of the young generation at the Golden Rule. This classification partitions economies into two types, with the dynamical behavior of an economy within each type having the same qualitative properties.

DEFINITION 1 An exchange economy $\mathcal{E}(u, e, n)$ is said to have negative (positive) transfers at the Golden Rule if $s^* = e_0 - c_0^* < 0\ (>0)$.

To abbreviate, we will refer to these economies as negative transfer economies and positive transfer economies respectively. The hairline case where $s^* = 0$ is ignored since it is exceptional. For given preferences u and demographic structure n (the indifference curve and the slope of the line in Figure 11.1) an economy has negative (positive) transfers if the endowment in old age is sufficiently large (small) relative to that in youth – Figure 11.1(a) and (b) respectively. Because classical economists typically considered economies in which income is increasing, while Samuelson in his original paper (1958) studied the case where endowments in old age are zero, Gale (1973) refers to the cases of negative and positive transfers as the "classical" and "Samuelson" cases respectively.

The rate of impatience $r(c_0, c_1)$ of an agent at a consumption stream $(c_0, c_1) \in \mathbb{R}_+^*$ is defined by

$$1 + r(c_0, c_1) = \frac{u_0'(c_0, c_1)}{u_1'(c_0, c_1)}.$$

Let $r^* = r(c_0^*, c_1^*) = n$ be the rate of impatience at the *Golden rule* and let $r_A = r(e_0, e_1)$ be the rate of impatience at the *autarchic* steady-state (initial endowment). From the geometry of the indifference curves in Figure 11.1, note the following useful property: $r_A > r^*$ *in an economy with negative transfers and* $r_A < r^*$ *when the transfers are positive*.

Note that the notion of an efficient steady-state is different from that of a Pareto-optimal allocation, since the comparison is restricted to other steady-states. The difference comes from the awkward presence of the representative old agent at date 0. For example, the autarchic steady-state where agents consume their initial endowment is typically an inefficient steady-state. However, in the case of negative transfers, it is a Pareto-optimal allocation: as seen from Figure 11.1(a) the allocations which are preferred to the initial endowment by the representative agent involve consuming more at date 0. But then feasibility implies that the representative old agent at date 0 consumes less, so that a Pareto improvement is impossible.

11.1.1 Market structure and intermediation

So far we have analyzed the feasible allocations of the economy $\mathcal{E}(u, e, n)$: now it is time to study the allocations that can arise when the agents resort to decentralized trading on markets. Our objective is to study the consequence of introducing a financial market structure in which agents can borrow and lend so as to achieve a lifetime consumption stream $c_t = (c_{0,t}, c_{1,t+1})$ which is preferred to their initial endowment stream $e = (e_0, e_1)$. It is clear that the contemporary young and old generations at date t cannot *directly* enter into such loan contracts, since the old will be dead next period. To make intergenerational loans possible we assume that there is an infinite-lived financial intermediary[1]: if young agents want to borrow, they borrow from the intermediary which collects their payments (principal and interests) when they are old, and uses the proceeds to lend to the new young generation. To ensure that the model can start off with funds available to lend to the young, we apply a "symmetry" condition to the old generation, assuming that in their youth ($t = -1$) they borrowed in a way similar to the current ($t = 0$) young, so that they are willing to reimburse in their old age. A similar reasoning applies when the young want to lend: they give their savings to the intermediary which uses them to pay back the old who saved in the previous period. In this latter case, the intermediary can be replaced by an asset like "money" which can be carried over from one period to the next. The young buy it from the old and sell it to the new young generation when they themselves have become old.

Suppose therefore that a young agent born at date t can borrow or lend with an interest rate r_{t+1} to be paid at date $t+1$. The agent's sequential budget set is given by[2]

$$\mathbb{B}(r_{t+1}) = \left\{ c_t \in \mathbb{R}^2_+ \;\middle|\; \begin{array}{l} c_{0,t} = e_0 - s \\ c_{1,t+1} = e_1 + (1 + r_{t+1})s, \quad s \in \mathbb{R} \end{array} \right\} \tag{11.4}$$

Let $c(r_{t+1})$ denote the solution to the agent's problem of maximizing utility over the budget set $\mathbb{B}(r_{t+1})$

$$c(r_{t+1}) = \arg\ \max\{u(c) \,|\, c \in \mathbb{B}(r_{t+1})\},$$

$c(r_{t+1})$ is summarized by the optimal savings function $s(r_{t+1})$ satisfying $(c_0(r_{t+1}), c_1(r_{t+1})) = (e_0 - s(r_{t+1}),\ e_1 + (1 + r_{t+1})s(r_{t+1}))$. Note that $s(r_A) = 0$ and $s(r^*) = e_0 - c_0^*$ (see Figure 11.1).

Since at date t there are $(1+n)$ young agents for each old agent, the equilibrium on the market for loans (the budget constraint of the intermediary) imposes the conditions

$$(1+n)s(r_{t+1}) = (1+r_t)s(r_t) \quad \forall t \geq 0. \tag{E}$$

When the model begins at date 0, the interest rate r_0, or equivalently the consumption $c_{1,0}$ of the old agents at date 0, must be given as initial condition.

Definition 2 An intermediary equilibrium for the OLG exchange economy $\mathcal{E}(u, n, e)$, with initial condition r_0, is a pair $((\boldsymbol{c}, \boldsymbol{s}), \boldsymbol{r}) = (c_{1,0}, (c_t, s_t)_{t=0}^{\infty}, (r_{t+1})_{t=0}^{\infty})$ consisting of a sequence of consumption-saving decisions $(\boldsymbol{c}, \boldsymbol{s})$ and a sequence of (correctly anticipated) interest rates $\boldsymbol{r}$ such that $c_{1,0} = s(r_0)(1 + r_0)$, $c_t = c(r_{t+1})$, $s_t = s(r_{t+1})$, and (E) holds for all $t \geq 0$.

Equation (E) is the difference equation which, together with the initial condition r_0, determines the equilibrium path of interest rates. Its steady-state solutions are $r^* = n$ and r_A, where the latter is the only interest rate r such that $s(r) = 0$. Define the functions $\mathcal{D}$ and $\mathcal{S}$ by

$$\mathcal{D}(r) = (1+n)s(r), \qquad \mathcal{S}(r) = (1+r)s(r)$$

At date t, the current interest rate is r_{t+1} and $\mathcal{D}(r_{t+1})$ is the "demand" function of the young – a demand for loans if $\mathcal{D}(r_{t+1}) < 0$, and a demand for savings opportunities if $\mathcal{D}(r_{t+1}) > 0$ – while $\mathcal{S}(r_t)$ is the "supply" of funds forthcoming from the old, which is fixed at date t since it depends on the previous period interest rate r_t. The equilibrium conditions equation (E) can be written as

$$\mathcal{D}(r_{t+1}) = \mathcal{S}(r_t), \quad \forall\, t \geq 0.$$

An intermediary equilibrium is thus a sequence of interest rate $(r_t)_{t\geq 0}$ such that at each date t, the interest rate r_{t+1} adjusts so as to equate the current demand $\mathcal{D}(r_{t+1})$ to the fixed supply of funds $\mathcal{S}(r_t)$ determined by the previous period interest rate r_t.

Under Assumption U, $\mathcal{D}(\cdot)$ and $\mathcal{S}(\cdot)$ are monotone increasing functions: their graphs are shown in Figure 11.2(a) for the case $r^* < r_A$, and in Figure 11.2(b) for the case $r_A < r^*$. Since $s(r) < 0$ whenever $r < r_A$ and $s(r) > 0$ whenever $r > r_A$, the curve $\mathcal{D}$ must lie below the curve $\mathcal{S}$ when $r < r^* < r_A$ in case (a) or $r < r_A < r^*$ in case (b), and when $r > \max(r^*, r_A)$. Whenever at any interest rate r, $\mathcal{D}(r) > \mathcal{S}(r)$, the interest rate must be reduced to clear the loan market, while if $\mathcal{D}(r) < \mathcal{S}(r)$ the interest rate must be increased. As a result for every economy (with $r^* \neq r_A$) the low interest rate steady-state is stable, and the high interest rate steady-state in unstable. For a negative transfer economy ($r^* < r_A$), the equilibrium converges to the Golden Rule for all initial conditions such that $r_0 < r_A$, the equilibrium stays in the autarchic steady-state if $r_0 = r_A$, and if $r_0 > r_A$ then the interest rate increases to infinity and there is no equilibrium starting from r_0. Thus in an economy with negative transfers the presence of a financial intermediary, by permitting transfers from the old to the young, makes it possible for the economy to reach a long-run efficient steady-state, for all viable initial conditions, except $r_0 = r_A$.

For a positive transfer economy ($r_A < r^*$), the equilibrium converges to the autarchy steady-state for all initial conditions $r_0 < r^*$; the equilibrium stays at the Golden rule steady-state if $r_0 = r^*$, and if $r_0 > r^*$ there is no equilibrium. Unless

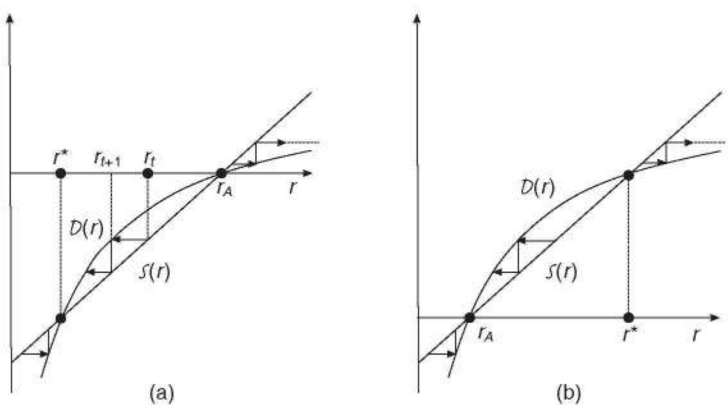

Figure 11.2 The equilibrium interest rate r_t changes to r_{t+1} to equate demand $\mathcal{D}(r_{t+1})$ to supply $\mathcal{S}(r_t)$ of loans. (a) for negative transfer economy r_t converges to r^*; (b) for positive transfer economy r_t converges to r_A.

the interest rate happens to be exactly right initially ($r_0 = r^*$), the economy is driven back to autarchy – an allocation which, in addition to being an inefficient steady-state, is not even Pareto-optimal.

What is very striking in this model is how the demographic factor – the population growth rate n – determines the outcome on the financial market, namely the behavior of equilibrium interest rates. To understand why it plays such a pervasive role, note first that by the definition of r_A and the monotonicity of each young agent's saving function, if $r < r_A$ young agents when faced with the interest rate r will want to borrow, while if $r > r_A$ they will want to lend (save). Now consider a negative transfer economy with an interest rate $r_t < n < r_A$. Then old agents have borrowed in their youth (at $t-1$) and are reimbursing at date t. However, since $r_t < n$ they do not collectively as a cohort reimburse enough to permit each young agent to borrow as much as they themselves had borrowed at date $t-1$: the sheer weight of the new cohort is too great to permit the loan market to be cleared at the same interest rate. Thus each young agent must be induced to reduce his borrowing, and the only way to do this is to raise the interest rate, $r_{t+1} > r_t$. On the other hand, if the interest rate r_t exceeds the rate of growth of the population ($n < r_t < r_A$), then the cohort of old agents reimburse in such a way that each young agent at date t must be induced to borrow more than the old agents had borrowed at the previous date, and the only way to do this is to lower the interest rate $r_{t+1} < r_t$. Thus for a negative transfer economy ($n < r_A$) it is the sheer weight of the cohort of young agents pouring out on the loan market (relative to the existing cohort of old agents) which drives the interest rate to equality with the rate of population growth n. The same reasoning applies in a positive transfer economy ($r_A < n$), except that in this case for an interest rate r_t close to n, young agents save instead of borrowing. Thus if $r_A < r_t < n$, the old agents must be reimbursed with the savings of the young. However, since $r_t < n$, the cohort of young agents is too large and young agents must be induced to save less than the previous generation and the only way this can be achieved is by lowering the interest rate $r_{t+1} < r_t$. If $r_t > n$, the young agents must be induced to save more than the previous generation and the interest rate has to increase, $r_{t+1} > r_t$. After a while however it is no longer possible to induce young agents to save more to reimburse the old generation, since the income of the young is bounded: adjustments in the rate of interest do not permit demand to be matched to supply and the system breaks down. Thus if $r_t > n$ in case (b) (or by the same reasoning if $r_t > r_A$ in case (a)) there is no equilibrium.

An important message that emerges from the OLG exchange model is that competitive financial markets can achieve long-run efficiency when the Golden Rule requires transfers from the old to the young (negative transfers), but cannot prevent the economy from reverting to autarchy in the case where agents save at the Golden Rule (positive transfers). For economies in this latter category, it seems that only direct transfers from young to old agents – akin to a pay-as-you-go social security system – can restore long-run efficiency.

11.2 OLG PRODUCTION ECONOMY

In this section we shall show that the insight obtained from the OLG model of an exchange economy regarding the role of intergenerational intermediation carries over in a natural and interesting way to a production economy. The canonical model of an OLG production economy was introduced in the classic paper of Diamond (1965). While the exact market structure underlying the model can be expressed in a number of equivalent ways, the basic idea is that certain agents of the young generation (who can be thought of as entrepreneurs) buy the capital stock of existing firms from the old generation entrepreneurs, and then decide on the new investment they want to make in their firms. The funds required to finance both the purchase of firms and the new investment is obtained by loans from fellow members of the young cohort: thus borrowing by firms from consumers is assumed to be *intragenerational*. In the resulting equilibrium, capital accumulates to an inefficient steady-state, called the Diamond steady-state.

We shall show that when intermediaries are introduced which make *intergenerational intermediation* possible, the dynamic properties of the equilibrium change in a radical way, for economies in which the Golden Rule level of capital requires substantial borrowing to purchase and maintain the efficient capital stock, relative to the willingness of the young to save (the analogue of a negative transfer economy in the production setting), in equilibrium capital accumulates to the long-run efficient steady-state. Thus long-run efficiency is restored by removing the constraint on borrowing imposed on the entrepreneurs of the young generation, and the increased borrowing power made possible by the intermediaries re-establishes the long-run efficiency of markets. In the opposite case where the borrowing required to finance the efficient capital stock is small relative to the willingness of the young to save (the analogue of a positive transfer economy), in equilibrium capital accumulates to the inefficient steady-state: for these economies, just as in the exchange setting, efficiency can only be restored by resorting to government intervention in the form of a social security tax which directly transfers from the young to the old. There is thus a complete parallel between the role of intergenerational intermediation and social security for the attainment of long-run efficiency in the exchange and production economy settings. In the production model the Diamond steady-state plays the role of autarchy in the exchange model – autarchy corresponding to the absence of intergenerational transfers. Whether or not intermediation plays an important role in facilitating the appropriate capital accumulation in the economy depends on the relation between two fundamentally different characteristics of the economy, the willingness of the agents to save and the technological and demographic factors determining the Golden Rule.

We show that the analysis can be pushed a step further. We introduce an additional element of realism into the modeling of firms by recognizing that, since the resources invested in a firm typically have to be adapted in a way that is *firm specific* to make them function smoothly and efficiently, the expenses involved in the purchase of such resources cannot readily be recovered on a resale market. Thus, to simplify, we consider the theoretical limit where capital, once installed in a given firm, has no value in a general market for second-hand capital. In order to maintain the value of its invested capital a firm must retain its identity as an income generating entity in the economy: the natural market for transmitting firms across generations then becomes a market for ownership rights to the future stream of income that it will generate, namely a stock market for ownership shares in the firm. Now, as in Tobin's q theory (Tobin, 1969), the *replacement cost* of a firm and its *stock market value* become two distinct valuations. We exploit this property to show that under certain conditions, the intergenerational intermediation required in an economy with "negative transfers" can be achieved through the stock market which transfers firms between the old and the new generation at a cost which is lower than the replacement cost of the firm. Thus we obtain a new insight into the role of a stock market as an instrument leading to long-run efficiency by permitting cheaper transfer of firms across generations. To show these results we begin by recalling Diamond's model of an OLG production economy.

11.2.1 Diamond's model

The demographic structure is the same as in the exchange model: agents live for two periods and at date t there are N_t young and N_{t-1} old agents with $N_t = (1+n)N_{t-1}$. There are three goods, a consumption good, a capital good and labor. Each agent has an endowment of one unit of labor when young and zero units when old. All agents have the same preference orderings over lifetime consumption represented by a utility function $u(c_0, c_1)$, with no disutility in the provision of labor. We will continue to assume that Assumption U is satisfied.

In the way Diamond originally presented the model, and in subsequent accounts given in the literature (for example, Blanchard and Fischer, 1989), there is no fully articulated micro-market structure which describes the complete functioning of the economy. Furthermore, there are a number of different, albeit equivalent, market structures within which the model can be embedded: we shall choose the one which provides the most natural and convenient reference point for presenting our generalized version of the Diamond model.

We consider the following market structure. At each date t there are four markets: a market for current output, a labor market, a second-hand capital market and a bond market. At each date there is a market on which the *current output* of firms is sold: this output can be used either as a consumption good or as a new capital good to add to the existing stock of capital. Since this is a real (as opposed to a monetary) model, the price of this good is normalized to be 1 in each period: this simply means that it acts as the numéraire in terms of which all valuations are expressed. In each period there is also a *labor market* on which the (homogeneous) services of the labor supplied by the young generation are sold to the firms, at the current wage rate w_t.

Since capital is durable, a market is needed for transferring the ownership of this capital between generations. Since previously installed capital is assumed to be a perfect substitute for new capital (much to the horror of vintage capital theorists), the price of *second-hand capital* is the same as the price of new capital, provided there is positive investment (i.e. young agents do not want to disinvest): we will restrict the analysis to this case.

The fourth market is the *bond market*, which permits young agents to save for their retirement and firms to finance their capital investment: the interest rate on a loan from date t to $t+1$ is r_{t+1}.

Some of the young agents born at date t are entrepreneurs: without loss of generality, the firm set up by each entrepreneur is a *sole proprietorship* and there are J_t such firms (we will see shortly that the number of these firms and the exact identity of the entrepreneurs does not matter, given that the assumption of constant returns to scale will be invoked). To create firm j the entrepreneur finances the capital input by borrowing from fellow members of his cohort on the bond market. This capital input, which we call K^j_{t+1}, because it becomes operational in the following period $t+1$, is purchased in part on the second-hand capital goods market and in part on the market for current output (the new capital goods component). When this capital K^j_{t+1} is combined at date $t+1$ with an amount L^j_{t+1} of labor services, it gives rise to an output $F^j_{t+1}(K^j_{t+1}, L^j_{t+1})$. The key simplifying assumption on which the Diamond and all growth models rest is that the production function F^j_{t+1} is time homogeneous, exhibits constant returns to scale and is the same for all firms.

Each firm, like the entrepreneur that owns it, lives for two periods: it is financed and created in period t and generates its output $F(K^j_{t+1}, L^j_{t+1})$ in the next period $t+1$. The entrepreneur must pay the wage bill $(w_{t+1}L^j_{t+1})$, reimburse the principal and interest on the loan $((1+r_{t+1})K^j_{t+1})$, and then sells the remaining used capital $((1-\beta)K^j_{t+1})$ on the second-hand capital goods market, where β is the depreciation rate of capital. The entrepreneur's profit is thus

$$\pi^j_{t+1} = F(K^j_{t+1}, L^j_{t+1}) - w_{t+1}L^j_{t+1} - (1+r_{t+1})K^j_{t+1} + (1-\beta)K^j_{t+1}.$$

Assuming that the prices (w_{t+1}, r_{t+1}) are such that profit maximization by firms has a positive solution, the first-order condition for the optimal choice of labor L^j_{t+1} is given by

$$F'_{\mathrm{L}}\left(K^j_{t+1}, L^j_{t+1}\right) = F'_{\mathrm{L}}\left(\frac{K^j_{t+1}}{L^j_{t+1}}, 1\right) = w_{t+1} \Longleftrightarrow f\left(k^j_{t+1}\right) - k^j_{t+1} f'\left(k^j_{t+1}\right) = w_{t+1}, \quad (11.5)$$

where $f(k^j) = F(k^j, 1)$ is the production function per unit of labor, and $k^j = K^j/L^j$. In the same way, the first-order condition for the optimal choice of K^j_{t+1} is given by

$$F'_{\mathrm{K}}\left(\frac{K^j_{t+1}}{L^j_{t+1}}, 1\right) = \beta + r_{t+1} \Longleftrightarrow f'(k^j_{t+1}) = \beta + r_{t+1}. \quad (11.6)$$

The first-order conditions equations (11.5) and (11.6) show that profit maximization for firm j only determines the capital–labor $k^j_{t+1} = K^j_{t+1}/L^j_{t+1}$ for firm j at date $t+1$, and that this ratio is the same for all firms. Note also that Euler's theorem for homogeneous functions of degree 1 implies that at the optimal capital–labor choice, firm j's profit is zero ($\pi^j_{t+1} = 0$).

In his youth, each agent must make a lifetime consumption plan: a wage w_t is received for the unit of labor supplied to one of the firms when young. A part of this income must be saved on the bond market to provide income for the agent in old age. Since entrepreneurs earn no profit (in equilibrium) from the ownership of their firms, the consumption-savings decision of every young agent is the same and involves choosing $(c_t, s_t) = (c_{0,t}, c_{1,t+1}, s_t)$ which solves the problem

$$\max \left\{ u(c_t) \;\middle|\; \begin{array}{ll} c_{0,t} = w_t - s_t, & s_t \in \mathbb{R} \\ c_{1,t+1} = s_t(1+r_{t+1}) & \end{array} \right\} \quad (11.7)$$

As in the exchange case, the solution is entirely characterized by the function $s : \mathbb{R}^2_+ \longrightarrow \mathbb{R}$ where $s(r_{t+1}, w_t)$ denotes the amount saved by a representative young agent faced with the interest rate r_{t+1} and the wage rate w_t.

At each date t there are four markets that need to clear: those for (current) output, labor, second-hand capital and bonds with prices $(1, w_t, 1, r_{t+1})$ respectively. The wage rate w_t must clear the labor market: since K_t^j has been inherited from the previous period, L_t^j is chosen so that the capital–labor ratio K_t^j/L_t^j satisfies equation (11.5) (at date t). Since all firms have the same capital–labor ratio, so that $k_t^j = \sum_{j=1}^{J_t} K_t^j / \sum_{j=1}^{J_t} L_t^j$ for all j, in order that the labor market clear $\left(\sum_{j=1}^{J_t} L_t^j = N_t\right)$, the aggregate capital stock of all firms at date t, $K_t = \sum_{j=1}^{J_t} K_t^j$, and the wage rate w_t must be linked by the conditions

$$f(k_t) - k_t f'(k_t) = w_t. \tag{11.8}$$

$$k_t = \frac{K_t}{N_t}. \tag{11.9}$$

Equilibrium on the bond market at date t requires that the savings of young consumers equals the borrowing by the entrepreneurs (of the same cohort), namely that

$$N_t s(r_{t+1}, w_t) = \sum_{j=1}^{J_t} K_{t+1}^j (r_{t+1}, w_{t+1}).$$

All entrepreneurs choose K_{t+1}^j, anticipating the same capital–labor ratio k_{t+1} at date $t+1$ satisfying

$$f'(k_{t+1}) = \beta + r_{t+1}, \tag{11.10}$$

so that $\sum_{j=1}^{J_t} K_{t+1}^j (r_{t+1}, w_{t+1}) = \sum_{j=1}^{J_t} L_{t+1}^j k_{t+1}$.

Given that their anticipations are correct $\left(\sum_{j=1}^{J_t} L_{t+1}^j = N_{t+1}\right)$

$$\sum_{j=1}^{J_t} K_{t+1}^j (r_{t+1}, w_{t+1}) = N_{t+1} k_{t+1}$$

so that equilibrium on the bond market will occur if

$$N_t s_t(w_t, r_{t+1}) = N_{t+1} k_{t+1} \Longleftrightarrow s_t(w_t, r_{t+1}) = (1+n) k_{t+1}. \tag{11.11}$$

Equilibrium on the second-hand capital market only requires that (net) investment defined by $I_t \equiv K_{t+1} - (1-\beta) K_t$ be non-negative. Entrepreneurs are indifferent between second-hand capital or new capital goods, so that we can assume that they begin by buying second-hand capital before drawing on the current output of firms to create "new" capital goods. In the analysis that follows we restrict attention to trajectories for which $I_t > 0$. The relation defining net investment when expressed in per-capita terms becomes

$$(1+n) k_{t+1} = (1-\beta) k_t + i_t,$$

where $i_t = I_t/N_t$. Note that by Walras' Law, equilibrium on the labor and bond markets implies that the market for the (current) output of firms clears. The zero profit condition for all firms implies that

$$\begin{aligned} Y_t &= w_t N_t + (1 + r_t)K_t - (1 - \beta)K_t \\ &= N_t(c_{0,t} + s_t) + N_{t-1}c_{1,t} - K_{t+1} + I_t \\ &= N_t c_{0,t} + N_{t-1}c_{1,t} + I_t \end{aligned}$$

so that supply equals demand for current output at date t.

Equations (11.8)–(11.11) together with an appropriate initial condition define an equilibrium of the model. Note that only aggregate variables enter into these equations, so that the model is in essence a *macro model*. As we have seen it is compatible with a micro-structure for consumers and firms but the exact structure of the firms (the number of firms, who the owners are and how capital and labor are allocated among firms) does not affect the equilibrium consumption streams of agents. In this sense the equilibrium of the model is invariant to the exact micro-structure of firms, and for this reason is usually studied directly at the aggregate level on the production side.

The model begins at date 0 with a capital stock K_0 inherited from the previous period $t = -1$: the condition that K_0 was chosen optimally at date -1 requires that

$$f'\left(\frac{K_0}{N_0}\right) = \beta + r_0 \tag{11.12}$$

and the per-capita savings of the date 0 old generation must have been such that $(N_0/1 + n)s_{-1} = K_0$. Thus

$$c_{1,0} = (1 + n)k_0(1 + r_0), \tag{11.13}$$

where r_0 is given by equation (11.12).

Definition 3 A Diamond equilibrium of the OLG production economy $\mathcal{E}(u, F, n)$ is a sequence $(\bar{\boldsymbol{c}}, \bar{\boldsymbol{w}}, \bar{\boldsymbol{k}}, \bar{\boldsymbol{r}}) = (\bar{c}_{1,0}, (\bar{c}_t, \bar{w}_t, \bar{k}_{t+1}, \bar{r}_{t+1})_{t\geq 0})$ such that

(i) $\bar{c}_{1,0}$ is given by (11.12) and (11.13)
(ii) $\bar{c}_t = (\bar{c}_{0,t}, \bar{c}_{1,t+1})$ is given by (11.7) with $s_t = s(w_t, r_{t+1})$ for all $t \geq 0$
(iii) $(\bar{w}_t, \bar{k}_{t+1}, \bar{r}_{t+1})$ satisfy (11.8)–(11.11) for all $t \geq 0$.

Given equations (11.8) and (11.10) the trajectory of the per-capita aggregate capital stock $(k_t)_{t\geq 0}$ entirely determines the equilibrium. Inserting equations (11.8) and (11.10) into (11.11) gives the first-order difference equation

$$s(r(k_{t+1}), w(k_t)) = (1 + n)k_{t+1}, \quad \forall t \geq 0, \tag{D}$$

which together with the initial condition k_0 determines the equilibrium path of capital. Assumption U implies the following properties:

- $0 < s'_w(r, w) < 1$
- $s'_r(r, w) \geq 0$

An additional assumption is needed to ensure that the equilibrium sequence $(k_t)_{t\geq 0}$ has a simple (monotonic) behavior.

Assumption S The function $k \to s(r(k), w(k))/k$ is decreasing with

$$\lim_{k\to 0^+} \frac{s(r(k), w(k))}{k} > 1 + n, \quad \lim_{k\to\infty} \frac{s(r(k), w(k))}{k} = 0.$$

Although this assumption is a joint assumption on preferences and technology, it can be decomposed into separate assumptions on the consumption and the production sides. For example, it holds if

- u is homothetic and satisfies Assumption U
- f is such that $w(k)/k$ is a decreasing function of k with $\lim_{k\to 0+} (w(k)/k) = \infty$ and $\lim_{k\to\infty} (w(k)/k) = 0$

These conditions are satisfied if both u and F are CES with elasticity of substitution greater than or equal to 1. The proof of the following result can be found in Magill and Quinzii (1999): it simply makes explicit the assumptions required to obtain Diamond's original result. The geometric interpretation is shown in Figure 11.3.

PROPOSITION 1 *Under Assumptions (U, S) the Diamond equilibrium trajectory $(k_t)_{t\geq 0}$ has the following properties*:

(i) *there exists an increasing function ϕ such that (D) is equivalent to $k_{t+1} = \phi(k_t)$*

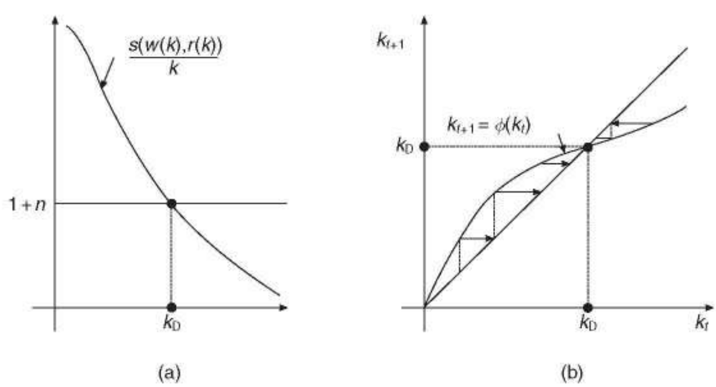

Figure 11.3 (a) the function $k \to s(r(k),w(k))/k$; (b) convergence to the Diamond steady-state.

(ii) *ϕ has a unique fixed point k_{D} such that*

$$k_{\mathrm{D}} = \phi(k_{\mathrm{D}}) \Longleftrightarrow s(r(k_{\mathrm{D}}), w(k_{\mathrm{D}})) = (1+n)k_{\mathrm{D}}$$

(iii) *If $k_0 > 0$, the sequence $(k_t)_{t\geq 0}$ determined by (D) converges monotonically to k_D.*

Is k_{D} an efficient steady-state? To answer this question let us consider the feasible steady-states of the economy, i.e. the feasible allocations $(\boldsymbol{c}, \boldsymbol{k})$ such that $c_t = (c_0, c_1)$ for all $t \geq 0$, $c_{1,0} = c_1$, $k_t = k$ for all $t \geq 0$, for some $(c_0, c_1, k) \in \mathbb{R}^3_+$. Feasibility implies

$$N_t c_0 + N_{t-1} c_1 + N_t i = N_t f(k),$$

where i is the (per-capita) investment which permits the (per-capita) capital to be constant i.e. $N_t(1-\beta)k + N_t i = N_{t+1} k \Longleftrightarrow i = (n+\beta)k$. The feasibility condition reduces to

$$c_0 + \frac{c_1}{1+n} = f(k) - (n+\beta)k$$

and the steady-state which maximizes the permanent per-capita consumption, called the *Golden Rule*, is characterized by

$$f'(k^*) = n + \beta.$$

Since the Golden rule is entirely determined by the technological (f, β) and demographic (n) factors while the Diamond steady-state depends in addition on agents' preferences, typically $k_{\mathrm{D}} \neq k^*$. Note that $r(k^*) = n$ is the interest rate which should prevail in order to induce firms to choose the capital–labor ratio k^*. A typical economy $\mathcal{E}(u, F, n)$ is such that either

$$\text{(i) } s(r(k^*), w(k^*)) < (1+n)k^* \quad \text{or} \quad \text{(ii) } s(r(k^*), w(k^*)) > (1+n)k^*,$$

economies for which the equality holds is exceptional. Let

$$Z(k_t, k_{t+1}) = s(r(k_{t+1}), w(k_t)) - (1+n)k_{t+1},$$

denote the transfers from the young generation to the old that would be needed if the young generation were to carry over to date $t+1$ the per-capita capital stock k_{t+1} when they receive a wage $w(k_t)$ and the interest rate $r(k_{t+1})$ justifies the choice of k_{t+1}. The economies $\mathcal{E}(u, F, n)$ can thus be classified by the sign of the transfers at the Golden Rule.

DEFINITION 4 A production economy $\mathcal{E}(u, F, n)$ is said to have *negative (positive) transfers at the Golden Rule* if $Z(k^*, k^*) < 0$ (>0).

Assumption S implies that economies with negative transfers are such that $k_{\mathrm{D}} < k^*$ (see Figure 11.3(a)) and $r_{\mathrm{D}} > n$ (since f is concave). The savings of the young agents at k^* are not sufficient to sustain the Golden Rule. Thus in order that at the Diamond steady-state the investment of the firms can be financed out of the savings of the young the interest rate has to be higher, to induce both a lower investment from firms and (weakly) more savings from the young agents. This case is referred to in the literature as the case of *underaccumulation*. Economies with positive transfers are such that $k_D > k^*$ and $r_D < r^*$: the savings of the young agents at k^* are too high for the capital stock needed at the Golden Rule. In order to absorb the savings, at the Diamond steady-state the interest rate is lower, both to induce firms to invest more and to discourage agents from saving. This is the case of *overaccumulation*.

11.2.2 Intermediation in a production economy

There is thus an interesting parallel between the OLG exchange and the production models, the Diamond steady-state playing the role in production economies of the autarchy steady-state in exchange economies. This suggests that the inefficiency of the Diamond steady-state comes from the constraint that the capital stock carried by the generation born at date t to date $t+1$ is entirely financed out of their savings. Let us show that when this constraint is removed by assuming the existence of an *infinitely-lived intermediary* which permits intergenerational transfers, then the Diamond steady-state is unlikely to occur in economies with negative transfers (underaccumulation), while it remains the stable steady-state in economies with positive transfers (overaccumulation). One way of seeing the difference between the two models is that in Diamond's model borrowing and lending goes through short-lived intermediaries (banks) which are created by the agents of the young generation and die with this generation in the subsequent period, while in the model that we now consider the intermediaries are infinitely-lived corporations transmitted from one generation to the next. While short-lived intermediaries restrict borrowing and lending within a given generation, long-lived intermediaries permit intergenerational transfers.

Consider the same market structure as in the Diamond model (markets for current output, labor, second-hand capital, borrowing and lending) but in which the borrowing and lending goes through an infinitely-lived intermediary; young agents, as consumers, give their savings to the intermediary and as entrepreneurs borrow from the intermediary; the old agents take out their savings and as entrepreneurs reimburse what they had borrowed for financing the investment of their firms. The output, capital market and labor markets function exactly as before: at date t consumers and firms face the prices $(1, w_t, 1, r_{t+1})$ and make their optimizing decisions in the same way. The choice of a consumer of generation t is described by the savings function $s(r_{t+1}, w_t)$, all firms choose the same capital–labor ratio, and the equilibrium equations (11.8), (11.9), and (11.10) have to hold. The only change is that the market-clearing equation (11.10) has to be replaced by the budget constraint of the intermediary

$$N_t s_t - K_{t+1} - N_{t-1}(1+r_t)s_{t-1} + (1+r_t)K_t = 0$$

or, in per-capita terms

$$(1+n)(s(r_{t+1}, w_t) - (1+n)k_{t+1}) = (1+r_t)(s(r_t, w_{t-1}) - (1+n)k_t). \qquad (11.14)$$

When the economy starts at date 0 all choices inherited from the past must be given as initial conditions, in particular the initial capital stock k_0, which as in the previous section determines the interest rate r_0 which prevailed at date -1. But (k_0, r_0) no longer determines $c_{1,0}$: one needs to know how much the old have saved in their youth, which depends on k_{-1} through $s(r_0, w_{-1})$. Thus the initial conditions must be (k_{-1}, k_0). Then

$$r_0 = f'(k_0) - \beta, \quad w_{-1} = f(k_{-1}) - k_{-1}f'(k_{-1}), \quad c_{1,0} = (1 + r_0)s(r_0, w_{-1}). \tag{11.15}$$

DEFINITION 5 An *intermediary equilibrium* of the OLG production economy $\mathcal{E}(u, F, n)$ is a sequence $(\bar{\boldsymbol{c}}, \bar{\boldsymbol{w}}, \bar{\boldsymbol{k}}, \bar{\boldsymbol{r}}) = (\bar{c}_{1,0}, (\bar{c}_t, \bar{w}_t, \bar{k}_{t+1}, \bar{r}_{t+1})_{t \geq 0})$ such that

(i) $\bar{c}_{1,0}$ is given by (11.15)
(ii) $\bar{c}_t = (\bar{c}_{0,t}, \bar{c}_{1,t+1})$ is given by (11.7) with $s_t = s(w_t, r_{t+1})$ for all $t \geq 0$
(iii) $(\bar{w}_t, \bar{k}_{t+1}, \bar{r}_{t+1})$ satisfy (11.8), (11.9), (11.10) and (10.14) for all $t \geq 0$.

An intermediary equilibrium, like a Diamond equilibrium, is determined by the sequence of per-capita capital stocks $(k_t)_{t \geq 0}$. This sequence is determined by the second-order difference equation

$$(1+n)(s(r(k_{t+1}), w(k_t)) - (1+n)k_{t+1}) = (1 + r(k_t))(s(r(k_t), w(k_{t-1})) - (1+n)k_t)$$

or

$$(1+n)Z(k_{t+1}, k_t) = (1 + r(k_t))Z(k_t, k_{t-1}), \quad \forall t \geq 0 \tag{E$'$}$$

with initial condition (k_{-1}, k_0). Equation (E$'$) is the second-order version of the first-order difference equation (E) which determines the interest rate in the exchange model. It expresses the equality of the demand for funds by the young to the supply of funds by the old. The steady-state equilibria of equation (E$'$) are k^*, defined by $r(k^*) = n$, and k_D defined by $Z(k_D, k_D) = 0$.

Under Assumption U, the difference equation (E$'$) can be written as

$$k_{t+1} = \psi(k_t, k_{t-1}), \quad \forall t \geq 0,$$

where ψ is a differentiable function for $(k_t, k_{t-1}) \gg 0$. If we define the new variable $v_t = k_{t-1}$ then this second-order difference equation can be written as the following equivalent first-order system in (k_t, v_t)

$$\left.\begin{aligned} v_{t+1} &= k_t \\ k_{t+1} &= \psi(k_t, v_t) \end{aligned}\right\} \quad \forall t \geq 0 \tag{11.16}$$

with initial condition $(k_0, v_0) \gg 0$. The simplest way of getting a feel for the qualitative properties of the trajectories defined by equation (11.16) is to examine the associated phase portrait shown in Figures 11.4(a) and 11.5(a) for economies with underaccumulation and overaccumulation respectively. To construct the phase diagram, we need to construct the curve $\mathcal{V}$ consisting of the points (k_t, v_t) in the plane such

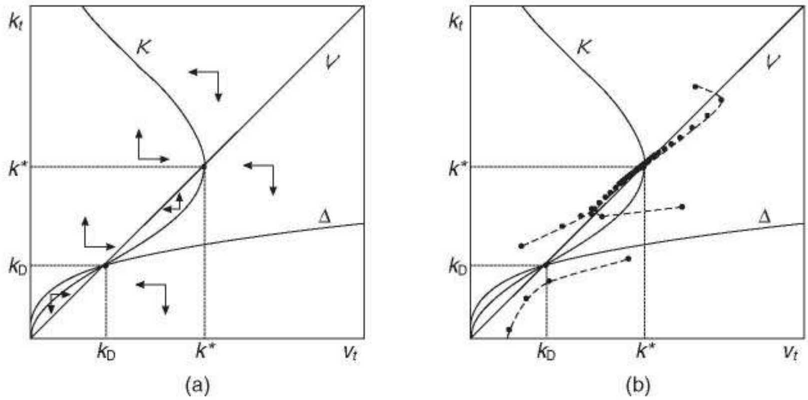

Figure 11.4 Intermediary equilibria converging to the Golden Rule in the case of underaccumulation.

that $v_{t+1} = v_t$, and the curve $\mathcal{K}$ of points (k_t, v_t) such that $k_{t+1} = k_t$. Depending on which side of the $\mathcal{K}$ and $\mathcal{V}$ curves, a point (k_t, v_t) of a trajectory is located, k_{t+1} will be greater or smaller than k_t and v_{t+1} will be greater or smaller than v_t. As indicated in Figures 11.4(a) and 11.5(b), $\mathcal{V}$ is just the diagonal ($v_{t+1} = v_t$ is equivalent to $k_t = v_t$), while the curve $\mathcal{K}$ is bell-shaped: there is a value $\bar{k} > \max(k_D, k^*)$ such that if v_t (i.e. k_{t-1}) is less than $\bar{k}$, then there are two values of k_t such that $k_{t+1} = k_t$, and if v_t is greater than $\bar{k}$, there is no value of k_t such that $k_{t+1} = k_t$.

Just as we found that in the exchange model the dynamics of equilibrium was such that the smallest interest rate steady-state was stable, so here in the model with production the phase diagram suggests that the steady-state with the highest level of capital (and hence the lowest interest rate) is the stable steady-state.

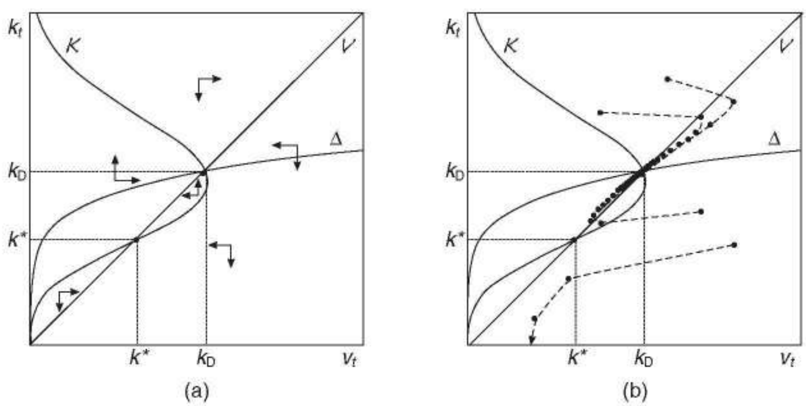

Figure 11.5 Intermediary equilibria converging to the Diamond steady-state in the case of overaccumulation.

The third curve Δ in Figures 11.4(a) and 11.5(a) is the curve on which the sequence of points (k_t, v_t) of any Diamond equilibrium must lie

$$\Delta = \{(k_t, v_t) \in \mathbb{R}^2_+ \mid Z(k_t, v_t) = 0\},$$

namely the pairs (k_t, v_t) satisfying equation (D). Note that if $(k_0, v_0) \in \Delta$ then for all $t \geq 0$, $Z(k_t, v_t) = 0$ and the trajectory is just the Diamond equilibrium converging to (k_D, k_D). If (k_0, v_0) lies above (below) Δ, then equation (E′) implies that $Z(k_t, v_t) < 0$ (resp > 0) for all $t \geq 0$: along the equilibrium trajectory the total capital on hand at any date t exceeds (is less than) the savings of the young.

As is well-known, for a difference equation system, the phase diagram is not sufficient to establish the dynamical properties of the trajectories, even though it suggests the qualitative behavior of the trajectories. A global result for the system equation (E′) can be obtained for economies with overaccumulation (see Tirole, 1985; Magill and Quinzii, 1999). For economies with underaccumulation a global result seems more difficult to obtain – although the trajectories starting above the curve Δ converge in all examples with Cobb Douglas and CES functions we have tried (see Figures 11.4(b) and 11.5(b)). In both cases, a local analysis can be carried out around the two positive steady states (k_D, k_D) and (k^*, k^*), and the nature of the two eigenvalues can be established: one root inside and one outside the unit circle for (k_D, k_D) (i.e. locally saddle point), and both inside the unit circle for (k^*, k^*) (i.e. locally stable) for an economy with underaccumulation; both roots inside the unit circle for (k_D, k_D), and one inside, the other outside for (k^*, k^*) for an economy with overaccumulation. Thus the following proposition can be established.

Proposition 2 *Consider OLG production economies $\mathcal{E}(u, F, n)$ satisfying Assumption (U, S). An intermediary equilibrium of $\mathcal{E}$ is characterized by (E′) and has the following properties.*

(i) *For an economy with positive transfers (overaccumulation) the Diamond steady-state is locally stable and the Golden Rule steady-state is saddlepoint stable. All trajectories with initial conditions satisfying $Z(k_0, k_{-1}) < 0$ converge to the Diamond steady-state. If $Z(k_0, k_{-1}) > 0$ either the trajectory converges to the Diamond steady-state, or there is no equilibrium, or the trajectory converges to the Golden Rule if (k_0, k_{-1}) lies on the one-dimensional stable manifold leading to k^*.*

(ii) *If in addition $kf'(k)$ is an increasing function of k then for an economy with negative transfers (underaccumulation) the Golden Rule steady-state is locally stable and the Diamond steady-state is saddlepoint stable. A trajectory converges to the Diamond steady-state if and only if $Z(k_0, k_{-1})$. The trajectories are such that $Z(k_0, k_{-1}) < 0$ converge to the Golden Rule or oscillate around it. If $Z(k_0, k_{-1}) > 0$ there is no equilibrium.*

An OLG model embeds demography into an equilibrium model in an essential way, the key demographic parameter being the rate of population growth n. As we have seen, for both a production and an exchange economy this parameter determines the Golden Rule interest rate. In Section A we showed how, in an exchange economy, the force of population growth also determines the long-run dynamics of the economy: the adjustment of the interest rate to cope with the constant inflow of new agents essentially explains why the "low interest rate" equilibrium is stable and the "high interest rate equilibrium" is unstable. The inflow of new agents onto the capital markets has a similar effect in a production economy, although the way in which it affects the equilibrium is more involved since the interest rate influences both the investment decisions of firms and the savings decisions of the consumers, and the savings decisions are in addition influenced by the endogenously determined wage rate.

However, it is still possible to get the intuition for the long-run properties of the equilibrium by considering some simple cases. For example, consider an economy with underaccumulation and suppose that $k_D < k_{t-1} < k_t < k^*$. Then $Z(k_t, k_{t-1}) < 0$ and its absolute value is the excess per-capita reimbursement of firms over the amount needed to reimburse old agents. Since $k_t < k^*$, $r_t > n$, and the new young generation must be induced to "overspend" on investment more per capita than the previous generation ($|Z(k_{t+1}, k_t)| > |Z(k_t, k_{t-1})|$) despite the fact that they have higher income ($k_{t-1} < k_t \Longrightarrow w_{t-1} < w_t$). Thus, to clear the capital market, r_{t+1} must be lower than r_t, implying that k_{t+1} is greater than k_t. If, on the other hand, $k_D < k^* < k_t < k_{t-1}$ then $Z(k_t, k_{t-1})$ is still negative, but since $r_t < n$ the young generation must be induced to overspend less per-capita than the previous generation ($|Z(k_{t+1}, k_t)| < |Z(k_t, k_{t-1})|$) even though they have a lower income. Thus the interest rate must rise, so that the capital stock decreases. Thus, once again, for a negative transfer economy ($Z(k^*, k^*) < 0$) it is the weight of the cohort of young agents which flows onto the loan market which drives the interest rate into equality with the rate of population growth, thereby inducing a process of capital accumulation which converges to the Golden Rule k^*.

Now consider an economy with overaccumulation and suppose that $k_t < k_{t-1} < k^* < k_D$. Then $Z(k_t, k_{t-1}) > 0$ and the firms reimburse less than is needed to pay back the principal and interest on the savings of the old agents: the difference must be made up by the excess savings of the young. Since $k_t < k^*$, r_t is greater than n and the young agents must be induced to have higher excess savings per capita than the previous generation, even though they have a smaller income: thus the interest rate must increase, leading to a decrease of the capital stock. If $k^* < k_{t-1} < k_t < k_D$ a similar reasoning shows that the capital stock must increase away from k^*. Thus for a positive transfer economy ($Z(k^*, k^*) > 0$) the requirement that the more numerous cohort of young agents balance the positions of the old leads the Golden rule to be unstable.

The intermediary equilibrium that we have presented in this section is not the only way in which an equilibrium can be obtained for which the capital stock trajectory is given by equation (E′): if there is a government which runs a deficit or a surplus without adding to it, or if there are assets in the private sector in which agents can invest which pay no dividends (bubbles), or if money (with no explicitly modeled transactions services) can be transmitted from one generation to the next, then the same equation (E′) summarizes the equilibrium. These cases have been extensively studied in the economic literature (see e.g. Tirole, 1985; Weil, 1987; Blanchard and Fischer, 1989; Azariadis, 1993). However, the government debt, or bubble, or money interpretations have never led emphasis to be placed on the property of convergence to the Golden Rule in an economy with underaccumulation: in order that the equilibrium capital stock converges to the Golden Rule rather than the Diamond steady-state, either the government has to run a surplus *at all times*, or agents have to invest in negative bubbles, or they must carry "negative" money – all of which are highly implausible situations for the models in question.

On the other hand, the possibility that long-lived intermediaries permit intergenerational transfers, although nowhere discussed except in Gale's paper (1973), seems to be a highly likely situation. A possible objection to the concept of an intermediary equilibrium is that it is impossible to initialize because there can never be an old generation which lends to the young, without the agents of this generation having had access to loans when they were young. This argument, while at first sight is compelling, underestimates the ability of intermediaries to create loanable funds. Banks began as depositary institutions for wealthy agents and exploited the stochastic nature of the deposit–withdrawal process to accumulate balances which they used to make loans to businesses. In the same way, other financial institutions such as insurance companies are able to create loanable funds out of the normal process by which they receive premia in advance of having to reimburse random claims. Thus it seems both natural and realistic to assume that financial intermediaries can indeed create a situation such that firms reimburse more than is needed to reimburse agents who have deposited their savings with them. Note furthermore that the size of the initial surplus needed to initialize the process can be very small, and that once the process has got underway, it is self sustaining.

Finally in the next section, we show that the bond market is not the only financial market that can generate the intergenerational transfers which lead to more investment than in the Diamond equilibrium. The stock market can play the same role provided that the financial value of firms on the stock market can differ from the replacement value of their capital.

11.2.3 The stock market and intergenerational transfers

Let us return to the Diamond model in which the bond market is restricted to intragenerational loans: thus the purchase of firms by the young and the new investment they undertake must be financed out of their savings. However, instead of assuming that capital is a homogeneous and malleable good, let us assume that, once installed, capital is firm specific and cannot either be transformed back into the consumption good or be transferred from one firm to another without incurring significant "adaptation costs" which, for simplicity, we take to be infinite. Note that the assumption of firm specificity of capital does not introduce any new imperfection into the model, as long as gross investment is positive at every date: since we assume that all firms are equally productive and that new investment is needed at every date, there is no reason to incur the adaptation costs involved in "unbolting" the capital installed in a firm. However, the nature of firms and the nature of the markets on which they are transferred from one generation to the next must now change. Firms need to become infinitely-lived *corporations* whose ownership is transferred through the sale of equity shares on the stock market.

We thus consider a model in which, in addition to the spot markets for current output and labor services, there are two financial markets – the equity and the bond market. As before, the current output of firms can be channeled either into consumption or used for new investment. However, while *ex ante* one unit of investment in any firm costs one unit of current output, *ex post*, once the newly invested capital is installed, it is a sunk cost. Let $(1, w_t, (Q^j_t)^J_{j=1}, r_{t+1})$ denote the two spot prices, and the prices and interest rate associated with the firms' equity and the bond at date t. Note that since the firms are infinitely lived, there is the same number J of firms at each date.

The representative agent of the young generation at date t purchases a portfolio of securities

$$(z_t, \theta^1_t, \ldots, \theta^J_t),$$

consisting of an amount z_t of the bond and a share θ^j_t of firm j (for $j = 1, \ldots, J$) so as to maximize lifetime utility $u(c_{0,t}, c_{1,t+1})$, subject to the two budget constraints

$$\begin{aligned} c_{0,t} &= w_t - \sum_{j=1}^{J} \theta^j_t Q^j_t - z_t \\ c_{1,t+1} &= \sum_{j=1}^{J} \theta^j_t (D^j_{t+1} + Q^j_{t+1}) + z_t(1 + r_t), \end{aligned} \tag{11.17}$$

where D^j_t is the dividend paid by firm j at date t. The agent takes the prices $(w_t, (Q^j_t)^J_{j=1}, r_{t+1})$ as given and correctly anticipates the next period dividends and equity prices $(D^j_{t+1}, Q^j_{t+1})^J_{j=1}$. The maximum problem of the agent has a solution if and only if the no-arbitrage condition between the stock and bond market

$$Q^j_t = \frac{1}{1 + r_{t+1}} \left(D^j_{t+1} + Q^j_{t+1} \right), \quad j = 1, \ldots, J, \tag{11.18}$$

holds for the equity price of each firm. Since by equation (11.18) the rate of return on the bond and each of the equity contracts is the same, the agent is indifferent between investing in any firm or investing in the bond market: all that matters is the total sum invested in the capital markets, namely the agent's total savings s_t. When equation (11.18) holds, the budget equations (11.17) can be written as

$$\begin{aligned} c_{0,t} &= w_t - s_t \\ c_{1,t+1} &= s_t(1 + r_{t+1}), \end{aligned}$$

where

$$s_t = \sum_{j=1}^{J} \theta^j_t Q^j_t + z_t. \tag{11.19}$$

Thus as in the previous version of the model the agent's optimal choice is characterized by the savings function $s(r_{t+1}, w_t)$. The precise decomposition of the savings between stocks and bonds will be determined by the equilibrium conditions.

Consider the investment decision of firm j. To maintain the symmetry assumption among agents, suppose that every young agent has an ownership share in firm j, and that the young agents assemble at a board meeting to decide on firm j's investment plan for date t, the investment being financed by borrowing. The present value of firm j's profit is

$$\frac{1}{1+r_{t+1}}\left(F((1-\beta)K_t^j + I_t^j, L_{t+1}^j) - w_{t+1}L_{t+1}^j - (1+r_{t+1})I_t^j + Q_{t+1}^j\right).$$

Assuming free disposal of firms, i.e. assuming that the old generation can destroy the capital $(1-\beta)K_{t+1}^j$ if they cannot sell it satisfactorily, Q_{t+1}^j must be non-negative. On the other hand, Q_t^j cannot exceed $(1-\beta)K_t^j$ since otherwise the young agents would be better off creating a new firm, purchasing the requisite capital $(1-\beta)K_t^j$ on the current output market. Thus at each date we must have $0 \le Q_t^j \le (1-\beta)K_t^j$. We will show how to find an equilibrium of the model assuming that these inequalities are not binding: this will imply that the analysis must be restricted to an appropriate subset of parameters and initial conditions, as we shall see in the following paragraph.

The optimal hiring decision with respect to labor, L_{t+1}^j, satisfies equation (11.5) as before, and the optimal choice of investment plan I_t^j satisfies

$$F_K'(K_{t+1}^j, L_{t+1}^j) - (1+r_{t+1}) + \frac{\partial Q_{t+1}^j}{\partial I_t^j} = 0. \tag{11.20}$$

In order to choose I_t^j optimally, the young agents must thus anticipate Q_{t+1}^j as a function of I_t^j. We assume that they correctly anticipate the path of future interest rates and wages and that the next generation (and all subsequent generations) will take optimal decisions once they have bought the firm from them. They thus anticipate that

$$\begin{aligned} Q_{t+1}^j = &\frac{1}{1+r_{t+2}}\left(F((1-\beta)^2K_t^j + (1-\beta)I_t^j + I_{t+1}^j, L_{t+2}^j) - w_{t+2}L_{t+1}^j\right. \\ &\left. - (1+r_{t+2})I_{t+1}^j + Q_{t+2}^j\right). \end{aligned} \tag{11.21}$$

By successive substitutions, Q_{t+1}^j is the infinite sum of dividends given by firm j from date $t+2$ on, plus possibly a bubble component $\lim_{t\to\infty} Q_t^j$. Thus equation (11.20) can also be written

$$F_K'(K_{t+1}^j, L_{t+1}^j) - (1+r_{t+1}) + \frac{1}{1+r_{t+2}}\left((1-\beta)F_K'(K_{t+2}^j, L_{t+2}^j) + \frac{\partial Q_{t+2}^j}{\partial I_t^j}\right) = 0. \tag{11.22}$$

Note that from date $t+1$ on, when $I_{t+1}^j > 0$ one unit of investment at date t is the perfect substitute for $(1-\beta)$ of investment at date $t+1$. Thus restricting attention to trajectories on which investment in firm j is positive at all date, by optimality of I^j_{t+1} (using equation (11.20) at date $t+1$)

$$\frac{\partial Q^j_{t+2}}{\partial I^j_t} = (1-\beta)\frac{\partial Q^j_{t+2}}{\partial I^j_{t+1}} = (1-\beta)\big((1+r_{t+2}) - F'_K(K^j_{t+2}, L^j_{t+2})\big),$$

which, substituted into equation (11.22) gives the same first-order condition equation (11.6) as in the Diamond model.

Thus as in the Diamond model, all firms choose the same capital–labor ratio and equations (11.8–11.10) must hold for all t in equilibrium to ensure that firms optimize and that the labor market clears. Consider the financial markets: in order that equilibrium can exist, equation (11.18) must hold for each firm, or equivalently, equation (11.21) must hold at each date. In aggregate per capita terms $\big(q_t = \big(\sum_{j=1}^J Q^j_t\big)/N_t\big)$

$$q_t = \frac{1+n}{1+r_{t+1}}\left(f(k_{t+1}) - w_{t+1} - (1+r_{t+1})\frac{i_t}{1+n} + q_{t+1}\right) \tag{11.23}$$

must hold. Then equilibrium on the stock market requires $\theta_t = 1/N_t$ and equilibrium on the bond market requires $z_t = i_t$. Since agents are indifferent between bonds and stocks, these conditions will hold if

$$s(r_{t+1}, w_t) = q_t + i_t. \tag{11.24}$$

Combining equations (11.8), (11.10), (11.23) and (11.24) leads to the same second-order difference equation (E$'$) as in the model with intermediaries, and thus to the same dynamics. However, some restrictions on the characteristics of the economy are needed to ensure that the inequalities $0 \le Q^j_t \le (1-\beta)K^j_t$ and $I^j_t > 0$ are satisfied along the trajectories. To understand the restrictions involved, note first that from the analysis of an intermediary equilibrium we know that $Z(k_{t+1}, k_t)$ has the same sign for all t as $Z(k_1, k_0)$. Since

$$Z(k_{t+1}, k_t) = s(r(k_{t+1}), w(k_t)) - (1+n)k_{t+1} = s(r(k_{t+1}), w(k_t)) - (1-\beta)k_t - i_t$$

and since in a stock market equilibrium $s(r(k_{t+1}), w(k_t)) = q_t + i_t$, it follows that

$$Z(k_{t+1}, k_t) = q_t - (1-\beta)k_t.$$

Thus if the initial conditions are such that $Z(k_1, k_0) \le 0$ (i.e. $q_0 \le (1-\beta)k_0$) then at all subsequent dates $q_t \le (1-\beta)k_t$. If $Z(k_1, k_0) = 0$ then the stock market price of each firm is equal to the replacement value of the capital which is transferred and the equilibrium is the Diamond equilibrium. If $Z(k_1, k_0) < 0$ then $q_t < (1-\beta)k_t$ at every date: the discounts at each period on the equity price of each firm are the transfers from the old (who own the firms) to the young (who buy them), and it is these discount-induced transfers which lead to the Golden Rule level of capital k^* rather than the Diamond capital stock k_D, in economies with underaccumulation.

However, in order that the investment of firms be positive at every date as we have assumed, the sequence of discounts on firms' prices must not become too large. A precise analysis of the firms' optimal investment decisions can be found in Magill and Quinzii (1999). It can be summarized as follows: in order that the young agents who have bought a firm decide to undertake new investment, they must anticipate that they will recover the (depreciated) cost of the investment when they sell the firm next period – this is the property which lies behind the first-order condition $f'(k_{t+1}) = r_{t+1} + (1 - \beta)$. In order that these expectations are fulfilled it must be the case that $(1+n)q_{t+1} \geq (1-\beta)i_t$ at every period. Thus in order that trajectories with positive investment exist and converge to the Golden Rule, this inequality must be satisfied at k^*. Recalling that $q^* = s(r(k^*), w(k^*)) - i^*$, this requires that

$$s(r(k^*), w(k^*)) \geq i^* + \frac{(1-\beta)}{1+n} i^*.$$

Thus we must restrict attention to economies for which the savings of the representative young consumer when faced with the interest rate and wage rate $(r(k^*), w(k^*))$ is sufficient to pay for the current (per-capita) investment i^* and the depreciated investment of the period before $(1-\beta)/(1+n)\, i^*$. Thus while the stock market can achieve intergenerational transfers, it is more limited than a bond market with infinite-lived intermediaries in the amount of transfers that it can achieve.

NOTES

1 More accurately we assume that there is a large number of these intermediaries so that perfect competition prevails and the intermediaries make zero profit on their constant-returns activity.

2 Note that by a standard argument, which involves multiplying the data t constraint by $(1 + r_{t+1})$ and adding it to the constraint for date $t + 1$, the budget set $\mathbb{B}(r_{t+1})$ is equivalent to the budget set $B(r_{t+1}) = \{c_t \in \mathbb{R}^2_+ | (1 + r_{t+1})c_{0,t} + c_{1,t+1} = (1 + r_{t+1})e_0 + e_1\}$.

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12 The sequential indeterminacy problem

Michael Mandler

General equilibrium models generically are determinate and have a finite number of equilibria. But, with linear activities and some inelastically supplied factors of production, the economies that arise endogenously as perfect-foresight equilibria and proceed through time can robustly be indeterminate. During the initial period of an intertemporal model, the perfect-foresight equilibria typically are well-behaved, but they can generate later-period endowments for agents, such that the equilibria that validate perfect-foresight expectations lie amid a continuum of other equilibria. Since later-period equilibria are not continuous functions of endowments and other parameters, agents (even if small relative to the market) have an incentive to manipulate market prices; the assumption that agents are price takers therefore breaks down. Sequential indeterminacy is related to Sraffa's indeterminacy results in that (1) smooth "neoclassical" technologies eliminate indeterminacy; and (2) the dimension of sequential indeterminacy matches the dimension of indeterminacy in Sraffa's model. Despite the link to Sraffa, long-run equilibria, where relative prices are constant through time, are generically determinate.

12.1 INTRODUCTION

Factor pricing was once one of the most contentious subjects in economics. From the late nineteenth through the mid-twentieth century, champions and critics of marginal productivity argued forcefully about the nature of technology and whether the infinite array of the factor substitution possibilities implied by the differentiable, "neoclassical" production function is realistic. Although the neoclassical production function remains the most common model of production, arguments about the nature of technology seem puzzling today: Arrow–Debreu general equilibrium theory does not impose differentiability assumptions on production functions or production sets. Premodern theorists thus seem to have been debating an irrelevancy.

I argue that earlier worries about factor substitution were well-justified. In the absence of sufficient substitutability, factor demand can be inelastic. If factor supply is also inelastic, equilibrium factor prices will not be determinate. Indeterminacy is not just a technical nuisance; it undermines the price-taking assumption of competitive equilibrium theory. In the indeterminacy under study, arbitrarily small reductions in factor supplies can discontinuously increase a factor's price, and consequently factor owners will not take prices to be parametric.

Standing opposed to the possibility of indeterminacy, the regularity literature of general equilibrium theory shows that competitive equilibria generically are determinate; for almost every configuration of parameters, general equilibrium models have only locally unique equilibria. Since this result holds for models of production with limited or even no possibilities for factor substitution and when factors are inelastically supplied, the regularity literature implicitly contends that factor price indeterminacy is not a problem.

I show, however, that in intertemporal models in which agents trade at multiple dates (instead of once-and-for-all at the beginning of economic time), it can occur robustly that the equilibrium behavior of agents endogenously generates the parameters at which indeterminacy occurs. At the start of an intertemporal model, the perfect-foresight equilibria generically are determinate, but those equilibria can generate endowments for agents such that in later periods the equilibrium that validates perfect-foresight expectations is contained within a continuum of other equilibria. This is the sequential indeterminacy problem. Although at each period t the equilibria of the economy that begins at t would generically be determinate if we could perturb endowments at t and later, economies are driven over time to precisely the measure-zero set of endowments that cause trouble.

I begin by reviewing the standard determinacy theorems of the regularity literature, both in general and in a two-factor example, and then show how an intertemporal equilibrium can be decomposed into an equilibrium with sequential trading. For simplicity, our sequential trading equilibria involve trading at just two dates. The decomposition of each intertemporal equilibrium defines a set of economies that operate during the second time period. It can turn out that almost all of these economies have indeterminate equilibria. Technologies with limited factor-substitution possibilities, such as linear activities, are indispensable to sequential indeterminacy; as we will see, the perfect-foresight equilibria of models with differentiable neoclassical technologies typically are determinate during their later periods of operation. The importance of the differentiability of technology hints at a link to Sraffa's famous indeterminacy claims. The connection turns out to be tight, except for one important proviso: sequential indeterminacy will not arise in the favored Sraffian environment of long-run equilibria.

12.2 DETERMINACY IN GENERAL EQUILIBRIUM THEORY

Despite the considerable attention theorists paid to determinacy prior to World War II, postwar general equilibrium theory initially ignored the subject. But after 1970, when Gerard Debreu published his path-breaking article on the number of equilibria in exchange economies, the modern literature developed rapidly. For models with a finite horizon (and as long as assets are real, not financial), the regularity literature has yielded remarkably sharp conclusions. Typical, or generic, models have a finite number of equilibria. And these equilibria have appealing comparative static properties: equilibrium prices and allocations change smoothly as a function of the parameters of the model. Consequently, in large economies, agents can have only a small effect on equilibrium prices and therefore have little incentive to manipulate markets. For example, as the number of agents becomes large, the effect on equilibrium prices of an agent withholding a portion of his or her endowment from the market becomes small. In the limit, agents take prices to be parametric and act competitively (Roberts, 1980). The study of determinacy thus not only resolved a technical issue – whether the number of equilibria is finite or infinite – but also shored up the foundational story of when markets can operate competitively.

For production economies, Mas-Colell (1975) and Kehoe (1980, 1982) established generic determinacy results for constant-returns-to-scale technologies and for the linear activity analysis model in particular.[1] From the perspective of the history of production theory, the Mas-Colell/Kehoe results are remarkable. The original popularity of the differentiable production function, after all, was due to the fact that it ensured that factor demand is elastic with respect to price; hence, even if factor supply is inelastic, factor price indeterminacy will not occur. But although differentiability secured this important theoretical goal, it was also attacked by many as empirically unrealistic (see Mandler (1999a) for a more detailed history). The modern regularity literature, in contrast, seems to show that earlier assertions of indeterminacy had been incorrect: determinacy is generic with virtually any description of technology and whether or not factors are inelastically supplied. The decades-long debate between the "fixed coefficients" and "differentiable production function" camps thus appears to have been pointless.

A model in the spirit of Mas-Colell and Kehoe will serve as our reference point. We assume there are L goods that provide agents with utility and M goods that agents supply inelastically. The L goods may be either pure consumption goods or factors, such as labor time, whose demand varies as a function of prices, while the M goods are pure factors of production. We suppose that there are N activities, $a_1, \ldots, a_N$, each $a_j \in R^{L+M}$, indicating the quantities of the $L+M$ goods necessary to run activity j at the unit level, and summarized by the $L+M$ by N technology matrix $A = [a_1 \cdots a_N]$. The ith row-jth column element of A, a_{ij}, is the quantity of good i used (if $a_{ij} < 0$) or produced (if $a_{ij} > 0$) by activity j. Letting $y \in R^N_+$ indicate the vector of activity or usage levels, aggregate production equals Ay. The production set that arises from this technology is $Y = \{\gamma \in R^{L+M}: \exists y \in R^N_+ \text{ s.t. } \gamma \leq Ay\}$. We make the standard assumption that production of positive amounts of all goods is impossible: Y intersects the positive orthant only at $\{0\}$.

We assume there are a finite number of agents. A typical agent k is described by (1) a twice continuously differentiable utility function $u^k: R^L_+ \to R$ that is differentiably increasing and differentiably strictly concave and where no indifference surface through any $x^k \in R^L_{++}$ intersects any coordinate axis; and (2) an endowment $(e^k_c, e^k_f) \in R^{L+M}_{++}$ of goods and factors.[2] These assumptions can be weakened considerably. In particular, agents could be endowed only with natural resources, and not with producible goods as well.

Let $(p, w) \in R^{L+M}_+ \backslash \{0\}$ denote the prices of goods and factors. Each agent k's budget set is then $\{x^k \in R^L_+: p \cdot x^k \leq w \cdot e^k_f + p \cdot e^k_c\}$. Maximizing u^k subject to this budget set generates the excess demand function $z^k(p, w)$, and summing, the aggregate excess demand $z(p, w) = \sum_k z^k(p, w)$. The excess demand for the M pure factors is simply $-e_f = -\sum_k e^k_f$. The aggregate excess demand function for the $L+M$ goods taken together is therefore $(z(p, w), -e_f)$. Let $e_f(i)$ denote the aggregate endowment of factor i.

An *equilibrium* is a (p, w, y) such that $(z(p, w), -e_f) \leq Ay$ and $(p, w)'A \leq 0$. Using Walras' law, it is easy to confirm that if any of the market-clearing inequalities is strict, then the corresponding price equals zero, and that if any of the no-positive-profits inequalities is strict, then the corresponding coordinate of y equals 0. Under our assumptions, an equilibrium exists.

The regularity literature takes the view that properties of a model that obtain only at exceptional, "nongeneric" combinations of parameters can be dismissed as unlikely. This principle should not be applied naively, but, if modified so as to not rule out seemingly unlikely configurations of parameters that in fact arise endogenously, the underlying precept is sound. Indeed, in order to show that a property arises systematically in the later periods of an intertemporal model, one must ensure that it arises for a non-negligible set of intertemporal economies. So, whether from the regularity literature's vantage point or from a sequential point of view, an explicit space of economies or parameters is indispensable.

We now specify our parameter space and define a "regular" economy. Roughly speaking, the key feature of a regular economy is that each vector of equilibrium relative prices and activity levels is locally unique and varies differentiably as a function of the economy's parameters. The pertinent equilibrium variables of a regular economy can in fact be viewed as the solution of a simple system of equations: the market-clearing conditions for all the goods not in excess supply (less one, due to Walras' law) and the zero-profit conditions of activities that make exactly zero profits. Regular economies, moreover, comprise a generic subset of the space of economies. The exact concepts needed to define regularity are a little involved; the reader may wish to proceed to the example discussed in the next section.

The parameter space will be the cross product of the open set of endowments that meet the assumptions imposed above and an open set of technology matrices to be defined momentarily. Since technology matrices and endowments can each be identified with finite-dimensional vectors of real numbers, the concepts of openness, density, and full-measure sets will have their familiar Euclidean meanings. We must take a little care with technology matrices. If each entry in $L + M$ by N technology matrix is a free parameter, then almost every technology matrix will specify that each good is either an input or an output in each activity. This would be absurd in any model, and physically impossible once we distinguish goods by the date at which they appear. Therefore, given an arbitrary, fixed technology matrix A meeting our assumptions, let A' be *admissible* if, for all i and j, $\text{sgn}(a'_{ij}) = \text{sgn}(a_{ij})$. Our parameter space for technology matrices will be the set of admissible matrices. A *generic* set is a subset of our space of economies that is open and whose complement has measure zero.[3]

To analyze equilibria (locally) as the solution of a system of equations rather than inequalities, consider the subset of the conditions $(z(p, w), -e_f) \leq Ay$ and $(p, w)'A \leq 0$ that "bind" at some equilibrium $(p, w, y)^*$ the market-clearing conditions where demand exactly equals supply and the no-positive-profits conditions for those activities that both exactly break even and utilize or produce at least one good not in excess supply. All of the market-clearing conditions for consumption goods are included in this subset, but some factors may be in excess supply and some (unused) activities may make strictly negative profits or utilize/produce only factors in excess supply. For $(p, w, y)^*$ near $(\overline{p}^*, \overline{w}^*, \overline{y}^*)$ the excluded factors will continue to be in excess supply and the excluded activities continue either to make strictly negative profits or to utilize/produce only factors in excess supply. Consequently, in a neighborhood of $(p, w, y)^*$ we may ignore these excluded equilibrium conditions: they are automatically satisfied. In addition, we may use Walras' law, which implies that if all but one of the economy's market-clearing equalities is satisfied then so is the remaining one, to eliminate one of the equalities in $(z(p,w), -e_f) \leq Ay$. We choose to omit the first consumption good. Finally, given the homogeneity of $z(p,w)$ and $(p,w)'A$, we may set the price of one of the consumption goods – say the first – equal to 1 without restricting the set of equilibrium allocations.

To specify our new system of equations and unknowns explicitly, we define $\overline{p}$ by setting the first coordinate of p equal to $1, \overline{w}$ and $\overline{e}_f$ by omitting the coordinates of w and e_f that correspond to the factors in excess supply, $\overline{z}(\overline{p},\overline{w})$ by omitting the first coordinate of the range of z, $\overline{A}$ by omitting the rows of A that correspond to the factors in excess supply and the columns that correspond to the negative-profit activities or activities that utilize/produce only factors in excess supply, $\overline{y}$ by omitting the same activity level variables, and finally, $\tilde{A}$ by omitting the row of $\overline{A}$ that corresponds to the first consumption good.

Keep in mind that these definitions are assembled relative to a particular equilibrium: depending on which factors are in excess supply and which activities break even, a different set of equilibrium conditions will bind. We will now typically use *equilibrium* to refer to a $(\overline{p},\overline{w},\overline{y})$. To say that $(\overline{p},\overline{w},\overline{y})$ is an equilibrium means simply that there exists an equilibrium in our previous sense where the prices omitted from $(\overline{p},\overline{w})$ are set to 0, all money-losing activities omitted from $\overline{y}$ have usage levels equal to 0, and the remaining omitted activities can be set to be consistent with market-clearing.

Given an equilibrium $(\overline{p},\overline{w},\overline{y})$, equilibria near $(\overline{p},\overline{w},\overline{y})$ are characterized by the equations $(\overline{z}(\overline{p},\overline{w}), -\overline{e}_f) = \tilde{A}\overline{y}$ and $(\overline{p},\overline{w})'\overline{A} = 0$. These equations contain as many equations as there are variables in $(\overline{p},\overline{w},\overline{y})$. The basic requirement of regularity is that this system of equations has "full rank," which will imply the local uniqueness of $(\overline{p},\overline{w},\overline{y})$. To this end, define $F(\overline{p},\overline{w},\overline{y}) = ((\overline{z}(\overline{p},\overline{w}), -e_f) - \tilde{A}\overline{y}), (\overline{p},\overline{w})'\overline{A})$. An equilibrium is *regular* if (1) DF, evaluated at $(\overline{p},\overline{w},\overline{y})$, is nonsingular (i.e. has rank equal to $L-1$ plus the number of factors in $\overline{w}$ plus the number of activities in $\overline{y}$); (2) each activity j whose zero-profit condition is exactly satisfied has $y(j) > 0$; and (3) each factor i not in excess supply has $w(i) > 0$.

If an equilibrium is regular, equilibrium prices and quantities are locally a differentiable function of our parameterized set of economies. To see this, consider an economy (e^*, A^*) with a regular equilibrium $(\overline{p}^*,\overline{w}^*,\overline{y}^*)$ and adjoin the parameter space to the domain of F. The implicit function theorem implies there is a continuously differentiable function of the parameters in a neighborhood of (e^+, A^+) that identifies the unique $(\overline{p},\overline{w},\overline{y})$ in a neighborhood of $(\overline{p}^*,\overline{w}^+,\overline{y}^*)$ that satisfies $F(\overline{p},\overline{w},\overline{y}) = 0$. In a small enough neighborhood, these $(\overline{p},\overline{w},\overline{y})$ must indeed be equilibria: (i) by holding fixed the usage levels of any activity that utilizes/produces only factors in excess supply at $(\overline{p}^+,\overline{w}^*,\overline{y}^+)$, any market-clearing condition omitted from the range of F must still be satisfied; (ii) any activity's profit level that is omitted from the range of F and that makes negative profits at $(\overline{p}^+,\overline{w}^+,\overline{y}^*)$ will continue to make negative profits; (iii) any activity that utilizes/produces only factors in excess supply at $(\overline{p}^*,\overline{w}^+,\overline{y}^*)$ will plainly continue to break even; and (iv) given (2) and (3), $(\overline{p},\overline{w},\overline{y})$ will be non-negative. Since $F(\overline{p},\overline{w},\overline{y}) = 0$ has a locally unique solution for the parameters (e^+, A^+), any regular equilibrium is locally isolated.[4]

An economy is defined to be regular if all of its equilibria are regular. Under our assumptions, it can additionally be shown that a regular economy has a finite number of equilibria.

Generic determinacy theorem (Mas-Colell/Kehoe). The regular economies form a generic set.

We have taken care to include in the model goods that do not give agents utility. The above theorem therefore establishes generic determinacy in the important case where technology is described by linear activities and factors are inelastically supplied.

12.3 REGULARITY AND DETERMINACY IN A TWO-FACTOR EXAMPLE

To see an example of how different parameter configurations can generate regular economies or the troublesome cases that cause indeterminacy, let there be one consumption good and two factors ($L = 1$, $M = 2$), suppose that each activity produces only the consumption good, and normalize the activities so that the unit level of each activity produces one unit of that good. Label the pure factors so that they have commodity indices 1 and 2. Setting the price of the single consumption good equal to 1, and using Walras' law to ignore the market-clearing condition for the consumption good, equilibria are characterized by:

$$-\sum_j a_{ij}y_j \leq e_f(i), \qquad i = 1, 2, \tag{12.1}$$

$$1 + w(1)a_{1j} + w(2)a_{2j} \leq 0, \quad j = 1, \ldots, N. \tag{12.2}$$

Due to the sign convention, each a_{ij} in (12.1) and (12.2) is negative.

For generic technology matrices, at most two activities are used in equilibrium and all unused activities will satisfy (12.2) with strict inequality. When exactly two activities are in use, the corresponding two equalities in (12.2) determine unique values for $w(1)$ and $w(2)$, and for generic technologies both of these numbers will be strictly positive in equilibrium. Thus, in this regular case, neither factor can be in excess supply. The usage levels of the two activities are determined by (12.1), both of which are equalities. At equilibria where only one activity is in use, on the other hand, there is only one equality in (12.2) to determine both $w(1)$ and $w(2)$. If one of the factors is in excess supply, its price must equal zero and regularity and determinacy still obtain. But if both factor market-clearing conditions are satisfied with equality (and thus neither factor price is constrained to equal 0), and if, as with regular equilibria, the inequalities in (12.2) for the unused activities are strict, the equilibrium is indeterminate. A continuum of values for $w(1)$ and $w(2)$ will obey the one equality in (12.2) while still satisfying the strict inequalities in (12.2). And since no prices appear in (12.1), any of these ($w(1)$, $w(2)$) can serve as equilibrium factor prices.

Summing up, indeterminacy occurs if neither factor is in excess supply and just one activity is in use, while prices are determinate (for generic activity analysis matrices) if two activities are in use or if one of the factors is in excess supply.

The generic determinacy theorem therefore implies that equilibria with just one activity in use but with two fully employed factors occur only at a zero-measure set of economies. This fact is easy to confirm directly. With neither factor in excess supply, (12.1) contains exactly two equalities. But since only one activity is in use, only one of the components of y, say y_j, is non-zero. Except for a zero-measure set of the endowment vectors e_f, therefore, there exists no value for y_j that satisfies the two equalities in (12.1). Hence, generically, indeterminacy does not occur. The regular, determinate equilibria, in contrast, arise robustly and vary differentiably as a function of e_f. For example, when two activities are in use, (12.1) contains two equalities and there are two non-zero activity levels; for generic activity analysis matrices, the equalities in (12.1) in this case have a unique solution for every e_f.

Figure 12.1 graphs the set of possible endowment points. The figure supposes that there are two activities, each represented by the ray of factor combinations that the activity can fully employ. The interiors of the three cones of endowments lead to determinacy: in the interior of the two outside cones, one activity is in use and one of the two factors is in excess supply, while in the interior of the inner cone, two activities are in use and both factors are fully employed. Only at the lower-dimensional set of endowments that lie on the two rays, where only one activity is in use and both factors are fully utilized, does indeterminacy occur.

Two representative isoquants, which links points on the activity rays that produce the same quantity of the consumption good, are also pictured. Along the activity rays, where indeterminacy occurs, a continuum of price lines support the isoquant; marginal products for the factors consequently are not well-defined. In the cone interiors, in contrast, only a single price line supports the isoquant and marginal products are defined. The relationship between determinacy and marginal productivity generalizes: with arbitrary numbers of activities and factors and a single consumption good, marginal products are well-defined at the same generic set of factor endowments at which equilibria are locally unique. Thus, generically at least, marginal productivity

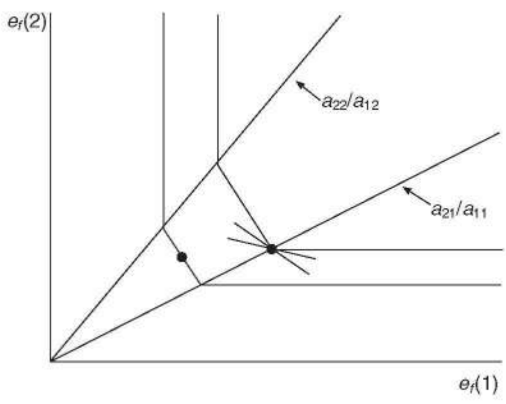

Figure 12.1 Endowments leading to determinacy and indeterminacy

does not require that differentiable production functions are posited as primitive; linear activities can serve as their foundation.

12.4 DETERMINACY VS. SEQUENTIAL DETERMINACY

The standard Arrow–Debreu model supposes that agents meet once and contract for delivery of goods at all moments of economic time. The determinacy that the regularity literature establishes, therefore, is the determinacy of equilibria for these sorts of markets. The standard literature simply does not address whether the economies that endogenously emerge through time have determinate or indeterminate equilibria.

As an intertemporal economy proceeds through time, the endowments of agents, and other parameters, endogenously evolve. The endowments that generate indeterminacy in one of the economy's later time periods, even though they form a measure-zero subset of the endowment space of that period, might therefore arise systematically. This outcome is more than an abstract possibility; optimizing agents accumulate capital goods only in specific configurations. We will see that in equilibrium the production of capital goods can generate precisely the endowments that lead to indeterminacy.

12.5 DECOMPOSING AN INTERTEMPORAL EQUILIBRIUM INTO A SEQUENCE OF MARKETS

The simplest way to model an economy proceeding through time is to define the intertemporal equilibria, where agents trade at only one date, and then reinterpret those equilibria as occurring via trading at a sequence of dates. See Radner (1972) for a general theory of such reinterpretations or "decompositions." To generate sequential indeterminacy, it will suffice to consider models with trading at just two periods. But each date might represent a multi-period composite, with the two dates forming a partition of a larger underlying set of time periods.

We begin by rewriting the model of Section 12.2 as a two-period intertemporal model. In the second period, there are L_2 consumption goods, M_2 pure factors, and N_2 activities that commence within the period. There are N_1 activities that commence within the first period, and L_1 first-period goods. The first-period goods could be a mixture of consumption goods and pure factors, but to minimize notation we assume they are all consumption goods. The intertemporal activity analysis matrix has the form

$$A = \begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \\ A_{31} & A_{32} \end{bmatrix} \cdot \begin{matrix} L_1 \\ L_2 \\ M_2 \end{matrix}$$

The second subscript of each A_{ij} indicates the period during which the activity commences. The 0 submatrices in A indicate that we are assuming, again for notational simplicity, that activities in each period do not directly produce or utilize consumption goods that belong to the other period.

Each agent k of the intertemporal model is described by a utility function $u^k(x_1^k, x_2^k)$, where (x_1^k, x_2^k) is k's first- and second-period consumption and by endowments $(e_{c1}^k, e_{c2}^k, e_{f2}^k)$ of first-period goods, second-period consumption goods, and second-period factors.

The primitives of the intertemporal model – the utility functions, the endowment vectors, and the technology matrix – are merely two-period versions of the primitives of the model of Section 12.2, and we assume that the current primitives obey the assumptions imposed there. An intertemporal equilibrium, which is a $(p_1, p_2, w_2, y_1, y_2)$, therefore always exists, and the generic determinacy theorem applies. Throughout the remainder of this chapter, we consider only regular two-period economies. This restriction ensures both that second-period endowments are determined endogenously and that those endowments are not unlikely from the perspective of the overall two-period model.

We now reinterpret the intertemporal equilibria as equilibria with sequential trading and perfect-foresight. In order that the two-time periods are properly linked, agents must be able to purchase or sell assets in period 1 for delivery during period 2. We assume that the assets in the model consist of those second-period pure factors that are outputs of some first-period activity and that there are $M_\alpha > 0$ such factors. These goods will be indexed as the first M_α of the second-period factors. Let α^k represent agent k's purchases of assets; α^k will be an element of R^{M_2} but the last $M_2 - M_\alpha$ coordinates of α^k are constrained to equal 0. Similarly, $q \in R^{M_2}$ will have the prices of the assets as its first M_α coordinates and will equal 0 elsewhere. Let p_1 denote the prices of the first-period goods, p_2 the prices of the second-period consumption goods, and w_2 the prices of the second-period factors.

Each agent k maximizes $u^k(x_1^k, x_2^k)$ subject to the budget constraints

$$p_1 \cdot x_1^k + q \cdot \alpha^k \leq p_1 \cdot e_{c_1}^k \text{ and } p_2 \cdot x_2^k \leq w_2 \cdot (\alpha^k + e_{f_2}^k) + p_2 \cdot e_{c_2}^k.$$

In order for agents to be willing to hold all M_α assets, the first M_α coordinates of w_2 must be proportional to the same coordinates of q: hence in equilibrium there must exist a $R > 0$ such that, for $i = 1, \ldots, M_\alpha$, $Rq(i) = w_2(i)$. Using this substitution, agent k's budget constraints reduce to

$$p_1 \cdot x_1^k + \left(\frac{1}{R}\right) p_2 \cdot x_2^k \leq p_1 \cdot e_{c_1}^k + \left(\frac{1}{R}\right) w_2 \cdot e_{f_2}^k + \left(\frac{1}{R}\right) p_2 \cdot e_{c_2}^k.$$

Utility maximization subject to this constraint generates the excess demand functions $z_1^k(p_1, p_2, w_2, R)$ and $z_2^k(p_1, p_2, w_2, R)$ for the first- and second-period consumption, and, summing, the aggregate excess demand functions are $z_1(p_1, p_2, w_2, R)$ and $z_2(p_1, p_2, w_2, R)$.

Letting y_1 and y_2 denote the activity levels for the two periods and defining $e_{f_2} = \Sigma_k e_{f_2}^k$, the market clearing requirements appear as

$$\begin{aligned} &z_1(p_1, p_2, w_2, R) \leq A_{11} y_1, \\ &-A_{32} y_2 \leq A_{31} y_1 + e_{f_2}, \\ &z_2(p_1, p_2, w_2, R) \leq A_{22} y_2, \end{aligned} \tag{12.3}$$

while the zero-profit conditions take the form

$$p_1' A_{11} + \left(\frac{1}{R}\right) w_2' A_{31} \leq 0 \quad \text{and} \quad p_2' A_{22} + w_2' A_{32} \leq 0. \tag{12.4}$$

A *sequential trading equilibrium* is a $(p_1, p_2, w_2, R, y_1, y_2)$ that satisfies (12.3) and (12.4). It is immediate that $(p_1, p_2, w_2, R, y_1, y_2)$ is a sequential trading equilibrium if and only if $(p_1, (1/R)p_2, (1/R)w_2, y_1, y_2)$ is an intertemporal equilibrium. Indeed, the only difference between the models is that in the sequential model agents make asset choices and face two budget constraints. But since each agent's reduced-form budget constraint replicates the budget constraint of the intertemporal model, the two equilibrium concepts coincide.[5]

In defining a sequential trading equilibrium, we did not need to specify the α^k. But to model second-period behavior, we must do so. At the beginning of the second period, each agent k owns the endowments $e^k_{c_2}$ of second-period consumption goods and $e^k_{f_2} + \alpha^k$ of factors. Using the sequential-trading-equilibrium values of p_2 and w_2, agent k's budget constraints imply that α^k must satisfy

$$p_2 \cdot z^k_2(p_1, p_2, w_2, R) = w_2 \cdot (\alpha^k + e^k_{f_2}). \tag{12.5}$$

And in the aggregate, total portfolio holdings of the produced factors must equal total production of factors:

$$\sum_k \alpha^k = A_{31} y_1. \tag{12.6}$$

Unfortunately, unless there is either a single agent or a single produced factor, equations (12.5) and (12.6) do not determine a single set of values for the α^k. Typically, therefore, a sequential trading equilibrium will generate a multi-dimensional set of second-period economies, each of which can be identified with one set of values for the α^k. The set of second-period economies that correspond to a sequential trading equilibrium can therefore be viewed as a finite-dimensional Euclidean space.

Once we stipulate the α^k, all of the primitives of a standard Arrow–Debreu economy are specified. The technology matrix is given by the second-period activities of the original intertemporal economy's technology matrix, endowments are as defined above, and each k maximizes the utility function $u^k(x^k_1, x^k_2)$ subject to $p_2 \cdot x^k_2 \le p_2 \cdot e^k_{c_2} + w_2 \cdot (\alpha^k + e^k_{f_2})$, where $x^k_1 = z^k_1(p_1, p_2, w_2, R) + e^k_{c_1}$ is now an exogenous variable determined by the first period of the sequential trading equilibrium. We will say that an intertemporal equilibrium *induces* this second-period economy.

Letting $z_2(p_2, w_2)$ denote the aggregate excess demand function for the second-period economy (a function that is different from the z_2 used to model the intertemporal economy), a second-period equilibrium is a (p_2, w_2, y_2) that satisfies $z_2(p_2, w_2) \le A_{22} y_2$, $-A_{32} y_2 \le A_{31} y_1 + e_{f_2}$, and $p_2' A_{22} + w_2' A_{32} \le 0$, where y_1 is fixed at its sequential-trading-equilibrium value.

12.6 SEQUENTIAL INDETERMINACY

Given a sequential trading equilibrium $(p_1, p_2, w_2, R, y_1, y_2)$, it is easy to confirm that (p_2, w_2, y_2) is an equilibrium, which we call the *continuation equilibrium*, of any of the induced second-period economies. At the continuation equilibrium, the expectations of second-period prices anticipated in the sequential trading equilibrium are fulfilled. But are continuation equilibria determinate? When, for each intertemporal economy in some non-empty open set, there exists an equilibrium where the continuation equilibrium of almost all of the induced second-period economies lies amid a continuum of second-period equilibria, we say that *sequential indeterminacy* occurs.

Analogously to the original intertemporal model, we use bars to indicate that factors in excess supply and activities that earn negative profits (or that utilize or produce only factors in excess supply) at the continuation equilibrium have been omitted from $w_2, e_{f_2}, y_1, y_2, A_{22}, A_{31}$, and A_{32}, that the first coordinate of p_2 has been set to one, and that the first coordinate of z_2 has been omitted. (As in Section 12.2, Walras' law allows us to ignore one market-clearing condition, which we choose to be the first of the second-period consumption goods.) The matrix $\tilde{A}_{22}$ omits the row of $\overline{A}_{22}$ that corresponds to the first of the second-period consumption goods. Finally, let $\overline{M}_2$ indicate the number of factors where demand exactly equals supply at the continuation equilibrium, and $\overline{N}_2$ denote the number of activities that make zero profits and utilize or produce some positively priced good. Equilibria are then locally characterized by the equations:

$$\overline{z}_2(\overline{p}_2, \overline{w}_2) = \tilde{A}_{22}\overline{y}_2, \tag{12.7}$$

$$-\overline{A}_{32}\overline{y}_2 = \overline{A}_{31}\overline{y}_1 + \overline{e}_{f_2}, \tag{12.8}$$

$$\overline{p}_2'\overline{A}_{22} + \overline{w}_2{}'\overline{A}_{32} = 0. \tag{12.9}$$

Conditions (12.7) to (12.9) consist of $L_2 - 1 + \overline{M}_2 + \overline{N}_2$ equations in the same number of unknowns – the variables, $\overline{p}_2$, $\overline{w}_2$, and $\overline{y}_2$.

Recall from the example of Section 12.3 that indeterminacy occurs at equilibria where a single activity fully employs two inelastically supplied factors. The example generalizes to cases where m positively priced (and hence fully employed) factors are used by $n < m$ activities. Mimicking the argument from the example, suppose we fix the pertinent n coordinates of $\overline{y}_2$ at their continuation equilibrium values. Independently of the value of the other $L_2 - 1 + \overline{M}_2 + \overline{N}_2 - n$ endogenous variables, the market-clearing conditions for the m factors will remain satisfied. The remaining equalities therefore constitute a system of $L_2 - 1 + \overline{M}_2 + \overline{N}_2 - m$ equations in $L_2 - 1 + \overline{M}_2 + \overline{N}_2 - n$ variables: the difference between the number of variables and equations is $m - n > 0$. This excess of variables over equilibrium conditions suggests that the system is indeterminate. To reach this conclusion formally, via the implicit function theorem, a rank condition must be satisfied. (That is, the derivatives of the remaining equalities with respect to the remaining endogenous variables must be a matrix of rank $L_2 - 1 + \overline{M}_2 + \overline{N}_2 - m$.)

Of course, for almost every value of the right-hand side of equation (12.8), there will be no set of m fully employed factors that are used by $n < m$ activities: the m equalities in equation (12.8) that correspond to the factors in question would have fewer than m endogenous variables (the n relevant coordinates of y_2). But consider again the intertemporal equilibrium that generates the second-period economy. From this perspective, equation (12.8) is also an equilibrium condition, but the additional variable $\overline{y}_1$ is endogenous. In the intertemporal equilibrium, therefore, the same m equalities may well have m or more endogenous variables. Hence, these m equalities can robustly have a solution. In short, from the vantage point of the intertemporal equilibrium, the m second-period factors can be used by or produced by at least m activities in the two time periods taken together, and hence the intertemporal equilibrium can be regular, but, from the vantage point of the second-period equilibrium, the factors may well be used by fewer than m second-period activities, and hence the continuation equilibrium can be indeterminate.

Summarizing the discussion so far, we conclude that if a regular intertemporal equilibrium has a set of m positively priced second-period factors used by fewer than m second-period activities and if the rank condition mentioned above is satisfied, then a robust case of sequential indeterminacy occurs. Curiously, the regularity of the intertemporal equilibrium is key to establishing robustness: it implies that the same factors continue to be fully employed and have positive prices and the same activities continue to be in use as the parameters of the overall two-period model change slightly.

To see a simple example of how second-period indeterminacy arises, suppose there is one consumption good per period and two factors in the second period. The first-period consumption good may either be consumed directly or used as the sole input in an intertemporal activity that produces the first of the second-period factors. The two second-period factors are then used as inputs in a single activity that produces the second-period consumption good. If, at some intertemporal equilibrium, both of the second-period factors are fully employed, (12.8) takes the form

$$
\begin{aligned}
-A_{32}(1)y_2 &= A_{31}(1)y_1 + e_{f_2}(1)\\
-A_{32}(2)y_2 &= e_{f_2}(2),
\end{aligned}
$$

where $A_{32}(i)$, $A_{31}(1)$, and $e_{f_2}(i)$ denote, respectively, the quantity of factor i used by the second-period activity, the quantity of factor 1 produced by the intertemporal activity, and the aggregate endowment of factor i. Independently of e_{f_2}, these two equations always have a solution (y_1, y_2) (and can robustly have a positive solution) if $A_{32}(2)$ and $A_{31}(1)$ are non-zero. But during the second period, when y_1 is fixed, the two fully employed factors are used by only a single activity. We are thus in the indeterminate case discussed in Section 12.3. The isoquants in the second period are L-shaped; when both second-period factors are fully employed, the second-period factor endowments – $(A_{31}(1)y_1 + e_{f_2}(1), e_{f_2}(2))$ – lie exactly at an isoquant vertex, analogously to the fact that the indeterminacy endowments in Figure 12.1 lie along the activity rays. Just as in that case, the marginal products of second-period factors are not well defined.

If there are multiple activities in the second period, the production possibilities frontier (PPF) – the boundary of the set of feasible aggregate first- and second-period consumptions – appears as in Figure 12.2. The segments in the PPF with a strictly negative slope (excluding the kinks) occur when two second-period activities are in use (except possibly in the rightmost negatively sloped segment, where it could be that one activity is in use and the non-produced factor is in excess supply): as each additional unit of first-period consumption is sacrificed, the mixture of second-period activity levels will shift slightly towards whichever activity uses the produced input more intensively (or more of the nonproduced factor is drawn into production). The kinks between the negatively sloped segments occur at the switch points when only one second-period activity is in use and thus correspond to second-period indeterminacy.

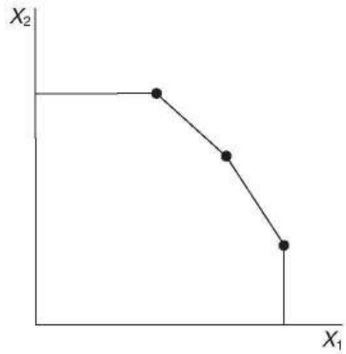

Figure 12.2 The intertemporal production possibilities frontier.

If the model contains only a single agent (or many agents with identical homothetic preferences), the intertemporal equilibrium can be represented by adding indifference curves for first- and second-period consumption to Figure 12.2. Equilibrium is determined by a tangency between an indifference curve and the PPF. Evidently, tangencies can occur robustly at the kinks in Figure 12.2; if an intertemporal equilibrium at a kink is regular, small changes in the two-period economy's parameters do not shift the equilibrium to one of the flat segments of PPF. Tangencies at the flat segments are equally robust. Thus, both sequential indeterminacy and regularity are normal events.

Sequential indeterminacy is easy to analyze when there is only one second-period consumption good (as in the above example): indeterminacy then necessarily occurs if some set of m fully employed factors is used by fewer than m activities. Due to the Walras' law omission of the market-clearing condition for the consumption good, equation (12.7) consists of no equations in this case. With multiple second-period consumption goods, we must use the implicit function theorem to establish indeterminacy. In exceptional cases, the rank condition mentioned earlier may not be satisfied and a formal proof of indeterminacy will not go through. Fortunately, the rank condition is satisfied for almost every induced second-period economy. Define an intertemporal equilibrium to be *potentially degenerate* if there is a set of m positively priced second-period factors used by fewer than m activities. A slight variant of the following theorem is proved in Mandler (1995).

Sequential indeterminacy theorem. There is a generic set of economies such that (1) if an intertemporal equilibrium is potentially degenerate then the continuation equilibrium of almost every induced second-period economy is indeterminate; and (2) if an intertemporal equilibrium is not potentially degenerate then the continuation equilibrium of almost every induced second-period economy is regular.

Thus, generically, all that matters for second-period determinacy is whether or not the number of second-period activities using each set of positively priced second-period factors is at least as large as the number of factors in that set.

12.7 THE SIGNIFICANCE OF SEQUENTIAL INDETERMINACY

By itself, the number of equilibria is a narrow concern. The disturbing implication of sequential indeterminacy is that the second-period equilibria need not vary continuously as a function of the second-period parameters. Consider the case again when there are two factors, one activity, and one consumption good in the second period, and suppose the continuation equilibrium prices ($w(1)$, $w(2)$) are strictly positive. If any quantity of factor i is withdrawn from the market, the other factor will be in excess supply, sending its price to 0, and raising $w(i)$ to $1/a_{1j}$. Consequently, no matter how small agents are as a proportion of the market, it is their interest to manipulate their factor supplies. If there is more than one second-period activity, the jump in factor prices need not be as large, but the discontinuity will remain, and agents will still have an incentive to manipulate market prices.

Price-taking behavior consequently becomes implausible, and some non-competitive mechanism must pin down factor incomes. As an example, suppose factor incomes in the second period are set by Nash bargaining among factor owners. Investors in capital goods would then anticipate that their returns will deviate from competitive levels and thus would supply suboptimal quantities of capital goods. The institutional response to sequential indeterminacy can thus induce a hold-up problem.

Even if factor markets in the second period do operate competitively, it is difficult to see how a sequentially indeterminate equilibrium would proceed through time: there is no mechanism to lead second-period markets to equilibrate at the continuation equilibrium prices. The continuation equilibrium is just one element of a continuum of equilibria; only the expectations that agents formed during the first period, and not any feature of markets narrowly construed, distinguish the continuation equilibrium from the rest. Moreover, since agents will foresee this difficulty in the first period, they will not anticipate any price vector with certainty; even in the first period, therefore, behavior will differ from competitive equilibrium predictions.

Intertemporal models with linear activities thus cannot operate via anonymous competitive markets. In order to replicate the competitive outcome, factor owners would have to sign long-term contracts that commit them to trade factors at prearranged prices. Although it is by no means unusual, nowadays, for factor markets to be modeled as long-term contracts, most analyses attribute the need for contracts to information asymmetries or to bilateral monopoly. The present account ascribes contracts to the very nature of technology and markets.

12.8 WHAT DRIVES SEQUENTIAL INDETERMINACY?

Three features of an intertemporal economy are crucial for sequential indeterminacy. First, technology must be modeled as a discrete set of activities, not as a continuum of techniques. Second, a long-run or steady-state equilibrium concept must not be in place. Third, at least some factors must be inelastically supplied.

We discuss the first two points in Sections 12.9 and 12.11 below. As for inelastic factor supply, it should already be clear that our indeterminacy arguments hinge on factor supply being unresponsive to price changes. Observe though that we could allow factor supply to be only locally, not globally, inelastic. As long as price changes in the neighborhood of a continuation equilibrium do not induce a factor supply response, our earlier sequential indeterminacy arguments apply unaltered.

12.9 CHOICE OF TECHNIQUE AND THE NEOCLASSICAL PRODUCTION FUNCTION

The neoclassical production function, differentiable at all points in its domain, implies that factors always have well-defined marginal products. Even the smallest change in factor prices then induces a change in factor demand, and consequently robust cases of sequential indeterminacy cannot occur.

Consider a constant-returns-to-scale production set $Y = \{x \in R^n: g(x) \leq 0\}$ that gives the aggregate production possibilities available for some set of n technologically related goods. The elements of Y use the standard sign convention for outputs and inputs, and we assume that g is convex and differentiable and that $Dg(x) \gg 0$ for all x. In equilibrium, producers will choose aggregate quantities x so as to maximize $\overline{p} \cdot x$ subject to $g(x) \leq 0$, where $\overline{p} = (1, p(2), \ldots, p(n))$ is a normalized price vector. Define $\overline{p}$ to be *normalized equilibrium prices* for x if x solves this maximization problem at prices $\overline{p}$. At a full competitive equilibrium, analogous profit-maximization conditions must be satisfied for other technologically related sets of goods and markets must clear.

Suppose, in the manner of our earlier indeterminacy arguments, that we fix x and that both $\overline{p}$ and $\overline{p}^*$ are normalized equilibrium prices for x. The Kuhn–Tucker theorem implies that there exist $\lambda > 0$ and $\lambda^* > 0$ such that $\overline{p} = \lambda\ Dg(x)$ and $\overline{p}^* = \lambda^*\ Dg(x)$. Hence, $(1/\lambda)\overline{p} = (1/\lambda^*)\overline{p}^*$. Since $\overline{p}(1) = \overline{p}^*(1)$, $\lambda = \lambda^*$ and $\overline{p} = \overline{p}^*$. Thus, there can be only one vector of normalized equilibrium prices for any set of aggregate quantities.

This reasoning does not show that competitive equilibria with differentiable technologies are locally unique – not even if Y were the aggregate production set for the economy as a whole. Local uniqueness of a competitive equilibrium with prices $\overline{p}$ and aggregate quantities x requires in addition that there are no other competitive equilibria with prices $\overline{p}'$ and quantities $x' \neq x$ that are arbitrarily close to $(\overline{p}, x)$. The standard regularity literature assures us that this property holds generically. To be convincing, however, a generic local uniqueness theorem must be set in an intertemporal framework and should not rely on arbitrary endowment perturbations. See Mandler (1997) for a proof of sequential determinacy – the determinacy of both the intertemporal equilibria and the second-period equilibria they endogenously generate – when technology is differentiable.

12.10 SRAFFA AND SEQUENTIAL INDETERMINACY[6]

Several parallels between sequential indeterminacy and Sraffa's (1960) indeterminacy argument in *Production of Commodities by Means of Commodities* have already surfaced. First, sequential indeterminacy occurs with linear activities but not with differentiable "marginal productivity" technologies. Second, Sraffa fixes the aggregate quantities produced and finds multiple equilibrium prices for those quantities. Our indeterminacy arguments follow the same method: we fix a subset of quantities and show that a continuum of price vectors support those quantities. Third, in line with Sraffa's emphasis on the production of commodities by means of commodities, it is only in dynamic models that indeterminacy arises systematically.

Sequential indeterminacy also sheds light on some of Sraffa's more elusive remarks. Sraffa was critical of theories that treat the economy as a "one-way avenue that leads from 'factors of production' to 'consumption goods.'" From the vantage point of sequential indeterminacy, the difficulty with "one-way" theories is not that they assume that final output is limited by society's factor resources, but that they let endowments be arbitrary rather than determined by ongoing investment decisions. Taking endowments to be in an arbitrary or generic set ensures that marginal products are well-defined even when technology is described by linear activities. (Recall from Figure 12.1 that marginal products are defined at and only at the generic endowment points at which factor prices are determinate.) Given that the endowments at which marginal products are ill-defined arise systematically in a sequential setting, Sraffa's suspicion of the device of an arbitrary starting point to economic activity accurately points to a serious difficulty in the foundations of marginal productivity theory.

But there are also clear differences between Sraffa's position and sequential indeterminacy. Our emphasis on inelastic factor supply has no parallel in Sraffa or the subsequent literature. More significantly, we will see sequential indeterminacy does not arise when equilibria are long-run.

Consider the following variant of Sraffa's model. Suppose n material inputs and labor produce the same n material goods, with the outputs appearing one period after the inputs are applied. Each commodity j is produced by one linear activity, represented by an n-vector $a_j = (a_{1j}, \ldots, a_{nj}) \geq 0$ of material input requirements and a scalar $l_j \geq 0$ of labor required. Capital invested in each activity earns the same rate of profit r. Assuming as Sraffa does that labor is purchased when output is sold, profit rate equalization implies, for each produced good j, that

$$p(j) = (1 + r)(p(1)a_{1j} + \cdots + p(n)a_{nj}) + wl_j,$$

where $p(j)$ is the price of good j, and w is the wage. Letting $A = [a_1, \ldots, a_n]$, $l = (l_1, \ldots, l_n)$, and normalizing by defining $\overline{p} = (1, p(2), \ldots, p(n))$, we can rewrite the above equations as

$$\overline{p} = (1 + r)A'\overline{p} + wl. \tag{12.10}$$

Since equation (12.10) constitutes n equations in the $n + 1$ unknowns $(\overline{p}, w, r)$, the equations have one degree of freedom. Hence, one variable can be varied exogenously while still allowing equation (12.10) to be satisfied. Sraffa lets r be this exogenous variable and concludes that competition leaves the interest or profit rate indeterminate.

Neoclassical critics, most thoroughly Hahn (1982), have faulted Sraffa's model on two main counts. First, Sraffa omits any mention of demand and supply. Possibly, therefore, some of the price vectors that solve (12.10) might be inconsistent with market-clearing. Second, input and output prices are constrained to be equal in Sraffa's model: equation (12.10) uses the same $\overline{p}$ on both left and right hand sides. According to the critics, a proper equilibrium model should allow prices to change through time. Indeed, except at particular combinations of endowments and preferences, equilibria will not exist if prices across time periods are required to be equal.

The literature on Sraffa presumes that once these two flaws are rectified, Sraffa's model necessarily becomes determinate. Our earlier indeterminacy results suggest, however, that determinacy is not guaranteed. Notice that the two objections to Sraffa work in opposite directions. Including market-clearing equations diminishes the potential for indeterminacy, but distinguishing prices by date adds new price variables and makes indeterminacy more likely. These two corrections turn out to offset each other exactly.

To meet the neoclassical objections, we distinguish between input prices $\overline{p}^1$ and output prices $\overline{p}^2$, and posit an aggregate demand function for the n outputs $x(\overline{p}^1, \overline{p}^2, w, r)$. We assume that the n material inputs are inelastically supplied at the level $e \gg 0$ and that labor is inelastically supplied at the level $e_l > 0$.[7] An equilibrium is a $(\overline{p}^1, \overline{p}^2, w, r, y)$ such that

$$x(\overline{p}^1, \overline{p}^2, w, r) \leq y, \tag{12.11}$$

$$\overline{p}^2 \leq (1 + r)A'\overline{p}^1 + wl, \tag{12.12}$$

$$Ay \leq e, \tag{12.13}$$

$$1 \cdot y \leq e_l. \tag{12.14}$$

We suppose that conditions (12.11) to (12.14) hold with equality.

We now apply the indeterminacy arguments of Section 12.6. At an equilibrium $(\overline{p}^1, \overline{p}^2, w, r, y)$ where each price, activity level, and $(1 + r)$ is strictly positive, the total number of factors with positive prices $(n + 1)$ will be larger than the number of activities in use (n). Assuming the appropriate rank condition is satisfied, the arguments of Section 12.6 imply that the equilibrium must be indeterminate. It is just as easy to reason directly. If we fix y at an equilibrium value, equations (12.13) and (12.14) remain satisfied at all values of $(\overline{p}^1, \overline{p}^2, w, r)$. The remaining equations consist of the $n - 1$ independent equations in equation (12.11) – we lose an equation due to Walras' law – and the n equations in equation (12.12). With y fixed, the endogenous variables consist of the $2n$ prices $(\overline{p}^1, \overline{p}^2, w, r)$. With one more unknown than equation, indeterminacy typically obtains.

Interestingly, the dimension of sequential indeterminacy tracks the dimension of indeterminacy in Sraffa's theory. Suppose, in the second-period model of Section 6, that each factor is fully employed and has a positive price and that the indeterminacy-inducing set of factors is the economy's entire set of second-period factors. The dimension of indeterminacy then equals M_2, the total number of second-period factors, minus $\overline{N}_2$, the number of activities in use. Hence, multi-dimensional indeterminacy – indeterminacy of dimension greater than 1 – occurs when $M_2 - 1 > \overline{N}_2$. In Sraffa's model of joint production, where activities can produce more than one output, the number of activities in use can fall below n even while all n goods are produced. Sraffa recognized that in this case the dimension of indeterminacy can expand beyond the single dimension claimed for equation (12.10). In Sraffa's words, multidimensional indeterminacy obtains when there are "more [goods] prices to be ascertained than there are processes" in use. Since the number of goods prices, n, equals the total number of factor prices (the material inputs plus labor) minus 1, Sraffa's condition for multidimensional indeterminacy is the same as ours. Curiously, although he recognized the possibility of multidimensional indeterminacy, Sraffa assumed it away by supposing that the number of activities in use always equals the number of goods. Also, Sraffa failed to report that indeterminacy can disappear when the number of activities in use is greater than n.

We have departed markedly from Sraffa in not making the "long-run" assumption that relative prices remain constant through time. If relative prices are to remain constant, the economy must be placed in an infinite-horizon setting. However, while sequential indeterminacy can easily arise in an infinite-horizon model, the long-run equilibria where relative prices are required to be constant are determinate. We sketch a simple model of the long run to establish this point. More details can be found in Mandler (1999b). The key additional ingredient is a steady-state demand function, $x(\overline{p}, w, r)$, which we let originate from overlapping generations of agents. The n goods are partitioned into a set of consumption goods and pure factors, and the subscripts c and f respectively denote the rows of the input requirement matrix and the entries of y that correspond to the two types of goods. Assuming for simplicity that all equilibrium conditions hold with equality, a *long-run equilibrium* is a $(\overline{p}, w, r, y)$ such that

$$x(\overline{p}, w, r) + A_c y = y_c$$
$$A_f y = y_f$$
$$l \cdot y = e_1$$
$$\overline{p} = (1 + r)A'\overline{p} + wl.$$

Since none of the market-clearing equations are redundant in a long-run OLG setting, we have $2n + 1$ independent equations in the $2n + 1$ variables $(\overline{p}, w, r, y)$.[8]

One might suspect that sequential indeterminacy could arise in this model, since, with no restrictions on A, some subset of m material pure factors and labor may well be used by fewer than m activities. Moreover, the endowment of material inputs in any given period, y_f, is an endogenous variable; hence, it can robustly occur that the market-clearing conditions for these m factors are satisfied with equality. Nevertheless, we have the following result from Mandler (1999b), which extends Kehoe and Levine's (1985) analysis of exchange economies to models of production.

Long-run determinacy theorem. There is a generic set of labor endowments, technology parameters, and demand functions such that each long-run equilibrium is locally unique.

The reason for the determinacy of the long-run model is not hard to grasp. Although in any given period, some set of m factors may be used by fewer than m activities, each factor's price as an input is constrained to equal the price of the same good currently being produced. These additional equilibrium conditions are enough to ensure generic determinacy. More general models with more non-produced pure factors in addition to labor are also generically determinate.

In sum, our results support neither side of the debate over Sraffa. If relative prices can vary through time, Sraffa had the right building blocks of a coherent indeterminacy argument. He identified the close link between linear activities and indeterminacy and rightly emphasized the significance of not imposing an arbitrary starting point for economic activity. But in a long-run setting of constant relative prices, the neoclassical case for determinacy is convincing. Each side to the Sraffa debate thus backs an equilibrium concept inconsistent with its determinacy claims.

12.11 CONCLUDING REMARKS: INDETERMINACY AND LONG-RUN EQUILIBRIA

The indeterminacy associated with inelastic factor supply and linear activities was well recognized in pre-World War II economic theory. Hicks (1932) and Robertson (1931), for example, acknowledged that factor prices in the short run may be indeterminate. They recognized that highly specialized forms of labor and capital might not allow any leeway for factor substitution and hence factor demand might not respond to price changes. Although they did not dismiss the short-run problem, Hicks and Robertson argued that equilibria would at least be determinate in the long-run. That is, even if deviations in factor prices from their long-run equilibrium values do not immediately disturb any equilibrium condition, persistent deviations will ultimately be incompatible with market clearing.

Hicks and Robertson, and most other prewar economists, analyzed factor markets in terms of aggregate *basic factors*, land, labor, and capital. Intermediate inputs and ultimately final output as well were viewed as reducible to the quantities of basic factors needed to produce them, and competitive equilibria were understood in terms of the demand for and the supply of basic factors. A set of basic factor prices determines cost-minimizing prices for final output; given these output prices, consumers choose their final output demands and thus indirectly determine the demand for basic factor inputs. Full long-run equilibrium occurs where basic factor demand, calculated in this way, equals basic factor supply.

Hicks and Robertson argued that if the economy begins at a position of long-run equilibrium then a permanent shift in basic factor prices will eventually change the demand for basic factors. Even if given stocks of intermediate inputs must be used in fixed proportions, opportunities for factor substitution emerge as new intermediate inputs become available. For instance, if the wage were to fall permanently, labor-intensive intermediate inputs will eventually become cheaper and will therefore be adopted in more production processes, ultimately raising labor demand. In addition, a fall in the wage will lower the price of labor-intensive final outputs, raising demand for these outputs and thus again raising labor demand. See Mandler (1999a) for an explicit model.

This long-run case for determinacy is considerably more convincing than direct factor substitution arguments. Since the birth of marginal productivity theory in the 1890s, a steady stream of economists have balked at applying factor substitution to specialized intermediate inputs. Explaining the elasticity of factor demand as a consequence of the switch to distinct intermediate inputs is vastly more plausible. Of course, as Hicks or Robertson acknowledged, at any given point in time, current-period prices of basic factors and intermediate inputs may still exhibit indeterminacy; only sustained changes in factor prices are likely to induce disequilibrium changes in factor demand.

Our determinacy and indeterminacy theorems bear out the prewar understanding of factor pricing. The determinacy of equilibria in the Hicks–Robertson sense finds formal expression in the long-run determinacy theorem of Section 12.10. And Hicks' and Robertson's claim that factor prices may nevertheless be indeterminate in the short-run is nothing more than an assertion of sequential indeterminacy. The only difference is that Hicks and Robertson did not dwell on changes in factor prices that persist for only a brief period of time; they shrugged off short-run indeterminacy and focused on the long run. But in both the Hicks–Robertson description of the short run and in the formal theory of sequential indeterminacy, the equilibrium that validates perfect-foresight expectations (long-term expectations in Hicks–Robertson) is surrounded by other equilibria.

The prewar approach to factor demand appears suspicious today in its cavalier aggregation of factors. In fact, the analytical difficulties that vex factor aggregation seem to have led Sraffa to believe that without aggregation, long-run models would be indeterminate. Certainly, Sraffa's most telling criticisms of neoclassical theory pertain to its aggregation claims. But as the long-run determinacy theorem indicates, an indeterminacy indictment of the neoclassical theory of long-run equilibria cannot be sustained. Hicks and Robertson happened to reason in terms of aggregated basic factors, but the long-run determinacy theorem (and its extensions) show that their analysis can be cleansed of any aggregation assumptions. Indeed, even the Hicks–Robertson confidence in long-run opportunities for factor substitution is unnecessary; the long-run determinacy theorem relies only on the fact that long-run consumption prices (and hence consumption demand) change in response to shifts in long-run factor prices. Again, see Mandler (1999a) for details.

The determinacy of long-run equilibria hardly leaves the factor price theory in satisfactory shape. As we mentioned, long-run equilibria suffer from the same sequential indeterminacy problems as do standard intertemporal equilibria; the only difference is that the equilibria near a long-run equilibrium do not have constant relative prices and therefore are not themselves long-run. And, just as with non-steady-state intertemporal equilibria, no pure market mechanism can force markets to equilibrate in each period at the long-run equilibrium prices and agents have a strong incentive to manipulate factor markets. Long-run equilibria thus also require an accompanying theory of factor markets contracts and contract enforcement.

NOTES

1 Mas-Colell and Kehoe's work was preceded by Fuchs (1974) and Smale (1974), which analyzed decreasing-returns-to-scale technologies. Mas-Colell (1985) provides a definitive overview of the regularity literature.

2 "Differentiably increasing" and "differentiably strictly concave" mean that, for all $x^k \in R^L_{++}$, $Du^k(x^k) \gg 0$ and $D^2u^k(x^k)$ is negative definite.

3 For the *Sequential Indeterminacy Theorem* of Section 6, we will need to supplement the basic parameter space with a space of utility functions. Given a utility u^k for agent k that meets the assumptions stated above, let $u^k + \epsilon f$ be an admissible utility for k, where f is any quadratic function on R^L_+ and where ϵ is a scalar in an open set small enough that $u^k + \epsilon f$ satisfies the same assumptions when restricted to the set of feasible consumption vectors that deliver at least as much utility for k as e^k_c. Since quadratic functions can be parameterized by a finite set of real numbers, our space of economies can still be seen as an open set of finite-dimensional vectors of real numbers.

4 The usage levels for any activities that utilize or produce only goods that are in excess supply will be indeterminate. This fact leads to no corresponding indeterminacy of relative prices or agents' welfare, however.

5 Since there are two budget constraints for each agent in the sequential model, an uninteresting nominal multiplicity of equilibria appears that is absent from the intertemporal model: given sequential-trading-equilibrium prices (p_1, p_2, w_2, R) and a scalar $\lambda > 0$, $(p_1, \lambda p_2, \lambda w_2, \lambda R)$ are also sequential-trading-equilibrium prices. The multiplicity only involves a rescaling of second-period prices, however, and so allocations of goods do not change.

6 This subject is treated more thoroughly in Mandler (1999b).

7 We require that the model obeys Walras' law: for all $(\bar{p}^1, \bar{p}^2, w, r)$, $\bar{p}^2 \cdot x(\bar{p}^1, \bar{p}^2, w, r) = (1 + r)\bar{p}^1 \cdot e + we_l$.

8 We have modeled equilibria as steady-state. But if the endowment of labor, e_l, were to grow at some fixed rate through time (or if each input requirement were to diminish at the same fixed rate), the model could with minor notational adjustments describe a growing economy instead.

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13 General equilibrium and the destiny of capitalism à la Schumpeter*

Michio Morishima

1. The subject assigned to me by the programme committee of the Siena Summer School was "Schumpeter and General Equilibrium". But I extend it in this chapter so as to include his Hauptwerk, *Capitalism, Socialism and Demoracy* (Schumpeter, 1942), in addition to his general equilibrium books (Schumpeter, 1908, 1934). The reason for this is that although *Capitalism, Socialism and Democracy* (Schumpeter, 1942) is a macroeconomic book, it may be taken as a second part of his "Tale of Entrepreneur-Innovator" that had begun with his famous, now classic, general equilibrium drama (Schumpeter, 1934).

We begin with a short survey of the development of general equilibrium theory (GET). The theory was given in a complete form when the fourth edition of Walras' *Elements* was published in 1900. Afterwards an advancement was made towards dynamization of the theory, when La Volpe and Hicks published their doctrines in 1936 and 1939, respectively. The third stage was built by Arrow and Debreu in 1954 and the fourth by Debreu alone in 1962. And then, the eventual destiny of the economy was discussed in a form of conjecture by Schumpeter (1942).

The stability of equilibrium was originally provided by Walras as the theory of tâtonnement. As it will not be discussed in the text below, I make a brief survey of works on the stability of economic equilibrium in this introductory section. First, Samuelson formulated the process of tâtonnement in terms of differential equations. It was then found by Arrow and Hurwicz (1958) and by Arrow, Block and Hurwicz (1959) that Samuelson's rather mechanical and computational stability results are obtained wherever the excess demand functions satisfy a certain qualitative economic condition such as the absence of complementarity together with either of the following two mild economic conditions pointed out by Negishi and Hahn respectively:

(i) the excess demand functions are homogeneous of degree zero, and
(ii) Walras' law is fulfilled.

It was then noted, rather recently, that the stability theories in these studies have to be re-examined in the framework of differential equations as long as excess functions are non-linear. That is to say, one must recognize the fact that non-linear excess demand

* I am grateful to F. Petri and B. Schefold for helpful comments at the conference.

functions do not necessarily produce a satisfactory stability result if the price adjustment functions are of difference equation form.

More precisely, Morishima (1996) (Article IV) has examined this remaining case; i.e. the case of non-linear excess demand functions with the adjustment functions being difference equations. In this case, the time interval h required for one round of tâtonnement assumes any finite number. Then if h approaches zero, the adjustment of prices becomes continuous, and we obtain a movement of prices that is valid for the case of instant adjustment with no time lag. From this system complicated results follow. If we assume absence of complementarity, the equilibrium exists and is unique. If we further assume that the adjustment time h is short enough, prices eventually approach the equilibrium point. Stability is thus assured as Negishi and Hahn claimed.

We cannot determine the dynamic path of prices theoretically, but can obtain it in a computative way by using the price adjustment equation as the iteration formula. Then for some fixed numerical value of h, the iteration eventually results in the prices visiting eight points repeatedly in a certain definite order in the case of a two-commodity economy that I examined. The same numerical model produces a sequence of prices which undergoes a series of period-doublings which afterwards turns to a series of period-halvings, according as the parameter h increases further. This means that the stability of equilibrium which Negishi and Hahn found is obtained only for those h's which are sufficiently small; that is to say, it is true only when the adjustment equations of the difference equation type are sufficiently close to the corresponding differential equations.

In addition, we find that for some particular values of h, the time path of the prices is unpredictable in the sense that two paths which are very close at the start will later diverge substantially, so that the prices $p(t)$ in the future cannot be predicted from the initial state of affairs. That is to say, first, there is an area of p such that the path of $p(t)$ repeatedly comes very near to any point in the area. Second, two paths of p starting from two close initial positions, however close they may be, will eventually separate from each other, at any given value of t, by at least some positive distance during the course of iteration. Namely, there is an area of h's in which $p(t)$ becomes chaotic.

However, we can show that the chaotic movements are confined in a limited area that does not usually stretch over the whole price domain. This means that the price movement is eventually made only within an area, whereas it endlessly fluctuates therein. We may thus observe a certain kind of stability; that is the convergence of the price movements to some limited area. This type of stability may be referred to as quasi-stability.

2. Now we turn to the works by Schumpeter. The first book on the GET by him was published in 1908, so that it was essentially a work on Walras' *Elements*, 1900; it may, in fact, be regarded as a reinforcement work on the Lausanne theory from the Austrian point of view. The *Elements* concentrates its attention in Part V on the theory of capital formation and credit and in Part VI on the theory of circulation and money of the *Elements*. Schumpeter reinforced Walras' capital theory by von Böhm-Bawerk's *Positive Theory of Capital*.

Neglecting Schumpeter's contribution to the work on *Zurechnungstheorie*, we concentrate our attention upon his conclusion that there is no entrepreneurial profits in the state of static or stationary equilibrium. To this effect, it is shown that we have the following equation from the equilibrium conditions of capital formation:[1]

> Value of total output of consumption goods and capital services − value of the supply of primary factors of production − depreciation of capital goods (including insurance premium of capital goods) = rate of profit × value of capital services used.

If the left-hand side of this equation is positive, the output produced exceeds that amount which goes to the suppliers of the primary factors of production, after having paid the depreciation charge and insurance premium for various capital goods. It stands for the net accumulation of the total output. Thus the above equation implies Schumpeter's proposition that the accumulation is positive if and only if the rate of profit is positive, so that there are no profits or *Zinz* in the stationary state equilibrium.

In his second book on the GET (1934) Schumpeter clarified the role of the entrepreneurs in the economy. The importance of entrepreneurs was stated by Walras as: "The entrepreneur is . . . a person (natural or corporate) who buys raw materials from other entrepreneurs, then leases land from land-owners on payment of a rent, hires the personal faculties of workers on payment of wages, borrows capital from capitalists on payment of interest charges and, finally, having applied certain productive services to the raw materials, sells the resulting product on his own account." (Walras, 1954: 227.) "The capitalist accumulates his capital by successive savings and lends money to the entrepreneur for a given period; the entrepreneur converts this money into capital proper and at the expiration of the contract he returns the money to the capitalist." (Walras, op. cit., p. 228.) "In a state of equilibrium of production, entrepreneurs make neither profit nor loss. They make their living not as entrepreneurs, but as land-owners, labourers or capitalists in their own or other businesses." (Walras, op. cit., p. 225). In the actual world, it is of course true, as Walras recognizes, that the entrepreneurs also assume the jobs of land-owners, capitalists, or workers, so that they live, in the state of equilibrium, on that income which they earn as the holder of these jobs. Note that Walras extends the concept of workers so as to include those charged with the special task of managing firms.

It has sometimes been said that Schumpeter's entrepreneurs are different from Walras'. I am, however, of the opinion that there is no inconsistency between them. The former emphasizes entrepreneurs as the persons who invent, propose and carry out innovations, but when they have no such idea, they are no more than simple managers of the firms. The latter considers entrepreneurs as promoters of business. They are not, however, always involved in some innovative projects. During the period of intermittence of the craze of enterprising activities they remain ordinary managing directors, that is, entrepreneurs of the Walrasian type. Thus, regarding these sleeping Schumpeterian entrepreneurs as Schumpeterians, the two definitions are identical with each other.

As far as the above aspect of the character of entrepreneur is concerned, there are no significant obstacles between Walras and Schumpeter. In addition to it, however, there is another aspect, which distinguishes Schumpeter's entrepreneur from Walras'. That is to say, while the latter is "the fourth person" of the economy, "whose role it is to lease land from the land-owner, hire personal faculties from the labourer and borrow capital from the capitalist", as I have quoted above, the former always appears in combination with a banker. The entrepreneur who introduces new combination needs a special capitalist (a banker) who provides purchasing power to him. Otherwise he will not be able to establish an enterprise of enormous impact. A creation of credit by a banker thus produces a promising entrepreneur. In this way innovator and banker always appear in combination, so that we may say that the combination gives them power.

A similar example may be pointed out with regard to Keynesian policy. When the government decides to make a spending policy, it would take time to sell out the whole necessary amount of government securities to the public; whereas in the same circumstances, the government alternatively sells the whole amount to the central bank at once and then the latter may sell the government securities thus obtained to the public gradually. It is evident that the total effect is immediate in the latter case, while it is gradual in the former. Thus the combination of the government and the central bank gives the government policy an extra power.

As will be seen later, Schumpeter insisted in private discussion with Yasuma Takata that economic theory or pure economics should not be involved in any analysis of effects of political and sociological elements upon the economic performance of society. Nevertheless what I have said above suggests that Schumpeter has at least been concerned with phenomena relating with social elements, such as power, that amplify their total effect if they work together: Joint behaviour of independent economic agents creates economic power. Once this is observed, then these agents tend to work together consciously; there are consequently certain types of economic activities which are intended for the realization of their full effects, that is to say, including not only the additive total of the effects created by single agents separately but also the additional effect created by their joint work. This type of GET of his has been developed in his second volume on general-equilibrium (Schumpeter, 1934), that is his famous theory of economic development. For this reason, it occupies a distinctive position in the GET literature.

3. Walras' theory of economic development consists of two parts. The first formulates his theory of capital formation and credit. The second deals with price movements in a progressive economy. A paper which I published in 1960 is included in *Equilibrium, Stability and Growth* (Morishima, 1964). It discusses Walras' model of capital formation consisting of the following seven sets of equations:

(1) The price–cost equations for consumption goods,
(2) the price–cost equations for capital goods,
(3) the supply–demand equations for consumption goods,
(4) the supply–demand equations for primary factor of production,
(5) the supply–demand equations for capital services,
(6) the equations implying that the net prices of capital services obtained after subtracting the depreciation charges and the insurance premiums from their gross prices are proportional to the prices of capital goods,
(7) the value of capital goods produced = the amount of savings.

The proportionality factor of equation (6) is referred to either as the rate of interest on capital or as the rate of profits.

Morishima (1964) shows that, by using the fixed-point approach, these seven sets of equations have solutions. The prices of consumption goods, capital goods, and the primary factors of production, together with the rate of profits are all determined; the outputs of consumption goods and capital goods are also determined. By (3) the demands for consumption goods equal their respective supplies. By (5) the total amount of capital services needed for the production of consumption goods and capital goods equals the amounts of services which the existing stocks of capital goods can supply. However, it is noted that this Walrasian system of equilibrium equations does not include the set of equations which state that the amounts of capital goods to be produced have to be equal to the increases of their amounts that the firms desire. That is to say, the Walrasian model pays no attention to:

(8) the supply–demand equations for capital goods.[2]

In spite of this, Walras extends his model of capital formation so as to be able to discuss the equilibrium of money and circulation. This model consists of a single equilibrium equation namely

(9) the supply–demand equation for money.

This determines the money price of the numéraire, so that all prices of goods are given in terms of money.

As the Walrasian equilibrium requires Say's law in the sense that equations (8) hold automatically, it is not necessarily realized in the actual world in which Say's law is not true. That is to say, there is no equilibrium satisfying all of equations, (1)–(8), simultaneously. Such a point may at best be taken as a target point at which the economy will eventually settle. Throughout this process of approaching to the long-run equilibrium point, equations (8) are satisfied, but equations (6) are not fulfilled; that is to say, the rates of profit are not equalized. The fourth edition of his *Elements* (Walras, 1954) was published in 1900; but he did not make any reconsideration on this point.

General equilibrium theory, as far as the aspect which I am discussing is concerned, had a rather long period of vacuum until finally La Volpe appeared in 1936 and then Hicks in 1939. During this period, there was formed a common conviction that the economy would eventually settle at a point where the rate of profits or "rate of net income" becomes equal for all goods.

This belief is shared not only among *pre* La Volpe–Hicks general equilibrium theorists but also by Marx and the Marxists. It is said that the work preparing Sraffa's famous 1960 book (Sraffa, 1960) started during the 1920s; then naturally its main sets of equations, the price–cost and the input–output equations are Walrasian, so that it belongs to the regime where the rates of profit are equalized; Say's law is implicitly assumed. Also, von Neumann's famous article (von Neumann, 1945–46), though it was published in English in 1945–1946, was originally made available in German in 1937, only 1 year after the publication of La Volpe (1993). It too, accordingly, naturally belongs to the world where the equality of the rates of profits is accepted without serious examination. Moreover, I have to add that Schumpeter's (1908, 1934) view of general equilibrium is mainly based on his study of Walras (1954) and von

Böhm-Bawerk [1914]. We must notice that when he examines dynamic effects of economic innovations upon equilibrium, he is mostly concerned with the problem of how the static equilibrium of the Walrasian type is disturbed and restored eventually. Innovations stimulate investment, as we shall discuss later.

We now extend the above argument to an area which Walras neglected. In any period t firms are provided with some initial endowments of capital goods. These are regarded as data when the equilibrium of capital formation is found out. If we may assume that this equilibrium is quickly and smoothly established, the economy will move to the equilibrium state. Unless gross investment equals depreciation for each capital good, initial endowments of capital goods for the next period are different from those in the previous period. We thus have a different equilibrium of capital formation in period $t+1$. A sequence of general equilibria created in this way is compared, later in Section 5, with the sequence of temporary equilibria due to La Volpe (1993) and Hicks–Morishima (Hicks, 1939) and (Morishima, 1996) the comparison will lead us to the yet unsolved problem of stability of motion.

Let us now turn to the second part of Walras' theory, that is a kind of superstructure built on the basis of the equilibrium theory of capital formation, which produces his Ricardo-like theory of economic growth. He calls it "the law of general price movements in a progressing economy".

In his capital formation theory the number of persons living in the economy is one of data to be taken as given. When it changes, the state of equilibrium to be established is induced to shift. The prices of commodities, the wages, the price of land (rent) and the prices of capital services (the interest charge) are all influenced. In case these changes stimulate a further increase in population, then the capital formation equilibrium is stimulated to shift further. The process of progress continues, as long as population growth continues.

Walras' analysis of this process of moving equilibria is not rigorous but only roughly outlined. Nevertheless, he does not hesitate to summarize it as: In a progressive economy, wages do not change substantially, rent will rise appreciably and the interest charge (or the rate of profits as we may call it) will fall appreciably. Entrepreneurs have thus to pay higher rents, but will employ less land services per unit of output; they will pay lower interest but will employ more capital services. Thus he obtains a view of economic progress that is very similar to Ricardo's.

In the analysis of the second part of the capital formation equilibrium Walras assumes implicitly that the equilibrium is stable and entirely stays away from a detailed description of the movement towards the new capital formation equilibrium. No one, however, can blame Walras severely for the incompleteness of his analysis in this respect; although in so far as we assume that individuals and firms carry out their economic intentions through perfectly organized markets, we may usually conjecture the stability of the equilibrium, still we must query and examine whether the equilibrium exists in case the population increases too rapidly. Takata (1995) points out that if the given amount of population is very large, full employment is obtained only at an extremely low level of wages, which is far lower than the subsistence level. In such a case workers prefer to be unemployed rather than to work. Workers' resistance against low wages will be unavoidable; the wages will settle at a minimum, that is, at the subsistence level.

An excess of population over the demand for labour corresponding to the one at the subsistence wage creates unemployment of labour, so that the Walrasian full employment equilibrium of capital formation does not exist if population grows very rapidly. This is the unemployment which Marx (1965: 631–632) calls the relative surplus population.

This shows that the population and its growth cannot be taken as arbitrarily given data; unemployment is inevitably created where its rate of growth is too high and exceeds the limit of labour that the existing stock of capital goods of the economy can accommodate within the economy. Thus Marx is more careful than Walras in this respect. But in order to sustain the subsistence wages Marx needs to explain the effectiveness of power. It is important to explain how social power works to create the downward rigidity of wages at the level of subsistence in spite of the excess supply of labour.

On the other hand, if workers' power is ineffective, the wages are pushed down so as to maintain full employment in the labour market. As the equilibrium wages are extremely low, there is no necessity for fixed capital equipment, so that an excess supply of fixed capital is inevitable. Again we do not obtain the Walrasian equilibrium of capital formation as long as Walras remains pure with no introduction of the power element into his model; otherwise his law of general price movement in a progressive economy holds only for the cases of the population growing at a certain admissible rate (see Takata, 1995: 170–172).

Moreover, as I have shown in my book on Walras (Morishima, 1977: 95), his model of capital formation is incomplete unless we assume Say's law, that is to say, unless investments are flexibly adjusted. This is because his model ignores the demand–supply equations of newly produced capital goods, (8) above. It obviously determines the equilibrium values of D_k's, but they are, according to his definition, the quantities of capital goods k to be newly produced, not their quantities demanded as the notation D_k suggests. Then we must accept the incompleteness of his model that results in a fatal weakness of his theory: the model has no demand–supply equations for capital goods to be produced. Therefore, the model needs Say's law which assures that the capital goods produced D_k, however large their amounts may be, are entirely demanded and invested.

If we reject this law because it is unrealistic, then the prices of capital goods to be determined in the capital goods markets will differ from those which are obtained from Walras' own (incomplete) model. Moreover, it is seen that the equal rates of profits that his model assures will be violated under the market prices of capital goods. In this way, we realize that the study of general equilibrium of capital formation has to be drastically re-examined. But this work has been done by those young theorists who appeared after Schumpeter had published his two general equilibrium books, *Das Wesen und der Hauptinhalt der theoretische National-ökonomie* (Schumpeter, 1908) and *The Theory of Economic Development* (Schumpeter, 1934).

4. We have already said that there are two kinds of general equilibrium theory: the Walras type and the La Volpe and Hicks type. The former, though it has some shortcomings just mentioned above, has been concerned with a sort of long-run equilibrium where the rates of profits are equalized through all firms and all industries, while the latter deals with a temporary equilibrium that is obtained when adaptation of the economy is not perfect. In this state of affairs the rate of profits differs from one firm to another, so that capital moves from less profitable firms to more profitable ones. A temporary equilibrium in week t does not last but will be replaced by a new temporary equilibrium in week $t + 1$; thus we obtain a sequence of temporary equilibria through time.

To see more precisely the matter, we explain the La Volpe–Hicks approach in detail. For this purpose let us concentrate our attention upon my earlier work as a graduate student, which was later published with the title of *Dynamic Economic Theory*, in 1950 in Japanese. This was afterwards translated into English and was finally made available as *Dynamic Economic Theory* (Morishima) in 1996. It may be taken as a revised version of Hicks, *Value and Capital*, as it removes his fatal assumptions: (a) output = supply for all products; and (b) input = demand for all factors. With these assumptions, there is no change in stock of commodities, so that investment is nil for all capital goods.

In spite of his model's inability to generate a dynamic movement of the economy because of this defect, Hicks still considers that it can accommodate savings and investment with no modification of his production model.

Hicks (1939: 158) writes the demand for cash of a firm as:

(10) Acquisition of cash = Value of output − Value of input − Interest on debts −Dividends + Value of securities issued (or sold).

This implies that he makes assumption (a) and (b) above and obtains (10) from the true equation,

(11) Acquisition of cash = Value of supply of products − Value of demand for factors −Interest on debts − Dividends +Value of securities issued (or sold).

holding regardless of (a) and (b). Introducing into (11) the definitional equations:

Value of output = Value of supply of products + the increase in the stock of products,

and

Value of input of factors of production = Value of demand for factors − the increase in the stock of factor,

we obtain for firms an equation:

Acquisition of cash = the output of products − the input of intermediate goods and primary factors − the increase in the stock of commodities + A

where

A = −Interest on debts − Dividends + Value of securities issued (or sold).

This, together with the equation for individuals,

Acquisition of cash = Receipts (including interest on securities owned) −Expenditure − Value of securities acquired, (see Hicks, 1939: 157), enables us to obtain the following equation which holds for the whole economy:

Acquisition of cash = (Value of net output − Expenditure by individuals
−Increase in the stocks of final and intermediate products)
+(Receipts of individuals − Payment for primary factors of production − Interest on debts − Dividends)
+(value of securities issued − Value of securities acquired

where, of course, the net output stands for the output of products minus the input of intermediate products. We refer to this as the Hicks–Morishima equation, for it is corrected so as to avoid Hicks' assumption (a) and (b). Denoting net output, individuals' expenditure and the increase in the stock of commodities by Y, C and I, respectively, the part in the first pair of parentheses stands for Keynesian excess supply of total output, $Y - C - I$. Under Hicks' assumptions (a) and (b) it reduces to $Y - C$, so that strictly speaking Hicks (1939) cannot discuss Keynesian problem.

Thus our firms, unlike Hicks', are concerned with two sets of planning: demand and supply planning and input and output planning. The latter is subject to technical limitations for production which are assumed to be described by a single implicit function. The deviations of the demand and supply from input and output are filled up by adjusting the stocks of products and producer's goods. The profit or the net income of the firm is defined as the remainder of its proceeds after retaining some amount of money for purchasing the factors of production and the producer's goods.

To determine the optimum streams of the stocks of commodities, as well as the holdings of money and securities, I introduced in *Dynamic Economic Theory* (Morishima, 1996) the idea of the firm's indifference surface of liquidity, that is given as a function of the stream of holdings of money, that of securities, and the streams of the stocks of commodities. The firm maximizes the total discounted value of the stream of net revenues over the planning period, subject to the technical production function and the indifference surface of liquidity. In the plan thus determined, the net revenues obtained will mostly be distributed among the firm's shareholders or executives as dividends or bonuses, respectively. The rest, together with the amount of cash the firm releases and the proceeds of products and the amount of shares it sells, limits the maximum amount of the factors of production the firm can buy (see Morishima, 1996: 161–164).

Let us assume that the firm is now concerned with the plan over coming weeks. The function of technical limitation has variables of inputs and outputs over these weeks. The firm maximizes the total value of the sequence of profits, each one of which, say profits in week t, is discounted by an appropriate factor called the discounting factor that is a function of the rate of interest per week for loans of week t.

As is the case in Hicks' original analysis, we may easily observe that the maximum profits of the firms are generally different from each other in their magnitudes as well as in their rates of profits. This property remains unvaried even though the temporary equilibrium in the Hicks sense is established. We may thus conclude that the equal rate of profits is not realized in the state of temporary equilibrium, provided that the general equilibrium economy is of the Hicks–Morishima type.

La Volpe published his short but compact monograph (1993) 3 years before Hicks (1939). As a predecessor or at least a competitor of Hicks (1939), this should have attracted much more attention in the post-war academic world than it did; unfortunately, La Volpe remained almost an unknown fallen soldier left in the battle field. I noticed a reference to La Volpe (1993) in an article of H. Aoyama, my teacher, while I was a first year undergraduate student in Japan, but soon I was called up to the War. As I was entirely ignorant of Italian and did not know the content exactly and accurately, I could not use it in my works until I finally included it in the *Classics in the History and Development of Economics* Series, Macmillan, edited by myself, as its first volume.

5. Now you may find that La Volpe (1993) and Hicks–Morishima, are of very similar significance, in spite of the mathematical techniques used by them being entirely different; the calculus of variations is used by La Volpe (1993) whereas Hicks (1939) and Morishima (1996) stick to differential calculus. Certainly, La Volpe (1993) is one of the first economic theory monographs which have used the calculus of variations; its novelties and achievements are quite detailed and comprehensively described in the foreword, written by me. It is important to observe that La Volpe does not make Hicks' assumptions (a) and (b), so that his model is compatible with Keynes' idea regarding investment as a key variable in the analysis of economic dynamism.

La Volpe's dynamic general equilibrium to be established at moment t has the same effect as Hicks' temporary equilibrium plays in *Value and Capital* (Hicks, 1939) and *Dynamic Economic Theory* (Morishima, 1996). I must mention particularly that the rate of profits is not equalized throughout the set of firms existing in the economy. Comparing Walras' model of capital formation and credit that has been revised in the last section, with the La Volpe–Hicks–Morishima equilibrium the latter is indeed temporary as rates of profits remain to be adjusted and eventually equalized. Is there a theory bridging these two sorts of equilibrium? Or, however often groupings and formations of temporary equilibrium are repeatedly made through time, will the historical motion of the economy that is traced out not reach the equilibrium state where no differences in the rates of profits are found in every corner of the business world.

This is a question we meet herewith: That is whether a series of temporary equilibria converges to the sequence of equilibria of capital formation that are generated by both accumulation of stocks of capital goods and increase in the population. For the sake of convenience, we may call the former the temporary equilibrium path and the latter the capital formation equilibrium path. We now have to examine whether these paths converge to each other. This is a problem of *stability of motion*, which has scarcely been discussed so far, because most economists of general equilibrium concentrate their attention upon the cases of *stability of equilibrium point*. I have discussed a case of stability of motion in *Dynamic Economic Theory* (Morishima, 1996), but I am not satisfied with it. There is no satisfactory work yet which has discussed that the path of temporary equilibrium converges, or does not converge, to the path of equilibrium of capital formation. Of course no rigorous analysis is yet available.

There is, however, a conjecture with which many economists will agree the Walrasian equilibrium of capital formation is stable, the sequence of temporary equilibria will eventually be led to it. We shall probably accept the conjecture if we reason in the following way. If there are discrepancies between the rates of profits of a number of firms, then an entrepreneur or a capitalist who uses money in order to finance a firm of lesser profitability will transfer it to another firm which is superior in profitability. We then assume that the rate of profit of any firm diminishes with respect to the amount of money spent on it.

Although we must have a more strict and precise mathematical argument before we conclude, it seems that we are able to argue in the following way, once the above assumption is accepted. Where the rates of profits are not equalized, money (or capital) moves from less-profitable firm i to more-profitable firm k. Firm k's activity will then be expanded, so that its rate of profits will decline, while firm i's level of activity will be lowered; its profit rate rises. The gap between the two will be reduced, so that the state of equal rates will be closer. Once the economy reaches a certain state of Walrasian equilibrium and stability is assumed to prevail, then the sequence of equilibria of the Walrasian type will continue, as long as the initial stocks of capital goods change and the expansion of population continues. The La Volpe–Hicks–Morishima type of temporary equilibrium analysis eventually merges with Walras' theory of shifting general equilibria as long as land can be cultivated intensively. The economy continues to be "progressive".

Probably Walras would accept this conjecture. The convergence that he discusses in *Elements of Pure Economics* (Walras, 1954: 380) is not the one that I have examined above, that is the convergence of the sequence of temporary equilibria upon the path of equilibrium of capital formation, but a convergence of a path generated by an adjustment mechanism peculiar to him, that is the path of tâtonnement adjustment. Nevertheless, this is an evidence for the premise that Walras would take his equilibrium as being stable with respect to a certain kind of adjustment.

6. Let us now turn to the effects of the appearance of Schumpeter. As I have already mentioned he entered the stage just after Walras had completed the fourth edition of his *Elements*. Two major works on general equilibrium by Schumpeter[3] (1908, 1934) are, in terms of the years of publication, rather close to the time of completion of Walras' work on general equilibrium. It took another 20 years and more for Schumpeter to see the publication of La Volpe (1993) and Hicks (1939). Moreover, he greatly respected Walras' analysis; he accepted it with no substantial amendment. He only let his favourite personality such as "entrepreneurs" and "bankers" go onto the stage of the Walrasian drama.

He describes the roles and performances of entrepreneurs and bankers as follows:

> If anyone in an economic system in which the textile industry produces only with hand labour sees the possibility of founding a business which uses power looms, feels equal to the task of overcoming all the innumerable difficulties, and has made the final decision, then he, first of all, needs purchasing power. He borrows it from a bank and creates his business.
>
> (Schumpeter, 1934: 129–130)

> New businesses are continually arising under the impulse of the alluring profit. A complete reorganisation of the industry occurs, with its increases in production, its competitive struggle, its super-session of obsolete businesses, its possible dismissal of workers and so forth. . . . Only one thing interests us here: the final result must be a new equilibrium position, in which, with new data, the law of cost again

> rules, so that now the prices of the product are again equal to the wages and rents of the services of labour and land which are embodied in the looms plus the wages and rents of the services of labour and land which must still cooperate with the looms in order that the produce may come into existence.
>
> (Schumpeter, 1934: 131–132)

> Consequently, the surplus of the entrepreneur in question and of his immediate followers disappears. Not at once, it is true, but as a rule only after a longer or shorter period of progressive diminution.
>
> (Schumpeter, 1934: 132)

This state at which the economy eventually settles is a new Walrasian equilibrium to be established at the end of a very long process that continues after the original entrepreneur has introduced a new combination and his followers have taken part in the struggle of distribution of the profits. That is to say, Walras' equilibrium is stable with respect to the Schumpeterian shock, the entrance of an innovative entrepreneur.

7. It has been repeatedly emphasized by many economists that Schumpeter's economic system is a dynamic world where "entrepreneurs" play the role of a driver or pilot and decide the course of progress of the society by pouring a stream of "new combinations" into its economy. He calls a productive activity or a method of production a combination, because a firm combines various producer's goods to produce an output. A new combination is a new method of production; it creates a demand for producer's goods which are different, in proportions, from the existing demand; and it produces products more efficiently or makes a new kind of products available, so that the economy changes direction of progress. More precisely, Schumpeter defines an individual who carries out a new combination as an entrepreneur and distinguishes him from a person working on an old combination.

However, his classification of a production process between new and old combinations is subtle and dubious. It takes a certain amount of time to establish a new combination. After the time of its completion and its operation having started, it is no longer new, in the rigorous sense, but old. The entrepreneur who carries out this kind of "old" activity is not an entrepreneur if the Schumpeterian definition is strictly applied. A concrete person who has been classified as an "entrepreneur" will soon lose this qualification, though he may restore it in the future. This fragile character of his definition of entrepreneur is not recognized well by Schumpeter.

When the entrepreneur becomes "old" as advantages from new combinations have exhausted, he will become an ordinary entrepreneur of the Walrasian type. Regardless of the newness or, oldness of an entrepreneur, Walras emphasized the role of entrepreneurs as he writes:

> The entrepreneur is a person (natural or corporate) who buys raw materials from other entrepreneurs, then leases land from land-owners on payment of a rent, hires the personal faculties of workers on payment of wages, borrows capital from capitalists on payment of interest charges and, finally, having applied certain productive services to the raw materials, sells the resulting product on his own.
>
> (Walras, 1954: 227)

> The capitalist accumulates his capital by successive savings and lends money to the entrepreneur for a given period; the entrepreneur converts this money into capital proper and at the expiration of the contract he returns the money to the capitalist.
>
> (Walras, 1954: 228)

Apart from the fact that Walras says nothing about the combinations introduced by the entrepreneur being of "new" or "old" type, the above quotations describe the entrepreneurial activities almost in the same way as Schumpeter (1934) in *The Theory of Economic Development*. The only difference between them concerns how the entrepreneur gets the money necessary for the business. Walrasian entrepreneurs get the money from capitalists, as shown above; Schumpeterian entrepreneurs obtain it from bankers. In this case, as is well known and emphasized by Schumpeter, the effect is immediate, because bankers are not only able to advance money currently deposited to entrepreneurs, but also able to lend large sums by creating credit. Consequently, bankers permit new combinations of a larger scale than do capitalists. The entrance of bankers on the stage is decisive for the drama. Obviously, Walrasian entrepreneurs helped by bankers will perform much more effectively than Schumpeterian entrepreneurs working on the business of new combination in association with capitalists!

The importance of bankers in Schumpeter's economics was emphasized in my joint article with Catephores (Morishima and Catephores, 1988). It was read by Catephores at the first conference of the Schumpeter Society but, unfortunately, I could not be there. Prof. E. März, a discussant of the paper, was critical of our view. He deducted in *Evolutionary Economics, Applications of Schumpeter's Ideas* (Morishima and Catephores, 1988: 57), from our statement in that "in the Walrasian system there is no place for entrepreneurs who make (innovative) investment decisions" his conclusion to the effect that the Walrasian and the Schumpeterian entrepreneurs are two very different animals. But it seems to me that his argument misrepresents ours. In *Evolutionary Economics, Applications of Schumpeter's ideas* (1988: 40), we write:

> There are no independent decision makers, but rather instruments of the market who bring into effect whatever investment is determined on at the moment, such that general equilibrium is established. Throughout the volume of *Elements*, Walras repeatedly stresses in chapter after chapter the importance of the entrepreneur as an independent entity: yet he accepts the perfect flexibility of investment, turning the entrepreneur quite simply into a *kuroko*, a sceneshifter rather than a leading actor.

This means that entrepreneurs make such investments that excess demands (positive or negative) are automatically cleared in the respective capital markets, so that Say's law prevails. Such entrepreneurs may be characterized as *kuroko* in *kabuki* because in Japanese classical dramas the *kuroko* is an individual dressed in black clothes and wearing a black head-dress who appears on the stage to arrange it by such functions as picking up clothes discarded by the actors. As long as the entrepreneurs play the role of *kuroko* they clean up the markets of capital goods and Say's law holds.[4] On the other hand, if there are other persons who stop entrepreneurs from being mere *kuroko*s, then Say's law does not prevail in Walras' system of capital formation. These persons are bankers (or merchant bankers in the contemporary British economy). Entrepreneurs who are coupled with bankers are entirely different from those without. Thus the essential personality which makes the entrepreneur an active actor rather than a mere *kuroko* is the banker. It is clear that Schumpeter's entrepreneurs are all coupled with bankers and the absence of them in Walras' economy is fatal for his conclusion of the existence of capital formation equilibrium. This is the reason why we assign to bankers the most important role in Schumpeter's dynamic theory of economic expansion.

If an entrepreneur and a banker collaborate with each other to realize an innovative plan, they must hold a number of secret meetings outside of the open capital market. They sit face to face and talk of the truth of their plan and intention with sincerity and honesty. How strongly they can trust each other is the most important element for the success of the team play. It is an entirely new type of economic behaviour that is not found in the general equilibrium economy of the Walrasian type, where each agent is supposed to behave independently. Nevertheless, we observe at the final stage of implementation of Schumpeterian innovation that intimate collaboration between the entrepreneur and the banker is no longer needed, the former easily gets a necessary amount of money from the open capital market, rather than from his trustful banker. Then at the final stage of implementation of projects of new combination, Schumpeter's entrepreneurs become similar to Walras', and the banker's role reduces to an insignificant one. Except for the fact that Schumpeterian entrepreneurs will enjoy monopolistic profits in a beginning part of innovative adventure, the path of the Schumpeter economy will smoothly approach a Walrasian equilibrium.

This is carried out in the following way. As Schumpeter's entrepreneurs have been provided with an enormous amount of purchasing power, through bankers' support, the economy will be shifted sufficiently to be put in a state of disequilibrium of noticeable magnitude. Prices will change and a number of followers stimulated by the original innovators will change them and create keen competition in that area of economic activities where the innovations have started. The monopolistic profits of the originators will then sooner or later disappear; thus the competitive equilibrium of capital formation will restore itself. Schumpeter's argument for the approach of his dynamic paths towards the Walrasian general equilibrium, though this argument has not been rigorously and precisely established by himself, is outlined by him in the manner of conjecture.

8. For Schumpeter who belonged to the generation between Walras on the one hand, and La Volpe and Hicks on the other, general equilibrium theory always meant the one established by Walras and supplemented by von Böhm-Bawerk. Economists of the new generation began to construct dynamic movements of the economy in terms of a sequence of temporary equilibria. In each state of affairs of the sequence prices have been adjusted such that demand equals supply for each good (each consumers' and producers' good), but rates of profits remain to be different from one firm to another. Then the problem of adjustment of the economy from one temporary equilibrium to the next must be how to equalize the rates of profits throughout all firms. This should not be made by means of adjusting prices at the markets, because all goods and factors have settled their prices at their respective temporary equilibrium values at each point of the sequence of temporary equilibria. According to La Volpe and Hicks, a different temporary equilibrium appears in the next period (week), because the initial assets of individuals and firms in the coming week differ from those in the present week; these result from the performances and achievements of the individuals and firms in the present week.

There occur, however, adjustments which are not general equilibrium theoretic, wherever rates of profits differ among firms as we usually observe in the actual world. Merchant bankers, as are seen in the UK, will advice relevant firms on mergers. Merchant banks initially distanced themselves from industrial issues in Great Britain, but opportunities were offered in the 1920s and 1930s to make merchant bankers turn to industrial issues. They became very professional in handling these matters. In the 1950s, they were involved in the business of sale of formerly private companies to the public. They acted as advisers of the vendors and underwriters in guaranteeing the sale price. They also participated in carrying out public takeovers. They played a powerful role in the economic transformation from private enterprises to public ones, as well as in the opposite transformation at the time of the 1980s privatizations. Thus, in the business of takeovers and mergers merchant bankers were high profile figures, and general equilibrium theory has not been concerned with these activities.

These business experiences provided merchant bankers themselves with opportunities for working out innovations. They obtained a significant position as institutional shareholders. They became very powerful opportunistic investors. Still, in the field of corporate financial transaction, they remained major players of merger and acquisition activities. Their reputation and their network of connections were sources of their success; based on these they made financial innovations. Although, after the "Big Bang" in the mid-1980s, the major clearing banks acquired the right for merchant banking business, the success of the British merchant bankers has originated from their reputation, professionalism and opportunism. In the Walrasian way, behaviours of such agents as merchant bankers, a historic product of meritocracy and elitism, cannot be well dealt with. Similarly, Schumpeter's composite personality yielding from collaboration of entrepreneur and banker cannot be dealt with satisfactorily by the Walrasian technique of analysis. They are combined by the belt of trust; but the trust is a concept that is outside of the field of traditional general equilibrium theory. Here is a limit of this theory. Schumpeter too could only handle the problem of entrepreneur–banker cooperation heuristically. No rigorous treatment that satisfies contemporary theorists is found in his work (Schumpeter, 1934).

9. Schumpeter points out the following characteristics of industrial innovations. First, he says: "the vast majority of new combinations will not grow out of the old firms or immediately take their place, but appear side by side, and compete, with them" (see Schumpeter, 1934: 226). Second, innovative entrepreneurs do "appear, not continuously, singly in every appropriately chosen interval, but in clusters" (Schumpeter, 1934: 228). Even so, effects of an innovation will sooner or later die out; the economy will eventually settle at a certain new equilibrium of capital formation of the Walrasian type.

The entrepreneur who carries out a new combination enjoys a favourable position as a monopolist for a certain length of time. The patent right for the new combination, however, ceases to be effective in due course of time. Then competitors appear in that corner of the industry where the new combination plays a dominant role. Then the originator loses a substantial part of the monopolist profits. Competition will sooner or later calm down and the queue of new entrants comes to an end where the profits from the new combination become equal, in terms of the rate, to the profits of other firms. Thus effects of a single innovation are eventually exhausted, though in the new state of equilibrium the old combination is replaced by the new one. Therefore, in order to create a long-run perpetual movement of the economy, we must have a succession of new innovations that are introduced and realized. This view of Schumpeter is not new, because we find the following statement in Walras: the permanent market[5] "is perpetually tending towards equilibrium without ever actually attaining it, because the market has no other way of approaching equilibrium except by groping, and, before the goal is reached, it has to renew its efforts and start again, all the basic data of the problem, e.g. the initial quantities possessed, the utilities of goods and services, the technical coefficients, the excess of income over consumption, the working capital requirements etc. having changed in the meantime" (Walras, 1954: 380).

Evidently, the adoption of new combinations is taken as a change in the data, so that the above quotation holds true in Schumpeter's economy as well. It then follows that Walras' passage below also holds true for Schumpeter:

> Viewed in this way, the market is like a lake agitated by the wind, where the water is incessantly seeking its level without ever reaching it. But whereas there are days when the surface of a lake is almost smooth, there never is a day when the effective demand for products and services equals their effective supply and when the selling price of products equals the cost of the productive services used in making them.
>
> (Walras, 1954: 380)

However heuristic and only impressionistic but not necessarily correct analytically these passages may be, it is true that Schumpeter has not developed a more stringent and precise argument than Walras did in the above. Therefore, we may conclude that there is no further analytical development in him in the treatment of the dynamic phases of the economy than the extent Walras has done in the *Elements*. Although it is true that Schumpeter's verbal argument appeals to the sense of economists, it seems that there are analytical gaps in various places of his literary reasoning.

In any case, it is beyond doubt that Schumpeter has introduced into pure economics such sociological elements as collaboration between entrepreneurs and bankers, and concepts such as authority, personal weight, and so forth. Nevertheless, we have to admit that his reasoning is only intuitive. At first sight, it may seem to fit our common sense understanding of the working of the economy well. We then soon find that these concepts are not precisely defined, so that it is very difficult to develop a dynamic theory which is in strictness at least on the level required in the post La Volpe–Hicks age. He has proposed an outstanding ingenious conjecture, but has left actual scientific operations to others.

Since the War time, the Japanese "main bank" system has produced a large number of couples of enterprise and bank. The bank which is responsible for financing an enterprise is called its main bank. It has once been regarded as one of the powerful weapons for the Japanese economy's expansion. The responsibility of the bank is not the one defined legally in accurate terms. It is no more than moral responsibility supported by the nationalistic sentiment in the War period: Banks must support enterprises.

After the War this arrangement was considered as a powerful guard while enterprises were growing well, where as it became nominal, once the enterprises were weakened. In the period of shortage of money, no main banks want to take their responsibility for the enterprises. Then the system becomes only nominal and has now almost collapsed. This shows that such sociological concepts and relationships as have been introduced by Schumpeter cannot play the role of solid analytical tools unless they are rigorously defined and properly legalized.

Finally, it must be added that the trust between an entrepreneur and a banker is impossible to measure, so that an exact analysis in terms of it is very difficult.

10. This is related with discussions between Schumpeter and Takata, my teacher and my boss at Osaka University, which they had in 1931 when Schumpeter visited Japan. Takata has maintained that sociological elements such as power and prestige influence the performances of the economy. Our society is not constructed of atoms each possessing the same power. Each individual succeeds to the power possessed by his or her parent; his or her capacity in the economy is different and abundant if he or she comes from a good family. There is no classless society even in the communist world. Of course, Schumpeter agrees with Takata on all these points, but they disagree over the crucial point concerning how to treat these social elements in economics.

Both of them are strong anti-Marxists. But Schumpeter's outlook of economics is much nearer than Takata's to the view of economics Marx held. Economics is that discipline of social sciences which plays the most significant role among them. It is pure and self-sufficient. All other social scientific phenomena are affected by the stage of development of the society's economy. The subjects dealing with these problems are the studies on superstructures built on the foundation of economics. They respond to its movement, but not vice versa. When one has to make a comprehensive view of the society, he must ask various subjects of the social sciences for assistance but throughout this process of synthesis the foundation is firm and solid and does not change. The synthesis of various sorts of disciplines is only additive and does not affect the content and the character of any of the superstructures as well as those of the foundation.

This is a view whose holders are not confined among Marxists; it seems to be widely held by Western scholars. Evidently, the pure economics for Marx is the Marxian economics. In the case of Schumpeter this idea has been obtained from von Böhm-Bawerk who insists that the pure economics (i.e. the economics of the Austrian school in his case) must be the most important tool elucidating the laws of the economy, however significant the political and the sociological elements such as power, trust and so on might appear to be. It is clear from the personal records of Takata on the discussion that Schumpeter has supported the neoclassical theory of distribution and has rejected Takata's approach.

Takata holds a different view as is seen in *Power or Pure Economics* (Schumpeter and Takata, 1998). He is an outright supporter of the neoclassical economics or Walras' general equilibrium theory in the same way as Schumpeter is. But he considers it a first approximation to the economy. Depending upon the distribution of social power among agents, the actual distribution of income deviates from the neoclassical pure distribution. Consequently, a second approximation is needed to obtain a more satisfactory theory of distribution. The "foundation" metamorphoses itself through the course of approximation. The analysis of economic phenomena contaminated by sociological elements becomes, through the process of approximation, the task of serious economists.

Thus, for Takata, there is no synthetic social science absorbing separate, individual sciences, which is to be built-up on the fixed foundation of economics. He considers that economics itself should be expanded and amalgamated with other disciplines, sociology, politics, history, anthropology, and so on, so as to make economics a powerful subject that is a theoretical approximation to the economy so that it can deal with effects of non-economic elements upon the economy. On the other hand, Schumpeter takes the same problem, as *Capitalism, Socialism and Democracy* (1942) clearly shows, by forming a multi-disciplinary group of subjects and consolidating the results of the respective disciplines. His approach is more pragmatic and rather easily leads one to clear conclusions. But there is no feedback to economics of the effects of superstructural subjects. Takata's approach is more orthodox and anti-Marx than Schumpeter's; but it is a difficult approach and will never be popularized.

Moreover, there are phenomena which must be clearly discussed by economics, such as those of relative wages, enterprise groupings, enterprise takeover and mergers, collaborations between entrepreneurs and bankers on the basis of mutual trust, and so on. But because of the Marx–Schumpeter worship of pure economics, the matters concerning these sociological elements are left outside of the vision of the orthodox economist, in spite of the existence of brave pioneering works which may be regarded as being in the direction towards the second approximation. I have concluded in (Schumpeter and Takata, 1998: xiv):

> If the value of economics, and the confidence placed in it, is to be increased, it is absolutely imperative that the pure economists move away from their isolated stance and make a positive advance into the areas covered by broad economics.[6]

Schumpeter himself moved out of pure economics in his imperialism paper (Schumpeter, 1919). He takes the view that the capitalist nations were not so blind that the disjuncture between economic conflict and military action was filled up by the national emotion of jingoism. Capitalists normally want to become richer by having steady good business and the workers of the capitalist countries always remain anti-imperialist. This view has naturally led him to regarding imperialism as a subject of sociology, rather than treating it as a subject of economics or broad economics as most Marxists do. He is thenceforth interested in multi-disciplinary subjects. We may observe a big change in his academic interest that is from the pure economics line to a broader social science line, that peaks in *Capitalism, Socialism and Democracy* (Schumpeter, 1942); it is evidently his greatest and most popular work. It is an orchestral concert of various social scientific instruments. Beautiful, overwhelming, and deeply moving; but for the reason stated in the section below it is not a science but an art.

11. How should we characterize *Capitalism, Socialism and Democracy* (Schumpeter, 1942)? Is it to be taken as a retreat of Schumpeter from the front against Takata who considers the effects of social elements upon the economy to be significant? Or, it may be based upon Marx's view that the foundation of the society, economy, is independent and moves autonomously; superstructures such as ethos, culture, political order, etc. change corresponding to the movement of the foundation, but have no substantial feedback effect upon the foundation.

This last possibility, although it looks strange because Schumpeter is strongly anti-Marx, cannot be denied as will be seen in the following paragraph. In *Capitalism, Socialism and Democracy* Schumpeter emphasizes the role of innovation as a most powerful factor stimulating economic development. However, where the economy continues to develop for a long time, the gain from a new innovation starts to decline; innovative works would bring few rewards. Ambitious talented young people leave business; they will commit themselves to the work of building a powerful welfare sector. Developed countries turn away from capitalism, making an advancement towards socialism. Thus in the superstructure there begins a peaceful replacement of the existing regime of capitalism by socialism.

This kind of social evolution is entirely antagonistic against Marx's revolutionary transmutation towards socialism. In his view, the foundation will suffer a bipolar division between rich and poor classes which follows a violent structural change in the foundation. Thus the conversion of the regime occurs outrageously in the foundation in the case of Marx, while it happens peacefully in the superstructure in the case of Schumpeter. In this sense, Schumpeter is anti-Marx. In another respect, however, he follows Marx's footstep because in *Capitalism, Socialism and Democracy* he has not been involved in the examination of effects of the change in the superstructure upon the foundation. Of course a shift from capitalism to socialism will cause various structural changes in the economy. It is absurd to suppose that Schumpeter has considered these to be negligible. I am rather of the opinion that he has assumed that no change is required in the reasoning of pure economics in spite of great social transformations.

On the other hand, Takata considers that changes in sociological elements bring about second approximations in economics. Consequently, the transition from capitalism to socialism gives rise to a big increase in the problems which economics must be able to manage. Moreover, as has been mentioned earlier, there remain economic, but not general equilibrium theoretic, elements whose economic consequences have not been analysed yet. These works direct economists into the area outside general equilibrium analysis and make economic theory richer. Therefore, Takata sees that the foundation, pure economics, moves and expands.

Takata's economics of second approximation, which I have called broad economics can be made multi-disciplinary, in the same way as Schumpeter combined his economics with other disciplines of social sciences, but Takata would have opposed this idea. Of course, he should know that the popularity of economics is greatly increased by the multi-disciplinarization but he was ascetic in keeping the spirit of science strictly.

In spite of this disagreement between Schumpeter and Takata, they both consider the substantial character of the logic of economics to remain unchanged, even though the economy becomes that of socialists. In *History of Economic Analysis* (Schumpeter, 1954: 1156), Schumpeter wrote:

> In Italy, we find a similar situation [as in Germany], only much more pronounced. The Fascist regime resented criticism of the measures as much as or, still more than was the case in Germany. It also insisted either on a sympathetic attitude of economists or else on neutrality. ... A few leading men – such as Ricci and

> Bresciani-Turroni – expatriated themselves, but most were not seriously disturbed. Purely scientific work was not interfered with at all. … Barring war effects there was no break and neither was there one after the fall of the regime.

As for Japan, Schumpeter writes in *History of Economic Analysis* (1954: 1153n):

> Japan and Spain never were 'totalitarian' in any meaningful sense of the term. But as regards Japan, it should be observed that the interruption of contacts during the war and my ignorance of the language have created a lacuna which in the time at my disposal, I have been unable to fill.

In fact, however, the most popular lecture course at Kyoto University during the War was a course on advanced economic theory given by Professor Kei Shibata entitled "Japanese logics of the economy". He stayed at Harvard for several months just before the War and it is said in Japan that he was warmly welcomed by Schumpeter when he arrived at Boston. He returned from America through Britain as an ultra-nationalist; a long controversy and quarrels continued between him and Takata until the end of the War. The latter of course insisted: "General equilibrium theory is the core of economics. There is no logic of economics specific to Japan besides orthodox economics."

Finally I must point out that in *Capitalism, Socialism and Democracy* Schumpeter made a grave error: he derived a Marx-like conclusion, that capitalism will eventually be transformed into socialism. In 1979, when Britain should have virtually completed the structural transformation towards socialism as he would see so, Mrs. Thatcher came to power and started to demolish the work of constructing the welfare state. Once such a possibility of U-turn has been recognized, then a UU-turn, a UUU-turn and so on can be expected to follow, and this sequence of U-turns may converge somewhere or may not converge anywhere. We cannot, therefore, say definitely that the economy will eventually settle at socialism. It may turn back to capitalism, or may converge on a mixture of capitalism and socialism. We may even have a case where the economy will continue to float indefinitely within the range having capitalism and socialism as its extremities.

In any case no one denies that his book (Schumpeter, 1942) is one of the monumental works in the territory of the comprehensive social science that embraces economics, sociology and other fields of study in order to make it truly interdisciplinary. It is a great encyclopaedic and artistic masterpiece. Nevertheless, it is my regret to say that it is not very outstanding if it is seen from the viewpoint of economic theory, because there is no rule in the comprehensive social science that stipulates which disciplines should be used or should not be used in order to explain such and such states of affairs. Choice of disciplines depends on the scholar's taste or his artistic sense.

No one can deny that Schumpeter is one of the greatest verbal expositors or storywriters; one sees this throughout his major works. Nevertheless one would be surprised to find that in inverse proportion to the enormous amount of his knowledge concerning academic works of economic analysis, the list of analytical tools named after him is rather short.

12. Now we review his *Capitalism, Socialism and Democracy* (Schumpeter, 1942) from a somewhat different angle. Although, Schumpeter was, with respect to a number of points, not happy to accept Pareto's sociology, he seems to have been influenced by

Paretian residue analysis (see Schumpeter, 1952: 134–142). Especially, in *Capitalism, Socialism and Democracy* which may be regarded as his work corresponding to Pareto's "circulation des aristocracies" (Pareto, 1991), his argument may be better understood if we regard his entrepreneurs or bourgeoisie as the aristocrats or the elite of the modern society. Pareto defines them as individuals embodying the residue of instinct for "combination" or innovation.

So we now begin by explaining Pareto's analysis of human sentiment that is carried out in terms of basic elements of human behaviour that Pareto calls residues. They are classified as:

Residue I: Instinct for "combination",
Residue II: Instinct for group-persistence,
Residue III: Instinct for expressing sentiment by external acts,
Residue IV: Instinct for sociality,
Residue V: Instinct for integrity of the individual and his appurtenances,
Residue VI: Instinct for sex.

It is clear that residues III, IV and VI are necessary for the people to form a persistent society, in which there are two subgroups: one consisting of those who are richly endowed with residue II, whilst the other group consists of those who are abundantly endowed with residue V. The former, called subgroup I, includes the members who render services to the whole society, while the latter, subgroup II, is formed by the people who are self-supporting. Then residue I comes up. As the translators of Pareto's *Trattato* into English (*Mind and Society* is the title of the translation) wrote, the Italian word "combinazione" was translated into English as "combination" though it embraces a much broader meaning than the English one, with the phrase: "the instinct for combination" suggesting the "inventive faculty", "ingeniousness", "originality", "imagination" and so on (see Pareto, 1935: 519). If we may accept this, residue I can be taken as the instinct for innovation, as I do throughout the following. Obviously we may presume that the members of subgroup II work in the business and industrial world, whereas those of subgroup I are in the service in the political, public, welfare and community service sectors.

Schumpeter's argument in *Capitalism, Socialism and Democracy* does not explicitly proceed in terms of this framework, but we may take it for granted that he implicitly assumes an appropriate distribution of the individuals having the residue I of instinct of combination between two social subgroups I and II. Then it may be taken for granted that the business and industry subgroup II is prosperous in the peak period of capitalism, while subgroup I of the political, public and welfare sectors is dominant in the socialist age. Schumpeter then infer that from this historically given initial circumstance the economy will develop into the stage of mature capitalism and finally decline into socialism. This is the main thesis of *Capitalism, Socialism and Democracy*. He writes as follows: "the actual and prospective performance of the capitalist system is such as to negate the idea of its breaking down under the weight of economic failure, but that its very success undermines the social institution which protects it, and 'inevitably' creates conditions in which it will not be able to live and which strongly point to socialism as their apparent." (Schumpeter, 1942: 61.)

To establish this conclusion, he wrote:

> there is what may be described as the 'material' the capitalist engine feeds on, i.e. the opportunities open to new enterprise and investment. The theory under discussion puts so much emphasis on this element as to justify the label we have affixed to it. The main reasons for holding that opportunities for private enterprise and investment are vanishing. ... (We have) the circumstance that many existing investment opportunities belong to the sphere of public rather than of private investment.
>
> (Schumpeter, 1942: 113)

In terms of our model of distribution of the residues in the society, this may be described as an autonomous shift of residue I from subgroup II to subgroup I, or the residue I of subgroup II being shrivelled and that of subgroup I being stimulated in the final epoch of prosperous capitalist age. This of course gives rise to expanding investment opportunities belonging to the sphere of public investment. Corresponding to this move, he points out that the process of making investment decision is depersonalized and automatized and that the ingenuity of innovative individuals is replaced by bureau and committee work as he sees in *Capitalism, Socialism and Democracy* (Schumpeter, 1942: 133).

Nevertheless, we may be able to say, as Schumpeter has said, that national and municipal investment is usually or unavoidably expected to expand in the modern capitalist society; so it is very difficult to establish a purely economic theory which convinces us of an inevitable break down of the regime. To convince us he directs our attention to what would happen within the "superstructure" of the advanced stage of capitalism. In the choice of one from structural changes in the superstructure there is some arbitrariness due to the author's personal preference. This is why I have put *Capitalism, Socialism and Democracy* (1942) in the category of the social-scientific art. Anyway he points out emphatically among others an internal cause for the decomposition of the capitalist regime that is the disintegration of the bourgeois family. He (Schumpeter, 1942: 157) says:

> To men and women in modern capitalist societies, family life and parenthood mean less than they meant before and hence are less powerful moulders of behaviour; the rebellious son or daughter who professes contempt for "Victorian" standards is, however incorrectly, expressing an undeniable truth. ... It is wholly attributable to the rationalization of everything in life, which we have seen is one of the effects of capitalist evolution.

It is true of course that this drastic change in the bourgeois mind brings about a substantial decrease in the standard of luxuriousness that the bourgeois style of life of the previous generation was associated with. In this way, businessmen lose the capitalist ethics that enjoys working for the future; this would diminish the importance of the functions of entrepreneurs and capitalists.[7]

This reasoning of Schumpeter, though it may be able to explain a decline of the functioning of residue I in subgroup II, does not necessarily result in a recognition that the same residue becomes more active in the public subgroup I. It is powerless to explain any shift of the residue from one subgroup to the other. Even though the persons of the individualistic egotistic-type become less enthusiastic over their business activities, this does not mean the persons working in the public and charities sectors start to work more energetic and imaginatively. Although Schumpeter's argument reminds us of Pareto's theory of circulation of aristocrats based on residue analysis, it does not develop into a theoretical analysis of transformation of the economic system in terms of residues. It may only be accepted as a historical description of the movement of the capitalist system in modern ages.

For the purpose of constructing a theoretical residue model of society, let us consider a society having subgroup II for business and industry, and subgroup I for politics, public administration and welfare. Schumpeter's theory of development (Schumpeter, 1934) applies itself to studying how the industry of subgroup II works. On the other hand, in subgroup I the politics sector is on the top of the group and is directed by the individuals having plenty of residue I. They are political entrepreneurs. Forming political parties they compete with each other in general and local elections. Their weapons are their own policy manifestos, which are declared, at the time of major elections, to the public. They are concerned with their political principles and the public policies of the respective parties. During the period of election campaign the manifestos are examined by electors for their feasibility, integrity and consistency. After the election the majority party forms the government and the people keep vigilance, throughout the period of sway of that party, over whether their manifesto is respected. If a serious breach of promise is found, the party will lose votes in the next election.

This is the idealized procedure of democracy. Though it may be an unrealistic picture for many countries, it is a reality in postwar Britain. Political innovations are as important as economic innovations there. Then public finance is implemented along the general guidelines of the political manifesto. Some of the major parties support free-enterprise activities for subgroup II, while others are keen on promoting social welfare and education policies which are appreciated by the members of subgroup I. These differences in favourite sectors of the parties are reflected in their policies of taxes and rates.

In the case of political power being handed over from a party supporting subgroup II to another being favourable to subgroup I, or vice versa, we may observe a turn, a U turn, or a UU turn of the direction of the progress of society. As far as Britain is concerned, however, the trend of the progress does not suggest that the society will eventually be metamorphosed into a regime of socialism. Thus Schumpeter's Marx-like conclusion that capitalism will not be able to survive but will be replaced by a socialist regime does not hold true as far as Britain is concerned. In that society subgroups I and II fluctuate in their performances depending upon the effectiveness of economic and political innovations. We may take this as a truly Schumpeterian conclusion derived from his theory of innovations though it clearly differs from his own conclusions as well as reasoning.

NOTES

1 The equation is obtained, for example, from equations (6) to (11) of Note to Chapter III of Morishima (1964).
2 Our conclusion that effective capital goods market are absent in Walras' economy has been taken as wrong or treated with scepticism by many economists studying Walras (1954). This may be due to Jaffe's inappropriate translation of "capitaux" into "capital goods". On p. 267 of *Elements of pure Economics*, Jaffe puts the original sentence, 'pour determiner les prix des capitaux, il nous faut considére un marché que nous appellerons marché des capitaux et sur lequel se vendront e s'achèteront ces capitaux', into "we must contemplate a market which we shall call a *capital goods market*, where capital goods are bought and sold." This translation may lead one to think of the existence of a capital goods market, but more accurately "a capital goods market" and "capital goods" in his translation should be replaced by "capital market" and "capital", respectively. Then it is clear that this market refers to the single equation (7), rather than to the set of equations (8). (8) is an identity rather than an equation, so that it is not mentioned explicitly by Walras. My *Walras' Economics* (1977) discusses this problem of Say's law in Walras' framework.
3 In *Das Wesen und der Hauptinhalt der theoretische National-ökonomie* (1908) Schumpeter combined the *Zurechnungstheorie* that is peculiar to the Austrian school led by C. Menger and culminating at the time of E. von Böhm-Bawerk with the marginal productivity theory due to Walras and others.
4 The microeconomic Say's law referred to in this place is exactly the same as Say's law discussed in previous sections of this paper. I must say that it should be distinguished from the law by Say (i.e. Say's law in "Anti-Say's law versus Say's law: a change in paradigm" (Morishima and Catephores, 1988)). The former is a microeconomic version of the law that is useful for assuring the existence of Walras' capital formation equilibrium. On the other hand the latter is the usual assumption for macroeconomic analysis. The former implies the latter, but not vice versa.
5 W. Jaffe translates "permanent" of the "marche permanent" into "continuous" rather than "standing".
6 I call economics in the narrow sense or pure economics, "economic theory", while broad economics is used for expressing the economics of the second approximation to the economic reality, which incorporates the influences of sociological elements.
7 Schumpeter in fact writes: "Consciously or unconsciously they analysed the behaviour of the man whose views and motives are shaped by such a home and who means to work and to save primarily for wife *and children*. Thus there is inherent in the capitalist system a tendency toward self-destruction...; things and souls are transformed in such a way as to become increasingly amenable to the socialist form of life.... In the end there is not so much difference as one might think between saying that the decay of capitalism is due to its success and saying that it is due to its failure." (Schumpeter, 1942: 160–162.)

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14 A 'Sraffian' critique of general equilibrium theory, and the classical-Keynesian alternative

Fabio Petri

14.1 INTRODUCTION

The purpose of this Workshop is to discuss from different viewpoints the problems of the (neoclassical) general equilibrium approach to value, distribution, and outputs; the prospects of overcoming them within fundamentally the same approach or 'vision' of the basic forces at work; and the possibility that alternative approaches may offer better hopes to overcome them. I shall try to contribute to this task by presenting a viewpoint inspired by the work of Sraffa and of other scholars who have taken up his attempt to revive a classical approach to value and distribution. Many of the claims I shall advance[1] are, I think, often very imperfectly understood, and therefore I give priority to the task of clarification (even at the cost of some overlapping with already published papers) over the presentation of new results.

The chapter is in two parts.

Part I presents some reasons, connected with the debates on capital theory, why economics needs an alternative to the neoclassical, or marginalist, or supply-and-demand approach to value and distribution (and to employment and growth). The argument proceeds by

1 briefly remembering why the older versions of general equilibrium theory which relied on a notion of capital somehow as a single factor are indefensible;
2 explaining why, contrary to a common misunderstanding, the reliance of economic analyses on the more recent versions of general equilibrium theory – where capital is, at least apparently, not treated like a homogeneous factor – betrays a continuing implicit faith in the legitimacy of the traditional conception of capital as a single factor, and is therefore not immune to the capital-theoretic criticisms, although these criticisms take somewhat different forms, given the different form taken by the theory to be criticized.

The implications of these arguments, I will contend, are such that a fundamental reorientation of the theories of distribution, employment and growth is needed. In terms of the title of this Workshop, I shall therefore argue that the *problems* with GE theory are such that there is no *prospect* of a solution fundamentally within the same general approach or 'vision'; therefore we need an *alternative*.

Part II sketches one such alternative which is being advocated by a growing number of economists, with some comments on other alternative directions which appear to me less fruitful than the one I shall advocate. I shall argue in favour of a reconstruction of the theory of value, distribution and employment along lines which comprise:

1 in the theory of value, a return to the tradition, not only classical but also marginalist until fairly recently, which gives a central role to *long-period normal prices*;
2 in the theory of outputs and employment, the adoption of Keynes's principle of effective demand (it is variations of aggregate output and employment which bring about the equality of aggregate supply and aggregate demand); this principle will be shown to imply that in the long run as well as in the short run it is aggregate demand and its rate of growth which determine the growth of output and of productive capacity;
3 in the theory of distribution, a return to the classical approach. This approach will be argued to be superior to the 'Cambridge' attempts (Kaldor, Joan Robinson) to derive a theory of distribution from the extension of Keynes's principle of effective demand to the long run. It will also be argued that this approach avoids certain shortcomings of the 'shirking' version of the theory of efficiency wages.

14.2 PART I: CRITIQUE

> the adjustments needed to bring about equilibrium take time
> (J.R. Hicks, *Value and Capital*, 1946: 116)

(1) By the end of the 1960s, it was universally admitted that 'reswitching' and 'reverse capital deepening' are perfectly possible phenomena, only excludable on the basis of highly restrictive hypotheses, and that therefore the predictions, on the long-period relationship between capital–labour or capital–output ratios and income distribution, derivable from neoclassical models where capital is treated as a single factor homogeneous with the product, do not extend to heterogeneous-capital models. Sraffa's claim was confirmed, that it may happen that, as the rate of interest decreases, the economy switches in the long period from a technique A to a technique B, but that when the rate of interest decreases further, the economy 'reswitches' to technique A, so that it is impossible to argue that a lower rate of interest always causes the adoption of more capital-intensive techniques of production, if the capital intensity of a technique is to be ascertained independently of income distribution; it was also shown that, except under very restrictive hypotheses, long-period technical choices do not guarantee that a lower rate of interest brings about a higher capital–labour or capital–output ratio in value terms.

The relevance of these results has not been adequately grasped. The dominant reaction on the neoclassical side has consisted of the claim that one-good neoclassical models are only simplified versions of a theory about whose *rigorous* versions – the disaggregated models of general equilibrium – have no need for capital aggregation and are therefore not undermined by the non-existence of a scalar index of capital. In spite of the caution of some general equilibrium specialists,[2] most neoclassically-trained economists appear to have concluded that their approach to the explanation of value, distribution and growth is still valid, and that therefore there cannot be much wrong in presenting the qualitative aspects of the approach in the simplest possible way i.e. through homogeneous-capital models: thus we find that homogeneous-capital models still dominate macroeconomics, growth theory, international economics etc.

In this first part of my lecture I try to explain why these opinions are mistaken. The critique, I shall argue, has undermined the entire marginalist, or neoclassical, approach to distribution, employment, and growth. One important reason why this is not generally appreciated is because the neoclassical, or marginalist, theory of value and distribution has undergone since the 1940s a radical shift in its general equilibrium foundations, from *long-period* formulations attempting the determination of long-period equilibria *where the composition of the capital stock is determined endogenously* (as made very clear by Wicksell), to *very-short-period* formulations (the intertemporal and temporary equilibria to which nowadays the expression 'general equilibrium' refers, and which I shall call *neo-Walrasian*) where *the initial composition of the capital stock is a datum of the equilibrium.*[3] As I shall explain, in the former, traditional versions, the conception of capital as a single factor, embodied in the several capital goods and capable of changing 'form' (i.e. composition) without changing in quantity, was *indispensable* for the specification of the equilibrium's factor endowments; the generally acknowledged illegitimacy of this traditional notion of aggregate capital accordingly undermines these versions. In the latter versions there appears no explicit notion of capital as a single factor of production, and this is why it has been thought that they are left unscathed by the Sraffian criticisms. But in fact the inconsistencies in them reappear in different forms, and I shall discuss in particular two of them, the *impermanence problem*, and the *arbitrariness of the full employment assumption* (deriving from the arbitrariness of the assumption that investment will adapt to full-employment savings). Unfortunately, the shift itself, by obscuring the logic of traditional analyses, made it very difficult for modern neoclassical economists to grasp the nature of the shift, its implications, even that such a shift ever occurred. After 1975 a number of writings published in English have insisted on these points (e.g. Garegnani, 1976, 1978, 1990; Petri, 1978, 1991, 1999; Eatwell, 1979, 1982; Milgate, 1979, 1982; Schefold, 1985), but to this day this second wave of critical articles has received no reply from mainstream theorists.[4] I try to summarize their message.

(2) It is nowadays often forgotten that the Walrasian treatment of the equilibrium's given capital endowment as a given *vector* was for many decades not representative of the approach of the vast majority of marginalist economists, who took as given the endowment of capital specified as a single number, in accordance with a conception of capital as a single factor incorporated in the several capital goods, and capable of changing 'form' without changing in 'quantity'. Wicksell is a particularly clear example. One may distinguish two roles of this concept of capital.

The first role was that of allowing marginalist analyses to determine long-period prices and thus not to abandon the fruitful way of connecting theory and explanation of reality, which had characterized economic theory since Adam Smith, and which Garegnani (1976) has called the 'method of long-period positions'. Authors like Marshall, Jevons, J.B. Clark, Böhm-Bawerk, Wicksell, Robertson, Pigou etc. agreed with the earlier classical authors on the central role, in the theory of value and distribution, of the relative product prices associated with a uniform rate of return on supply price (URRSP): which is the notion of prices which appears as *natural prices* in Smith and Ricardo, as *prices of production* in Marx, as *long-period normal values* in Marshall, as simply *equilibrium prices* in Wicksell or Walras,[5] and which I shall refer to as *long-period prices*. They thought that the details of disequilibrium, influenced as they were by accidental and transitory circumstances, were both unpredictable, and ultimately uninteresting, the important thing being the averages of prices and quantities and their trends. Luckily, according to them, competition caused relative market prices to gravitate toward central values: the long-period prices, which because of this gravitation could be taken to indicate with reasonable approximation the averages of the actual, fluctuating and non-uniform market prices actually observed, and whose shifts could therefore be viewed as explaining and predicting the trends of the observable market averages. These long-period prices, and the corresponding long-period quantities, determined as they were by persistent forces making themselves felt through the vagaries and accidents of disequilibria, were what according to these authors economic theory could hope to determine, and starting from which in some cases one could try and develop more short-period analyses of particular markets or periods, taking into account the peculiarities of each situation.[6]

This notion of long-period price is of course what students are introduced to in any economics textbook, in the chapter illustrating the partial-equilibrium analysis of the tendency, in competitive conditions with free entry, of the short-period price of a product toward the long-period price corresponding to zero 'profits'.[7] In these analyses the long-period price, equal to the minimum average cost, is determined on the basis of *given* input prices. But the moment capital goods are admitted among the inputs of the good in question, the same tendency should be admitted to be simultaneously at work for *their* prices, and (by altering their quantities) to be influencing the rentals to be paid on them and thus the average cost of the good in question; so, a consistent determination of the long-period price of a product requires the simultaneous determination of the long-period prices and rentals of all capital goods directly or indirectly entering its production, i.e. the determination of the URRSP relative prices I have been speaking of.

Now, as the same textbooks make clear in the same chapter, the gravitation of the price of a product toward its long-period level *takes time*. This is an important key to the way capital was treated in traditional marginalist analyzes. During the time taken by the gravitation toward long-period prices, production and consumption go on; so the data determining these centres of gravitation cannot include the amounts in existence of the several capital goods, because these amounts can be quickly altered by production and productive consumption, and will be so altered unless they have themselves reached an equilibrium. And the time required in order to considerably alter the amounts in existence of most capital goods is of the same order of magnitude as the time required for, say, the supply of and the demand for a produced consumption good to tend to equality; so it would be contradictory to assume equilibration on the markets for produced consumption goods, and not to admit the variability of the relative amounts in existence of the several capital goods. Thus in the founders of the marginalist approach (with the only exception of Walras, on whom more will be said below) the admission that the adjustments toward equilibrium take time went hand in hand with the conception of general equilibrium as a situation in which the amounts in existence of the several capital goods were *endogenously* determined.

To determine them endogenously, traditional marginalist economists relied on two ideas. The first one was the already mentioned tendency to a *uniform rate of return on supply price*. If the rate of return obtainable by purchasing (at a price equal to the supply price, i.e. to the cost of production) a capital good were higher than for other capital goods, investors would scramble to buy that capital good, inducing a rapid increase in the flow of production of that capital good, and thus also in its relative endowment. Only when the rate of return on supply price was the same as for the other capital goods, could the endowment of a capital good be taken to have reached an equilibrium. The prices that analysis had to determine were therefore the relative prices guaranteeing a uniform rate of return on the supply prices of all capital goods with positive equilibrium endowments.[8] On this aspect there is no difference between these marginalist authors, and the older classical economists. Thus Wicksell's or Walras's[9] equilibrium prices, if the assumptions about the type of technology are the same – e.g. circulating capital goods – satisfy *exactly the same* price equations as Sraffa's modern reformulation of Ricardo's or Marx's natural prices or prices of production.[10,11] Thus the classical approach and the marginalist approach do not differ on the issue: how are normal long-period prices determined, once the quantities are produced (if the non-substitution theorem does not hold) and either the real wage or the rate of profits are determined? They differ on the forces determining the distribution of income between wages and profits (interest), and the quantities produced.

The second idea, specific to the marginalist approach, was the conception of the several capital goods as embodying a single factor 'capital', a substance of which the changing and passing capital goods were only transitory embodiments, and whose services were rewarded by the rate of interest – the conception which will be indicated in the sequel with 'capital' in inverted commas. The conception of 'capital' as a factor of given 'quantity' but variable 'form' made it possible to determine the rate of interest via the condition of equilibrium between supply of and demand for 'capital', while at the same time leaving the endowments of the several capital goods free to adapt their *relative* proportions so as to satisfy the condition of a uniform rate of return on supply price. In this way, the determination of long-period prices was rendered compatible with the conception of the income from the property of capital as the reward of a factor of production, analogous to the rentals of non-reproducible factors like labour or land. The logic of the approach can be sketched as follows (cf. the appendix to the present paper for a more formal discussion). For each rate of interest, and for each composition of demand, the assumption of a uniform rate of return on supply price determines the cost-minimizing methods of production – and thus the capital goods desired by firms – simultaneously with relative prices and with the real wage;[12] thus, under the assumption of long-period general equilibrium on product markets and on markets for non-produced factors, one can derive the long-period vector of capital goods' endowments necessary for equilibrium, as a function of the rate of interest. The remaining degree of freedom is 'closed' through the condition that the vector of capital goods desired by firms must 'embody' the amount of 'capital' of which the economy is endowed.

'Capital', in this view, is destroyed and re-created, and can change its 'form' in the process, much like an amount of carbon dioxide congealed into pieces of dry ice of different shapes, which gradually evaporate, and are then re-formed by congealing more CO_2: the total amount of dry ice can change its 'form' without changing in 'quantity' (here weight), if the newly-congealed CO_2 is equal to the amount lost through evaporation, but is congealed into pieces of dry ice of shapes different from the old ones. The process through which the change in the 'form' of 'capital' was thought to operate was, of course, the utilization of the resources, which might have produced the capital goods necessary to replace the worn-out capital goods, for the production of different capital goods.[13] Thus the 'form' of 'capital' was left free to adapt itself to the one desired by firms, and equilibrium only required that the total amount of 'capital' demanded by firms be equal to its endowment.

To sum up: the conception of the several capital goods as embodiments of a common factor 'capital' made it possible to reconcile the view of distribution as determined by the equilibrium between supply-and-demand for factors, with the need for an endogenous determination of the amounts of the several capital goods present in the equilibrium, a need deriving from the admission that the adjustments toward equilibrium take time.

(3) Before coming to the second role of the traditional marginalist notion of 'capital', it is useful to notice that the conception of equilibrium as a situation where there is a uniform rate of return on supply price also explains why all the authors, who adhered to that conception of equilibrium and hence adopted the notion of 'capital' as a single factor, measured the endowment of 'capital' as an amount of *value*. In equilibrium, different 'chunks' of the same factor must earn rewards proportional to the amount of the factor they contain: e.g. if there are two fields A and B of land of the same quality, and if in equilibrium field A earns a total rent twice as big as field B, we know that A's surface must be twice the surface of B. If now A and B are two capital goods, with capital good A earning a rental (net of depreciation) twice as big as B, and if we want to see these net rentals as earned by the productive contribution of a common factor 'capital' embodied in them, we must then conclude that A contains twice as much 'capital' as B; but, in a URRSP situation, capital good A will also have a cost of production, and a value, twice as great as B, because the rental is the interest, and the rate of interest on the value of capital goods will be uniform. So necessarily the equilibrium relative values of different capital goods *must* be a measure of the relative amounts of 'capital' embodied in them. It suffices to measure 'capital' in units such that a capital good of unitary value embodies one unit of 'capital', and then the value of capital goods will also measure the endowment of 'capital'.

Thus there was a logical necessity behind the conception of 'capital' as measurable in the same units as income (and as savings). This fact may have made it easier for some applied neoclassical economists to consider it acceptable to represent economies via aggregate production functions $Y = F(K, L)$ where Y (the net product, obviously an amount of value, because only aggregable in terms of value) and K are treated as if made of the same single commodity; but the use of aggregate production functions must be seen as only an attempt at simplification of a theory which was in fact a disaggregated, general equilibrium theory. Thus Wicksell (1934), for example, formulates a completely disaggregated system of general equilibrium, and needs nonetheless the conception of 'capital' as a single factor, whose given endowment is measured in terms of value, because he treats the *composition* of the capital endowment as determined endogenously by the tendency toward equilibrium.[14] Contrary to a frequent misunderstanding which sees 'aggregate capital' as a notion motivated by the convenience of aggregate production functions, the motivation for the conception of capital as a single factor was not the *convenience* of aggregation, it was the *need* to leave the composition of capital as endogenously determined, because it was admitted that adjustments to equilibrium took time. The equilibrium endowments of the several capital goods were not made to disappear into an aggregate treatment of production, but they were not *data*, they were treated as variables to be determined by the equilibrium. Capital 'aggregation' (the specification of the 'capital' endowment as a single quantity) was not a simplifying device, it was *indispensable* to the theory, given the notion of equilibrium these authors were trying to determine.[15]

(4) The second essential role, played by the conception of 'capital' as a single factor with the rate of interest as its 'price', was that of making it possible to view the average economy-wide 'capital'–labour ratio as a decreasing function of the rate of interest, and thus to view the rate of interest as the 'price' bringing the demand for 'capital' into equality with its given endowment, or, more concretely, as the 'price' bringing investment into equality with full employment savings. As is well known, the rate of interest is seen in this traditional marginalist approach as determined by the equilibrium between supply of and demand for loanable funds; but (if we leave aside possible disturbances due to malfunctioning of financial intermediaries) the supply of loanable funds is simply the manifestation of decisions to save, while the demand for loanable funds comes from firms which want to utilize those funds to buy capital goods i.e. to invest;[16] so equilibrium between supply of, and demand for, loanable funds means equilibrium between savings and investment decisions. Now, the reason why, in this approach, the rate of interest can act as the price bringing supply and demand for loanable funds into equilibrium is, fundamentally, that the demand for loanable funds is seen as a decreasing function of the rate of interest, because aggregate *gross investment* is viewed as a decreasing function of the rate of interest: the reason being that gross investment decisions are seen as the concrete manifestation of the demand for 'capital', reflecting the desire to maintain or to bring capital stocks to the desired levels; and therefore gross investment increases above the replacement level, i.e. net investment becomes positive, when in the aggregate firms want to increase their capital stocks, i.e. when their demand for 'capital' increases, owing to a decrease of the rate of interest. The demand for a certain stock of 'capital' manifests itself concretely as a succession over time of demands for loanable funds (Garegnani, 1978). If the rate of interest maintains supply and demand for 'capital' in equilibrium over time, then it also maintains in equilibrium the *variations over time* of supply and demand for 'capital' i.e. net savings and net investment, and therefore it also keeps in equilibrium the supply and demand for loanable funds; conversely, by establishing and maintaining equilibrium between supply and demand for loanable funds, the rate of interest also establishes and maintains equilibrium between savings and investment, and thus between supply and demand for 'capital'. (Obviously all this is under an assumption that financial intermediaries do not interfere with this process: their potential disturbing role was in fact seen by traditional marginalist authors as a main

reason for the trade cycle.) In this way, the conception of 'capital' as a factor of production, the demand for which manifested itself concretely as a demand for savings, made it possible to provide a foundation for Say's Law, and thus to remove the possible obstacles posed by aggregate demand to the reaching of the full employment of resources.

(5) On the basis of what has been said above in **(3)**, it is easy to point out a grave problem of such a conception of capital. Since, whatever substance it is conceived as being made of, the 'capital' embodied in different capital goods must be proportional to their equilibrium relative values, any change in distribution, by altering the relative values of commodities, implies a change in the relative amounts of 'capital' contained in different capital goods; thus the 'quantity of capital' embodied in any given vector of capital goods depends on the choice of numéraire and on the prices (and hence on the income distribution) assumed to be ruling at the time of measurement; therefore the 'capital' endowment of an economy changes as relative prices change, even when the capital stock is given as a physical vector; it is therefore impossible to take the endowment of 'capital' of an economy (a single number) as given without arbitrariness.[17] A long-period marginalist general equilibrium is accordingly impossible to determine: the datum relative to the endowment of 'capital' is logically indeterminable.

(6) This problem did not go unnoticed by marginalist economists. Knut Wicksell, the first economist to attempt the writing down of the complete system of equations of a long-period disaggregated general equilibrium where the amounts of the several capital goods were variables to be determined endogenously, grew clearly uneasy with the need for an endowment of 'capital' measured as an amount of value; indeed, he wrote: 'But it would clearly be meaningless – if not altogether inconceivable – to maintain that the amount of capital is already fixed before equilibrium between production and consumption has been achieved. Whether expressed in terms of one or the other, a change in the relative exchange value of two commodities would give rise to a change in the value of capital' (Wicksell, 1934: 202), and he admitted a few lines later that this implied an 'indeterminateness' of the endowment of capital.[18] His pupil Lindahl openly admitted in 1929 (but an English translation only became available 10 years later) that the notion of a 'quantity of capital' was indefensible because it was indeterminable independently of relative prices (Lindahl, 1939: 316–317). Friedrich Hayek adopted a similar attitude in about the same period (Milgate, 1979); the rejection of the measurability of the capital 'fund' in terms of a single number is particularly clear in Hayek's controversy with Knight culminating in Hayek (1936). Shortly afterwards John Hicks too (Hicks, 1935) became unhappy with capital as a 'fund', under the impact of Shove's harsh objections to his uncritical use of that notion in *The Theory of Wages* (Hicks, 1932; cf. Garegnani, 1976).

How come, then, did they not abandon the marginalist approach? A detailed historical study of the question does not exist yet, but on the basis of what is already known I would propose the following answer. By the time these doubts were expressed, the supply-and-demand approach to value and distribution had become so exclusively dominant and so deeply ingrained into the economists' minds that its conclusions tended to be taken as obvious aspects of reality, rather than as the results of a complex chain of deductions, whose validity became doubtful once one of its pillars was recognized to be untenable.[19] Thus neither Hicks, nor Lindahl, nor Hayek were prompted by their rejection of capital as a single factor to doubt the validity of the forces, which marginalist theory had argued to exist *on the basis of that conception of capital*. Indeed, it may be argued that they never really abandoned that conception: without it, as will be argued in the sequel of this paper, they would have found it impossible to go on believing, for example, in the tendency of investment to adjust to savings. It is, in this respect, highly indicative that in the Commentary added to the reprint of *The Theory of Wages* which Hicks finally authorized in 1963, he reaffirmed his faith in the possibility of conceiving capital as a single factor in some *physical* sense (1932: 345), and in the existence of capital–labour substitution induced by variations in income distribution (ibid., p. 366).

On this basis, the direction these authors took is not surprising. In the formulations of marginalist theory they criticized, the conception of 'capital' as a factor capable of changing 'form' obviously went together with the admission that at each moment the 'capital' of an economy would be, so to speak, 'crystallized' in specific capital goods, and could take the 'form' best adapted to changed demand or technical conditions only gradually, as the existing capital goods were consumed and the 'capital' embodied in them became 'free' to be reinvested in different capital goods (through the employment of the resources, which might have reproduced the scrapped capital goods, to produce different capital goods). This meant that the 'marginal product of capital' could only concretely manifest itself, and be adapted to the rate of interest, in the investments in new plants, where the flow of 'free' 'capital' would meet the flow of labour 'freed' by the gradual closure of plants reaching the end of their economic life,[20] and where therefore the 'capital'–labour ratio could be chosen as the one dictated by the current income distribution and demand conditions.[21] Indeed, in the most influential formulation of the approach for decades, the one due to Alfred Marshall, one does not find an explicit general-equilibrium determination of the equilibrium between supply and demand for 'capital' as in Wicksell, one only finds a determination of the rate(s) of interest on the savings-investment market(s) in a 'short-period' situation in which the productive equipment existing in the various industries is taken as given (and earning a residual 'quasi-rent'), real wages are determined by the marginal product of labour on the existing plants, and the rate of interest reflects the marginal product of 'free' capital.[22] In other words, it was implicit in the approach that the long-period forces pushing toward the full employment of resources could be 'seen' in operation in the short period as well, as determining the 'capital'–labour ratio in new investments.[23] It cannot therefore cause great surprise that when Lindahl, Hayek, and Hicks looked for a way to dispense with a value endowment of 'capital', they turned to a reformulation of the theory of general equilibrium in terms of very-short-period (intertemporal or temporary) equilibria where the endowments of the several capital goods appear among the data of the equilibrium. What made it possible to believe that these very-short-period reformulations did not alter the conclusions on the tendencies of market economies, was the continuing faith in the presence, even in these very-short-period equilibria, of those supply-and-demand forces, capable of bringing about the full employment of labour and the equality between investment and full-employment savings, which are the distinguishing feature of the marginalist/neoclassical approach.

But these forces had been traditionally derived, and – I shall now argue – could only be derived, from long-period analysis (in which 'capital' played an essential role); the faith in their presence even in very-short-period analyses shows an implicit faith in the fundamental correctness of the traditional conception of 'capital' as a 'fund', a single factor of production whose ratio to labour is a decreasing function of the rate of interest: this conception therefore comes out to be only apparently dispensed with.

I shall try to prove this contention by showing: (i) that by themselves neo-Walrasian equilibria permit no conclusion as to the behaviour of actual economies; (ii) that therefore the only way to argue that neo-Walrasian equilibria provide indications on the behaviour of actual economies is by *independently* arguing that real economies tend to approximate the behaviour of neo-Walrasian equilibrium paths; (iii) that such an argument must rely on the validity of neoclassical 'macroeconomics';[24] (iv) that the latter validity presupposes the validity of the traditional analysis of 'capital'–labour substitution, without which neither the decreasing demand curve for labour, nor the decreasing investment function – two indispensable pillars of neoclassical macroeconomics – can be defended.

(7) In the neo-Walrasian intertemporal (Arrow–Debreu–Radner) or temporary (Hicks–Grandmont) equilibria, the notion of 'capital' as a single factor is, at least at first sight, absent: the endowments of the several capital goods are among the *data* of the equilibrium, and therefore no need arises for an endowment of capital as a single quantity. This is what has made it possible to retort against the Sraffian critics that 'rigorous' neoclassical theory has no need for the notion of capital as a single factor of production.

But the formal disappearance of that notion from these formulations of general equilibrium theory has hidden a continuing faith in its substantial validity. Otherwise the entire neoclassical approach to value and distribution would have been recognized to have lost its foundations.

As remembered above, the traditional marginalist notion of 'capital' had two roles: (1) by making it possible to treat the composition of capital as endogenously determined, it made it possible to admit time-consuming disequilibria and thus to remain within the method of long-period positions; (2) by making it possible to view investment and the demand for loanable funds as decreasing functions of the rate of interest, it provided a justification for Say's Law. If one really drops that notion, either role becomes impossible to fulfill.

Let us show it for the issue behind the first role: the connection between the theoretically determined equilibrium and the explanation of reality.

(8) For reasons of time, I shall dwell in some detail on only one of the difficulties arising in this respect owing to the given vector of endowments of capital goods, a difficulty which may be called the *impermanence problem* (cf. Garegnani, 1976; also Petri, 1991). Because their data lack the necessary permanence or persistence, these neo-Walrasian equilibria cannot have the role, traditionally assigned to equilibria, of indicating the situation around and toward which market variables gravitate: in order to have that role, an equilibrium must have data which are not affected (or only negligibly affected) by what happens during the trial-and-error disequilibrium processes; on the contrary, in neo-Walrasian equilibria, disequilibrium decisions can alter the relative endowments of the several capital goods to nearly any extent;[25]

in temporary equilibria, also the shapes of expectation functions can change to nearly any extent during the disequilibrium adjustments; then, the data no longer being the initial ones, the economy cannot reach, nor gravitate around, the equilibrium corresponding to the initial data; that equilibrium cannot therefore be the basis for predictions on the tendencies of markets.

This difficulty is occasionally admitted by mainstream economists; thus Christopher Bliss has written:

> Does it not take time to establish equilibrium? By the time equilibrium would be established will we not have moved on to another 'week' with new conditions, new expectations, etc.?
>
> (Bliss, 1975b: 210)

And Franklin M. Fisher has later declared, with admirable clarity (Fisher, 1983: 14, my italics):

> In a real economy, however, trading, as well as production and consumption, goes on out of equilibrium. It follows that, in the course of convergence to equilibrium (assuming that occurs), endowments change. In turn this changes the set of equilibria. Put more succinctly, the set of equilibria is path dependent [...][This] path dependence... *makes the calculation of equilibria corresponding to the initial state of the system essentially irrelevant*. What matters is the equilibrium that the economy will reach from given initial endowments, not the equilibrium that it would have been in, given initial endowments, had prices happened to be just right.

But the full import of these admissions appears not to have been grasped. '[T]he equilibrium that the economy will reach from given initial endowments' is unknowable, because, being modified during the disequilibrium in a way which the theory cannot predict, the vector of endowments of the several capital goods when the economy finally reaches an equilibrium (assuming it does) cannot be known. Neo-Walrasian equilibrium theory cannot give indications as to the behaviour of real economies, unless reasons can be found to believe that the extent, of the divergence of the actual behaviour of an economy from the behaviour predicted by the neo-Walrasian intertemporal equilibrium or sequence of temporary equilibria, will be small. For that, a theory of what happens in disequilibrium is needed, but this theory must be something else from the neo-Walrasian theory, because the latter cannot admit time-consuming disequilibria capable of altering the data of equilibrium, and so it cannot explore the working of economies in disequilibrium (the only 'disequilibria' it can discuss are the hypothetical ones in 'logical' time associated with the mythical auctioneer, or analogous fairy tales). Therefore on the sole basis of neo-Walrasian theory one cannot exclude a cumulation over time of divergences of the actual path of an economy from the equilibrium path. For example, one cannot exclude multiplier-accelerator interactions, bankruptcies causing further bankruptcies, etc.; nor can one exclude that it is the endowments of capital goods which adjust to aggregate demand, rather than aggregate demand adjusting to productive capacity, what would mean that growth is not determined by savings, and that maintaining the full employment of labour does not determine income distribution (cf. Part II); no basis appears to be left for preferring a neoclassical view of the determinants of distribution and growth to other views.

This conclusion is confirmed by Franklin Fisher's (1983) results. He is able to prove that the economy will converge to *some* rest point only by assuming the extremely restrictive hypothesis of No Favourable Surprise, which in a discrete-time formulation would amount to assuming perfect foresight (cf. Madden, 1984); and anyway even under this assumption the situations the economy may be converging to remain very indeterminate, and may be drastically different from the equilibria of neoclassical theory (in other words, nearly anything can happen). The lesson it seems necessary to draw from his analysis is that the reaching of conclusions as to the behaviour of real economies requires locating *persistent* forces making themselves felt through the vagaries of time-consuming trials and errors,[26] and that these persistent forces are not found in neo-Walrasian equilibrium theory.

The drastic but inevitable conclusion appears to be, that neo-Walrasian equilibrium theory cannot by itself tell us *anything at all* on the behaviour of actual economies.

(**9**) On this basis, I have claimed (Petri, 1999) that neo-Walrasian theory is as liable to a charge of logical inconsistency as long-period equilibria which measure the given 'capital' endowment as an amount of value. The argument goes as follows. In either case, it is not a formal inconsistency of the mathematical model that one is talking about: the inconsistency is between the data relative to the endowment of capital, and the way the equilibrium determined by the model is used in order to explain and predict. In long-period equilibria, mathematically there is no contradiction caused by the introduction of a given (e.g. the observed) value of capital into the model; the logical inconsistency arises when one argues that prices and distribution will tend to (and their trend or average is, because of this, explained by) those determined by the model: this cannot be argued if some of the data of the model cannot be presumed to remain unaltered as prices and distribution change, and this is precisely what will happen to any observed value of capital. In neo-Walrasian equilibria too, the inconsistency is not a formal contradiction of the model; it is an inconsistency between the requirement that the equilibrium determines the situation the economy tends to,[27] and the given vector of endowments of capital goods, which will not remain unaltered during any adjustment process (except fairy-tale ones).[28] So if the neo-Walrasian equilibrium aims at determining what the economy tends to,[29] then it cannot be based on a capital endowment vector equal to the initially observed one; nor does one know what capital endowment vector to put in its place. Conclusion: if the aim is to determine what the economy tends to, the equilibrium's capital endowment is as indeterminable when measured as a vector of physical capital goods (as in neo-Walrasian equilibria), as when measured as an amount of value (as in long-period equilibria).

(**10**) An implication of the criticism of (**8**) is that, since neo-Walrasian equilibria cannot by themselves indicate the tendencies of real economies, no conclusion can be derived from neo-Walrasian theory as to whether actual economies tend to the full employment of resources. Such a tendency must be argued to exist for real economies, and therefore it cannot be based on the study of the stability of the instantaneous, or timeless, adjustment processes which are the only ones compatible with the neo-Walrasian framework.

In order to find the real microfoundations of the present-day applications of the neoclassical approach, in particular, of neoclassical macroeconomics, one must turn therefore to the older versions of marginalist theory, the ones based on the conception of capital as a single factor, an amount of value, embodied in the several capital goods.

That the conception of capital as a single factor is what in fact lies behind standard macroeconomic theory is made clear by reflection on the two schedules which in that theory make it possible to argue that the 'real' forces at work in market economies push them toward full employment: the labour demand curve, a decreasing function of the real wage; and the aggregate investment schedule, a decreasing function of the interest rate. Let us discuss them in turn.

(11) What is taken as given, as far as capital employment is concerned, when one draws an aggregate labour demand curve in a macromodel?[30] This curve aims at showing, for each level of the real wage, the labour employment toward which one may expect actual employment to gravitate (assuming price flexibility and stability in all other markets) if the real wage stays fixed at the given level. It is therefore the curve implied by the corresponding solutions of a general equilibrium model where the equation 'demand for labour = supply of labour' has been eliminated and in its place the real wage is treated as a parameter, and where the income, from which the demand for final goods comes, is the income of the employed factors only.[31] The question is, what does the given endowment of capital consist of, which is maintained constant and fully employed as the real wage changes? Clearly, what is taken as given cannot be the endowments of each different capital good: most of these endowments would quickly change if the change in the real wage induced changes in productive methods or in the composition of the demand for consumption goods, so a labour demand curve based on them would have no validity for assessing the effects of changes in the real wage, effects which are universally recognized to take some time (often months, sometimes years) to manifest themselves; also, the curve might easily be extremely inelastic, suffering from the nearly total absence of substitutability between labour and capital goods once the 'form' of capital is completely specified,[32] so it would easily generate unplausible equilibrium levels of the real wage. There remain only two possibilities. The first one is that what is taken as given is the amount of 'capital' of which the economy is endowed, treated as a single factor of variable 'form': this appears to be the only way to derive the standard decreasing labour demand curve, when the possibility is admitted of significant changes in the composition of capital: then, if that traditional conception were not in the background, it would be unclear why the change in the composition of capital associated with a changed real wage should always entail a change of the demand for labour of opposite sign to the change in real wages. But we know that the endowment of 'capital' is indeterminable: so the labour demand curve is also indeterminable. The other possibility is that the analysis be confined to a Marshallian or Keynesian short period, in which the fixed plants, or more generally the durable capital stocks, are treated as given in physical terms, while the amounts of circulating or intermediate capital goods (work-in-progress, in Keynes' terminology) are endogenously determined (thus admitting that, in order for additional labour to produce more, more intermediate products must also be present). In this way the need for an endowment of 'capital', an amount of value, can be argued not to arise, and the inelasticity due to a completely specified 'form' of capital can be argued to be somewhat reduced. The labour demand schedule then indicates the value of the *net* marginal product of labour, i.e. its value marginal product minus the cost of the additional work-in-progress necessarily associated with the increase in product.[33] But such a construction is also subject to decisive criticisms, which for space reasons I only discuss very succinctly (cf. Petri, 1997, for more details).

First, there appears to be no single clear-cut division between kinds of capital goods as to the speed with which their endowments can change, but rather a continuum, so that any separation of capital goods into two categories, one with given endowments and one with endogenously determined endowments, appears arbitrary.

Second, as is well known from the literature on full-cost and mark-up pricing and from the studies on the real wage in the trade cycle, if fixed plants are given and what varies with labour employment is the degree of their utilization, then the net marginal product of labour is in most cases not regularly decreasing, but rather constant or even increasing up to a seldom reached full-utilization level,[34] after which it falls very abruptly.

Third, any result reached on the basis of the assumption of given endowments of some capital goods is bound to have at most temporary validity, and to be modified to a greater and greater extent, as time passes, by the increasing influence of long-period choices. So even if it were possible to demonstrate that the demand for labour is decreasing if based on given fixed plants, the possibility that long-period choices may go in the opposite direction to short-period choices would render the short-period analysis, relevance doubtful, because the short-period choices, being admittedly of more limited elasticity, would be quickly dominated by long-period choices.

In conclusion, *we have no right to draw the standard downward-sloping labour demand curve*.

(12) The faith in the traditional conception of 'capital' is even clearer in the treatment of aggregate investment. The recent *Journal of Economic Literature* survey on the theory of investment states:

> The Benchmark Model is based on a demand for capital and, with the addition of dynamics, a demand for investment. The demand for capital is derived from elementary economic principles, and is determined by the equality between the expected marginal benefits and costs from an additional unit of capital.
>
> (Chirinko, 1993: 1877)[35]

Chirinko here is simply following the standard approach to investment, where the traditional conception of capital as a single factor is uncritically accepted, and investment demand, a flow, is variously derived from the demand for the stock of capital, its speed of depreciation, and its variations.

Here we find still fully operative the second role of the traditional conception of 'capital': to ensure that the investment schedule (the demand for savings) be a decreasing function of the interest rate, thus providing a foundation for Say's Law. The discovery of the possibility of reverse capital deepening undermines this foundation.

The belief that the long-period value of capital per unit of labour is a decreasing function of the rate of interest appears to have been derived simply from the *presumption* that 'capital' could be treated like a 'technical' factor in spite of being a quantity of value; when on the basis of Sraffa's analytical advances the question has finally been examined, that belief has emerged to have no foundation.[36] The view of the rate of interest as determined by the tendency to an equilibrium between supply and demand for 'capital', i.e. concretely, between savings and investment, thereby loses its foundation.

The recourse to the short period without reference to 'capital' is again of no avail, because then the reaction of investment to changes in the rate of interest would depend on a myriad of elements, for example, on whether changes in the rate of interest will induce anticipated scrapping of fixed plants, or on how they will influence the investors' expectations: this would make it impossible to reach any general unambiguous conclusion on the effects of changes of the rate of interest on investment. It might, for example, happen that the price changes, induced by an *increase* of the interest rate, make it convenient to anticipate the replacement of part of the existing durable capital, inducing an *increase* in investment; or it might happen that *a decrease* in the interest rate causes expectations of further decreases, inducing a postponement, i.e. a *decrease*, of investment. Therefore the only way to argue that even in the short run aggregate investment is a decreasing function of the rate of interest would appear to be, by admitting that, even in the short run, the main influence on investment decisions comes from long-period forces, i.e. from the 'capital' demand function. But then investment would be a decreasing function of the interest rate – if at all – *in spite of* the short-period nature of the analysis, in so far, that is, as the complications and accidents of the short period do not counterbalance the long-period effect of variations of the interest rate, effect being based, however, on the indefensible notion of 'capital'–labour substitution.

(13) This argument became available in English only with Garegnani (1978), and thus, for the reasons briefly mentioned in **(1)**, it has had less impact on macroeconomics than it deserved. But it is being reinforced by three recent contributions.

The first contribution I want to mention is a paper of mine (Petri, 1997), which deals with the theories of aggregate investment as a function of the rate of interest from Keynes onwards. I must be very brief. In this paper (Petri, 1997), it is argued that not all the modern derivations of a negative elasticity of investment with respect to the interest rate rely on the traditional conception of 'capital', but that, even when they do not, they still rest on unacceptable premises.

Thus Jorgenson (1967) and the adjustment-costs approach need, besides other questionable assumptions, (1) the assumption of a given number of firms[37]; and (2) the assumption that the net returns to investment are independent of the level of the interest rate, i.e. that product prices do not decrease when a decrease of the interest rate decreases costs of production: both assumptions go against all traditions in value theory.

The derivation based on Tobin's q rests either on adjustment costs, or on the increasing-supply-price approach of Keynes, Lerner and Ackley, which is empirically doubtful, and anyway needs the traditional notion of 'capital'–labour substitution to explain why a lower interest rate makes the economy desire an increase of the capital stock.

The conclusion is that no theoretical reason exists, why aggregate investment should be considered a negatively elastic function of the rate of interest; it is then not surprising that – as confirmed by Chirinko's recent survey quoted above, which does not differ in its conclusions from older studies – econometric enquiries should find it very difficult to confirm this negative elasticity.[38]

(14) The other two contributions are present in this volume: they deal with intertemporal equilibria and argue that even in an intertemporal-equilibrium framework, there emerge problems with the assumption of equilibration between savings and investment. I am referring to the contributions by Schefold and by Garegnani. They converge in arguing that, even conceding for the sake of argument the legitimacy of the neo-Walrasian determination of the *endowment* of capital as a given vector, the illegitimacy of the traditional conception of capital reappears, so to speak, *on the demand side*: the dependence of the relative prices of produced goods on distribution can cause 'perverse' behaviours of investment or of the demand for labour in the intermediate periods of intertemporal equilibria, for reasons strictly analogous to those causing reverse capital deepening in long-period analyses. (Turning to *temporary* equilibria would not improve matters, because expectations could be an additional source of problems.)

Together with my paper on investment theory, these papers confirm that the traditional faith in the equilibrating role of the rate of interest is without foundations, not only in long-period but also in short-period or very-short-period (neo-Walrasian) analyses.

Thus the abandonment, in the now-dominant versions of general equilibrium theory, of the notion of capital as a single, somehow homogeneous factor of production is illusory: that notion no longer appears explicitly among the data of the equilibrium, but it is still implicit in the hypothesis that effective demand poses no obstacle to the reaching of full employment – a necessary hypothesis if one wants to see the equilibrium as, not just a benchmark with no connection with the actual behaviour of market economies, but instead as reflecting the tendential result of the actual working of market economies.

(15) Summing up then: Sraffa has opened the way to a radical critique of the marginalist, or neoclassical, approach to value, distribution and employment, by showing the illegitimacy of the conception of capital as a single, somehow homogeneous factor of production, and by opening the way to a demonstration of how radically different from traditional beliefs can the behaviour of the 'amount' of capital per unit of labour be. The attempt by the neoclassical side to do without a value *endowment* of capital by turning to neo-Walrasian versions has resulted in the theory becoming, if possible, even weaker: in the first place, because enormous doubts arise as to the connection between these equilibria and the behaviour of real economies;[39] and second, because the given vector of capital goods endowments does not prevent the occurrence of the 'perverse' phenomena whose possibility had been highlighted – in a long-period framework – by Sraffa and the Cambridge controversy; in particular, the assumption of stability of the savings-investment market remains as illegitimate in neo-Walrasian analyses, as in traditional long-period marginalist analyses based on 'capital' as the single factor. The belief that neo-Walrasian equilibria, or sequences of such equilibria, may with reasonable approximation describe the working of market economies betrays therefore a continuing implicit faith in the traditional adjustment mechanisms derived from the conception of capital as a single factor. The untenability of this conception of capital undermines neoclassical theory in all its versions.

14.3 PART II: SOME SUGGESTIONS TOWARD AN ALTERNATIVE

(16) In looking for an alternative to the marginalist/neoclassical approach, we are far from starting from scratch.

One implication of the Sraffian criticism of neoclassical economics is that the reasons why the traditional *method* of long-period positions has been abandoned in mainstream work on value and distribution, have nothing to do with weaknesses of that method, but have their origin in the increasing realization of the problems of the marginalist/neoclassical *theory of distribution*, a theory which could not do without an endowment of 'capital' the value factor unless it gave up the attempt to determine long-period prices. That the founders of the marginalist approach all shared the method of long-period positions is in itself, *prima facie* evidence that, the method offered little reason for dissatisfaction. Since the marginalist theory of distribution must be rejected, and since the theory of long-period prices has been shown by Sraffa to be solid,[40] there appears to be little obstacle to putting long-period relative prices back at the centre of the stage, and thus to reconcile the theory of value not only with the long-dominant tradition in economics, but also with the practice of applied economics which has never abandoned that traditional method.[41]

Thus, what the building of a convincing alternative to the marginalist/neoclassical tradition essentially requires is an alternative theory of distribution, and an alternative theory of the general level of output, and hence of employment and growth.

On the latter issue, the Keynesian principle of effective demand (it is variations of aggregate output which bring about the equality of savings and investment) is a fundamental analytical advance. Indeed, its extension to the long run – its application to the theory of growth – has been the foundation of an attempt to develop a theory of distribution, different both from the neoclassical and from the classical one: the Cambridge approach of Kaldor and Joan Robinson. I now want to summarize a recent literature which argues that, precisely because of its extremely important and seldom fully realized implications for the theory of growth, the Keynesian principle cannot be a sufficient foundation for a theory of distribution (Ciccone, 1990; Garegnani, 1992; Trezzini, 1995).

(17) It is well known that, in market economies, there is considerable adaptability of production to demand, not only downwards – what no one would deny – but also upwards, owing to the fact that firms maintain reserves of unused productive capacity to be used if demand increases, thus making it possible for production to be superior to the normal one (i.e. to the average one in the expectation of which plants had been built) even for long periods. There is still much to be explored on this subject, but it appears possible to say that firms want to maintain spare capacity because that allows them to meet expected demand peaks when demand seasonally fluctuates; or in order to be ready not to lose market shares in case of unexpected increases of demand; or because a growing trend of demand is expected; and finally, because very high rates of utilization cause higher average costs, above all because overtime-labour and night-labour are more expensive. The first three reasons for spare capacity mean that firms will produce more at no extra average cost if only demand is higher on average than expected; and even the last reason is usually not sufficient to prevent an increase of production if the demand for the product increases, even if the price of the product remains constant, because in imperfectly competitive markets (the most frequent market form) the fear of losing market shares will make a higher degree of capacity utilization than the long-period optimal one becoming convenient in the short run – and this without any redistribution of income away from labour; rather the opposite is the more likely case because of the increased share of overtime wages.

This spare capacity, together with the existence of inventories of intermediate goods, makes it possible to adapt production to demand not only for single industries, but also for entire sectors, in particular, for the capital-goods-producing sector, and – the moment one admits that the supply of labour is not usually fully utilized and can usually be increased (in the short period perhaps by overtime work) – also for the entire economy. The initial run-down of inventories is made up by the increase itself of production, and the higher level of production is therefore sustainable for long periods.

But then – concentrating now on the capital-goods sector, i.e. the sector whose production creates productive capacity – the production of productive capacity, and therefore the rate of growth of productive capacity, must be considered as being determined by demand, so the evolution over time of the overall productive capacity of an economy must be considered, in an analysis of growth, the result of demand, rather than a determinant of the growth of production.

(18) Examples may indeed easily be built,[42] showing that a variation in some exogenous component of demand (e.g. state expenditure, or exports) will induce a variation of the average degree of utilization of capacity, which will influence investment decisions and thus the utilization of capacity in the capital-goods sector, and will thus cause a faster, or slower, rate of growth of output and of productive capacity than otherwise; the exogenous change in the rate of growth of output causes an adaptation of the rate of growth of productive capacity, not through a change in the share of consumption in output, but rather through a change in the degree of capacity utilization. Even a 2 per cent faster rate of growth of output over many years can be shown generally to run against no bottlenecks (as long as there is sufficient labour available). Obviously, slowdowns in the rate of growth meet even less obstacles; thus an insufficient growth rate of demand may be responsible, after a few years, for a very considerable loss of productive capacity compared with potential alternatives, a loss easily resulting in structural unemployment, but otherwise not easily perceptible, because not visible.

The historical observation that productive capacity is not greatly underutilized for very long periods can then be explained as due to the fact that, if productive capacity is excessive relative to demand, then net investment becomes negative. Thus productive capacity shrinks, possibly together with a temporary further decrease of aggregate demand due to the fall in investment, until it reaches again a rough adjustment to demand. The multiplier–accelerator process thus set in motion may cause instability, especially if simultaneously at work in a number of major countries, but then haven't we observed just something of the kind in the Great Crisis of the 1930s?

(19) What is of interest here is that the change in the growth of the autonomous component of aggregate demand can change the growth rate of the economy without any need for changes in the propensity to consume. This may help one to explain the fact that one does not observe that the nations with the higher propensity to save always have higher growth rates (Palumbo, 1996). It becomes possible that a considerable part of the differences in growth rates among industrialized nations may be explained by differences in the degree of capacity utilization, to be themselves explained, at least in part, by differences in the stimulus to a high utilization of capacity coming from effective demand. There is here room for interesting empirical work.

These same considerations are also relevant for an evaluation of the 'Cambridge' approach to the determinants of income distribution, associated with Joan Robinson and Nicholas Kaldor. In such an approach it is argued that in the very long run one must assume that the degree of utilization of productive capacity is on average the normal one, because of the tendency of the degree of utilization to gravitate toward the normal one; then Harrod's growth formula

$$g = I/K = S/K = (S/Y)/(K/Y) = s/v$$

(where I and S are now *net* investment and *net* savings, s is the average propensity to net savings, and v is the capital–output ratio) is used to argue that, since v can be taken in a first approximation as constant (the neoclassical conception of capital–labour substitution is rejected), an increase in the long-run growth rate of an economy requires an increase of the average propensity to save. If then one assumes that the average propensity to save is higher for capitalists than for workers, and if one furthermore assumes that the long-run rate of growth is given (by state policies maintaining the full employment of labour, or by the 'animal spirits' of entrepreneurs), then one obtains the well-known Cambridge (Kaldor–Pasinetti) equation $g = s_c r$, where s_c is the propensity to save out of profits or interest, and r is the rate of profits or of interest: the rate of profits is accordingly determined by the rate of growth and by the propensity to save out of profits.

If it were so, then it would be true that, in order to increase the average growth rate of an economy over long periods, it is necessary to increase r, i.e. to redistribute income from wages to property incomes. But such a conclusion forgets that the degree of utilisation of capacity may remain far from normal for very long periods, even decades.

This is easily conceived the moment one admits an important role for exports. But a similar role can be held by public expenditure, or by autonomous investment (e.g. railways at the end of last century). Or one may think of the following scenario. Let us imagine a slowdown, for any reason, of an economy until then on the warranted path. The downward Harrodian instability would entail a further slowdown of investment and growth. But if the state steps in so as to avoid a deepening of the recession and stimulates aggregate demand (e.g. keeps investment up via investments in nationalised industries) it might happen that capacity utilization remains lower than normal but without further decreases, and this situation may, again, go on indefinitely, with the spontaneous tendency of capacity utilization further to decrease being checked by continuous state intervention.

So one should be ready to admit that the average degree of capacity utilisation of an economy may remain different from the normal one even for decades. And were it to go back to normal, there would be no reason to expect that, in subsequent decades, the divergence would be of opposite sign so as to compensate.

Now, when the rate of utilization of capacity varies, the Cambridge equation no longer implies that a higher rate of growth implies a lower real wage, because the r in the formula will no longer be interpretable as the *normal* rate of profits, which is the one inversely connected with the real wage, and which is associated with the normal utilization of capacity (this rate will continue to be fundamental, because it is the one realizable, at the given real wage, on new investments, which will be planned so as to realize the normal utilization of capacity; this suggests that, because of entry-prevention pricing, the long-period prices associated with the given real wage may be generally presumed to be the centres of gravitation of market prices even in an economy with an average degree of capacity utilization different from the normal one; this is an important topic for research). The r in the Cambridge formula may now at most be interpretable as an average ex-post ratio of profits to the *normal value* of existing capital (i.e. to the value the existing capital would have if normally utilized: the *actual* value of underutilized capital will be different from its normal value, being determined by capitalisation of the *actual* profits at the *normal* rate r, if long-period prices continue to determine price formation as suggested above).

Another reason to doubt the validity of the Cambridge approach to distribution is the dependence of the normal degree of utilization of capacity itself on institutional elements which may be modified by political decisions. For example, changes in the legal length of the working week, or in the number of legal holidays, may considerably affect the normal degree of utilization of fixed plants. The normal degree of utilization of plants is also affected by the differences between normal wages, overtime wages, and night-shift wages. According to statistics of some years ago, workers in Western Europe worked on average about 20 per cent fewer hours per year than in Japan: this suggests a lower utilization at least of single-shift plants.

(20) Thus the admission of a Keynesian influence of the growth of aggregate demand on the growth of an economy does not eliminate the need for a theory of distribution. A given rate of long-run growth[43] does not suffice to determine income distribution.

The obvious non-neoclassical alternative in the theory of income distribution is the approach of the classics, from Adam Smith to Marx, which is based on the notion of a customary 'subsistence', from which the real wage may depart to some extent owing to changes in the balance of force between classes, a 'subsistence' which itself slowly incorporates those lasting changes into the standard of living, which through habit come to be felt as being necessary to a decent living, and accordingly enter into the customary conception of a 'fair or necessary wage'. It is worthy of notice that Robert Solow (1990), definitely not a classical economist, has nonetheless insisted that the evidence is overwhelming that this is how the labour market actually works.

This may seem overly vague to economic theorists, who nowadays usually want economic theory to produce definite predictions on the outcomes of economic interactions (at least, definite qualitative predictions on the direction of change) on the basis of few universal principles. This ambition, the result of the long dominance of the marginalist approach, appears excessive for issues so influenced by political elements as income distribution. The classical perspective argues that the political element in income distribution makes income distribution the result of a complex interaction of historically changing forces, which make its explanation not very different from the explanation of, say, the status of women or the advent of universal suffrage or of Fascism; so that one should not expect to be able to discover a simple way of explaining income distribution, based on few relevant regularities, valid for all nations and decades since the birth of capitalism: history will all the time be throwing up novelties, for example, the welfare state, the Russian revolution, world wars, neo-corporatism, student protest. In such a perspective the question why in certain places and periods – e.g. in Italy in the so-called 'hot autumn' of 1969 – real wages jump up by very big amounts and in other places and periods – e.g. the USA in the last decades – they stagnate in spite of increases in labour productivity, is not very different from questions such as, why the French Revolution happened, and why not before or later; or from questions such as, why the welfare state developed, and why differently in different nations, and why now it is being partly torn down. Which does not in the least mean giving up the attempt to explain real wages, it only means that the explanations will have to combine economic, sociological and historical elements.

I conclude with some brief remarks which attempt to indicate that without this enrichment some currently influential non-neoclassical theories of wages remain unsatisfactory.

Let us take first the Marx–Bauer–Goodwin idea (Goodwin, 1967) that, when growth causes labour unemployment to decrease, the resulting greater bargaining power of workers causes real wages to accelerate and profits to decrease, causing a decrease of investment, hence a slowdown of the economy and an increase of unemployment, which causes wages to slow down; so profits and investment pick up again, and growth resumes, in a cycle where technical progress allows the profit squeeze (and the downturn) to happen at a higher and higher level of the real wage at each successive cycle: so real wages secularly increase, while the rate of profit oscillates around a constant value. This is at first sight an attractive explanation of an indubitable phenomenon, the secular increase of real wages in the industrialised nations, an increase which has prevented the rate of profits from increasing, or at least from increasing considerably. But such a mechanical explanation does not explain the shifts, historically observed, in the capacity of unemployment to affect the rise of real wages; and it assumes Say's Law, or at least that a higher rate of profits will mean more investment, what is accepted by many Marxist economists but, I think, is far from being solidly established.[44]

Another currently fashionable theory is the theory of efficiency wages, which is usually adopted by non-neoclassical economists in its 'shirking' variety: the efficiency wage is then determined by the tastes of labourers, by the rate of unemployment, and by the fall-back income, which is usually identified with unemployment subsidies.[45] I see a basic weakness of this approach in that the reason why, in different historical periods, the same rate of unemployment recurs but associated with different real wages remains fundamentally unexplained.[46] To explain this fact through an unexplained change in tastes would mean to explain nothing. To explain it through changes in unemployment subsidies, besides being insufficient (unemployment subsidies have often been non-existent), would mean having to explain why unemployment subsidies have historically increased – what would send us back to those historical and political elements whose indispensability I am defending. To explain it through notions of a fair wage, below which workers refuse to work properly, and which gradually incorporates the historically realised increases in standard of living, would mean that 'efficiency wages' is only a misnomer for the classical approach; in other words, one would be admitting that the reason why firms do not lower wages in the presence of unemployment is, not that workers would individually shirk more because at the lower wage they care less about being fired, but rather that firms do not dare to lower wages because that would give rise to conflict – whose forms may include shirking too, but as a conscious form of protest or struggle, a struggle to resist reductions of the wage below the level felt as 'fair and necessary' (what the classics called 'subsistence') and in fact reflecting the prevalent balance of bargaining power until then. (This view appears to be confirmed by the occasions in which workers accept reductions of the wage with little resistance: firms in trouble, nations with grave problems of external balance of trade, i.e. when workers can be persuaded that a reduction of the real wage is not simply a redistribution in favour of property incomes, but is necessary in order to avoid worse troubles.)

So, it would seem, struggles motivated by notions of social justice, and 'social accords' (the realisation on the part of the ruling class that continuing economic expansion needs an expansion of consumer markets which requires an increase of labour incomes more or less in step with productivity) would appear to be a necessary part of the explanation of the secular increase of real wages, but certainly there is still much research to be done here.

14.4 APPENDIX: THE DERIVATION OF THE DEMAND CURVE FOR 'CAPITAL' IN LONG-PERIOD MARGINALIST ANALYSES

This Appendix shows, in the context of a simple (but generalizable) economy, how one might derive the 'demand curve' for 'capital' from the conditions for long-period marginalist equilibrium. It therefore also shows the need for an endowment of 'capital' expressed as a single number for the determination of a long-period equilibrium.

Let us first remember how the demand curve for a factor can be derived in the simplest marginalist economy: only one good, produced by labour and land. Let us assume a constant returns to scale production function, imposed by competition to all firms (which are then aggregable); a given, fully employed supply of land; no savings. For each level of the real wage, profit maximization will imply a demand for labour such as to equal the marginal product of labour to the given real wage. The curve of the marginal product of labour in the economy as a whole coincides therefore with the demand curve for labour. If one assumes that the only income spendable on the product market is the income of the employed factors, then a real wage higher than the equilibrium one will imply disequilibrium on the labour market only (in only apparent contradiction with Walras' Law), because on the product market there will necessarily be equilibrium since the income of the employed factors equals costs and costs correspond to the value of the product. Thus the demand curve for labour can also be seen as the locus of equilibrium values of the real wage, derived by assuming equilibrium on the labour market and performing the comparative-statics exercise of letting the (rigid) supply of labour vary. Therefore, the demand curve for a factor thus derived tells us what the supply of that factor should be at each level of the factor rental, in order to have equilibrium on that factor's market, assuming equilibrium on all other markets.

Let us apply the same idea to the demand for 'capital' in a heterogeneous-capital economy.

Let us assume an economy with four produced goods: goods 1 and 2 are circulating capital goods with no direct utility for consumers; goods 3 and 4 are pure consumption goods; good 4 is the numéraire. Production is in yearly cycles, wages w are paid at the end of the year. a_{ij} ($i = 1, 2, L$; $j = 1, 2, 3, 4$) is the technical coefficient of input i in the production of good j, a technical coefficient which can be assumed to be a function of relative prices and of the rate of interest r if one wants to admit technical choice (in the sequel for brevity the explicit indication of this functional dependence is omitted; it is assumed that technical choice results in a unique value of the technical coefficients for each value of r). Long-period prices are determined by the following equations:

$$p_1 = (1 + r)(a_{11}p_1 + a_{21}p_2) + a_{L1}w \qquad (14.1)$$

$$p_2 = (1 + r)(a_{12}p_1 + a_{22}p_2) + a_{L2}w \qquad (14.2)$$

$$p_3 = (1 + r)(a_{13}p_1 + a_{23}p_2) + a_{L3}w \qquad (14.3)$$

$$p_4 = (1 + r)(a_{14}p_1 + a_{24}p_2) + a_{L4}w. \qquad (14.4)$$

Let us assume for simplicity that consumers are aggregable into a single consumer. This aggregate consumer decides how much labour to supply, how much to demand of the two consumption goods, and how much to save, on the basis of his endowments of labour (a datum) and of 'capital' K (an unknown, an amount of value to be determined endogenously as a function of r); of income distribution, r and w; and of the prices of consumption goods (the prices of capital goods are irrelevant to him, all he cares for is the rate of return on the investment consisting in purchasing them, and in equilibrium the rate of return on supply price is the same for both). Again for simplicity, let us assume a rigid supply of labour, normalized to 1 unit. Thus from consumer choice we obtain the savings function $S(r, w, p_3, K)$ and two demand functions for consumption goods, $D_3(r, w, p_3, K)$ and $D_4(r, w, p_3, K)$. In the sequel, for brevity, I shall drop w and p_3 from the variables influencing these functions because they are uniquely determined by r, from equations (14.1) to (14.4); thus I shall simply write $S(r, K)$, $D_3(r, K)$, $D_4(r, K)$.

Let I_1, I_2 indicate the demand for (i.e. gross investment in) new capital goods.

Assuming equilibrium on product markets (productions equal to demands) one can derive the demands for stocks of existing capital goods k_1 and k_2, and for labour D_L, as functions of the demands for new capital goods and for consumption goods:

$$k_1 = a_{11}I_1 + a_{12}I_2 + a_{13}D_3(r, K) + a_{14}D_4(r, K) \qquad (14.5)$$

$$k_2 = a_{21}I_1 + a_{22}I_2 + a_{23}D_3(r, K) + a_{24}D_4(r, K) \qquad (14.6)$$

$$D_L = a_{L1}I_1 + a_{L2}I_2 + a_{L3}D_3(r, K) + a_{L4}D_4(r, K). \qquad (14.7)$$

The total value of investment must equal the value of savings:

$$S(r, K) = I_1p_1 + I_2p_2. \tag{14.8}$$

Let us now assume that the consumer invests by purchasing himself the capital goods, to be then lent to firms, and that he demands new capital goods so as to maintain the composition of the stock of capital unaltered.[47] The endowments of capital goods are variables to be determined, so let us assume them to be equal to the demands for them. Then k_1 and k_2 also indicate the *endowments* of capital goods necessary for the equality between supply of and demand for the capital goods already in existence. Then the composition of investment is determined by

$$I_1/I_2 = k_1/k_2. \tag{14.9}$$

The full employment of labour imposes

$$D_L = 1. \tag{14.10}$$

The last equation determines K, the value of the endowment of 'capital' demanded by firms, as equal to the value of the endowments of the two capital goods demanded by firms:

$$K = p_1k_1 + p_2k_2. \tag{14.11}$$

Equations (14.1)–(14.4) have the sole role of determining w, p_1, p_2, p_3 once r is given, so we may drop them after deriving from them $w(r)$, $p_1(r)$, $p_2(r)$, $p_3(r)$ as functions of r rather than as variables: we drop four variables and four equations, and we remain with seven equations in the seven variables r, I_1, I_2, k_1, k_2, D_L, K. But only 6 of theseequations are independent equations, because of Walras' law: e.g. if there is equilibrium on all product markets, since $(1 + r)K + w = (1 + r)(p_1k_1 + p_2k_2) + w$ is the consumer's income which must equal the value of sales of products because of the consumer's budget constraint, the equality must hold: $I_1p_1 + I_2p_2 + D_3p_3 + D_4p_4(1 + r)K + w$; but from equations (14.5) to (14.7) and the price = cost conditions (14.1) – (14.4) one also obtains $I_1p_1 + I_2p_2 + D_3p_3 + D_4p_4 = (1 + r)(p_1k_1 + p_2k_2) + wD_L$; so the demand for labour necessarily satisfies equation (14.10). If we treat r as a parameter, these six independent equations in six unknowns determine – assuming them to have a solution, and a unique one – what the 'capital' endowment K of the consumer must be[48] in order for the equilibrium to obtain at each rate of interest, and simultaneously determine the physical capital stocks $k_1(r)$, $k_2(r)$ corresponding to each such equilibrium. The curve $K(r)$ thus obtained is the demand curve for 'capital' the value factor, the curve traditionally believed to be decreasing and which on the contrary can have nearly any shape, as shown by Garegnani (1970) and other authors.

If r is *not* taken as given, the system of equations has one and only one degree of freedom. The conception of the several capital goods existing at any moment in an economy as embodying a given endowment of 'capital' authorizes the addition of an equation, establishing that the demand $K(r)$ for 'capital' must equal the actual endowment K^*:

$$K = K^*. \tag{14.12}$$

This determines the equilibrium corresponding to the given 'capital' endowment of the economy, and in this way endogenously determines the equilibrium physical capital stocks k_1, k_2.

NOTES

1 It is *not* my aim to present the views of a united and consistent 'Sraffian school' or 'neo-Ricardian school', what – in view of the considerable differences between the authors often grouped under these labels both on the evaluation of what is wrong with the neoclassical approach and on the best way forward – would be impossible. The bunching together of very diverse positions under the 'Sraffian' or 'neo-Ricardian' label has spread confusion; it has for example made it possible, in Hahn (1982), to present the 'Cambridge' approach to distribution of Joan Robinson, Kaldor and Pasinetti as the sole alternative proposed by the 'Cambridge' critics, what is far from the truth as I shall try to make clear in Part II. I feel nonetheless I can call 'Sraffian' the views I shall advance, because they attempt to carry forward Sraffa's project, of recuperating and rehabilitating the classical approach to value and distribution, and of showing the existence of fundamental flaws in the marginalist/neoclassical approach. In this attempt I am not alone, and some of the points I shall argue are shared by (when not taken from) other economists; but other points are my own, so the views advanced here should not be taken as representative of a 'school', nor attributed also to any other economist without a previous careful check of her/his writings.

2 For example, Frank Hahn has admitted that, owing to reswitching, 'various neoclassical adjustment theories . . . are certainly at risk' (Hahn, 1982: 373).

3 Walras was the first to adopt this specification of the endowment of capital as a given *vector* of endowments, but he was simply contradictory because he was still attempting to determine a long-period equilibrium (cf. endnote 9 below), and this is why the subsequent, more consistently very-short-period versions should be called neo-Walrasian rather than just Walrasian.

4 Some of the neoclassical assessments of the Cambridge controversies (e.g. Blaug, 1974; Stiglitz, 1974; Bliss, 1975b) antedate these writings, but other ones do not (e.g. Dougherty, 1980; Burmeister, 1980, 1991; Hahn, 1982) and yet contain no reference to any of the post-1975 critical writings mentioned in the text. No wonder that considerable misunderstandings persist.

5 The notion of equilibrium price in Wicksell or Walras includes, besides the element of uniform rate of return on supply price, also the additional element, which is not part of the definition of long-period product prices, that the distributional variables (real wage, interest rate, land rentals) are the equilibrium ones. To marginalist authors it was natural to consider these two elements together, since in general competitive conditions the gravitation of prices toward their long-period normal levels was seen by them as simultaneous with the gravitation of distributive variables toward their equilibrium levels; but the separability of the two elements would become clear even in marginalist analyses the moment the real wage or the rate of interest were taken as given, e.g. fixed by law or by non-competitive elements.

6 It is indicative of the complications which were admitted to arise with more short-period analyses that (as stressed by Ciccone, 1999) Marshall had to admit that the analysis of normal short-period prices could not reach the same definiteness of results as long-period analysis: Marshall admitted that the fear of producers of 'spoiling the market' makes the short-period supply curve of doubtful use for market prices below the long-period normal value.

7 Zero 'profits', in the marginalist sense of what is left of revenue after paying all costs including interest (gross of a risk allowance) on the capital employed. The classical authors did not include interest among the costs to be subtracted from revenue in order to obtain profits, so that the term 'profits' has a different meaning: the tendency to zero 'profits' in the marginalist sense is expressed by the classical authors as the tendency of profits to become the normal ones i.e. to guarantee the normal 'rate of profits' (the same rate of return on the capital employed as in other industries – once account is taken of risk).

8 Obviously some capital goods, being less convenient than other ones (e.g. because technologically obsolete), will not be demanded and hence in the long-period equilibrium their endowments will be zero.

9 Walras was the only one, among the founders of marginalist theory, to take the endowments of the several capital goods as given, without initially realizing that his conception of equilibrium, which was the usual one of a centre of gravitation of time-consuming adjustment processes (the 'tâtonnement' was described in the first three editions of Walras's treatise as going on in real time and including the actual implementation of disequilibrium production decisions) and accordingly comprised the standard condition of a uniform rate of return on supply price, was incompatible with the inclusion of given endowments of the several capital goods among the equilibrium's data. He only realized it some time between the third and fourth edition of his *Eléments d'Économie Politique Pure*, probably in 1899 (Walker, 1996): only in the fourth edition he introduces the provisional 'tickets' or *bons* in the tâtonnement, which is now imagined as going on in a situation of suspended economic activity; in that same edition he admits that the given vector of capital endowments will make it impossible to satisfy the condition of uniform rate of return on supply price, and therefore not all capital goods will be produced (cf. Garegnani, 1990). In spite of these admissions, the equilibrium is still defined as if one could assume that relative prices are not changing over time, what shows Walras's difficulty with abandoning the traditional conception of equilibrium as a long-period position.

10 Cf. Petri (1989, Chapters 7 and 8). Walras assumes (for simplicity I leave insurance costs aside, and adopt different symbols) that the supply price (equal to cost of production) P_i of the i-th capital good be equal to its perpetual rental net of depreciation, $v_i - d_iP_i$ (where v_i is the gross rental and d_i the radioactive rate of depreciation), divided by the interest rate: $P_i = (v_i - d_iP_i)r$. Let us assume production in yearly cycles with rentals and wages paid at the end of the year, and circulating capital goods: then $d_i = 1$ and $v_i = (1 + r)P_i$. Also, 'price = cost of production' means $P_i = a_{1i}v_1 + a_{2i}v_2 + \cdots + a_{ni}v_n + a_{Li}w$, where a_{ji} is the technical coefficient of input j ($j = 1, \ldots, n, L$) in industry i. In this equation we can replace v_j with $(1 + r)P_j$ and we obtain Sraffa's formulation. As to Wicksell, his price = cost equations in the *Lectures* are derived from Sraffa-type equations by 'reducing' costs of production to dated wages and rents with compound interest on them.

11 And, just as in the classical authors, so also in Wicksell the slow changes that long-period prices may be undergoing over time in spite of the endogenous determination of the composition of capital – changes due, e.g. in the marginalist approach, to the slow change of distribution over time if the growth rate of labour supply is different from the growth rate of capital – are neglected in the determination of equilibrium (i.e. the equations of equilibrium are formulated as if relative prices were constant over time), precisely because of their slowness. The neglect of the changes that relative prices may be undergoing over time does not, therefore, mean that analyses determining long-period prices are only concerned with, or applicable to, a stationary state or steady-growth states; cf. Wicksell who explicitly states that his stationary state assumption is only a 'simplifying assumption' (1934: 155) and that 'The application to non-stationary conditions offers no difficulty in principle' (ibid., p. 154); for a fuller discussion cf. Petri (1999: 27–37). Walras too, as noticed in endnote 9, neglects in his equations the changes that relative prices may be undergoing over time, what shows that his notion of equilibrium was a long-period one. The assumption of constancy through time is anyway not essential to the determination of long-period relative prices, it is only a first step, from which one may go on to more complex analyses if the case so requires (cf. Petri, 1999: 36–37).

12 For simplicity I am assuming the conditions for the validity of the non-substitution theorem (no scarce natural resources and no joint production).

13 The legitimacy of the assumption that one may abstract from net accumulation while allowing the 'form' of 'capital' to change rested on the fact that the speed, with which an economy can alter the relative proportions between the amounts in existence of the several capital goods, could be presumed to be generally much greater than the speed with which the total stock of 'capital' was altered by net accumulation: this made the treatment, of the endowment of 'capital' as given, as legitimate as the analogous treatment of the endowment of labour (which is also generally not strictly constant, being altered e.g. by population growth), when determining long-period prices and quantities. So (cf. Garegnani, 1976) the notion of long-period equilibrium should not be confused with the notion of a *secular* equilibrium or steady state, in which the accumulation of capital per unit of labour has come to a halt.

14 Cf. Wicksell (1934: 204) where after specifying the conditions corresponding to URRSP and to the full employment of labour and land he finally, and somewhat hesitantly, adds the condition that 'in equilibrium the sum total of capital shall have a certain exchange value'. The way labour and land are distributed among their possible dated employments, and therefore the (implicit) equilibrium amounts of intermediate products (capital goods), are variables in Wicksell's model. The same endogenous determination of the composition of capital is present in Wicksell's *Value, Capital and Rent* (1893) where Wicksell adopts Bohm-Bawerk's average period of production as a measure of the capital intensity of production processes. Wicksell and some of his pupils (the most recent example is Bent Hansen, 1970) are the only marginalist economists who explicitly formalized the notion of long-period equilibrium and the endogenous determination of the composition of capital, implicit in the conception of capital as a single 'fund' of variable 'form' shared by the generality of the early marginalist authors: cf. e.g. J.B. Clark: 'Where there is a capital of five hundred dollars for each worker, that fund is in one set of forms; and where there is a capital of a thousand dollars per man, it is in a different set' (1925: 159); 'As we take away labourers, we leave the capital everywhere unchanged in amount; but we change the forms of it in every one of the industries, so as to make it accurately fit the needs of the slightly reduced working force' (ibid., p. 170).

15 This point is not grasped by Bliss (1975: 162) when he argues that there is no 'notable, particular and distinct problem posed by capital aggregation' because the problem is the same as with labour or output aggregation: on the contrary, the problem with capital *is* different, because, if adjustments are admitted to take time and long-period equilibria to be therefore the only sensible notion of equilibrium, then capital 'aggregation' (the determination of its endowment as a single quantity of endogenously determined 'form') is indispensable, differently from the case with different types of labour or of land, whose endowments can be treated like so many different factors because not altered by the disequilibrium adjustments. Bliss concludes that 'the widespread belief that there is a notable, particular and distinct problem posed by capital aggregation is at best an ill-formulated idea, and at worst is based simply on ignorance' (ibid.); on the contrary it is Bliss who in this way exhibits his ignorance, of the history of his own theory and of the central role in it of the notion of long-period equilibria, from which follows his inability to understand that the capital aggregation problem posed by the Cambridge criticism concerned long-period equilibria. (This way of showing the importance of capital 'aggregability' relies only on the first, supply side role of the traditional notion of capital as a single factor; the second role supplies another reason – to avoid instabilites of the savings-investment market – which holds even conceding instantaneous adjustments, as made clear in the contributions by Garegnani and Schefold in this volume.) For discussions of the equations of long-period general equilibria cf. Garegnani (1990), Petri (1978, 1999), and the Appendix at the end of the present paper.

16 If one leaves aside possible disturbances coming from the malfunctioning of financial intermediaries, the flow of supply of loanable funds equals net savings plus depreciation plus consumption of intermediate goods, while the flow of demand for loanable funds equals net investment plus the same depreciation and consumption of intermediate goods.

17 The conditions for the Gorman–Fisher technical aggregability of heterogeneous capital goods into a single factor in the production functions, if appropriately extended to encompass all the production functions of the several industries, would make it logically possible to determine the endowment of 'capital' and its variations in technical terms, but are extremely restrictive (Bliss, 1975, Chapter 7): the production functions must be such that it must be as if, in *all* firms, the several capital goods (unassisted by labour) instantaneously produced a single intermediate good K which then, together with labour, produced the firm's product, and furthermore, the 'production function' producing the fictitious good K must be the same in all firms. It is easy to see that then, to all relevant effects, it is as if there were only one capital good in the economy (the several actual capital goods formally have a role identical to that of temporarily existing intermediate products in a stage of the process producing K through the employment of K and labour, cf. Petri, 1999: 44–46).

18 In spite of this admission, in the immediately following formulation of the general equilibrium equations Wicksell takes as given 'the total exchange value of the capital employed' (1934: 204); and he gives no indication of why his previous admission does not render this procedure unacceptable. It is therefore not surprising that his pupil Lindahl rejected the measurement of capital as a 'fund', an amount of value, and moved in the direction of temporary equilibrium.

19 It must be noticed that the marginalist approach became dominant on the basis of the mistaken faith that it was able to consistently determine a *long-period* equilibrium. At the end of the nineteenth century it was nearly impossible to believe otherwise: the specialist works on this issue, Walras' *Eléments* in its first three editions (1874–1896), and Wicksell's *Uber Wert, Kapital und Rente* (1893), the sole attempts at the time to treat the problem of capital within a general equilibrium setting, had both claimed to have shown that the approach was able to determine a long-period equilibrium. Nor was it easy to realize, later, that the changes in Walras's 4th edition of his *Eléments* (1900) implied defeat in this respect: Walras did not openly admit it, and only with Garegnani (1960) did the thing become clear. As to Wicksell, his rejection of the average period of production and hence of the 1893 analysis in the *Lectures* (1901–1928) became available in English only in 1934, and the reticent admission in that book of some problems with determining the endowment of capital also went unnoticed. The history of economic theory might easily have taken a different turn, if Walras' and Wicksell's changes of mind had happened earlier and had become common knowledge before the supply-and-demand approach had had the time thoroughly to permeate the economists' minds.

20 The existing plants would normally keep employing most of the supply of labour, since, once in existence, it would be convenient to go on operating them as long as they yielded non-negative residual quasi-rents. Cf. Garegnani (1978).

21 Thus one finds e.g. Knight writing in 1946: 'Under conditions of perfect competition, or in an economic system in the position of the theoretical equilibrium (stationary or moving), all sources would yield a uniform rate of return on their cost of production, which would be equal both to their cost of reproduction and their market value … Under real conditions, this rate 'tends' to be approximated at the margin of new investment (or disinvestment), with allowance for the uncertainties and errors of prediction.' (Knight, 1946: 396.)

22 Cf. Marshall, 1970, VI, ii, 4 (p. 443): '…the income derived from capital already invested in particular things, such as factories or ships, is properly a quasi-rent and can be regarded as interest only on the assumption that the capital value of the investment has remained unaltered…the phrase "the general rate of interest" applies in strictness only to the anticipated net earnings from new investments of free capital.…Thus then interest, being the price paid for the use of capital in any market, tends towards an equilibrium level such that the aggregate demand for capital in that market, at that rate of interest, is equal to the aggregate stock forthcoming there at that rate'; also cf. ibid., VI, vi, 6 (p. 492), where he adds that, since replacement investment is the larger part of gross investment: 'It is therefore not unreasonable to assume for the present that the owners of capital in general have been able in the main to adapt its forms to the normal conditions of the time, so as to derive as good a *net* income from their investments in one way as another. It is only on this

supposition that we are at liberty to speak of capital in general as being accumulated under the expectation of a certain net interest which is the same for all its forms'. It would seem therefore that Marshall only differs from J.B. Clark or Wicksell in that he eschews a careful description of the long-period demand for 'capital', and concentrates on the short-period demand for investible funds in each market, but within the same overall theory.

23 Obviously, in order for the long-period forces to be visible, one had to presume that their effect was stronger than that of the accidents of the short period, i.e. that even in the short period the decisions having effects over longer periods, such as the investment decisions, were taken on the expectation of an average value, of economic magnitudes over longer periods, determined by long-period forces.

24 That is, analysis of the overall tendencies of the economic system, in particular, when time-consuming disequilibria are admitted.

25 Indeed the total 'capital endowment' or productive capacity of the economy might also be relevantly altered; cf. Part II of the present paper.

26 This would appear to have been also Keynes' opinion, as expressed in a letter to Kalecki dated 12 April 1937: 'I hope you are not right in thinking that my *General Theory* depends on an assumption that the immediate reaction of a capitalist is of a particular kind. I tried to deal with this on page 271 [*Sic* – probably 261], where I assume that the immediate reaction of capitalists is the most unfavourable to my conclusion. I regard behaviour as arrived at by trial and error, and no theory can be regarded as sound which depends on the *initial* reaction being of a particular kind. One must assume that the initial reaction may be anything in the world, but that the process of trial and error will eventually arrive at the conclusion which one is predicting.' (Keynes, 1973–1979, vol. XII, p. 797.)

27 This role of equilibrium is admitted even in the latest neo-Walrasian advanced textbook, Mas-Colell *et al.* (1995: 579): 'The object of investigation in this chapter is the notion of Walrasian equilibrium, which we take as a positive prediction for the outcome of a system of markets in which consumers and firms are price takers and the wealth of consumers derives from their initial endowments and profit shares.' This highly esteemed textbook nowhere mentions the impermanence problem. Of a 'Marshallian' quantity tâtonnement (based on given factor endowments) it even says that 'we can interpret the dynamics as happening in real time' (ibid., p. 624), i.e. as entailing actual disequilibrium productions, evidently forgetting that the equilibrium under discussion was supposed to be interpretable as an intertemporal equilibrium with, among its data, endowments of capital goods which production would alter. The sole reference to Fisher (1983) is in a footnote and reads: 'For an extensive analysis of market adjustment procedures in real time, see Fisher (1983)' (ibid., p. 624, fn. 69); given the book's generally careful coverage of the literature on other topics, this short sentence cannot but induce readers to think that Fisher's book does not contain startling novelties; so graduate students trained on this textbook most probably do *not* go and read Fisher, and thus remain ignorant of the fact that Fisher's admission, that the equilibrium based on the initial data is irrelevant, makes it impossible to accept the interpretation of 'Walrasian equilibrium' as a 'positive prediction' of the behaviour of market economies.

28 'There is scarcely any period of time so short that it can give us temporary equilibrium (in Marshall's sense) for all commodities; there will nearly always be some products whose supply can be increased within the period' (Hicks, 1946: 122).

29 A partly different position is held by Frank Hahn, who has repeatedly admitted that the Arrow–Debreu notion of general equilibrium (the only well worked-out notion of general equilibrium in his opinion) is not a good guide to the actual behaviour of real economies, e.g. by writing (Hahn, 1981: 1036): 'I have always regarded Competitive General Equilibrium analysis as akin to the mock-up an aircraft engineer might build…theorists all over the world have become aware that anything based on this mock-up is unlikely to fly, since it neglects some crucial aspects of the world, the recognition of which will force some drastic re-designing. Moreover, at no stage was the mock-up complete; in particular, it provided no account of the actual working of the invisible hand'); but who has also argued (Hahn, 1973) that that notion of equilibrium has nonetheless usefulness as a benchmark, as a way of helping us understand what the world would have to look like in order for certain contentions

to be acceptable, e.g. that real economies are Pareto-efficient. In such a perspective, Arrow–Debreu equilibria do not aim at indicating the situation the economy tends to; the accusation of logical inconsistency advanced here is thereby avoided, but at the price of conceding my claim that neo-Walrasian equilibria cannot tell us how market economies actually behave. In other writings however Hahn seems to accept many standard neoclassical tenets, e.g. that the demand for labour is downward-sloping.

30 This section relies on Garegnani (1990: 57–58) and on Petri (1991, 1997, 1999). Here in order to concentrate on the problems caused by capital I assume homogeneous labour; but the same problems would arise for the demand curve for one type of labour.

31 So unemployed workers do not demand final goods (except with income from sources other than their labour); for each level of the real wage, the economy is in equilibrium on all markets except the labour market; Walras' law as normally intended does not hold (there is disequilibrium on only one market), because the demand for final goods is not based on the income consumers *count on* obtaining from their desired supplies of factors (as is on the contrary the case in the tâtonnement with 'tickets'), it derives only from the income they actually obtain. This is the curve which in neoclassical theory may allow predicting, for example, the change in real wage resulting from immigration, or the level of employment determined by an exogenously fixed real wage. (The spread of neo-Walrasian notions of equilibrium and of the habit of conceiving the equilibrium as reached by a tâtonnement evidently obscured this assumption implicit in the derivation of the labour demand curve, and in Keynes' analysis, to the point that Clower's re-discovery of it, under the name of 'dual-decision hypothesis', was hailed as a great analytical advance.) Clearly, if one cannot assume that all workers are identical, this traditional labour demand curve is somewhat indeterminate outside the full-employment equilibrium point, in so far as the composition of demand, and hence the demand for labour, are affected by precisely *which* workers remain unemployed if labour supply is greater than demand, or by which hypotheses one makes as to the tastes of the imaginary workers employed in excess of the supply of labour if labour demand (and hence employment) is greater than supply. But what is important for traditional analysis is, essentially, that the curve be decreasing, and this is hardly affected by the above indeterminacy.

32 Thus Hicks in *The Theory of Wages* had written: 'In the short period, therefore, it is reasonable to expect that the demand for labour will be very inelastic, since the possibility of adjusting the organization of industry to a changed level of wages is relatively small.[. . .] Since the whole conception of marginal productivity depends upon the variability of industrial methods, little advantage seems to be gained from the attempt which is sometimes made to define a 'short period marginal product' – the additional production due to a small increase in the quantity of labour, when not only the quantity, but also the form, of the co-operating capital is supposed unchanged. It is very doubtful if this conception can be given any precise meaning which is capable of useful application.' (Hicks, 1932: 21). The problem arises also for the several capital goods: as noticed by Garegnani (1990), changes in methods of production generally require, not different proportions among the same capital goods or between them and labour, but rather the employment of different capital goods; thus, the proportions in which capital goods must be combined with other capital goods and labour being very rigid, and their endowments being arbitrary, nearly certainly in the initial period of these very-short-period equilibria a very high proportion of equilibrium capital goods' rentals will be zero, the risk of a zero or implausibly low wage rate is very high, and very small changes in the relative endowments of capital goods may cause many equilibrium rentals, including perhaps the wage rate, to vary very considerably, often jumping from zero to positive or vice versa; the prices determined by these equilibria cannot therefore aim at being good guides to observed prices, which do not exhibit such variability.

33 This cost would be determined by cost of production, inclusive of the rate of interest. The latter would be determined by the ratio between value net marginal product and cost of production of those, of the durable capital goods, of which there were positive production.

34 Cf. Part II of this lecture on why the technical full utilization of fixed plants is seldom reached.

35 Many other quotations to the same effect could be produced. Junankar in his 1972 survey of investment theory is very candid on how aggregate capital is to be measured: 'There are several problems involved in measuring aggregate capital stock [. . .] Cambridge economists have argued very strongly that it is impossible to measure capital in value terms in a way that is independent of the rate of interest and wages. For the purposes of this survey I shall sidestep this controversy and *assume* that we can measure capital in value terms.' (Junankar, 1972: 12–13, my italics.) Nearly all other authors do not even bother to mention the Cambridge controversy and the questionable nature of the measurement of capital in value terms.

36 The freedom with which the value of capital per unit of labour may vary is illustrated by Garegnani (1970: 431–435). A strictly connected phenomenon is that the long-period net product per unit of labour (consumption per unit of labour, if the growth rate is zero) can be nearly any function of the rate of interest (except that it cannot be greater than at $i = 0$). (Mas-Colell (1989) confirms these results, although through an analysis lacking transparency: e.g. the multiplicity he shows to be possible, of values of consumption per unit of labour for a given rate of interest, has reasons which are better grasped when one realises that two or more wage curves may be tangent to one another on the outer envelope of the wage curves.)

37 Both Jorgenson's theory and the adjustment-cost approach try to determine the investment behaviour of a single firm. Aggregate investment is therefore indeterminate the moment the possibility of entry of new firms is admitted. This is admitted e.g. by Söderstrom (1976: 386), who writes that in adjustment-cost theories of investment 'market equilibrium . . . may be indeterminate under free entry': where rigour would require replacing 'may be' with 'is'.

38 'While there is clearly no uniformity in the results and the role of shocks remains to be assessed, it appears to this author that, on balance, the response of investment to price variables tends to be small and unimportant relative to quantity variables' (Chirinko, 1993: 1906). The survey by Junankar (1972) had reached the same conclusion. Even on the basis of these empirical conclusions alone, it would appear that macroeconomics ought to be built on the principle that it is variations of income which adjust savings to investment, and that the accelerator is a much more important influence on investment than the rate of interest.

39 It must be kept in mind, in this connection, that the general abandonment of one-good models, at least by 'high-brow' neoclassical theoreticians, in favour of neo-Walrasian equilibria would appear to have been a reluctant and recent move, essentially prompted by the Cambridge controversies. As admitted e.g. by Christopher Dougherty, the results on reswitching, on reverse capital deepening, and on the illegitimacy of aggregate or surrogate production functions were decisive, cf. Dougherty, 1980: 3: 'Since then [the mid-1960s] the general equilibrium model has been the undisputed core of neoclassical capital theory'. Thus the Cambridge controversies on capital theory have had an important role in forcing the adoption of neo-Walrasian equilibria as the sole possible rigorous foundation for neoclassical analyses.

40 Research continues on the problems raised by joint production; it must be kept in mind, in this connection, that the non-uniqueness of the technique to which the economy may converge in the long run, which appears to be a possibility in these cases (Bidard, 1997), does not endanger the approach because the determination of the real wage (or of the rate of profits, depending on which of the two is the variable one sees as more directly determined by the forces affecting income distribution) is not simultaneous with the determination of relative prices, as is on the contrary the case with the marginalist approach; all that would need to be admitted is some dependence of relative product prices on historical accidents.

41 Cf. e.g. the opinion of a highly esteemed industrial economist: 'In an economy subject to uncertainty, profits and losses signal the existence of excess demand or excess supply at long-run competitive price. If resources are free to respond to market signals, they should move into areas where profits are being earned and out of areas suffering losses. This movement of resources continues until returns are equalized across all markets (with appropriate adjustment for risk). Of course, each new period brings new uncertainties and new positions of profits and loss, so that a point in time when all firm or industry profit levels

are equal never obtains. But if the market is capable of responding to the signals of profits and losses, the long-run movement of individual firm and industry profit rates should be toward a common competitive level. All observed profits and losses should be short-run deviations around this trend .... Although most studies of profit rate determinants have focused on industry profit levels, the competitive environment hypothesis of convergence on a single competitive level should be equally valid for firm-level profits and for industry profits. For a homogeneous product, all firms in an industry should charge the same price under competitive conditions. Free entry and exit should ensure that only the most efficient firms survive, that all firms have the same average costs as well as price.' (Mueller, 1986: 8–9). These lines appear to reflect the general opinion in industrial economics, where therefore the traditional conception of normal competitive prices as long-period prices yielding a uniform rate of return on (the supply price of) capital and as centres of gravitation of market prices is still dominant, and has not been abandoned or forgotten in favour of neo-Walrasian notions of equilibrium prices. Let me remind the reader that long-period positions are not stationary nor steady-growth positions, as made clear by the Classical authors but also by e.g. Wicksell (cf. above, notes 11 and 13).

42 Cf. e.g. Garegnani (1992), Petri (2001).

43 The Cambridge way to determine the long-run average rate of growth is also open to criticism. The full employment of labour (Kaldor) is not something empirically observed, and anyway it would not be truly a constraint on growth because of the possibility (and historical reality) of migrations and of other processes of adaptation of labour supply to demand. As to Joan Robinson's determination of the rate of growth (well summarised in Marglin, 1984), it depends on the assumption that investment is an increasing function of the rate of profits, an assumption of unclear theoretical foundations once Say's Law is rejected, and not very solidly confirmed by empirical evidence. Indeed, the moment one admits – as one must – that, over the long run, in competitive conditions rate of interest and normal rate of profits must adapt to each other so that normal extraprofits (net of the appropriate risk allowance) are zero, there appears to be no reason why entrepreneurs should be more prone to invest when the rate of interest is 5 per cent than when it is 3 per cent. As to the empirical evidence, the latest study (Glyn, 1997) finds that there is a positive association between rate of profits and rate of growth, but only on average, there being several nations which constitute counter-examples; and anyway the result, even if empirically stronger, might reflect the tendency of the total world investment to go in greater proportion where the rate of return is higher, without implying that a higher world-wide average rate of return would stimulate world-wide investment.

44 Cf. the preceding endnote. Let us remember that the multiplier theory of income, when combined with different saving propensities out of wages and out of profits, tends to suggest that redistributions of income from wages (or from social expenditures addressed at the labouring classes) to profits tend to *depress* aggregate demand, what might well result, through the accelerator, in a *reduction* of investment (at least in a closed economy, or in the world economy).

45 Technology has no direct influence on the level of efficiency wages; only in a neoclassical framework it becomes indirectly relevant, because the real wage per unit of effort determines the demand for labour and thus unemployment; thus in a neoclassical framework unemployment and the efficiency wage must be determined simultaneously. In a non-neoclassical framework the level of unemployment will be determined in other ways.

46 Another weakness is that in most jobs it is not difficult to ascertain whether a worker shirks or not (cf. Petri, 1994).

47 This assumption is a plausible approximation in all cases in which the situation only changes slowly, as must be the case in a long-period equilibrium even when it is not stationary, as long at least as the kinds of capital goods in use are not changing (cf. Petri, 1999: 27–29).

48 Clearly, when consumers are not aggregable, as K varies some assumption must be made as to how its distribution among consumers varies. The simplest assumption is that this distribution does not vary, i.e. that the endowment of 'capital' of all consumers varies in the same proportion, starting from the quantities corresponding to the actual, observed endowment (here assumed determinable for the sake of argument).

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15 Competitive equilibrium and non-cooperative game theory

Noise and bounded rationality*,†

Hamid Sabourian

In this chapter I survey some of the issues that arise when a non-cooperative game-theoretic foundation is provided for the competitive behaviour in dynamic settings. It is often claimed that a market with a 'large' number of 'insignificant' agents result in a competitive outcome. One way of formalizing this in a dynamic game-theoretic context is to assume that there is a continuum of anonymous agents. Here, I consider the equilibria of two types of dynamic games – repeated games, and bargaining and matching models – with a large but finite number of players. Equilibria in these models can be shown to be radically different from those found in models with a continuum of anonymous players. The reason for this discontinuity at infinity in dynamic games with a finite number of players is that players can choose history-dependent strategies. The possibility of conditioning behaviour on histories allows one to construct a large number of equilibria in these settings. In this chapter, I discuss, in the context of repeated games and bargaining and matching models, some attempts at explaining away the above discontinuity. These attempts show that noise and/or some elements of 'bounded rationality' can provide some game-theoretic foundation for competitive behaviour in dynamic models with a finite number of agents.

15.1 INTRODUCTION

In this chapter I survey some of the issues that arise when a non-cooperative game-theoretic foundation is provided for the competitive behaviour in dynamic settings. In a competitive market agents take prices as given. Usually, this is justified by saying that there is a 'large' number of 'insignificant' agents. One way of formalizing this in a dynamic game-theoretic context is to assume that there is a continuum of anonymous agents. Such models induce the competitive outcome(s) under some regularity conditions. Here, I discuss the problems that arise in providing a competitive limit theorem. In particular, I consider the equilibria of two types of dynamic games – repeated games, and bargaining and matching models – with a large but finite number of players. Equilibria in these models can be shown to be radically different from those found in models with a continuum of anonymous players. While this discontinuity at

* Prepared for the International School of Economic Research XII Workshop on "General Equilibrium: Problems, Prospects and Alternatives," Siena, Italy, July 1999.

† The content of this lecture has been greatly influenced by Gale (2000), which addresses the issue of dynamic games and competitive behaviour in more detail.

infinity may seem narrow, the issue is important because it is critical in terms of providing a game-theoretic foundation to the competitive behaviour in a dynamic-setting. Clearly, economies with a continuum of anonymous agents are of limited value if their equilibria are radically different from those found in economies with a large but finite number of agents. The reason for this discontinuity at infinity in dynamic games is the following. With a continuum of anonymous agents, a single agent's actions cannot influence the information the others receive during the game, whereas with a finite number of players, at each information set players can condition their behaviour on ('payoff irrelevant') past history, and in particular on past plays of other players. This possibility of choosing history-dependent strategies allows one to construct equilibria with the property that at each stage every player has to consider the possible response of others to different moves. By selecting appropriate response rules to past behaviour of others, a large number of equilibrium outcome paths can be constructed.[1] In such equilibria a single agent has a large effect. As a result, even in a frictionless market with a large but finite number of agents, these equilibria induce non-competitive outcomes.

However, notice that in dynamic games it is assumed that a great deal of precise information is observed and remembered at no costs, and that the players do not make any mistakes. If there are observational errors, or if players make mistakes or if there are some costs in remembering past information then the equilibria of dynamic games may look very different. In this chapter, I discuss, in the context of repeated games and in the context of dynamic matching and bargaining models, how the above discontinuities disappear if there is noise in the model and/or if players, in taking decisions, care about the complexity of their rules of behaviour ('bounded rationality'). Noise and/or some elements of 'bounded rationality' can provide some game-theoretic foundation for the competitive behaviour and can ensure that in equilibrium players choose history-independent (sometimes referred to as stationary or Markov) strategies in dynamic models with a finite number of agents.

15.2 NON-COOPERATIVE REPEATED GAME

15.2.1 Oligopoly model

The earliest and best-known example is that of the Cournot model of oligopoly. This is a static one-shot game with K firms choosing quantities simultaneously. An inverse demand function, representing the behaviour of the consumers, determines the market clearing price as a function of aggregate output. When K is finite, the Nash equilibrium of this game results in a price above that predicted by the price-taking competitive model. However, one might conjecture that as K becomes unboundedly large any Nash equilibrium of this model converges to a perfectly competitive equilibrium. The reasoning behind this conjecture is as follows. With a large number of firms, each small firm could benefit from producing an extra unit of output if the market price is above the competitive one because the effect of an extra unit of output on the price is of a lower order of magnitude than the revenue from selling an extra unit. Roberts (1980) and others showed that this conjecture is correct under some strong regularity conditions.[2] He also provided a counter-example to this limit result by showing that there may be a discontinuity in the inverse demand function. Such a discontinuity arises from inverting a selection from the demand correspondence of the consumers. The strong regularity conditions ensure that this does not arise. Without such regularity conditions, a small increase in the aggregate output may result in a sharp (discontinuous) fall in the market price. As a result, at such a discontinuity point, a small increase in output may result in a loss of revenue on inframarginal units that is of a higher order of magnitude than the revenue from the additional output. Thus, at such discontinuity points, each firm has a non-negligible effect on the price, no matter how many firms and consumers are in the market.

The discontinuity in the inverse demand function pointed out by Roberts has been studied in a more general setting by Green (1984). In particular, Green looks at the limits of Nash equilibria in static games as the number of players increases without any bound. He shows that to achieve a limit result characterizing the limits of such equilibria, some continuity restrictions on the payoffs may be necessary. All these may appear to be purely technical and in static games we may assume them away by only considering games in which the payoff functions are 'well-behaved'. However, when one looks at dynamic games such discontinuities can arise naturally and no reasonable assumptions on the underlying parameter can remove the discontinuities that arise when a ('small') player changes its action by a 'small' amount. This is because in these games in equilibrium, players can choose complex history-dependent strategies. As a result, there is no guarantee that in equilibrium a change in a single player's strategy will not result in a large response by others.

Applying the Folk Theorem of the repeated game to the repeated Cournot oligopoly model, it is trivial to show that the monopoly outcome can be sustained as an equilibrium of the repeated Cournot model with an arbitrary number of firms if the firms are sufficiently patient and there is perfect monitoring (all firms observe the past output actions of every other firm).[3] Clearly, one could argue that perfect monitoring may be very costly in a large game. However, Green (1980) shows that the above result holds even if the game is anonymous. In particular, he demonstrated the result for the case in which the firms observe only some sufficient statistics (the past prices of the good) reflecting the past aggregate actions of all the firms. The basic idea of Green's result was to construct the following trigger strategy profile. This profile requires each firm to produce $1/K$th of the monopoly output (K refers to the number of the firms) at the initial period and at every other period if the past prices were the monopoly ones. If non-monopoly prices have been observed play Cournot–Nash output forever. The Cournot–Nash behaviour is the punishment that deters deviation from the monopoly behaviour. Clearly, this strategy profile constitutes a subgame perfect equilibrium if the players are sufficiently patient. Notice also that as K increases it is easier to support the monopoly outcome: as the number of the firms increases, the Cournot–Nash equilibria converge to the competitive outcome (given Robert's regularity condition) and therefore the threat of punishing a deviator by reverting to the Cournot–Nash equilibrium becomes more severe.

The message of Green's result is that even when K is large a single player can be 'informationally significant' because any change in his behaviour results in a non-monopoly price and this will trigger a punishment phase (of all players playing the Cournot–Nash equilibrium of the one-shot game). Putting it differently, in any finite game there is information about the past behaviour of the firms that past market prices reveal. In particular, past prices can reveal if any player has deviated from the monopoly output in the past, although the identity of the deviator is not revealed.

One way of reducing the informativeness of past prices is to allow prices to vary because of, say, random demand in the industry. If the scale of each firm's output is small relative to the size of the market, any stochastic demand disturbance should make the strategies involving punishment threats more difficult to enforce. Effectively, one is hoping that the introduction of exogenous noise will allow agents to become 'informationally insignificant' as the number of firms increases without any bounds. In the rest of this section, I shall make this idea precise in the context of an abstract repeated game.

15.2.2 The abstract game with random outcome (imperfect monitoring)

15.2.2.1 Stage game

Consider a stage game $H_K = (\mathcal{K}, \overline{A}, A, X, F, \pi)$ where
$\mathcal{K} = \{1, \ldots, K\}$ is a finite set of players;
$\overline{A}$ is the set of actions available to each player k;
$A = \overline{A}^K$ is the set of action profiles;
X is the set of publicly observed outcomes (signals);
$F : A \to \Delta(X)$ maps action profiles into the set of outcome distributions (for any set Q, $\Delta(Q)$ refers to the set of (Borel) probability measures on Q);
$\pi_k : \overline{A} \times X \to \mathcal{R}$ denotes player k's payoff function.

Thus in this set-up player k's payoff depends on the action taken by k and the realization of the outcome. Also, I assume that both $\overline{A}$ and X are subsets of Euclidean spaces (in fact, complete separable metric spaces will suffice) and that all mappings from measurable spaces into these spaces will be assumed to be Borel measurable.

Now for any action profile $a \in A$, define the expected payoff to k in the one-shot game H_K by

$$\Pi_k(a) = E_{F(a)}\pi_k(a_k, x) = \int_X \pi_k(a_k, x)dF(a)(x).$$

Definition 1 An ϵ-Nash equilibrium of H_K is a profile $a^* = (a_1^*, \ldots, a_K^*) \in A$ such that for any player k

$$\Pi_k(a_k, a_{-k}^*) - \Pi_k(a^*) < \epsilon \quad \text{for all} \quad a_k.$$

15.2.2.2 Repeated game with random outcome

H_K^∞ refers to the infinitely repeated game H_K with discount factor $\delta < 1$. Players are assumed to observe the outcomes of past play (and not the past actions); thus strategy for player k can be written as a sequence of functions $s_k = \{s_k^t(.)\}_{t=0}^\infty$ where $s_k^t : X^t \to \overline{A}$ and X^t is the t-fold Cartesian product of X. Let s be a profile of strategies and S the set of the strategy profiles in H_K^∞.

We shall also assume that the game is anonymous. Formally, this anonymity is defined as follows. Let $m : A \to \Delta(\overline{A})$ map profiles of actions into the distribution of actions; thus for any $a \in A$ and any $B \in \overline{A}$

$$m(a)(B) = 1/K|\{k \in \mathcal{K}| a_k \in B\}|$$

where for any set Q $|Q|$ refers to the cardinality of the set. Now H_K^∞ is anonymous if for any action profile $a \in A$ the outcome function $F(a)$ depends on the action profile a only through the distribution of actions $m(a)$. Formally, there exists a function $G: \Delta(\overline{A}) \to \Delta(X)$ such that $F(a) = G(m(a))$ for all $a \in A$.

Green demonstrates that the equilibria of the repeated game coincides with that of the one-shot game under the above anonymity condition for a model with a continuum of players. To obtain an equivalent result for a model with a 'large' but finite number of players, one needs to restrict the amount of information the publicly observed outcomes (signals) of the game carry even further than that implied by anonymity. Effectively, it is necessary to make the agents' actions 'informationally' insignificant as the number of players is increased. Clearly, the mapping G from the set of distribution over actions to the set of outcome distribution needs to have some continuity property. As a first attempt, one could assume G is continuous when both $\Delta(\overline{A})$ and $\Delta(X)$ are endowed with weak topology. It turns out that this is not sufficient to obtain the result on the equivalence of the equilibria of H_K and H_K^∞ as the number of players increases (see Green, 1980). Sabourian (1990) shows that the result holds for large but finite number of players if we assume that G is continuous when $\Delta(\overline{A})$ is endowed with the weak topology and $\Delta(X)$ is endowed with the total variation norm (TVN) topology.[4] (See the appendix for a basic exposition of these topologies.) Continuity of G with respect to the TVN topology (for $\Delta(X)$) turns out to be precisely the condition needed to ensure that each player becomes 'informationally insignificant' as the number of players increases without bounds (see below).

THEOREM 1 *Consider a family of one-shot games* $\{H_K = (K, \overline{A}, X, \pi_k, F, G\}_{K\in\mathcal{N}}$ *with their associate repeated games* $\{H_K^\infty\}_{K\in\mathcal{N}}$ *($\mathcal{N}$ refers to the set of natural numbers). Assume that the payoffs of* $\{H_K\}_{K\in\mathcal{N}}$ *are uniformly bounded and G is continuous when* $\Delta(\overline{A})$ *is endowed with the weak topology and* $\Delta(X)$ *is endowed with the TVN topology. Then for any* $\epsilon > 0$ *there exists a* $\overline{K}$ *such that for all* $K > \overline{K}$*, for any Nash equilibrium strategy profile s of* H_K^∞ *and after any history h of the game* H_K^∞ *that occurs with a positive probability (given s), the action profile prescribed by f after h is an* ϵ*-Nash equilibrium of the one-shot game* H_K.

Now I would like to provide some intuition for the above result and provide some explanation for the need for the continuity of G with respect to this stronger (TVN) topology.

Fix any repeated game H_k^∞. For any strategy profile $s = (s_k, s_{-k})$ in this game let $E\pi_k^\infty(s)$ be the repeated game payoff of player k. Using techniques that are by now standard in the literature on repeated games one can factorize the long-run payoff $E\pi_k^\infty(s)$ into the first period payoff and the continuation payoff. Formally, for each k there exists a bounded and measurable (continuation payoff) function $V_k: S \times X \to \mathcal{R}$ such that[5]

$$E\pi_k^\infty(s) = \Pi_k(s^0) + \delta \int_x V_k(s, x) dF(s^0)(x),$$

where s^0 is the action profile prescribed by s at the initial period. Given the anonymity condition, the above can be written as

$$E\pi_k^\infty(s) = \Pi_k(s^0) + \delta \int_x V_k(s, x) dG(m(s^0))(x).$$

Thus, a player takes into account the future reaction of others to his own action to the extent that a change in his action affects the second term in the RHS of the above expression. Thus, to establish the equivalence of the equilibria of H_K and H_K^∞ as the number of players increases, one needs to show that the effect of a player's action on the second term in the above expression goes to zero as the number of players increases. Formally, one needs to show that for large K and for any Nash equilibrium $s \epsilon S$

$$\left| \int_X V_k(s,x)dG(m(s^0))(x) - \int_X V_k(s,x)dG(m(a_k, s^0_{-k}))(x) \right| \simeq 0 \quad \text{for all} \quad a_k \in \overline{A}. \tag{15.1}$$

Clearly, for large K the action of any single player has a small effect on the distribution of actions; thus $m(s^0)$ is arbitrarily close to $m(a_k, s^0_{-k})$ for large K. This together with the continuity of G imply that for large K the distributions $G(m(s^0))$ and $G(m(a_k, s^0_{-k}))$ are close with respect to the TVN topology. Now, if a probability space is endowed with TVN topology then two probability measures are 'close' if the expected values of *any bounded* measurable function are close with respect to these two measures (and the closeness of the expected values hold uniformly for any set of uniformly bounded functions).[6] Since $V_k(s,x)$ is a bounded measurable (measurability is with respect to x) function, the expression in equation (15.1) follows from $G(m(s^0))$ being close to $G(m(a_k, s^0_{-k}))$ with respect to the TVN topology.

Finally, notice that if $\Delta(X)$ was endowed with the weak topology the expression in equation (15.1) would not necessarily be true. This is because weak topology is equivalent to claiming that two probability measures are close if the expected values of any bounded *continuous* function with respect to these two measures are close. (See the appendix for a formal statement of this result.) The continuation payoff function $V_k(s,x)$ is not necessarily continuous as function of x in repeated games. Therefore, weak topology does not limit the amount of information that the outcomes of the game carries.

In repeated games, the only assumptions one can impose on $V_k(s,x)$ are (uniform) boundedness and measureability. But closeness of two measures with respect to the TVN topology is equivalent to saying that expected values of any bounded and measureable function with respect to the two measures are close (with some uniformity). Thus continuity of G with respect to the TVN topology is exactly the condition needed to ensure that in a game with a finite but large number of players, a change in the action of a single agent has a small influence on the continuation payoff of that agent.

Before concluding this section, I like to briefly discuss how strong the assumption of the continuity of G with respect to TVN topology is. If the range of G contains degenerate distributions (no noise) then G may not be continuous with respect to this strong topology. For example, suppose the sequence of distributions of actions $\{m_n\}$ with $m_n \in \Delta(\overline{A})$ is such that m_n converges weakly to $m \in \Delta(\overline{A})$,

$G(m_n) = \{$the degenerate probability distribution that attaches probability 1 to $1/n$ and zero elsewhere$\}$

and

$G(m) = \{$the degenerate probability distribution that attaches probability 1 to 0 and zero elsewhere$\}$

Then $G(.)$ is not continuous with respect to TVN topology (see the Appendix).

On the other hand, if the range of G (and thus F) is any of the usual parametrized families of distributions (for example normal) and does not contain any degenerate distribution, then (weak) continuity of the parameters of these distributions as a function of the distribution of actions ensure that G is continuous with respect to TVN topology.[7]

Finally, I like to mention a new paper by Al-Najjar and Smorodinsky (1999) on this subject that has to come to my attention since I wrote the first draft of this paper. This chapter extends the result of Sabourian (1990) in Theorem 1 to repeated games with non-anonymous signal functions by imposing a stronger set of conditions on the underlying game (in particular, it assumes that the stage payoff functions are continuous). This result is established by using the notion of 'influence' (pivotal) that measures the impact of a change of a single player's action on the expected value of the collective outcome. Building on their earlier work (Al-Najjar and Smorodinsky, 1998), they show that for any given level of influence ϵ, the number of players who have more than ϵ influence on the collective outcome in any period is bounded by number $\gamma(\epsilon)$. This number, in their set-up, turns out to be independent of the number of players and it holds uniformly over all strategy profiles. This establishes that as the number of players increases to infinity the proportion of agents that have a positive influence on the continuation game becomes arbitrarily small. Thus, with a large number of players almost all players play the repeated game as if it is a one-shot game.

15.3 DYNAMIC MATCHING AND BARGAINING WITH A FINITE NUMBER OF PLAYERS

The theory of bargaining has been extensively studied. Dynamic matching models with explicit bargaining and a continuum of agents have been used to provide a game-theoretic foundation to the competitive equilibrium (see Osborne and Rubinstein (1990) and Gale (2000)). Rubinstein and Wolinsky (1990) – henceforth referred to as RW – consider a simple dynamic matching and bargaining model with a finite number of players. The predictions of this model turn out to be radically different from that of the competitive market even for the case in which there are no transaction costs or frictions. In this section, I shall first discuss RW's results and then discuss some recent work by Gale (2000) and Sabourian (1999) which shows that the predictions of RW's model are consistent with the competitive outcome if the agents are assumed to be 'boundedly rational' in specific ways.

15.3.1 RW's model with random matching

RW consider a market with B identical buyers and S identical sellers. Each seller has a single unit of an indivisible commodity. Each buyer would like to buy precisely one unit of the commodity. The utility that each buyer derives from consuming one unit of the good is one and the disutility of parting with a unit of the good (the reservation price) for each seller is zero. Assume, throughout, that

$$B > S.$$

Thus the competitive solution in this model is such that the sellers receive the entire surplus and all available units are sold at the price 1.

The play of the dynamic matching and bargaining game takes place in discrete time. Let $\delta \in [0, 1]$ be the common discount factor for each player. Thus if at any period $t = 0, 1, 2, \ldots$ a buyer and a seller agree to the sale of a single unit of the good at a price p then the seller and the buyer receive a payoff of $\delta^t p$ and $\delta^t(1 - p)$ respectively.

At each period the remaining agents in the market are matched in pairs of one seller and one buyer. The matching process is random and all possible seller–buyer matches are equally probable. When two agents meet, each has an equal probability (probability 1/2) of being chosen as a proposer. Once an offer is made by the proposer, the other agent accepts (A) or rejects (R) the offer. If the proposal results in an acceptance, the agreement is implemented and the parties leave the market. If the offer is rejected the match is dissolved and the parties return to the market to join the next matching period. In any period, unmatched buyers are required to remain inactive until the next matching stage.

At each period t each agent is assumed to know everything that has happened in the market up to the end of period $t - 1$, including all the past outcomes in matches in which he was not a party to. In addition, at each period t, each player knows the identity of his match at t, the selection of the proposer, and in the case of a responder, the proposal. However, at any period t, when players select their actions they do not know the identity of the other matches and what actions are being simultaneously chosen by other agents.

Notice that if $S > 1$ then there is more than one match in each period. Thus, the game is one of imperfect information when there is more than one seller. Therefore, the appropriate equilibrium concept, in this case, is sequential equilibrium (perfect Bayesian equilibrium). With $S = 1$, it is sufficient to use subgame perfect equilibrium as the solution concept.

15.3.2 Equilibrium characterization for the case of $\delta = 1$

The case for the competitive outcome(s) is often made for economies in which there are no transaction costs and/or frictions. In RW's model, the only possible transaction cost (friction) is due to the assumption of discounting. For the case of no discounting, RW's provide the following characterization result.

THEOREM 2 *If $\delta = 1$ then for every price $\overline{p}$ between 0 and 1 and for every one to one function β from the set of sellers to the set of buyers there exists a sequential equilibrium in which each seller s sells one unit of the good to buyer $\beta(s)$ for a price $\overline{p}$.*

Since in a competitive equilibrium all goods are sold at the unique price of 1 (because $B > S$), the above result demonstrates that, even when there are no transaction costs, a continuum of non-competitive prices can be sustained as a sequential equilibria in the above dynamic matching and bargaining game with a finite number of agents.

The intuition behind the proof for the case of $S = 1$ is the following. There is a distinguished buyer $\overline{b}$ who has the 'right' to buy the single seller's good at $\overline{p}$ ($\overline{b}$ depends on the past history of play). The equilibrium strategies are such that whenever the seller meets the distinguished buyer, which ever is chosen as the proposer offers a price $\overline{p}$ and the responder accepts. Whenever the seller meets a buyer $b \neq \overline{b}$, the seller as a proposer offers the good at a price $p = 1$ and the buyer $b \neq \overline{b}$ offers to pay a price $p = 0$. In both cases, the responders reject the offers. The outcome of these strategies is that the seller sells the good to the buyer $\overline{b}$ at $\overline{p}$.

To show that it does not pay players to deviate from the above, the strategies further specify the following responses to any deviations. If the seller proposes a price different from the equilibrium price ($\overline{p}$ to $\overline{b}$ and 1 to $b \neq \overline{b}$) to any buyer $\widetilde{b}$ then this buyer rejects and he has the 'right' to buy the good at a price $\widetilde{p} = 0$. Thus, the continuation strategy is the same as above with the price $\widetilde{p}$ in place of $\overline{p}$ and the buyer $\widetilde{b}$ in place of $\overline{b}$.

If one of the buyers deviates from their equilibrium strategies then the seller rejects and another buyer $\widehat{b}$ gets the right to buy the good at a price $\widehat{p} = 1$. The continuation strategy is the same as before with the price $\widehat{p}$ in place of $\overline{p}$ and the buyer $\widehat{b}$ in place of $\overline{b}$.

Further deviations can be treated in exactly the same way.

It is easy to check that it does not pay any player to deviate from the above strategy after any history. Clearly, any initial deviator is no better off from deviating given the punishments. Also after any deviation any responder is at least as well off rejecting the proposed deviation and following the punishments than from accepting the proposed deviation.

Notice that the strategies are quite complicated and the behaviour of each agent at any period depends on the history of play up to that period – there are potentially an indefinite number of potential deviations and for each deviation the above strategy profile specifies a tailor-made response in order to deter the deviation. Thus the agents need a large amount of information to implement the above strategy profile.[8] At the other extreme, we can assume that at any period the agents only have access to the history of play in that period and cannot condition their behaviour on the previous history of plays. Thus for the purpose of comparison, one can consider history-independent (stationary or Markov) strategies. RW show that the only stationary equilibrium outcome is the competitive one. In fact, their result is slightly stronger.

THEOREM 3 *If at any time each player's information consists only of the set of players that are present in the market at time t and the time itself then the unique sequential equilibrium price is the competitive price of 1.*

The above informational restriction prevents agents from punishing a deviator since the deviator is not remembered. For example, in the proof of Theorem 2, any deviation by the seller was rejected by the responder because the rejection led to a reward for the buyer. In Theorem 3 with stationary strategies the buyer could not be rewarded because the deviation of the seller could not be observed.

In the literature on dynamic games, it is often the case that only stationary/Markov equilibria are considered. There are very few formal attempts at justifying such equilibria (Piccione and Rubinstein, 1993; Maskin and Tirole, 2001; Chatterjee and Sabourian, 1999, 2000 are some exceptions). By appealing to some elements of 'bounded rationality', Gale (2000) and Sabourian (2000) show that the competitive price is the unique outcome in RW's model and thereby provide some justification for stationary/Markov equilibria in these class of games. However, these two papers formalise bounded rationality in different ways; here, I will briefly sketch their arguments. Before addressing the works of Gale (2000) and Sabourian (2000), I will briefly discuss the no discounting assumption and alternative matching technologies in RW's model.

15.3.3 Discounting and matching

With discounting and random matching, it is not possible to establish the existence of a continuum of sequential equilibrium prices as in Theorem 2. RW establish this theorem by constructing strategies that induce special 'relationships' between buyers and sellers after every history. As was described in the previous subsection, these relationships involve a buyer obtaining the right to buy a good provided by a specific seller at a particular price. With no discounting, the threat of forming new relationships deter deviations from the equilibrium strategies in RW's set-up. With discounting, the threat of such new relationships may no longer be credible. This is because, with random matching, after any history the expected time that elapses before the designated members of this new relationship meet each other may be very long (for example, this will be the case if S and B are large). This is not important when there is no discounting. However, with discounting, new relationship that take (on average) a long time to occur do not have sufficient deterrence value. As a result, with discounting such strategies may not constitute a credible equilibrium. In fact, RW establish the following result for the one seller case.

THEOREM 4 *(See RW) Suppose that* $S = 1$ *and* $\delta \in (0, 1)$. *Then the subgame perfect equilibrium outcome is unique and the agreement is reached in the first period. Moreover, the unique equilibrium price converges to the competitive price of 1 as* $B \to \infty$ *or as* $\delta \to 1$.[9]

The convergence of the equilibrium prices to the competitive price of 1 as $\delta \to 1$ seems to throw some doubt on the multiplicity result in Theorem 2. However, RW argue that with random matching discounting has an unrealistic feature. If players discount the future then holding a special relationship becomes costly. In particular, a pair of agents face a cost of maintaining their relationship even after they are matched. But why should staying with one's current partner be costly? Thus, they consider a new model of matching in which a matched pair of agents do not have to separate at the end of a bargaining session. With this new matching model with an endogenous choice of partner they establish the continuum of (non-competitive) equilibria even for the case in which the players discount the future.

THEOREM 5 *(See RW) Suppose that there is one seller s and in each period s chooses the buyer with which to bargain with. Then for each buyer b and any price* $(2 - \delta)/2 \leq p \leq 1$ *there exists a subgame perfect equilibrium outcome such that (i) s always chooses b in the first period and (ii) either s is selected as the proposer, in which case they agree on the price p, or b is selected as the proposer, in which case they agree on the price* $\delta p/(2 - \delta)$.

Thus, even with discounting, it is possible to demonstrate the existence of a large number of non-competitive sequential equilibrium outcomes, irrespective of the number of buyers.[10]

15.3.4 Some attempts at providing a justification for the competitive outcome in RW's model

First some notation. Let

- e be the outcome of all actions (plays) in all matches in a given period.
- $h^t = (e^1, \ldots, e^{t-1})$ be the history of outcome of plays in all matches up to period t.
- H^t be the set of all possible histories of plays (outcomes in all matches) up to period t.
- $H^\infty = \cup_{t=1}^{\infty} H^t$ be the set of all finite histories of plays.
- d_i be the information that any player i receives in any period of the bargaining; thus either d_i says that i has been selected to make a proposal to some player j or d_i consists of a price offer by some player j to i.
- D_i be the set of all d_is.
- f_i be the strategy of player i in the bargaining game; thus $f_i : H^\infty \times D_i \to A \cup R \cup [0, 1]$.
- F_i be the set strategies for i.
- $\langle f_i \mid h\rangle$ be the strategy induced by $f_i \in F_i$ after history $h \in H^\infty$; thus for any h and $h' \in H^\infty$

$$\langle f_i \mid h\rangle(h') = f_i(h, h').$$

- $\pi_i(f_i, f_{-i})$ be the expected payoff to player i if the strategy profile (f_i, f_{-i}) is chosen.

15.3.4.1 Gale (2000)

For any strategy profile f, let $I(f) \equiv \{f' \in F | f' = \langle f \mid h\rangle \text{for some } h\}$ be the set of all strategies induced by f after some h. Effectively $I(f)$ is the set of rules (strategies) within the strategy profile f. At the beginning of the game the players follow the strategy profile $f \in I(f)$; if the outcome e^1 occurs in the first period then the players follow the rule (strategy) $\langle f \mid e^1\rangle \in I(f)$ in the second period and so on. Thus, if the players choose a profile f then the players effectively make transitions between rules depending on the outcome in any period. For any outcome of plays e in a period, this can be formally described by a transition function

$$\Psi(f', e) = \langle f' \mid e\rangle \quad \text{for all} \quad f' \in I(f).$$

Now Gale (2000) introduces a small amount of randomness into the transitions between rules. In particular, he fixes the strategy profile f and some $\epsilon > 0$; then he assumes that if the players are following a rule $f' \in I(f)$ and an outcome e happens then they follow the rule (strategy) $\langle f' \mid e\rangle$ with probability $(1 - \epsilon)$ and with probability ϵ they uniformly follow all the rules in $I(f)$. Thus, for strategy profile f such that $I(f)$ is finite, the transition between members of $I(f)$, in Gale's set-up, is given by:

$$\Psi(f', e) = \begin{cases} \langle f' \mid e\rangle & \text{with probability} \quad 1 - \epsilon + \epsilon/N \\ f'' \neq \langle f' \mid e\rangle & \text{with probability} \quad \epsilon/N \end{cases}$$

where N refers to the cardinality of the set $I(f)$.

The above randomness ensures that there is always a small probability that in any continuation game all the rules within the strategy profile is chosen with a positive probability. Now, Gale's notion of an equilibrium consists of a profile of strategies $f = (f_i, f_{-i})$ that is a subgame perfect equilibrium in the game with the above ϵ transition trembles. Clearly, his equilibrium notion is the same as the standard subgame perfect equilibrium when $\epsilon = 0$. However, with $\epsilon > 0$, Gale shows that the unique equilibrium outcome is the competitive one in the case in which there is a single seller s. I shall now provide a sketch of Gale's arguments for the case of an equilibrium profile f for which $I(f)$ is finite and the seller always makes the offer.

Denote the elements of $I(f)$ by $f^1, f^2, \ldots, f^N$. Let $v_i(f^n)$ be the expected payoff of i in the game with transition error ϵ if the players choose f^n with probability $(1-\epsilon)$ and with probability ϵ they uniformly follow all the rules in $I(f)$. Thus

$$v_i(f^n) = (1-\epsilon)\pi_i(f^n) + (\epsilon/N) \sum_{f' \in I(f)} \pi_i(f')$$

Also, let

$$1 - z = \min_n v_s(f^n) \qquad (15.2)$$

Then,

$$\sum_{b \in B} v_b(f^n) \leq z \quad \text{for all} \quad n$$

where B is the set of buyers. Thus for each n there exists at most one buyer $b(n)$ such that $v_{b(n)}(f^n) > z/2$. But this implies that there exists a buyer b such that $v_b(f^n) \leq z/2$ for at least a fraction $(N-1)/N$ of the rules (otherwise, at some n more than one buyer have a continuation payoff of more than $z/2$). But then for any n

$$\begin{aligned} v_b(f^n) &\leq (1-\epsilon)z + \epsilon\left(\frac{1}{N}z + \left(\frac{N-1}{N}\right)z/2\right) \\ &= z\left(1 - \epsilon\frac{N-1}{2N}\right) \end{aligned}$$

Thus buyer b always accepts any offer $p < 1 - z\left(1 - \epsilon\frac{N-1}{2N}\right)$. But then z must be equal to 0. Otherwise, s can always guarantee himself more than $1 - z$ by making a price offer p such that $1 - z < p < 1 - z(1 - \epsilon(N - 1/2N))$. But this contradicts equation (15.2).

15.3.4.2 Sabourian (2000)

It is often argued that if players prefer simple strategies to more complicated ones then in equilibrium stationary strategies would be chosen.[11] Sabourian (2000) attempts to formalize this intuition, in the context of RW's model, by introducing complexity costs lexicographically with the standard payoff into the players' preference ordering as in Rubinstein (1986), Abreu and Rubinstein (1988), Piccione and Rubinstein (1993) and others.

The earlier papers of Rubinstein and others addressed the issue of equilibrium selection in a two-player repeated games by modelling players as finite-state automata.

Complexity costs in these papers was measured by the number of states of a machine. Here, since the game played in each period is itself an extensive form, there is not a unique way of specifying an automaton in the above matching and bargaining game. Sabourian (2000) formulates a particular specification of an automaton. With this specification, the minimum number of machine states needed to implement a particular strategy $f_i \in F_i$ turns out to be the cardinality of the set of induced strategy

$$I_i(f_i) = \left\{ f_i' \middle| f_i' = (f_i \mid h) \text{ for some } h \right\}. \tag{15.3}$$

But this measure of complexity does not measure fully the complexity of behaviour in each period. In the context of the random matching model, (Sabourian) introduce a different definition of complexity to measure the complexity of behaviour within a period. In particular, I assume the following complexity criterion. *A* strategy f_i is more complex than another machine f_i', denoted by $f_i \succ f_i'$, if the strategies f_i and f_i' are otherwise identical except that given some partial history (information) $d_i \in D_i$ in a single period, f_i' changes its action less often in response d than f_i (thus f_i' conditions less on the previous history of the game prior to d_i than f_i).[12]

This definition of complexity is a very weak concept and is sufficient to ensure that in equilibrium the competitive outcome is selected in the random matching model.

The basic equilibrium notion used in Sabourian (2000) is *Nash Equilibrium* of the game with complexity cost introduced lexicographically. Formally, a Nash equilibrium with complexity cost (denoted by NEC) is a strategy profile $f = (f_1, \ldots, f_n)$ that satisfies the following two conditions:

$$\pi_i(f_i, f_{\sim i}) \geq \pi_i(f_i', f_{\sim i}), \forall f_i',$$
$$\nexists f_i' \quad \text{such that} \quad \pi_i(f_i, f_{\sim i}) = \pi_i(f_i', f_{\sim i}) \quad \text{and} \quad f_i \succ f_i'.$$

Clearly, the concept of NEC does not put any restriction on the behaviour of the agents 'off-the-equilibrium' path. One way of ensuring credibility is to restrict attention to NEC profiles that are perfect Bayesian equilibrium (sequential equilibrium) of the underlying game. Sabourian (2000) does precisely this and refers to such a strategy profile as perfect Bayesian equilibrium with complexity costs (PBEC).[13]

The main selection result of Sabourian (2000) corresponds to this notion of equilibrium. In particular, for the random matching model of RW, I show that any PBEC-strategy profile f is stationary and induces the unique competitive price of 1, if set $I(f)$ is finite (finite memory).[14]

Here, to provide some intuition for the role of complexity I shall explain why the strategies that are used in the proof of Theorem 3.1 cannot constitute a PBEC. First, assume otherwise. Next, note that these strategies are non-stationary. In particular, the set of buyers $B' \equiv \{b \in B | b \neq \beta(s) \text{ for all } s \in S\}$ – this is the set of all those buyers who do not have any rights to any good – also follow non-stationary (complex) strategies. Third, note that any $b \in B'$ has a payoff of zero in the equilibrium constructed in the proof of Theorem 3.1. Since any $b \in B'$ can always obtain at least a payoff of zero by following a simpler strategy that always makes the same offer and accepts all offers, it follows that any such b can economize on complexity without sacrificing any payoff. But this is a contradiction.

The proof of the selection result for the case of a single seller in Sabourian (2000) is similar to above but applied to the continuation payoff. The selection result for an arbitrary number of sellers is demonstrated by applying an induction argument to the set of sellers.

Sabourian (2000) also considers RW's voluntary matching model. With voluntary matching, at the beginning of each period, each seller chooses his bargaining partner during that period. This introduces an additional element of complexity. In Sabourian (2000), I also show that the above selection result for the random matching model extends to models with voluntary matching if the notion of complexity is strengthened to include both the complexity of behaviour in a given period, as above, and the 'counting states' notion of complexity (the cardinality of the set $I_i(.)$ defined in (3.2)). Effectively, counting the number of states measures the complexity of conditioning the choice of one's partner at the beginning of each period on past history.

15.4 APPENDIX: WEAK CONVERGENCE AND TOTAL VARIATION NORM TOPOLOGY

For any measurable space (X, Σ), where X is an arbitrary set and Σ is the σ-algebra on X, I define the set of probability measures on (X, Σ) by $\Delta(X, \Sigma)$.

Definition 2 (Weak convergence) A sequence of probability measures $\{m_n\}_{n=1}^{\infty}$ in the set $\Delta(X, \Sigma)$ converges weakly to a probability measure $m \in \Delta(X, \Sigma)$ if and only if for all bounded continuous real-valued function $f : X \to \mathcal{R}$ the following holds

$$\lim_{n\to\infty} \int f dm_n = \int f dm$$

Next, I define the total variation norm.

Definition 3 Consider any measurable set (X, Σ). The total variation norm of any probability measure $m \in \Delta(X, \Sigma)$ is defined by

$$|| m || = \sup \sum_{i=1}^{n} | m(B_i) |$$

where the supremum is over all finite partitions of X into disjoint measurable sets $B_1, \ldots, B_n$ and for any arbitrary real number r, $| r |$ refers to the magnitude of r.

The above total variation norm defines a metric on the space $\Delta(X, \Sigma)$. This metric defines the total variation topology.

Definition 4 (Total variation norm topology) A sequence of probability measures $\{m_n\}_{n=1}^{\infty}$ in the set $\Delta(X, \Sigma)$ converges in the total variation norm to the probability measure $m \in \Delta(X, \Sigma)$ if and only if $\lim_{n\to\infty} || m_n - m || = 0$.

The following result (see Stokey and Lucas, 1989 Section 11.3) summarizes the properties of the total variation norm topology.

DEFINITION 5 For any sequence of probability measures $\{m\}_{n=1}^{\infty}$ in the set $\Delta(X, \Sigma)$ and for any probability measure $m \in \Delta(X, \Sigma)$, the following conditions are equivalent:

(i) $\lim_{n\to\infty} \| m_n - m \| = 0$,
(ii) $m_n(B)$ converges uniformly to $m(B)$ as $n \to \infty$ for any measurable set $B \in \Sigma$,
(iii) for any real-valued bounded measurable function f defined on (X, Σ)

$$\lim_{n\to\infty} \int f dm_n = \int f dm$$

and this convergence is uniform for all f such that $\sup_{x\in X} | f(x) | \leq 1$.

To illustrate how strong the TVN topology is relative to weak convergence consider the following deterministic example (also found in Stokey and Lucas, 1989).

Example 1 Let $X = [0, 1]$ and for any integer n let $m_n = \{$the degenerate probability distribution that attaches probablility 1 to $1/n$ and zero else$\}$. Clearly, the sequence $\{m_n\}$ is equivalent to the sequence

$$1, \frac{1}{2}, \frac{1}{3}, \ldots$$

and this sequence converges to zero. Now let $m = \{$the degenerate probability distribution that attaches probability 1 to 0 and zero else$\}$.

Does m_n converges to m either with respect to the TVN topolgy or with respect to weak topology? First, consider the real valued function $f : X \to \mathcal{R}$ given by

$$f(x) = \begin{cases} 1 & \textit{if } x = 0 \\ 0 & \text{otherwise} \end{cases}$$

Then $\int f dm_n = 0$ for all n and $\int f dm = 1$. Therefore, by (iii) of Definition 5, m_n does not converge to m with respect to TVN topology.

Next note that for any bounded real valued function $f : X \to \mathcal{R}$ we have $\int f dm_n = f(1/n)$ for all n and $\int f dm = f(0)$. Therefore, if f is continuous then $\lim_n \int f dm_n = \int f dm$. Thus m_n converges weakly to m.

NOTES

1 For example, in the case of the repeated games, the Folk Theorem holds for games with a finite number of (anonymous) players but not for the case of a continuum of anonymous players (see below).
2 See also Mas-Colell (1983) for a survey on the static Nash equilibrium foundation of the competitive equilibrium.
3 With perfect monitoring this result holds even for the case with a continuum of firms.
4 Green (1980) demonstrated this result by restricting players to trigger-type strategies. Sabourian (1990) generalizes it to all strategies.
5 Measurability of V_k is with respect to its second argument.
6 See the appendix for a formal statement of this result.
7 In the context of finite (three period) extensive form game, Levine and Pesendorfer (1995) establish a similar result as in Sabourian (1990) by assuming directly that the range of G are

distributions that have continuously differentiable density functions and by assuming that the level of noise disappears as the number of players increases, but not too rapidly.

8 RW also obtain the same result for the case in which each player observes only his own past history.

9 No equivalent result is known for the case of more than one seller.

10 Theorem 5 can be extended to the case in which there is more than one seller.

11 For example, Gul (1989) mentions simplicity as a reason for selecting stationary equilibria in the non-cooperative coalitional bargaining. (See also Osborne and Rubinstein (1990) chapter 3 for an argument against this view.)

12 Chatterjee and Sabourian (1999, 2000) use a similar notion of complexity to justify stationary equilibria in n-person alternating bargaining games.

13 Alternatively, as in Chatterjee and Sabourian (1999, 2000), one could ensure credibility by introducing noise into the system and consider extensive form trembling hand equilibrium with complexity costs.

14 If complexity costs enter the preference ordering of the agents as an arbitrary fixed positive cost (rather than lexicographically) then one does not need to assume that $I(f)$ is finite.

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16 Applications of the classical approach

Bertram Schefold

The applications of the classical approach in a broad sense have been many, starting with the classical authors themselves, and present-day economists often inadvertently use a methodology which is closer to that of the classicals than to that of the neoclassicals. The relationship between intertemporal general equilibrium and models of prices of production deserves first to be clarified in this context. More specific applications include the extensions of input–output models to long-run prices and distribution, the analysis of joint production, with special extensions to energy analysis, to exhaustible resources and others. Finally, historical applications concern the analysis of technical progress and of different economic systems.

16.1 CRITIQUE OF CAPITAL THEORY: RELATING STEADY-STATE COMPARISONS TO INTERTEMPORAL EQUILIBRIUM

Classical theory primarily is concerned with the long-period position of an economy in which the gravitation of market prices to prices of production has resulted in a uniform rate of profit. The gravitation in conditions of free competition without external shocks usually is postulated like an axiom of the theory, but it is also possible to model processes of gravitation (they can take very different forms) in order to analyse the conditions under which it takes place. A workshop on gravitation was held at the Certosa di Pontignano in 1990 in which most participants "agreed that prices of production represent in some sense a guide to the persistent components of competitive market prices" (Caminati and Petri, 1990: 9). A tendency towards long-period equilibrium is also inherent in Arrow–Debreu intertemporal equilibria; it has been formalised in turnpike-models like that by Epstein (1987). Epstein uses recursive utility functions with an infinite time horizon: the rates of return on capital in all lines of production and the rates of time preference of consumers all converge to one and the same rate, provided certain conditions are met. This process of gravitation is special in so far as perfect foresight is assumed, and it results in a special kind of long-period equilibria: the neoclassical theory of distribution holds (Schefold, 1997, Chapter 18.1).

Despite the special nature of intertemporal equilibrium, this process of convergence may help to elucidate the relationship between classical and neoclassical theory. Classical theory became a focus of attention in post-war economic theory when the problems of capital theory were discussed in the 'sixties in terms of steady-state comparisons. The reswitching debate showed that an inverse monotonic relationship between the rate of interest and the intensity of capital did not necessarily exist so that the existence of aggregate production functions in particular had to be doubted (for a summary, see Harcourt, 1972). But intertemporal equilibrium did not seem to be affected by this critique. Today, turnpike properties of intertemporal equilibrium offer one way to demonstrate that the problems of capital theory also surface in an intertemporal equilibrium context. In this section of my paper I shall provide a summary of this approach (for a more extensive version with proofs of the theorems see Schefold, 1997, Chapter 18.2), while the remaining sections will be concerned with more concrete and positive (as opposed to critical) applications of modern classical theory.

A simple and economically relevant example of the problems of capital theory is provided by the following parable[1]: Production in an economy is assumed to be represented by a well-behaved production function. There is a certain level of population L_1 and of capital accumulated K_1; both are fully employed. The ratio of the wage rate to the rate of interest is equal to the absolute value of the slope of the tangent to P_0 in Figure 16.1.

An immigration now takes place so that the equilibrium is disturbed in the short run and real wages fall relative to the rate of interest. Less mechanised techniques become profitable. If they are introduced, a new full employment equilibrium, with a higher level of output but at the same level of capital accumulation, is attained at P_1 in the long run. We now attempt to model the same scenario with heterogeneous capital goods in a comparison of stationary long-period equilibria. $\mathbf{A}_\alpha$ is an indecomposable

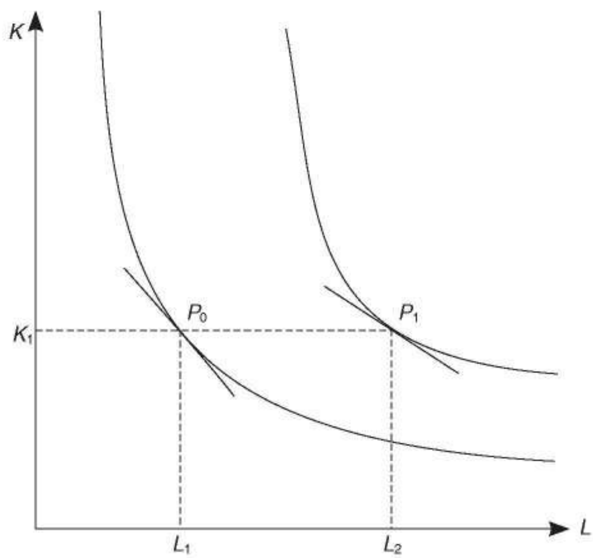

Figure 16.1 A parable: "immigration". Increase in population from L_1 to L_2, K_1 given.

and productive input–output matrix and $\mathbf{l}^\alpha$ the associated positive labour vector; long-run prices $\mathbf{p}^\alpha$ with rate of interest r and wage rate w_α are then given by

$$(1+r)\mathbf{A}_\alpha\mathbf{p}^\alpha + w_\alpha\mathbf{l}^\alpha = \mathbf{p}^\alpha.$$

These equations define a wage curve pertaining to technique α if some numéraire (usually the vector of net output) is given. If the wage curves of two techniques α and β for the production of the same goods were linear – as was assumed in Samuelson's construction of the surrogate production function – we could represent the parable as in Figure 16.2. As is well known, the slope of linear wage curves measures the capital–labour ratio. The immigration scenario implies a movement from a technique with a high capital–labour ratio – say, Q_1 on wage curve w_α – to a technique with a low capital–labour ratio – say, point Q_2 on wage curve w_β. We assume that the change of technique implies a change in the capital–labour ratio which is just sufficient to absorb the increase in the amount of the variable factor, given the amount of the constant factor. In this steady-state comparison, we ignore the problem of saying what it means to keep the constant factor ("capital") constant.

Reswitching in the simplest case is depicted in Figure 16.3. It is well known that wage curves are – except in fluke cases primarily associated with the labour theory of value – not straight. The capital–labour ratio is, for a wage curve such as w_β at a point such as Q_1^*, determined by

$$k_\beta = \operatorname{tg}\mu_\beta = \frac{w_\beta(0) - w_\beta(r)}{r} = \frac{P/L}{P/K} = K/L,$$

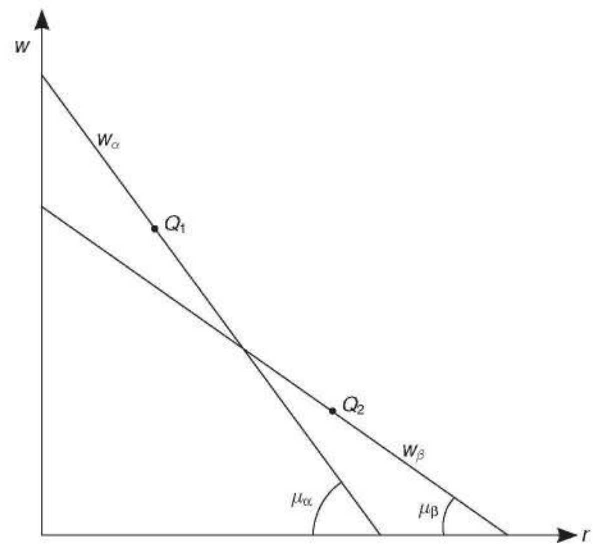

Figure 16.2 Linear wage curves, representing techniques with high capital–labour ratio $k_1 = \operatorname{tg}\mu_\alpha$, at Q_1, and low capital–labour ratio $k_2 = \operatorname{tg}\mu_\beta$, at Q_2. Transition Q_1 to Q_2 corresponds to "immigration".

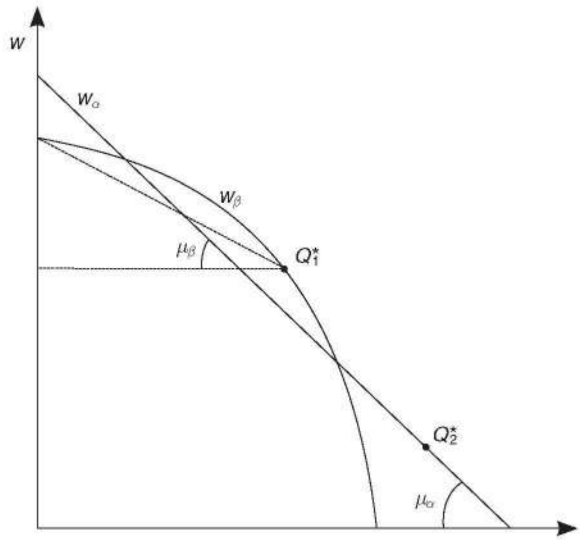

Figure 16.3 Two wages curves with reswitching. A transition from a higher to a lower capital–labour ratio ("immigration scenario") now involves a movement from Q_2^* to Q_1^*.

since $w_\beta(0)$ is the price of net output per head in the stationary state. The immigration scenario now is a movement like that from Q_2^* to Q_1^*. In order to preserve full employment by moving from a technique with a high capital–labour ratio to one with a low capital–labour ratio ($\text{tg}\,\mu_\alpha = k_\alpha > \text{tg}\,\mu_\beta = k_\beta$), the real wage has to be raised in reaction to the immigration of labour. This counterintuitive move of a factor price allows to restore full employment after the immigration if technique β is the only alternative to technique α and if the amount of capital is somehow given and somehow kept constant during the transition.

The main point in the debate about reswitching was that it involved such counterintuitive moves of factor prices. Our task is to represent the transitions in an intertemporal model. The intertemporal model is based on a spectrum of techniques $(\mathbf{A}, \mathbf{B}, \mathbf{l})$, where $\mathbf{A}$ is the input matrix (with semi-positive rows), $\mathbf{B}$ the output matrix (with semi-positive columns) and $\mathbf{l}$ the (positive) labour vector, composed of a finite number of methods of production, to produce n goods which are both consumption goods and capital goods. Time is divided into periods of production $1, \ldots, T$. Endowments of capital goods are available at the end of period 0. In each period of production, a positive amount L_t of labour is available. Activity levels are given by row vectors $\mathbf{q}^t, t = 0, \ldots, T$. Similarly, we have consumption vectors $\mathbf{c}^t$, $t = 0, \ldots, T$.

At the end of period $t-1$, an output of $\mathbf{q}^{t-1}\mathbf{B}$ is available. It divides into consumption, $\mathbf{c}^{t-1}$, and the inputs for production in period t, $\mathbf{q}^t\mathbf{A}$. At the end of period $t-1$, goods will be sold at prices $\mathbf{p}^t$. At the end of period t, a wage according to wage rate w_t will be paid.

At the beginning of the first period, a stock $\mathbf{q}^0\mathbf{B} > 0$ will be given. At the end of period T, a final stock of goods $\mathbf{f}$ will be required to exist: $\mathbf{f} \geqq 0$. The assumption of a positive final stock of goods will often be made in order to prevent the economy from shrinking to nil in the last period and to allow – albeit in an arbitrary manner – for the possibility of a stationary state with a finite horizon.

There is one consumer, characterised by a utility function $U = U(\mathbf{c}^0, \mathbf{c}^1, \ldots, \mathbf{c}^T)$, which is positive and strictly concave. We assume positive first and negative second partial derivatives, in each variable c_i^t; $t = 0, \ldots, T$; $i = 1, \ldots, n$.

An *equilibrium* is a programme $\mathbf{z} = (\bar{\mathbf{c}}^0, \bar{\mathbf{c}}^1, \ldots, \bar{\mathbf{c}}^T, \bar{\mathbf{q}}^1, \ldots, \bar{\mathbf{q}}^T) \geq 0$, together with prices and wage rates $\mathbf{u} = (\mathbf{p}^0, \ldots, \mathbf{p}^T, w_1, \ldots, w_T) \geq 0$ such that, with $\bar{\mathbf{q}}^0 = \mathbf{q}^0$,

(a) the following equilibrium conditions hold:

reproduction feasible	*rule of free goods*
$\bar{\mathbf{q}}^{t-1}\mathbf{B} \geqq \bar{\mathbf{c}}^{t-1} + \bar{\mathbf{q}}^t\mathbf{A}$,	$(\bar{\mathbf{q}}^{t-1}\mathbf{B} - \bar{\mathbf{c}}^{t-1} - \bar{\mathbf{q}}^t\mathbf{A})\mathbf{p}^{t-1} = 0$; $t = 1, \ldots, T$;
$\bar{\mathbf{q}}^T\mathbf{B} \geqq \bar{\mathbf{c}}^T + \mathbf{f}$,	$(\bar{\mathbf{q}}^T\mathbf{B} - \bar{\mathbf{c}}^T - \mathbf{f})\mathbf{p}^T = 0$,
$L_t \geqq \bar{\mathbf{q}}^t\mathbf{l}$,	$(L_t - \bar{\mathbf{q}}^t\mathbf{l})w_t = 0$; $t = 1, \ldots, T$;
competition	*maximisation of profits*
$\mathbf{B}\mathbf{p}^t \leqq \mathbf{A}\mathbf{p}^{t-1} + w_t\mathbf{l}$,	$\bar{\mathbf{q}}^t(\mathbf{B}\mathbf{p}^t - \mathbf{A}\mathbf{p}^{t-1} - w_t\mathbf{l}) = 0$; $t = 1, \ldots, T$;

(b) and such that the condition of the maximisation of utility of the household is fulfilled:

$$U(\bar{\mathbf{c}}^0, \ldots, \bar{\mathbf{c}}^T) \geqq U(\mathbf{c}^0, \ldots, \mathbf{c}^T)$$
$$\text{in the set H of all } (\mathbf{c}^0, \ldots, \mathbf{c}^T) \geqq 0$$
$$\text{such that } \mathbf{c}^0\mathbf{p}^0 + \cdots + \mathbf{c}^T\mathbf{p}^T \leqq \mathbf{q}^0\mathbf{B}\mathbf{p}^0 - \mathbf{f}\mathbf{p}^T + w_1L_1 + \cdots + w_TL_T,$$

i.e. such that utility is maximised among all consumption bundles which can be bought with the budget of the household.

It should be noted that overproduction of goods will not occur because goods are consumption goods as well as means of production. The wage rate, however, may be zero, if full employment (in the literal sense) is not attained; the unemployed retreat to a subsistence economy (this assumption is more plausible if a subsistence wage is included in the coefficients of the input matrix). The rule of free goods applies to the labour market.

THEOREM

1 *There is an optimum. It is uniquely determined. Each optimum is an equilibrium.*

2 *An equilibrium exists. It is uniquely determined. Each equilibrium is an optimum.*

We are interested in this relatively simple model of intertemporal equilibrium because it provides a link with the linear models familiar from the debate on capital theory. We shall soon see that the competitive price system described by conditions (a), together with the conditions of reproduction, contains stationary or steadily growing Sraffa systems as special cases. But we shall also see that the price paths described by conditions (a) converge under fairly general conditions towards solutions with a uniform rate of profit or interest so that it can also be argued that the intertemporal equilibrium then describes a process of convergence of "market prices" to the "normal prices" of the long-period. We shall use this property of intertemporal equilibrium in order to construct the transitions between techniques which involve reswitching, using a representative consumer.

The question now is whether utility functions exist which generate the kind of transitions we have in mind. Such a utility function only would have to fulfil the condition that marginal utilities are equal to prices, as determined by the conditions of reproduction. This follows from the following corollary:

COROLLARY *Let a strictly concave utility function U and a programme* $\bar{\mathbf{z}} = (\bar{\mathbf{c}}^0, \ldots, \bar{\mathbf{c}}^T, \bar{\mathbf{q}}^1, \ldots, \bar{\mathbf{q}}^T)$ *with prices* $\mathbf{u}$ *be given which fulfil the equilibrium conditions (a), and for which we have in* $(\bar{\mathbf{c}}^0, \ldots, \bar{\mathbf{c}}^T)$

$$\frac{\partial U}{\partial c_i^t} = p_i^t; \quad t = 0, \ldots, T; \quad i = 1, \ldots, n.$$

Then $\bar{\mathbf{z}}$ *is an optimum with respect to U and* $(\bar{\mathbf{z}}, \mathbf{u})$ *is an equilibrium.*

Example Let a programme $\bar{\mathbf{z}}$ with prices $\mathbf{u}$ be given such that the equilibrium conditions (a) are fulfilled, with $\sum p_i^t < 1$, $\bar{c}_i^t < 1$. The utility function (cf. Geary 1950/51)

$$U^* = \prod_{i,t} (1 + c_i^t - \bar{c}_i^t)^{p_i^t}$$

renders $\bar{\mathbf{z}}$ an optimum and $(\bar{\mathbf{z}}, \mathbf{u})$ an equilibrium.

The trick therefore is as follows: Suppose we know the prices and quantities of a programme, fulfilling conditions (a) of equilibrium. Conditions (b) will then also be fulfilled if these equilibrium prices and quantities are treated as parameters in the function U^*, and equilibrium is defined with U^* regarded as given.

We shall start our investigation by analysing the logic of price formation and of the choice of technique which is implicit in the equilibrium conditions (a). Because of competition, prices fulfill

$$\mathbf{B}\mathbf{p}^t \leqq \mathbf{A}\mathbf{p}^{t-1} + w_t\mathbf{l}.$$

We now assume single production and consider a programme with positive consumptions; reproduction is feasible. Prices are positive (interior solution) but the wage rate may be zero. For each good it is necessary to activate at least one process producing it. There therefore has to be at least one profitable process for each good, according to the profit maximising condition. For each good we choose one profitable process producing it. Taken together, those profitable processes form a square system $\mathbf{A}_\sigma, \mathbf{l}^\sigma$ such that

$$\mathbf{p}^t = \mathbf{A}_\sigma\mathbf{p}^{t-1} + w_t\mathbf{l}^\sigma.$$

This technique σ will then be called "temporarily dominant", i.e. dominant in period t. All other processes, not in technique σ, in the spectrum of techniques $(\mathbf{A}, \mathbf{B}, \mathbf{l})$, will then make losses and only exceptionally be as profitable as processes in σ. This means that any other square system forming a technique α chosen from the spectrum of techniques $(\mathbf{A}, \mathbf{B}, \mathbf{l})$, must fulfill

$$\mathbf{p}^t \leqq \mathbf{A}_\alpha \mathbf{p}^{t-1} + w_t \mathbf{l}^\alpha.$$

This formulation makes it clear that the dynamic of the choice of techniques and of the formation of prices appears to be determined entirely, once prices $\mathbf{p}^0$ "in the beginning" are known, and once distribution is given in the form of a fixation of wage rates in each period. Initial prices $\mathbf{p}^0$ and distribution are endogenous variables in the determination of equilibrium but we treat them as exogenous in much of what follows because we are interested in the inductive, forward-looking determination of prices according to the equilibrium conditions (a) in the case of single production. Once $\mathbf{p}^0$ and w_1 are known, $\mathbf{p}^1$ is determined. The procedure is analogous if wage rates are positive and given in each period. Technique σ is the cost-minimising technique in period t; it is convenient to combine the determination of prices and cost-minimisation by writing

$$\mathbf{p}^t = \mathbf{A}_\sigma \mathbf{p}^{t-1} + w_t \mathbf{l}^\sigma \leqq \mathbf{A}_\alpha \mathbf{p}^{t-1} + w_t \mathbf{l}^\alpha$$

for all rival systems α in the spectrum.

The system of quantities influences the dynamics of the choice of technique via distribution. The role of distribution will become easier to grasp once we introduce undiscounted prices. For each period, we define a new price vector and a new wage rate through $\tilde{\mathbf{p}}^t = \mathbf{p}^t/\mathbf{s}\mathbf{p}^t$, $\tilde{w}_t = w_t/sp^t$. Then we have $\tilde{\mathbf{p}}^t = (1 + r_s^t)\mathbf{A}_\sigma \tilde{\mathbf{p}}^{t-1} + \tilde{w}_t \mathbf{l}^\sigma$, where $\mathbf{s}$ is the basket of goods used as numéraire and where r_s^t is the own rate of interest in terms of $\mathbf{s}$: $1 + r_s^t = \mathbf{s}\mathbf{p}^{t-1}/\mathbf{s}\mathbf{p}^t$. If $w_t > 0$, we can also define $\hat{\mathbf{p}}^t = \mathbf{p}^t/w_t$ i.e. we can define prices in terms of the wage rate. It is to be noted that, conversely, discounted prices can be calculated from a sequence of undiscounted prices and own rates.

If techniques are compared in terms of long-term prices, at a given rate of profit, that technique will be superior which yields the highest wage in terms of any numéraire. It is called "dominant in the long-run". The competitive process therefore leads to the selection of the method which is dominant in the long run only after a number of periods which is sufficiently large for intertemporal prices to have approached to the long-run normal prices. Here, we only formulate a theorem which holds if the rate of interests is lower than the maximum rate of profits.

THEOREM *Let a sequence of undiscounted intertemporal prices be given which defines the temporarily dominant technique in each period. We assume the input matrices to be indecomposable, primitive and such that $w_t > 0, t = 0, 1, 2, \ldots$ (full employment). The own rate of interest in terms of* $\mathbf{s}$ *is assumed to be constant, therefore $r_s^t = r$, $t = 1, 2, \ldots$. The technique which is dominant in the long run at r, δ, is defined as the technique for which w in terms of long run normal prices*

$$\mathbf{p} = (1 + r)\mathbf{A}_\delta \mathbf{p} + w\mathbf{l}^\delta, \quad \mathbf{s}\mathbf{p} = 1$$

is largest (it is also assumed that $r < R_\delta, R_\delta$ being the maximum rate of profit of technique δ). This technique is known to minimise costs in terms of its own long run *prices. Then we have (if the same numéraire is used for all price vectors): Whatever the* $\tilde{\mathbf{p}}^0 \geq 0$ *in the sequence of intertemporal prices, there is a* t' *such that* δ *is temporarily dominant for all* $t > t'$ *and* $\tilde{\mathbf{p}}^t$ *converges to* $\mathbf{p}$ *and* $\tilde{\mathbf{w}}^t$ *to* $\mathbf{w}$ *(Morishima, 1964).*

A comparison in terms of long-run normal prices thus allows to decide at once which of two given techniques in the spectrum is superior, using the criterion of the higher wage rate at the given rate of profit. But if we consider a process of adaptation, using intertemporal prices, the adoption of the best technique will in general take place only after a certain time lag, for intertemporal prices may take many periods before they approach prices of production sufficiently closely. This "lagging-behind" may involve several changes of methods of production before the system eventually settles down, with intertemporal prices approaching long-run normal prices pertaining to the technique which is dominant in the long run.

A change of distribution may induce a change of technique. It is the central neoclassical idea that an increase of the rate of interest leads to the use of less capital and, with the accompanying reduction of the wage rate, the use of more labour. This is reflected in our system as well, if the change of methods of production is such that we can speak unambiguously of "more" or "less" capital, independently of relative prices. If we have an increase of r and a reduction of w, a transition from a method $\mathbf{a}_1$ to a method $\mathbf{a}_0$, both producing the first good, with $\mathbf{a}_0 < \mathbf{a}_1$ and with $l_0 > l_1$, will be unambiguous. Two such techniques have only one switch point in common (Schefold, 1997: 263). Consider the switch point between two techniques α, β where long-run prices $\mathbf{p} = \mathbf{p}^\alpha = \mathbf{p}^\beta, w = w^\alpha = w^\beta$, therefore

$$(1 + r)\mathbf{a}_0\mathbf{p} + wl_0 = (1 + r)\mathbf{a}_1\mathbf{p} + wl_1,$$
$$w(l_0 - l_1) = (1 + r)(\mathbf{a}_1 - \mathbf{a}_0)\mathbf{p}.$$

If r is raised and w is lowered, and if relative prices change little, a cost advantage will appear on the left hand side of both equations. This means that the switch implies a transition to a less capital intensive technique which can here be defined as such unambiguously (independently of distribution and prices).

Reswitching may occur only where $\mathbf{a}_1 - \mathbf{a}_0$ has both positive and negative components. The second equation shows that it presupposes a considerable change of relative prices. How likely is reswitching? This question is at last being dealt with in the literature (d'Ippolito, 1987; Mainwaring and Steedman, 2000; Schefold, 1997: 57–68). The papers referred to suggest that the measure of the set of techniques allowing reswitching is generically positive but not large compared to the measure of all conceivable productive techniques. Reswitching may be rather unlikely, but there are many related phenomena of capital theory some of which are more "likely" to occur.

We now construct intertemporal equilibria with reswitching; as a preliminary exercise, we first confirm that steady states of finite duration may be represented as intertemporal equilibria. Let only technique α be given. Net output is represented by $\mathbf{d} = \mathbf{c}^t; t = 0, \ldots, T$; therefore $\mathbf{q}^t = \mathbf{q} = \mathbf{d}(\mathbf{I} - \mathbf{A}_\alpha)^{-1}$ and the labour force must fulfill $\mathbf{q}^t\mathbf{l} = \mathbf{q}\mathbf{l} = L_t$; $t = 1, \ldots, T$. We define $\mathbf{f} = \mathbf{q}^T - \mathbf{c}^T = \mathbf{q} - \mathbf{d}$. We assume that the initial endowments are such that the stationary state is possible: $\mathbf{q}^0 = \mathbf{q}$.

Diverse price systems are compatible with this artificial stationary state, according to the utility function. We suppose that r is given, $0 \leqq r < R_\alpha$, and we choose undiscounted prices $\tilde{\mathbf{p}}^t = \mathbf{p}^\alpha(r), \tilde{w}_t = w_\alpha(r), \mathbf{d}\tilde{\mathbf{p}}^t = 1$. We transform them into discounted prices by putting $\mathbf{p}^0 = \mathbf{p}^\alpha, \mathbf{p}^t = (1+r)^{-t}\tilde{\mathbf{p}}^t, w_t = (1+r)^{-t}\tilde{w}_t$.

We then choose, e.g. according to the example provided, a utility function U for which we have in $(c^0, \ldots, c^T)$ that marginal utility equals price, i.e. $\partial U/\partial c_i^t = p_i^t$ for all commodities in all time periods considered. According to the corollary, the stationary state then is an optimum and an (intertemporal) equilibrium.

We next want to show under what conditions the central neoclassical idea of a substitutability of labour for "capital" may be represented in our framework. In this paper, we only provide an intuitive description of this and the following cases; an extensive formal treatment is to be found in Schefold (1997, Chapter 18.2).

To this end, let our economy be in a stationary state for some time as in the previous case, i.e. for $t = 1, \ldots, t'$. Suppose there is a less capital intensive technique $(\mathbf{a}_0, \mathbf{l}_0)$, with $\mathbf{a}_0 < \mathbf{a}_1$, $l_0 > l_1$; this means, for $t = 1, \ldots, t'$ we have

$$(1+r_1)\mathbf{a}_0\tilde{\mathbf{p}}^{t-1} + \tilde{w}_t l_0 > (1+r_1)\mathbf{a}_1\tilde{\mathbf{p}}^{t-1} + \tilde{w}_t l_1 = \tilde{p}_1^t.$$

Because of $\mathbf{a}_0 < \mathbf{a}_1, l_0 > l_1$, it is clear – independently of prices! – which technique employs less capital per man. At the end of period t' we still have $\tilde{\mathbf{p}}^{t'} = \mathbf{p}^\alpha$, but in the subsequent period, a change of distribution occurs, accordingly also a change of technique and of consumption, because our representative consumer will have more labour at his disposal than before: $L_{t'+1}$ exceeds $L_{t'}$. One expects a fall of wages and a rise of the rate of profit, and both will now be assumed. In an intertemporal equilibrium, the adaptation of intertemporal prices will be gradual; own rates of interest rise first. We assume that $r_s^{t'+1} = r_2$, corresponding to a move from Q_1 to Q_2 in Figure 16.2, and that r_s^t remains constant thereafter.

This "normal" reaction in distribution entails a "normal" reaction in the choice of technique. We consider the side of quantities, assuming that the technique changes between t' and $t'+1$. For simplicity, we keep gross outputs constant. Hence we have $\mathbf{c}^{t'} = \mathbf{q}^{t'} - \mathbf{q}^{t'+1}\mathbf{A}_\beta = \mathbf{q}(\mathbf{I} - \mathbf{A}_\beta)$. If the difference between any two techniques is not too large, we can be certain that consumption remains positive. On the other hand, we assume immigration to be such that $L_t = \mathbf{q}\mathbf{l}^\beta, t = t'+1, \ldots, T, \mathbf{q}\mathbf{l}^\beta > \mathbf{q}\mathbf{l}^\alpha$. This transition will take place at once if the rise of $r_s^{t'+1}$ relative to $r_s^{t'}$ is sufficiently large. The effect of "lagging behind" which we mentioned earlier is here not very likely to happen because less capital is being used in a physical sense and more labour such that the introduction of the new technique primarily depends on the change in the distributive variables themselves and not so much on a consequent change of relative prices. Prices will then start to converge towards a new stationary state. The path so constructed is again turned into an equilibrium by replacing the undiscounted prices by discounted prices and by choosing an appropriate utility function.

It is now clear how a transition involving reswitching, from Q_2^* to Q_1^* in Figure 16.3, must be constructed. However, a 'lagging behind'-effect requires an earlier start of the adaptation of prices in this case. For reswitching, contrary to the previous example of demechanisation, presupposes in an essential way that not only distribution changes but also relative prices. Reswitching cannot happen if relative prices are constant for given techniques and if wage curves are straight lines.

In consequence of the transition to the new method, the old method must appear to be unprofitable. We therefore need first what we shall call an "anticipated change in distribution", then an adaptation of relative prices, then the choice of the new method of production (this date is fixed exogenously through immigration) and finally an adaptation of prices to the new steady-state – an adaptation which will be the better the larger is T.

One has again to convert undiscounted prices into discounted ones and to apply the corollary in order to obtain the intertemporal equilibrium with reswitching. The increase in employment which corresponds to the transition which we have considered is possible because less capital is being used in terms of intertemporal prices.

Clearly, this equilibrium is highly unplausible. The assumption of perfect foresight is particularly difficult to sustain. The lagging behind implies that the market participants have to set the price signals to themselves, by changing distribution in anticipation of the immigration, so as to ensure that the change of technique occurs timely. It is not plausible from a common sense point of view, but also with regard to customary assumptions in stability analysis, that the real wage rises in consequence of an anticipated increase in the supply of labour. It is even more unplausible from the point of view of neoclassical theory that capital diminishes, measured in the short run prices, as the rate of interest is lowered. The equilibrium exists formally because the consumer accepts it. What the construction really means is that the underlying theory is flawed: If the equilibrium exists, it presupposes unplausible preferences and, if these are accepted, it is unstable. (A preliminary analysis of stability is proposed in Schefold, 1997: 500–501.)

The method here presented may be used to construct many other types of intertemporal equilibria. If we restrict our construction to capital theory, we may mention growth at a constant labour force with reswitching, i.e. such that the rate of profit paradoxically rises as more capital is accumulated. Or one may consider the case where the wage rate falls to zero (assuming that the means of subsistence of the labour force are contained among the means of production and that the unemployed can retreat to a subsistence economy). The rate of interest then rises to the maximum rate of profit. What will happen? Formally, a consumer may be constructed who then wants to reduce consumption, who accumulates and who thus allows a return to the employment of all labour supplied, but this again would involve the paradox of an acceleration of accumulation with a rise of the rate of interest.

To summarise: There are indications that effects such as reswitching may not be likely. On the other hand, it has been shown that the paradoxes of the debate on capital theory reappear in intertemporal equilibrium. The debate on the logical consistency of neoclassical theory therefore is not closed. Meanwhile, we must ask what modern classical theory achieves.

16.2 THE HISTORICAL SPECIFICITY OF THE APPLICATIONS OF CLASSICAL ECONOMICS

The revival of the classical approach inaugurated by Piero Sraffa (1960) usually is regarded to lead to a theoretical alternative to neoclassical theory. The theory is thought to concern the same object of cognition, since Sraffa's critique questions the logical coherence of the neoclassical theory of capital and distribution. One might conclude that Sraffa's theory should be capable of the same kind of application. However, there is not much around in so-called "Sraffian" economics that might count as applied work in the usual sense of the word. In fact, not only the theories differ, but also, to some extent, the objects of cognition and therefore the areas and the character of application. Neoclassical theory is concerned with abstract market systems with, in the case of general equilibrium theory, very specific properties. There are rational individuals with given preferences and profit maximising firms which produce material goods and services under conditions of perfect or imperfect competition, but there are no historically specific institutions. Or, at least, their historical specificity is hardly ever stressed. The new institutionalism could otherwise not be regarded as a new departure.

Classical theory, by contrast, did contain elements that were historically specific, even in Ricardo, who does not seem to have been interested in contributing to the historical embedding of his analysis of capitalism in the early nineteenth century, while Smith, Sismondi and even more the later adherents of the historical school in Europe – it was a European, not only a German phenomenon according to Pearson (1997) – placed the analysis of capitalism of their respective contemporaries into a historical context, with longer or shorter accounts of the genesis of the stage which had been reached and with confident or cautious guesses about its future development. The most diverse aspects of the theory were historically specific: the assumption of the subsistence wage and its justification, the assumed system of land tenure, the class-specific structure of consumption, various behavioural hypotheses concerning saving and the accumulation of capital, the institutions of the monetary system and the substitutes of money in circulation, etc. The modern world was subject to an evolutionary process. It had evolved out of feudalism. The main contrast with the present was provided by the world of antiquity, with its characteristic different moral standards e.g. with regard to the attitude to work and to the status of labourers. Then there was some knowledge about the different conditions in oriental empires, and elements of economic anthropology gradually were assembled later on in the nineteenth century. Economics in some sense was a moral science for opponents (Marx), reformers (John St. Mill) or defenders of the liberal system (Smith), and it was hoped that morals would improve with general development. Such optimism was voiced particularly by the German Historical School in the decades preceding the First World War (Schefold, 1996).

The demonstration of historical specificity is a form of application of the theory, but it may be argued that Sraffa's revival was to some extent successful because his published work was of such an abstract nature that all traces of historically specific applications seemed to be irrelevant. This high level of abstraction facilitated the comparison with advanced neoclassical theory – a comparison which resulted in the known challenge of the traditional theories of capital and distribution. It probably was a drawback of the chosen approach that it seemed to lock the classical approach in an ivory tower of its own. When I told a colleague that I had been asked to speak about "Sraffa and applied economics", she retorted: "You want to say that you are going to speak about 'Sraffa *or* applied economics'."

But, under the polished surface of "Production of Commodities by Means of Commodities", there *are* historically specific assumptions – how could it be otherwise, if the book was to revive the classical tradition? The degree of abstraction leaves room for a multiplicity of historical interpretations in some cases. For instance, the degree of freedom in the determination of distribution leaves room for several explanations of the rate of profit: as indirectly determined through the level of subsistence wage, as directly determined through the level of interest rates and others. More will be said about the historical specificity of these applications below. On the other hand, not all applications are necessarily historically specific; at any rate, some concepts are open to several applications. For example, the interpretation of distribution is not necessarily in terms of a consideration of classes as in Physiocracy or the Ricardian model, since profits may also accrue to firms and shareholders.

It is an open question whether Sraffa's approach should also be regarded as historically specific insofar as it concentrates on industrial capitalism and disregards the specificity of services. The very title of the book ("Production of Commodities") seems to suggest it. There is no explicit treatment of the services characteristic of the economies in the early period of industrialisation (the menial servants or the lawyers, mentioned by Petty, Smith and others, whose work was regarded as "unproductive"). There is no treatment of the modern services (banking, finance, advertising etc.) either. In a letter of 1971, concerned with the idea of symmetry between supply and demand, Sraffa wrote:

> You say, 'I don't see how demand can be said to have no influence on . . . prices, unless constant returns' . . . I take it that the drama is enacted on Marshall's stage where the claimants for influence are utility and cost of production. Now utility has made little progress (since the 1870s) towards acquiring a tangible existence and survives in textbooks at the purely subjective level. On the other hand, cost of production has successfully survived Marshall's attempt to reduce it to an equally evanescent nature under the name of 'disutility', and is still kicking in the form of hours of labour, tons of raw materials, etc. This, rather than the relative slope of the two curves, is why it seems to me that the 'influence' of the two things on price is not comparable.
>
> (Sraffa, quoted in Bharadwaj and Schefold, 1990: 342)

Although the letter was concerned with the critique of Marshallian partial analysis, it sheds light on the central idea of Sraffa's approach: A list of the physical costs of production ("hours of labour, tons of raw material"), together with results of the determination of distribution (the given rate of profit), allows to determine costs (if one also knows what is to be produced and how).

The work process transforms inputs (which are measurable) into outputs (which are also measurable, independently of the inputs). In particular, different kinds of work can be distinguished according to given standards. These assumptions provide the "objective" foundation of value in classical theory. The standardisation of commodities on methods of production is a social process, tied to institutions, like the private and governmental agencies that provide rules for measurement, safety standards, environmental standards, etc. The origin of such standards may follow some historical logic, but they are given in any period. I have attempted to identify and to describe the genesis of the institutions in the seventeenth century, like the forms of communication between merchants, on the basis of which the classical economists were able to speak of (socially given) "use values" (Schefold, 1999): something quite different from "utility" to individuals.

Similarly, classical authors took standards for labour as given. Sraffa assumed what I call a "weak homogeneity" of labour; relative wage rates are constant and used for the aggregation of different kinds of labour (Schefold, 1989: 252).

The relative remuneration of two forms of labour of different skills is thus determined by conventions which are assumed to be stable in the face of changes in distribution. The stability of a hierarchy of wage rates used to be a feature of industrial capitalism; the differentials could in part be explained in terms of costs of training. Most classical economists did not have to say more on this matter. The standards of work were exogenous and only the work to be performed was represented in the formal theory. The output of the work processes was measurable because the product was of a physical nature.

The assumptions underlying this approach are not necessarily fulfilled in the services where output often is not measurable independently of the work performed and where the standardisation of the quality of the service as an output cannot be separated from that of the effort. Modern classical economists seem inclined to leave this vast and important field of inquiry to sociological and historical description ("outside" the "core" of classical theory – indeed what is said about the services does not even yield a datum for the theory of production, in the way the theory of distribution yields a datum for the theory of production, i.e. the rate of profit).

The attempt to explain the problems of control involved through an interplay of subjective evaluations has been criticised (Currie and Steedman, 1993) because principal agent theory presupposes a cardinal measure of utility which I am inclined to interpret as a reflection of evaluations which are inevitably social. But this observation has so far not lead to an integration of a theory of prices in the services economy into the classical theory of long-run prices either; the services remained "outside".

One might conclude that Sraffa's critique has nothing to say on the development of an essential aspect of modern economic reality. I prefer to state that the classical way of posing the question still represents a challenge (what are the socio-economic characteristics of capitalism in the era of the service economy?), and it remains interesting to understand the classical object: agrarian and industrial capitalism, with its historically given standards for commodities and types of work, which is not entirely a thing of the past. At any rate Sraffa's theory clearly is historical, if it primarily refers to these specific forms of economic activity, and the applications of the theory therefore also are historically specific.

16.3 CLASSICAL METHOD AND MODERN APPLICATIONS

Most people think of applied economics as of a modern form of Political Arithmetic. The methods employed by good applied economists, even if they only have neoclassical training, can sometimes be justified better on the basis of classical rather than neoclassical theories while both predict similar results in specific contexts.

Input–output analysis provides a good example of an important tool of applied economics based on a classical methodology. Although Leontief has taken pains to emphasise the compatibility of his conception with the neoclassical tradition (Leontief, 1941, 1951: 37), it is quite clear that it fits in much better with the classical. For it is the point of input–output analysis to regard the methods of production in use as given independently of relative prices, to derive conclusions from the technological interdependence and to consider the influence of changes in relative prices and in technical progress on the coefficients of the input–output structure only subsequently. (Inputs and outputs of services are here measured *ex post*; there is no presumption to explain their prices.)

It is a fundamental principle of classical economics that *separates* (a) the determination of outputs; (b) the determination of distribution; and (c) the analysis of the relations between the distributive variables and between them and relative prices. It is this which makes the classical approach a better basis for applied economics. It provides direct links between the essential magnitudes in the system (the "short chains of reasoning", which Marshall was looking for) (Garegnani, 1983), while the countless relations of interdependence in a fully disaggregated general equilibrium system are a poor guide to any application. Input–output systems also serve to analyse "interdependence", but by treating the methods of production in use as given and the determination of prices and distribution as separate issues, the analysis of the dependence of activity levels on final output becomes manageable even if feedbacks between different industries have to be taken into account. The separation of the different parts of the theory requires formal links with a given productive single product system, represented by an indecomposable matrix **A**, and a given vector of net outputs, **d** (but often it is more meaningful to take gross outputs as given); activity levels are determined by

$$\mathbf{q}(\mathbf{I} - \mathbf{A}) = \mathbf{d}.$$

The vector **d** acts like a ring, linking two chains of reasoning: the determination of outputs (starting from needs and distribution; see Section 4 below) and that of investment and accumulation. Some remarks will be made on different theories of distribution in Section 16.5, while we here assume that the theory of long-run prices, in the case just considered given by

$$(1 + r)\mathbf{A}\mathbf{p} + w\mathbf{l} = \mathbf{p},$$

is known (the choice of techniques is based on minimisation of costs; for joint production see Section 16.4).

This method is (with modifications) used in many models. The strength of the classical approach is most visible in dynamic analysis. While it is hard to represent the evolution of the economic system even without technological change as a sequence of disaggregated Walrasian equilibria empirically, there are now several quite successful large econometric models which capture the process of macroeconomic dynamics and of structural evolution by means of a combination of an input–output system for the representation of technology with a macroeconomic model for the representation of the evolution of effective demand in its interaction with distribution and government policy, and a demand model based on *aggregate* (i.e. not individual) demand functions which may be differentiated according to socio-economic criteria. My education in classical economics proved very useful for the understanding of the true functioning of these models which one encounters in research on the economics of energy. However, one must admit that the eclectic character of most econometric models does not allow an unambiguous interpretation of their theoretical background (Schefold, 1997: 291–292).

Incidentally, it may be stressed that the method of classical theory facilitates, indeed invites, the consideration of institutions. Some of them are reflected directly in the theory and in applied models. For example, the use of certain methods of production may be imposed by authority or by tradition. The energy models just referred to provide examples. Other institutions are linked with the theory in a more indirect manner, e.g. through the conditions accompanying different states of distribution. One approach is to analyse the subjective side of these conditions and to take the complexity of motivations in their historical contexts into account, through an understanding of texts which document a mentality and through an analysis of the processes which let certain motivations prevail. The work ethic in a modern society, where wages are high and differentiated and in which professions provide an identification at a time when there are fewer family ties, is different from that in an economy where the mass of labourers live at or near subsistence wages in large rural families.

Theories develop along broad ideas which change in some of their aspects, but more slowly than the models which we formulate in our research. I found the differences between the theories far less important when I was doing empirical research than when I was working theoretically. For it is often possible to express an argument proposed in the language of one theory in that of another. To the neoclassical, the saving of energy induced by higher fuel costs is a substitution on the part of the consumer, given preferences; to the classical economist, it may be a substitution of energy-saving domestic methods of production in order to fulfill the same need, like having warm rooms or hot meals (see Section 16.4) – in this context, the concept of substitution is well-defined and belongs as much to the classical as to the neoclassical approach, while the difference concerns the substitution possibilities in the supply of factors (Schefold, 1997: 18).

16.4 COMPOSITION OF OUTPUT AND JOINT PRODUCTION

We now turn to the influences on the composition of output. In the case of single product systems, quantities are given in terms of social needs and expand with accumulation. We only consider the problem in the perspective of consumption, because this seems appropriate in order to demonstrate that the theory need not be based on utility "at the purely subjective level". As for a critique of the neoclassical theory of investment and some constructive suggestions of a different orientation, see Petri (1997).

If there are constant returns, prices are given independently of the levels of output so that various explanations of demand are compatible with a given structure of production and distribution. Morishima (1964) has shown how preferences may be introduced without affecting a Cambridge-type theory of income distribution with given savings propensities on a balanced growth path. His assumption of Engel elasticities equal to one precluded an influence of demand on factor prices and hence the operation of the neoclassical interrelation between demand and distribution. To represent demand by means of given needs in a given period is, from a formal point of view, then even more trivial.

But how are income effects associated with growth to be treated? If incomes change because of a change of the distributive variables such as a growth of the real wage rate associated with a change of productivity, wage earners get the opportunity to buy commodities which were not part of their habitual consumption basket. If they exercise their option, they acquire what at least initially must be considered as luxury goods; if they do not, they save. This shows that in this view the budget constraint must be assumed to be weak in that there is no necessity to spend incomes fully, and that there is a symmetric possibility of spending (slightly) more than current income. In the absence of perfect foresight there can be no definite commitment to the purchase of specific future consumption goods. On the other hand, there may be a pattern to the way in which people climb a social ladder as they receive higher incomes. Their behaviour can then be predicted on the basis of an ordering of consumers according to income classes. This yields an explanation of income elasticities of demand.

It is natural to order these three possibilities and to assume a social process by which rises of incomes are first saved, then spent for luxury goods, and these luxuries tend after some time in turn to be regarded as conveniences in a habitual standard of life (Schefold, 1997: 337–338). Such a hierarchy of consumption corresponds to a hierarchy of needs according to Maslow (1970).

The concept of needs may also be applied to the analysis of price changes. The needs are broadly defined, like having a warm house or eating fruit. There are then domestic activities to satisfy those needs, like using storage heating or eating apples, and changes in relative prices lead to the substitution of activities in the methods of domestic production to satisfy the needs.

If one wishes, therefore, one may list the inputs in terms of consumption goods to be bought on the market (and this includes, e.g. material for the insulation of houses) with the associated inputs of domestic labour. To each set of prices and a wage rate there corresponds a demand for inputs to the domestic processes of providing heat (or saving energy) of any given household under ideal conditions (absence of habits and ignorance, instantaneous adaptation etc.). This is a demand associated with prices. One can then work out how demand changes with changes in prices, taking complications such as the diversity of households, speeds of adaptation, etc. into account and thus arrive at short-run and long-run demand schedules (Schefold, 1997: 340).

The amount and the composition of consumption goods thus changes with the growth of incomes (people reach higher income classes) and with changes of prices. The price changes may be due to changes in distribution, to the availability of natural resources and to changes of technique. The formal analysis of the Sraffa system can still remain much the same as before, if the domestic methods of production are represented by linear activities. Specific questions arise as to which wage is to be ascribed to domestic labour, whether there is an explicit or implicit rate of interest, whether habits retard the adoption of new methods, etc. But we need not go into these details.

The crucial question is whether this methodology can be pursued also in those cases which have by neoclassicals traditionally been considered as their special domain: joint production and variable returns to scale. For the neoclassicals have (starting with Jevons's critique of Mill), assumed that under those circumstances the classical theory of value must fail and concluded (but it is a *non sequitur*) that "value depends on utility". They thought that relative prices of joint products could not be ascertained through a cost of production approach (as such they saw the classical theory) without taking into account what John Stuart Mill (1848) called "the antecedent forces of supply and demand". But we shall now show – and this is, analytically speaking, a testing ground for the idea of this paper – that Sraffa has found a way to extend the classical methodology. It has been explored elsewhere (Schefold, 1997, Chapters 6 and 13); here, I shall present a summary which stresses the link with classical views of consumption. The key idea is to analyse change sequentially: e.g. if technological change is to be considered, the vector of final demand (which includes domestically produced consumption goods) is regarded as given (Schefold, 1997, Chapter 5), while methods of production are regarded as fixed, if we want to analyse (small) changes in demand. Large changes in demand may necessitate technological changes (in the case of joint production), and this may be a more complicated matter especially if there result effects on distribution (Schefold, 1997: 341).

We limit our considerations here to a brief overview of problems regarding joint production, omitting the problem of variable returns. Sraffa made the assumption that the number of commodities produced at positive prices is equal to the number of processes used, i.e. that his system is 'square'. The squareness of the system results simply from the assumption that the composition of output is given. Flukes apart, as many processes will be activated to produce a vector of commodities of a given composition, as there are commodities which are produced with positive prices. Competition will ensure that not more methods are used, for otherwise the system of prices would be overdetermined.

We therefore have in the stationary state

$$\mathbf{q}(\mathbf{B} - \mathbf{A}) = \mathbf{d},$$

where $\mathbf{B}$ is a square output matrix and $\mathbf{d}$ the vector of consumption; prices are given by

$$(1 + r)\mathbf{Ap} + w\mathbf{l} = \mathbf{Bp},$$

where one parameter of distribution and the price of a numéraire can be fixed in the usual way so that the prices in terms of the wage rate become a function of the rate of profit, $\hat{\mathbf{p}}(r) = \mathbf{p}/w$:

$$\hat{\mathbf{p}}(r) = (\mathbf{B} - (1 + r)\mathbf{A})^{-1}\mathbf{l}.$$

It is important to realise that, strictly speaking, the composition of output here is given in terms of the needs of the population, for the needs can, with some plausibility, be given prior to prices, and the commodities sold in the market depend on the domestic activities used, which, in turn, depend on prices, the level of incomes and the level of employment. And these are endogenous variables in the classical approach. The rigid structure of needs does not imply a rigid structure in the demand for consumption goods, for consumption goods may be produced by means of domestic production. The combination used to satisfy a given need may change with prices since prices influence the choice of domestic methods of production. However, in order to simplify the exposition, we have assumed that needs can directly be represented by commodities i.e. by vector **d**. The system then can be shown to be square and to result in positive prices **p**. A full proof (Schefold, 1997, Chapters 5–7) is complicated but the mathematical argument may be explained in intuitive terms as follows.

For any given square system, the vector $\hat{\mathbf{p}}(r)$ is, apart from flukes, i.e. in the so-called 'regular' case, a curve in space with the property that n price vectors at n different rates of profit $\hat{\mathbf{p}}(r_1), \ldots, \hat{\mathbf{p}}(r_n)$ are linearly independent. The most typical irregular case occurs if $\hat{\mathbf{p}}(r)$ is a linear function; prices are then proportional to labour values. Barring such irregularities, the choice of techniques will yield different wage curves for different square systems which can intersect only at isolated points. A linear programme of the type

$$\text{Max } \mathbf{d}\hat{\mathbf{p}} \text{ s.t. } (\mathbf{B} - (1+r)\mathbf{A})\hat{\mathbf{p}} \leqq \mathbf{l},\ \mathbf{p} \geqq \mathbf{0},$$

$$\text{Min } \mathbf{ql} \text{ s.t. } \mathbf{q}(\mathbf{B} - (1+g)\mathbf{A}) \geqq \mathbf{d},\ \mathbf{q} \geqq \mathbf{0},$$

is considered for the golden rule case ($r = g$). At any $r = g$, there results (except for crossing wage curves), *one* square system that is feasible both in the primal and in the dual. ("Square system" means essentially that the number of positive prices in the solution of the programme is equal to the number of positive activity levels at any chosen $r = g$. Prices of overproduced goods – "outside" the solution – and activity levels of unused processes – "outside" the solution – are zero.) The solution maximises the real wage among systems that fulfill the quantity conditions, in accordance with the classical idea of a maximisation of the real wage, given the rate of profit. This golden rule wage curve falls monotonically, as the rate of profit rises, as in the case of single product systems. A fixed point argument allows to deal with the case $g < r$: The solution will still be square, but multiple solutions may arise, and the wage curve for the cost-minimising systems may not be falling throughout. It is generically continuous. The squareness rule may have to be modified with different assumptions about demand.

We return to the economically intuitive argument; it is simple: if wool and mutton are produced jointly, usually varying the age at which the sheep are slaughtered will suffice to adapt the amount of wool and mutton to the given composition of output by choosing an appropriate combination of old and young sheep. Hence there will be two processes to produce two commodities – some sheep are slaughtered when "old", some when they are young – and relative prices are determined on the output side.

However, if this cannot be done, one commodity is produced in excess, while the activity is such that the production of the other is at the required level. The overproduced good receives a zero price. The argument of the formal theory then is closed, since the resulting system is square (the number of commodities with positive prices equals that of activities used), but the story may be continued. The overproduced commodity now is free in the location where it is available in excess, but that part of it which is consumed will cost as much as is necessary to bring it to the market for consumption. The good brought to market hence is a commodity and has a positive price; bringing it there is an activity so that the system remains square. If the excess of this commodity cannot be disposed of at zero cost, because it is a pollutant, producers actually will pay a subsidy to the entrepreneurs who are willing to take it and to transform it into a consumable commodity. This often happens in chemical industry (Brägelmann, 1991).

Whether it is a pollutant or not: the excess supply of the good in question lowers its market price (possibly to zero), and the lowering of the price will induce new uses of it either in industrial or in domestic production or even in direct consumption. As a second continuation of the story, take one of many examples which have been given for such effects in a more general description.

It is possible that the additional process is introduced by the households themselves, and this is perhaps the less familiar case. Increased consumption of a commodity thanks to its reduced price does not have to lead to a change of needs or habits. If, for instance, the price of electricity at night is lowered because of new forms of power stations, the answer of households may be to introduce domestic storage heating which allows them to accumulate heat at night and to release it during the day. Since the cost of direct (central) heating by means of oil is given, this activity determines the value of night-time electricity to households. The answer to cheaper electricity at night is not to start cooking at bed-time. We do not have to postulate that needs are responsive to price changes.

It is useful for concrete applications to represent such possibilities symbolically. Consider three variants of a system involving nuclear power stations (*NPS*). The costs of nuclear and of coal-fired power stations and the costs of coal and of central heating are supposed to be known.

1 $NPS \rightarrow ED \oplus EN$

$C \rightarrow ED$

2 $NPS \rightarrow ED \oplus EN$

$CH \rightarrow H$

$EN \rightarrow H$

3 $NPS \rightarrow ED \oplus EN$

$EN \rightarrow ED.$

In case (1), *NPS* have been introduced which must run for weeks or months in a row for technical reasons. Peaks during day-time can be met by means of coal-fired power stations, determining the cost of day-time electricity (*ED*); the price of night-time electricity (*EN*) is residually determined. In system (2), central heating by means of coal (*CH*) determines the cost of domestic heating (*H*); the power of *EN* is determined if night-time electric storage heaters compete with central heating.

A third possibility (3) is that night-time electricity is converted into day-time electricity through pumping stations: *EN* is used to pump water on a mountain. It flows down during the day, producing *ED*, which can be more valuable because it has more applications.

The multiplicity of potential methods of production, which has been invoked here to explain how prices of production can be determined in the face of an apparent under-determination, is greatly enhanced if we remember the fact that most processes of production are not rigid in that some substitution between outputs is possible. In order to take this into account, one could allow the input–output coefficients to vary continuously in function of some parameter, but it seems more convenient to use a linear approximation, by means of a finite number of separate activities, to smooth transformation curves (where they exist).

It follows that the tendency towards an underdetermination of prices is replaced by a tendency towards an overdetermination, with many methods competing against each other, and with a differentiation of rates of profit. The square Sraffa system may be regarded as the result of the corresponding competitive process (Schefold, 1997: 349).

The system will not be exactly square even in the long run if there are different income classes, in particular if the composition of output required by capitalists is not the same as that required by workers. The variability of distribution introduces a degree of freedom in the adaptation of the quantity system, with the result that the number of activities used to produce the vectors of outputs required by the classes may be $n - 1$, if the number of commodities produced at positive prices is n. These details have been clarified in discussions with Paul Samuelson, Christian Bidard, Neri Salvadori and others (Bidard, 1991; Kurz and Salvadori, 1995). They only lead to a modification of a general principle, not to its abandonment. However, squareness is not a generic result in the neoclassical case, for, with indifference curves, it is obviously formally possible that only one process produces all commodities in the economy jointly, and that the rates of substitution determine relative prices. The problem then becomes to explain why there exists a – perhaps historically changing – tendency to have many processes; in the limit single product systems.

Specific applications concern fixed capital systems. By treating machines as joint products of the processes producing finished goods, one can derive formulas for amortisation and depreciation which may be compared with those used in accounting, and the stock of machines can be reduced to the flow of their services (Schefold, 1989: 172–178). Land also is a special case of joint production, since land, defined by the "indestructible powers of the soil", leaves the process of production unchanged. The counting of equations, applied to the theory of land rent, then yields a theory of the specialisation of lands, using the fact that the system of equations again is "square" (Schefold, 1989: 205–210). A comprehensive land-model is the following (Schefold, 1997: 365–374):

$$\begin{aligned} \mathbf{q}(\mathbf{B} - \mathbf{A}, -\mathbf{Z}) &\geqq (\mathbf{s} + t\mathbf{c}, -\mathbf{v}), \\ (\mathbf{B} - (1 + r)\mathbf{A})\mathbf{p} - \mathbf{Z}\mathbf{u} &\leqq \mathbf{0}, \\ r\mathbf{q}\mathbf{A}\mathbf{p} &= \mathbf{s}\mathbf{p}, \\ \mathbf{v}\mathbf{u} &= t\mathbf{c}\mathbf{p}. \end{aligned}$$

Here we have a choice of technique involving land (the quantities of various lands needed in production are denoted by **Z**, the total quantities available by **v**). Workers' consumption is at subsistence (included in **A**). Capitalists consume or invest a given vector of commodities **s** and they spend in a Kaleckian manner so that their profits become equal to their expenditure: $r\mathbf{qAp} = \mathbf{sp}$, while landlords consume their rents in proportion t to a given bundle **c**; they spend what they get: $\mathbf{vu} = t\mathbf{cp}$. Under suitable conditions concerning the availability of land, a square solution, except for flukes, can be shown to exist (the number of positive activity levels equals that of positive prices).

16.5 DISTRIBUTION, RATES OF INTEREST AND MONEY PRICES

We return to the historical specificity of the classical approach, now in the context of distribution. Examples will here be more useful than abstract assertions.

One of the main roots of classical theory is Physiocracy. Cartelier (1976) has proposed a modern interpretation of physiocratic theory in order to visualise the physiocratic account of the distribution of the surplus with its associated definition of manufacturing as 'sterile'. In the following formulae

$$(1+R)(a_{11}p_1 + a_{12}p_2) = p_1$$
$$a_{21}p_1 + a_{22}p_2 = p_2,$$

the first line describes the production of corn in agriculture, the second manufacturing in the city. The amounts of corn needed for production in the countryside and in the city are a_{11} and a_{21}; a_{12} and a_{22} are the necessary quantities of the manufactured commodity as a means of production. The corresponding prices are p_1 and p_2. A surplus is produced of both commodities, i.e. we have both $a_{11} + a_{21} < 1$ and $a_{12} + a_{22} < 1$. The equations determine the relative price.

The point is the 'rule of distribution': the surplus in monetary form is levied in agriculture and accrues to the owners of the land (the monarch, the church, the feudal lords) so that there is a rate of a surplus product R only in the first process. The smaller the needs of reproduction the larger R is. These may be reduced by means of technical progress. But it should be noted that the input coefficients do not only contain the means of production which are necessary for technical reasons. They also contain the necessaries of the workers who are largely dependent peasants on land while the workers are free labourers, artisans, unskilled workers, etc. in the city. Wages may remain at a subsistence level while the surplus product increases with productivity.

The rule of distribution therefore expresses the curious physiocratic doctrine of the sterility of manufacturing and of the genesis of the surplus product in the countryside. The rule may be justified by invoking the monopoly power of land owners and the competitive character of manufacturing. The formal representation allows to let land appear as solely productive in price terms without denying that manufacturing also contributes to the existence of a physical surplus. The labour theory of value does not allow to capture this structural characteristic of the pre-revolutionary French economy since the model implies that there are obstacles to the mobility of labour while there is trade in commodities. The free labourers in the city have no free access to land while the labourers on the land can produce only under the constraint that they must hand over the surplus to the owner of the land. Oppenheimer called this the "land barrier" (*'Bodensperre'*).

Sraffa's book clarifies the reasons why prices of production cannot be explained in terms of labour values for logical reasons. His approach may also be used to criticise the "historical" transformation of values into prices. As I have argued earlier (Schefold, 1989: 344–346), the simplest form of capitalistic competition is represented by

$$(1 + R)\mathbf{Ap} = \mathbf{p},$$

where $\mathbf{p}$ is the vector of prices and R the rate of profits.

The coefficients of the input matrix $\mathbf{A}$ again contain the necessaries of the workers; the entire surplus is appropriated by the owners of capital. Nothing is said as yet about the status of labourers – whether they are free, or whether they are serfs or slaves or whether there are only automata.

This general image of capitalistic production is different from that proposed by Marx. He spoke of capitalistic production only as soon as production was undertaken by means of free labourers, the proletariat. He was aware that capitalistic commerce was much older than this. But his scheme did not allow capitalistic production by means of slaves, of which examples are found in antiquity, especially in late antiquity and in Byzantium.

Of course, capital has never flown freely between all productive activities. In classical Athens, maritime trade was thought to be more profitable than agriculture, partly on account of risk – at any rate, more interest was charged on maritime loans. The formula for the physiocratic system is a formalisation of a specific monopoly of landowners. The formula for the simplest form of capitalist competition expresses the idea that capitalistic competition precedes the formation of a free and mobile workforce, which was in fact only the historical product of the later mercantilist period, as a result of a long process of transition, at the beginning of which there is the commutation of the forced labour of the serfs into rents and the migration of landless peasants to the cities. Such formulae are only of limited use in a concrete analysis of past economic systems but they help to establish the logical possibility that wage labour may be regarded as the product of capitalism rather than as a presupposition of it (Schefold, 1996a: 304–305).

It is my main contention that the modern theories of distribution which are related to Sraffa's work also are historically specific: each 'applies' only to certain phases of accumulation. Such a historicity of the theory of distribution is familiar in another case: The ideas that the wage rate is determined by the subsistence level and that the rate of profit determines distribution mutually exclude each other; they therefore must refer to different phases in the evolution of capitalism if both are not only logically coherent but also empirically valid. The difference between these phases need not be as great as that between antiquity and the middle ages or between any of the tradition-bound economies and any variant of the modern system, but it is a difference which involves far more than the determination of distributional variables, first of all because of the different status of the workers and associated institutional differences, as illustrated by the growth of the trade union movement, but also by indirectly linked transformations such as the development of the social security system or of schooling, and by the cultural transformations associated with the increasing demand for luxury wage goods like television (the cultural transformation has taken hold when such wage goods begin to be regarded as necessary).

As regards the theories explaining the rate of profit, we first have to consider the well-known Cambridge theory of income distribution. The propensity to consume out of profits is significantly larger than the propensity to consume out of wages. To simplify matters, the latter is assumed to be equal to zero; the propensity to consume out of profits is equal to s_p and assumed to be given. The equality between investment and saving then implies

$$I = S = s_p P$$

(P mass of profits), so that one obtains, after dividing by the capital stock K,

$$r = P/K = \left(1/s_p\right)(I/K) = g/s_p.$$

The rate of growth of the accumulation of capital therefore determines the rate of profit, r. The formula is tautologically true in the closed economy without activity of the state, but it has causal significance only in a long upswing with little turbulence. A high level of demand makes it possible for entrepreneurs to raise prices above costs. Moreover, there are quasi rents for advanced firms, due to dynamic efficiency gains. According to an older view, such quasi rents are eliminated in the long run through competition. However, a durable upswing will reproduce the quasi rents from period to period, so that a higher rate of profit is obtained that corresponds to the sum of interest on capital and managerial salaries and for what a self-employed entrepreneur might regard as a salary. It is true that the margin between profits, net of managerial salaries and interest, leads to investment and a growth of capacities. Insofar, one might expect that prices and profits fall because of the expansion of supply. But such investment is necessary in order to maintain growth at a constant rate. Scarcity rents can also be due to a lack of labour of adequate skill, and this in turn stimulates labour saving innovations which lead to quasi rents in the sense of dynamic efficiency gains. The coexistence of different bottlenecks in different (not necessarily all) industries in the labour supply and in capacity utilisation, together with some degree of imperfect competition, prevents a supply response which could be sufficiently elastic to lead to lower prices. Excess demand thus is reproduced from period to period and supports the level of prices and the prevailing rate of profit. Such a dynamic equilibrium need not be completely free of disturbances, it only requires some steadiness and will not easily develop spontaneously out of a state of slow growth or stagnation. An essential characteristic is a considerable margin between the rates of return of industry and monetary rates of interest. It is a matter of definition whether a rate of profit so determined is still to be called 'normal'; the rate of profit here is an average, like Marx's "average rate of profit".

On the basis of a different view (Pivetti, 1991), it is assumed that competition in a long period position eliminates all profits which are not in some sense necessary, and although this is not the only conceivable outcome in the long run as we have just seen, some reserve the name 'normal rate of profits' for this case. If firms are indebted or regard interest as an opportunity cost, the rate of profit must be at least as high as the rate of interest. Profits further will include a salary for the entrepreneur and a compensation for that kind of risk which cannot be reduced to a cost and hence be insured because it is connected not with external accidents but with inherent risks of the business itself. Even here, the long period position is not a completely tranquil state but an average of good and bad business conditions so that the risk element remains. If we now write

$$r = i + u,$$

u is this necessary entrepreneurial profit or normal profit of enterprise. We measure it as a rate on total capital, although it also makes sense in some contexts to relate the profit of the entrepreneur to the capital he owns. Here, the conception is that a certain specific profit is needed to operate a firm in a given sector of the economy, whatever the share of the financial capital in the property of the firm. This profit is a proportion of total capital; we abstract from differences in this proportion between sectors.

The equation is a tautology if it is read as a definition of the profit of enterprise. In order to have causal significance, the normal profit of enterprise must not only be given, but it must be explained how changes in the rate of interest cause corresponding changes of the rate of profit. The rate of interest is acted upon by the banking system as a whole; we may assume that the central bank is able to govern it (using the simplification that there is only one rate of interest). Usually, a rise in the rate of interest is believed to cause a contraction in demand, therefore to lead to changing quantities and, if anything, to a reduction in the rate of profit. It is here asserted that the central bank also can permanently raise the rate of interest, thus forcing a rise in the rate of profit, without significant changes in levels of activity, and the rate of interest is the real rate – a nominal rate of interest could even be higher than the maximum rate of profit.

It is probably best to think of such long period changes in the rate of interest as of specific scenarios. The economy of a nation may suffer from a structural deficit in the balance of trade which forces the central bank to raise the interest rate for some time. If it is done cautiously and confidence does not suffer, prices and the rate of profit may be forced up, while techniques and quantities do not change significantly.

Since the rate of entrepreneurial profit is a given element of cost and equal to the margin between the rate of profit and the rate of interest, investment cannot be driven by this margin. The growth of the system is likely to be low; investment is not induced by quasi rents but by the expected growth of demand.

This monetary theory of the rate of profit has diverse aspects. We consider only one in the remainder of this section: the price changes engendered by changes in the rate of interest, for the theory here leads to intriguing, and – in their precise quantitative form – novel predictions. The rise of prices here is relative to the wage rate. We use the standard commodity to measure it. It is defined as $\mathbf{q}(\mathbf{I} - \mathbf{A})$, where $\mathbf{q}$ is the positive eigenvector associated with $\mathbf{A}$, $(1+R)\mathbf{qA}=\mathbf{q}$, normalised so that $\mathbf{ql}=1$, one obtains the well-known linear relationship between w and r:

$$1=\mathbf{q}(\mathbf{I}-\mathbf{A})\mathbf{p}=\mathbf{q}(r\mathbf{Ap}+w\mathbf{l})=\frac{r}{R}\mathbf{q}(\mathbf{I}-\mathbf{A})\mathbf{p}+w\mathbf{ql}=\frac{r}{R}+w;$$

the expression

$$s=\mathbf{q}(\mathbf{I}-\mathbf{A})(\mathbf{p}/\mathrm{w})$$

denotes the labour commanded by the standard commodity or the standard commodity in terms of the wage rate. This is an index for prices, not for measuring price changes over time, but for price changes induced by changes in distribution.

It is a well-known result of the classical theory that prices in terms of the wage rate rise monotonically with the rate of profit in single product systems; we get, using Sraffa's linear wage curve

$$s=\mathbf{q}(\mathbf{I}-\mathbf{A})(\mathbf{p}/w)=1/w=R/(R-r).$$

We thus see that prices rise on average relative to the wage rate like a hyperbola and diverge to infinity at the maximum rate of profit. Taking the derivative,

$$\frac{\mathrm{d}s}{\mathrm{d}r}=\frac{R}{(R-r)^2},$$

we observe that this rise is proportional to $1/R$ even at low rates of profit. We can also calculate the price rise in percentage terms and obtain

$$\frac{\mathrm{d}s}{s}=\frac{\mathrm{d}s}{\mathrm{d}r}\frac{\mathrm{d}r}{s}=\frac{R}{(R-r)^2}\frac{R-r}{R}\mathrm{d}r=\frac{\mathrm{d}r}{R-r}.$$

The significance of these observations is best shown by means of a numerical example. The maximum rate of profit is equal to the productivity of capital of the standard system, hence it is an approximation to the inverse of the capital coefficient of the real system, which is a relatively stable macroeconomic magnitude, by and large equal to 4, and not very different between countries. With a rate of interest of 5 per cent ($r=5$ per cent), with a change in the rate of interest of one percentage point ($\mathrm{d}r=1$ per cent), and assuming cautiously that R is equal to 1/3, we get

$$\frac{\mathrm{d}s}{s}=\frac{1/100}{(1/3)-(1/20)}=\frac{60}{17\cdot 100}\approx 0.035.$$

Prices relative to wage rates therefore would have to rise in the long run by 3.5 per cent because of a multiplier effect, if the central bank raises the rate of interest by one percentage point. If this happened at a rate of interest of zero, prices already would rise by 3 per cent. If we take the component of the normal profit of enterprise into account (with $r = i + u$ and $\mathrm{d}r = \mathrm{d}i$), and assuming that this u is also equal to 5 per cent, we get

$$\frac{\mathrm{d}s}{s} = \frac{1/100}{(1/3) - (1/10)} = \frac{30}{7 \cdot 100} \approx 0.043;$$

the change of prices relative to money wages now is equal to 4.3 per cent, and it is 8.6 per cent if interest is raised by two percentage points. The effect is the more dramatic, the more r approximates R (the multiplier tends to infinity). On the other hand, we have at small rates of profit:

$$\frac{\mathrm{d}s}{s} = \frac{1}{R}\frac{\mathrm{d}r}{1 - (r/R)} \approx \frac{1}{R}\left(1 + \frac{r}{R}\right)\mathrm{d}r;$$

in approximation of order zero, the price change equals $(K/Y)\mathrm{d}r$ (Schefold, 1998: 18–24).

The classical theory of prices may be used to discuss various extensions: processes of inflation, the inclusion of joint production and of land. There will be two price levels if land or other kinds of permanent assets are introduced. Commodity prices in terms of the wage rate rise, asset prices fall as the rate of interest is increased (Schefold, 1998). The central bank therefore faces a dilemma: if interest rates are raised to fight inflation, the move contributes directly to the rise of commodity prices, as long as quantities are not changed and as long as the wage rate stays the same. Only the fall in asset prices and the downward pressure on activity then leads to falling market prices and to a fall in the wage rate. The fear of inflation (which almost always forces to take such unpalatable measures eventually) therefore is well founded.

We obtain the following result, regarding the determination of the rate of profit: The theory of distribution which explains the level of the rate of profit as determined by the level of the rate of interest plausibly is concerned with states of slow growth. For it is the margin between profits and interest which is relevant to the investment of enterprises, and this margin is taken to be constant as a "normal profit of enterprise" – indeed, the normal profit of enterprise is regarded as a cost, and the quasi rents (which are that part of the profit of enterprise which really stimulates investment) are assumed to have vanished under the influence of competition. The relationship between the rate of interest and the rate of profit can be causally significant only if the former influences the latter. The banking system and the central bank in particular can achieve long-term changes of the rate of interest only under special circumstances; the short run effects can be of a contrary nature. If the interest *is* raised permanently and the rate of profit *does* rise in consequence, the redistribution can be explained as a monotonic rise of money prices relative to money wages, as long as only prices of goods and rents are taken into account. A multiplier translates changes in the money rate of interest into price changes which is of the order of magnitude of the capital coefficient at low rates of interest and considerably higher at higher rates. A contrary effect concerns the land prices and, to a lesser extent bonds and shares, in the short run even machines; then, an index of all prices taken together as a function of the interest rate is U-shaped; prices will first fall and rise afterwards, as the rate of interest is raised from zero to the maximum rate of profits. The freedom of action of the central bank to change interest rates thus seems constrained on both sides (Schefold, 1998: 30).

16.6 CONCLUSION

We have encountered applications of the classical approach in diverse fields: The representation of the relation of interdependence of prices and quantities of the system, the change of the composition of output, joint production and distribution. More could be added, in some cases with econometric support, like measurements of the wage curve and of normal prices in relation to labour and to other costs. These applications of the classical approach can be distinguished from other applications of economic theory by the use of the classical method of separating the heterogeneous determinations of outputs and of distribution. Given both, long-run prices and techniques can be determined. The advantage of this separation is flexibility: It is convenient to analyse the effect of changing energy prices on the choice of methods using energy in industry and domestic production by separating it from related effects on changes in habits and tastes of the consumer. The latter are important but cannot be reduced to independent subjective responses with unchanged preferences, at least not, if the price changes are large and surprising so that they induce social learning processes. Similarly, it is instructive to observe the influence of changing interest rates on costs and prices, without considering simultaneously the changes in activity levels and employment, even if one is ultimately most concerned with effects on the growth of the economy.

The dominant method of singling out the interdependence of preferences, endowments and given techniques through an intertemporal equilibrium leads to the frequent neglect of effective demand, of technical change and of monetary influences in contexts where macroeconomic, microeconomic, evolutionary and institutional aspects are all nearly equally important. The classical method, even when it is used by a professedly neoclassical economist, allows a more flexible combination of the economic disciplines, in approaching a reality which is always historically specific. Some traits are fairly general, like the competitive nature of markets – hence the assumed uniformity of prices and of the rate of profit. But others are special, like the organisation of labour markets or the monetary institutions. Eucken (1940) wanted to use the tools of neoclassical economics in order to analyse different economic systems, but general equilibrium theory does not yield much more than an opposition of planned and market economies. The theories of imperfect competition add variety to this simple menu, but, as regards theories of development, of accumulation and distribution, the classical approach is richer in empirical content.

NOTE

1 The editor asked for an essay which was to summarise my work on applications of classical economics. In consequence, I have occasionally taken the liberty to paraphrase passages of previous papers of mine; I hope that the combination will nevertheless be recognised as an original contribution.

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17 General equilibrium: problems, prospects and alternatives

An attempt at synthesis

Alan Kirman

17.1 INTRODUCTION

In many people's minds Siena is associated with the Palio. This is a peculiarly ferocious and unforgiving horse race in which each area, or contrada, prepares its horse for the brief turns around the piazza in the centre of the town. In this competition no holds are barred and even the most dubious tactics are allowed. It is not uncommon for horses to die in the race. Each horse is meticulously prepared by its own contrada and a whole town is alive with the preparations for the race for several days before the event. Battles fought long ago are evoked and many of the previous races are rerun in the minds of the older spectators.

Taking part in a meeting on general equilibrium in Siena is very much like taking part in an intellectual Palio. Here again, proponents of different theories show remarkably few scruples in pushing their own point of view. Some of the dialogue between the participants was situated more at the level of trying to destroy the opponents' arguments rather than convincing him of the truth of alternatives. Echoes of great ideological battles from the past were present and we relived the battles between classical and neoclassical proponents and the great capital theory controversy involving the two Cambridges came back to the surface.

This experience was entertaining and enlightening and I believe we all learned some lessons from it. However, if I am to make any sense of the debate I am forced to go back to first principles. The first question that arises is why should we care about the state of general equilibrium theory today when so many people have consigned it, at worst to the waste bin, and at best, to the historical archives.

If we do care then from what point of view? A first and obvious answer is that we are simply interested in clearing up the history of general equilibrium analysis and so a clear vision of its evolution would certainly be worthwhile in this respect.

A second point of view is that of the methodology of economics. What role has general equilibrium played in the evolution of our subject and, in particular, has this role been positive or, as many would argue, and many did argue in this conference, has it had a stultifying effect on the development of our subject?

A third and perhaps somewhat sceptical position would be to suggest that economists keep building general equilibriuim models and theorising about them as a result of pure intellectual inertia. Many people, particularly those with a mathematical background, are in the habit of solving the sort of problems posed by general equilibrium theory and are unlikely to change their habits.

An alternative and a more positive view point is that even if, as some people suggest, general equilibrium theory has little to say about the real economy the problems raised within it are of genuine intellectual interest and this alone is a justification for this activity. As someone who spent quite a lot of time working on the relationship between the core and competitive equilibrium, I have, or at least, used to have, some sympathy for this point of view.

A last justification for being concerned with general equilibrium would be the belief that it does actually have something to tell us about real economic phenomena. This would certainly be a strong argument for maintaining this activity. Indeed, this would probably be the strongest argument. However, apart from those who use computational general equilibrium models this is not a widely used justification for the use of general equilibrium theory. At the Siena conference this was probably the only intellectual contrada without a horse.

Many of the above arguments for holding such a debate are overlapping and many of the contributions could be classified under several of them. To give some structure to this attempted synthesis let me specify the different arguments I will examine. The first of these is the nature of equilibrium and its definition. The second is the problem of the existence of equilibrium. The third is that of indeterminacy and the fourth concerns the stability of equilibrium. The next topic involves the analysis of production and capital, the corner stones of classical theory but only present in a very diminished form in general equilibrium analysis. Next, there are financial markets and the role that they play in general equilibrium theory. This subject is followed by that of game theory and the introduction of strategic behaviour. Then comes a particularly thorny problem, that of the beliefs of economic agents. If we then turn to look at how an economy evolves and how agents react to changing information we have to decide on which approach to base our analysis. Here, the roles of learning, adaptation and evolutionary approaches become very important. Finally, I will conclude by making some modest proposals for new avenues which should take us beyond the framework of modern general equilibrium theory.

17.2 EQUILIBRIUM: ITS NATURE AND DEFINITION

At the risk of trying the patience of some of my readers I have to go back to basics. As economists we are interested in characterising the "states" of an economy. Three things are important here:

1. The detail in which we wish to study these states.
2. What are the properties of certain states that we single out, for example, as equilibria
3. How do economies move from one state to another?

The first point is of great importance, for general equilibrium theory has adopted an uncompromising generality. When we describe a state of the economy we describe every detail of the situation of every individual at every point in time. Yet, this is far from being the concern of most economists. Macroeconomists, for example, are interested in very broad aggregates. Even those interested in specific sectors or markets are not preoccupied with individual details. So, the first problem is that of which level of aggregation we should choose. The standard approach to modern macroeconomics is that, provided we start with sound micro-foundations, we neednot be concerned with the problem of aggregation as such. Reducing the economy to "representative agents" enables us to move smoothly between the micro- and macro-level. In fact, this is illegitimate and leads to false conclusions about macro-behaviour (see e.g. Kirman, 1993). As Frank Hahn points out clearly in this volume, the situation is far from being simple. There are feedbacks from aggregate variables to individual behaviour which cannot and should not be overlooked. Furthermore, the behaviour of an interactive system cannot be reduced to the behaviour of its average member. Thus, the general equilibrium model in full generality is nothing more than an extreme case and to deduce macroeconomic relationships from it is an exercise which is doomed to failure.

However, even if we know at which level of aggregation we want to work, there remains the question as to what constitutes an equilibrium state of the economy at this level. There are several different answers to this sort of question. A first view and one which is consistent with the so-called neoclassical approach is that equilibrium is simply the solution of an appropriately specified set of equations. Second, equilibrium could be interpreted as a rest point of a system of dynamic equations, a steady-state of some ongoing process such as that found in overlapping generation models or a rest point in the very long-term, a position with which classical economists could identify. A third view is that equilibrium is a state of the economy from which no one, given the rules, has any incentive to deviate. This is the point of view of game theory and Debreu's (1952) early paper reflects this influence.

Another remark is in order here. It is very clear that one thing that separates classical analysis from standard general equilibrium theory and its extensions is the lack of concern in the former with consumers and hence a lack of interest in one of the main preoccupations with general equilibrium theory, that of Pareto optimality. It is worth noticing that those people who situate themselves in the intellectual contrada associated with the standard vision of general equilibrium pay a great deal of attention to Pareto optimum. Indeed, as Frank Fisher points out, the Fundamental Theorems of Welfare Economics are the cornerstones for the arguments in favour of economic liberalism.

In the general equilibrium setting the definition of equilibrium can be appropriately extended to cover models with a variety of different structures and there are several examples in this volume: general equilibrium models with incomplete markets (GEI) such as those treated by Magill and Quinzii, those which allow default by economic agents as treated by Geneakopolous and those with extensive treatment of expectations or "beliefs" as analysed by Kurz.

An alternative view of equilibrium is that offered by Foley in which he proposes a statistical concept which does not involve simple market clearing but which takes another approach and looks at the feasible Pareto improvement than can be achieved in the greatest number of ways. However, with the possible exception of this last approach all of the other visions mentioned, concern a very static and classical form of equilibrium. There is little place here for any genuinely dynamic theory. Indeed one could ask how one might set about defining an equilibrium notion in a world which was evolving? For example, consider an economy which evolved from one state to another with a certain probability. With rather unplausible assumptions this might be modelled as a simple Markov process and in that case, the appropriate equilibrium notion would be a limit distribution. Thus we would know how long, on average, the economy would spend in each state but would not expect it to converge to any particular state. Even this idea is rather far from the central preoccupation of economists who, in general, when they introduce time into their models look for convergence to some steady state. Yet, in making this step we have not gone very far. To model the evolution of the economy in this way is to assume that there is a finite number of states, known *a priori*, and that the economy simply moves amongst them. Yet, in a genuinely evolutionary setting the number of possible states would be constantly changing as would the dimensions used to describe them. Describing such a situation formally seems to be a heroic task.

The third point mentioned above, that of how an economy moves from one state to another, can either be thought of as a question about the economy of its equilibrium path or about how an economy can shift from one equilibrium to another, a question which has been around for a long time in the Classical contrada. In standard macroeconomic models the economy would have to be affected by an exogenous shock in order for it to be shifted from its trajectory onto another one.

17.3 EXISTENCE OF EQUILIBRIUM

In general equilibrium theory, in its standard form, we have this under rather general conditions. Nevertheless, as Magill and Quinzii point out, a lot of the recent developments in the theory have come as a result of adding "imperfections" to the model and each time one does this, the question of the existence of equilibrium has to be taken seriously. In this regard, some important contributions have been made by people here.

An interesting line of research that goes back to Cournot is the idea that if individuals are sufficiently different, and if there are enough of them then equilibrium can be shown to exist under very general conditions, and even stability of equilibrium, which I will discuss later, may be restored.

Here Chichilnisky's contribution is of particular importance since her limited arbitrage condition shows why individuals cannot be too different. Thus diversity is not a panacea. She also reveals the common structure of a number of problems which might not seem similar at first glance. Geanakopulos extends the model to a genuinely intertemporal framework and then proposes a way to introduce a natural notion, absent in the standard model where all is perfectly anticipated – that of default. Here again the notion of equilibrium and its existence are important.

If we come back to an old problem that there is no natural time horizon which can be imposed on the model then, if we are to date all goods, we must introduce an infinite dimensional commodity space. Accinelli reflects on the problem of existence in this context and his results take us beyond those of Bewley (1973). Bewley's major contribution as far as I can see was to introduce the Mackey topology. In effect, using this topology is like attributing discount factors to individuals since the weight on distant events is greatly diminished. It is this that enabled him to produce his existence results in the infinite case. So paradoxically, it was something that has a natural economic interpretation that led to the solution of an essentially mathematical problem.

Existence is not usually directly posed as a problem by those who inhabit the Classical contrada. Yet, a problem I have with the classical solutions is that they appeal to an implicit adjustment process (see for example, Garegnani's and Schefold's contributions). So to prove existence in such a context is a rather different matter than in the standard general equilibrium framework. I am not sure that economists in the classical tradition regard the problem of existence, as such, as a matter of any great importance but their preoccupation with the possibility that there might be more than one equilibrium path that leads directly on to the next subject.

17.4 INDETERMINACY OF EQUILIBRIUM

This is a problem that has beset economics since Sonnenschein, Mantel and Debreu although was known long before their time and discussed at some length by Pareto (1906). In game theory, the situation is worse and the whole subject has been overwhelmed by proposals to select amongst different equilibria or to suggest refinements of equilibrium which reduced the possible set of such equilibria. Although Debreu's (1970) contribution showed us that, in the standard general equilibrium model, equilibria were locally unique; this was not of much help to macroeconomists interested in doing comparative statics nor to those who were hoping for some general stability results.

Once again this is a subject which seems to have a different significance for classical economists, for the possibility of switching from one steady-state to another, which, in a sense, reflects the same question, is something which has intrigued them for a long time. I find it particularly interesting that Heinz Kurz and Neri Salvadori integrate recent developments in growth theory with the classical long-term approach to economics. However, I find it difficult to understand why they would want to consider the endogenous growth model as a general equilibrium model. Furthermore, it seems to me that there is a considerable difference between growth theoretical models in general and the general equilibrium model as formulated by Arrow and Debreu. Indeed, Kurz and Salvadori emphasise that the Arrow–Debreu model does away with any direct consideration of evolution over time. It seems clear that the viewpoint adopted by Debreu deliberately avoided any discussion of temporal evolution and was concerned only with the simultaneous clearing today of all markets in the future. This was an ambitious technical project but for many people in other contradas than his, one which was of limited economic interest.

As I have already mentioned, uniqueness of equilibrium can only be bought at the price of extreme assumptions on individuals such as, for example, assuming that they all have Cobb–Douglas preferences or by some sort of distributional assumptions on characteristics. The latter approach developed recently by a number of authors such as Hildenbrand (1983) and Grandmont (1992) is a step away from the traditional line of reasoning in general equilibrium theory which is that only assumptions on individuals are legitimate. Once we allow assumptions on the distribution of characteristics we move to another level, one which has already been used elsewhere in economics, for example, in trying to resolve the difficulties posed by Arrow's impossibility theorem. However, in doing so we open a possible avenue to resolve the other problem posed by Sonnenschein, Mantel and Debreu, that of the stability of equilibrium.[1]

17.5 THE STABILITY OF EQUILIBRIUM

Perhaps, the most striking failure of general equilibrium theory, particularly in its purest form, has been that of establishing any tendency towards an equilibrium. There are different points of view as to the importance of this question. Morishima clearly believes that without any natural tendency towards equilibrium the concept itself is only of intellectual interest. Many have suggested that economists simply take such a tendency for granted and indeed Petri (forthcoming) says explicitly "The existence of such a tendency is something on which no famous economist since Adam Smith appears ever to have had doubts". The notion of equilibrium that Petri has in mind here is that of a common return on capital in the long run and not the standard Arrow–Debreu equilibrium. However, there is nothing in the Arrow–Debreu model to suggest that there should be any such tendency. Certainly the rate of interest will be different in equilibrium for different commodities in such an equilibrium. So Debreu is a major counter-example to this claim. This is consistent with the remark of Kurz who says "Hence in the Debreu analysis, as opposed to that presented by Walras with its long-period orientation, general equilibrium cannot be thought of as a centre of gravitation". My impression is that those most involved in the formalisation of general equilibrium theory were far from convinced that there was any sort of natural stability. Indeed, Gerard Debreu never skated on this thin ice whilst many of his distinguished contemporaries such as Frank Hahn, Takashi Negishi, Ken Arrow and Leo Hurwicz ventured into the analysis of the problem without much success in the end.

The end of this story was provided by the results, already mentioned, of Sonnenschein, Mantel and ironically Debreu himself. In showing that our standard assumptions on individual preferences in no way restrict the form of aggregate excess demand they also showed that without very different assumptions there is no hope for stability. Much earlier, a warning flag had appeared which seems to have passed largely unnoticed. In a series of results culminating in that of Jim Jordan (1982), the amazing parsimony of the informational requirements of Walrasian equilibrium were made explicit. These requirements are for the information required for the process to function when it is at equilibrium. However, the results of Saari and Simon (1978) show that if one is to guarantee convergence to equilibrium from any initial prices, then one needs an infinite amount of information. Despite efforts by Herings (1997) and others to produce clever price adjustment mechanisms which use less information the danger was present for all to see. Walrasian equilibrium is really a static notion and we have little to say about "The Invisible Hand Process". Frank Hahn's remarks are directly related to this problem. What happens as soon as an individual's choice is influenced by aggregate variables? In this case, the economic mechanism must convey the signals about these variables to the individuals in the economy. But this is now much more than could be handled by an adjustment mechanism of the tâtonnement type.

Fisher explains very lucidly where the tâtonnement process and its successors went wrong and explains what he thinks must be the basis for dynamics and stability in a full model. His work is a partial answer to the classical criticisms of general equilibrium. There is trade out of equilibrium and this must introduce "path dependence" in the sense that the final equilibrium will not be determined by the inital conditions alone.

As I have said, an alternative answer to the Sonnenschein–Mantel–Debreu problem has been suggested and this avoids all the difficulties of dealing with the specifications of the trading process and its dynamic evolution. This consists in sticking with the static model but expanding our assumptions to include ones on the distribution of agents and then to ask whether we can obtain stability or uniqueness. In this connection there has been considerable discussion of the problem of the heterogeneity of agents. Here it seems to me that there is a fundamental misunderstanding. The argument seems to be that there is not enough variety of characteristics in the standard model. This is not the case. General equilibrium theory allows for a large degree of heterogeneity of agents and only restricts them to be utility maximisers without specifying anything about the distribution of preferences. The problem is that the degree of heterogeneity is not specified. Whilst Grandmont and Hildenbrand sought greater variety, modern macroeconomists took exactly the opposite route. Their recourse to the "representative agent" model takes the same way out of the dilemma but makes an extreme and very different distributional assumption which is that the behaviour of the group can be summarised by that of its average member. The most restrictive assumption in this direction would be to assume that all the agents were the same. However, it should be understood that, far from being an integral part of general equilibrium theory, the introduction of the "representative agent" is an attempt by macroeconomists anxious to have micro-foundations, to circumvent the results of Sonnenschein, Mantel and Debreu. An interesting discussion of the role of the diversity of agents characteristics is given by Chichilnisky and it is worth comparing her approach with that of Grandmont.

One important observation here is that even when stability is an issue, price formation and adjustment in the Arrow–Debreu context involve the notion that prices are central signals and not variables determined by individuals. As Fisher says clearly in his contribution, what we need is decentralised adjustment processes and he was one of the first to try and spell out a theory of this sort. Before passing on to other aspects of general equilibrium, I have to deal with a fundamental aspect of economics which is central to the preoccupations of the Classical contrada.

17.6 PRODUCTION AND CAPITAL

Once one has ventured into the classical contrada one cannot avoid a discussion of production and capital. However, here, as someone brought up on standard general equilibrium theory, I am on very thin ice indeed. General equilibrium theory has remarkably little to say about production. Our definition of technologies and profit maximisation seem to me in no way to capture the reality of firms. This is clearly revealed by an old theorem of Trout Rader who showed that for any production economy satisfying the standard convexity assumptions on production technologies one could construct an "equivalent" exchange economy. Thus, moving from production to a fixed vector of endowments has no effect on equilibrium prices. One can transform inputs into goods idependently of the demand sector and thus production is a somewhat trivial appendix of the model. Worse, Kehoe and others have pointed out the difficulties of adding production to the general equilibrium model in terms of indeterminacy. Adding production actually adds force to the Sonnenschein–Mantel–Debreu problem. Furthermore, efforts to add more realism by introducing increasing returns (see Quinzii (1992), for example) have met with little success. Accinelli shows that we need extremely restrictive assumptions on production to obtain existence in infinite dimensional economies. Thus in standard theory, production is not only trivial but also somewhat troublesome. Trying to take the firm seriously in the general equilibrium framework poses all sorts of problems – witness the difficulties with reconciling the firm as a bundle of capital assets with its existence as a financial entity in Magill and Quinzii's contribution.

Yet in classical theory production is all important. The analysis of that production may not be more satisfactory than in neoclassical theory, but it gets a great deal more attention. Much attention is focussed on factor prices as can be seen from the contributions in this book by Schefold, Petri and Morishima. The latter builds bridges through Hicks from classical to neoclassical capital theory but is more interested in a Walrasian approach than in the more sterile Arrow–Debreu approach to production.

The whole reswitching controversy turns on certain specific features of the production technologies and one runs into the same technical problem as was encountered in demonstrating factor price equalisation in international trade theory.

The apparent differences of emphasis between the classical and general equilibrium or more popularly, neoclassical, approach led to many of the most vigorous confrontations in this Palio and I would be failing in my task if I did not take a glance at this battle.

17.6.1 Classical vs. neoclassical economics

Here, we have a division between the two oldest contrada. The long-standing controversies concerning the nature of capital and the very process which describes the economy all came out in the discussions. In particular, Petri and Garegnani were particularly insistent on what they have seen as the inherent contradictions in general equilibrium theory. Petri's "Sraffian" critique of general equilibrium theory shows clearly where the fundamental problems of comprehension arise. My own impression is that there is a deep and perpetual lack of communication between the two parties. Most of us were convinced that Cambridge, England had won the battle on capital and aggregate production functions but this was not as far as I can see a battle involving the appropriateness of general equilibrium theory. The basic problem was with aggregation, and this is a whole subject on its own. Capital goods, as such, make no sense in general equilibrium theory any more than a single rate of interest does. Goods are dated and distinguished by their dates. Aggregation across different inputs at different dates involves making assumptions which are not present in the Arrow–Debreu model.

The same is true for labour. Thus, an aggregate demand curve for labour is not a feature of the pure general equilibrium model and yet it is this which is the starting point for Petri's criticism of neoclassical general equilibrium theory. The answer seems to be that general equilibrium as it has evolved is not useful for studying the aggregate problems which interest and interested the inhabitants of the Classical contrada. Although Petri's contribution tells us a great deal about the evolution of classical thinking and is probably much closer to saying something about the growth of the economy than is general equilibrium theory, it does little to resolve the old and misplaced conflicts. This I think was Frank Hahn's (1981) basic position when he said

> I have always regarded Competitive General Equilibrium analysis as akin to the mock-up an aircraft engineer might build... theorists all over the world have become aware that anything based on this mock-up is unlikely to fly, since it neglects some crucial aspects of the world, the recognition of which will force some drastic re-designing.

As Petri himself says, taking this point of view means that general equilibrium theory cannot be accused of logical inconsistency but rather of irrelevance.

Once again, one has to repeat an observation at this juncture. One has only to note the attention paid to efficiency by a number of contributors from the Arrow–Debreu contrada and even some who would not so identify themselves with efficiency or Pareto optimality, to see how deep the divide between the classical and neoclassical approaches is. Many of us like Fisher, as I have said, would argue that the underlying justification for the prevalent free-market approach is the Fundamental Welfare Theorems. Yet such a justification, misplaced as it may be, simply does not have a role in the classical framework.

What is also absolutely clear is that the classical contrada has long been preoccupied with tendencies in the economy, and in particular, as Petri clearly points out, one cannot extract tendencies towards any particular state from Walrasian equilibrium theory. Once again, this is a criticism of the appropriateness of the Walrasian model and not a criticism of its logical structure. The preoccupation of economists in the classical tradition with long-term prices and the rate of profit are not directly relevant to general equilibrium ,since the removal of discrepancies in the rate of profit between different activities is not part of the model.

The notion of arbitrage across investment in different firms only makes sense if we specify who owns firms and how that ownership may be transferred. None of this is present in Arrow and Debreu, and as Magill and Quinzii point out in this volume, it was only with the adoption of sequential models and trade at each point in time that one could discuss this sort of problem. In Magill and Quinzii's work we see how the notion of ownership of firms can enter into general equilibrium analysis and the consequences of its introduction. Incidentally their work shows how a reconciliation between general equilibrium and the contracts contrada, not really represented in this Palio, might be possible. Furthermore, in a second paper in this volume they show how the stock market can provide a mechanism for the intergenerational transfer of funds with important consequence for the long-run efficiency of the economy. Their framework is that of overlapping generations and one could ask a simple question. Are they saving the structure of this particular form of the general equilibrium model by the introduction of an infinitely lived mechanism for the transfer of ownership or are they providing a reasonably realistic setting for the discussion of the ownership of income streams? What is certainly true is that their analysis brings us back to considerations of the "golden rule" and the preoccupation with the long run that characterises members of the Classical contrada.

Nevertheless, I find myself completely out of tune with the classical or Sraffian approach, not because it is not a possible view of the world, but rather since it seems to me to be attacking the wrong target. As far as I understand it, the central argument seems to be that general equilibrium fails because it does not answer the questions that the Sraffians or neo-Ricardians judge to be interesting. This may make it sterile or uninteresting from their point of view but it does not make it wrong in any sense. There is no requirement of zero profits in the general equilibrium model, and indeed, Debreu had to introduce artificial "shares" in firms to redistribute the positive profits made by firms at equilibrium. There is nothing in the Walrasian model which would guarantee zero profits, and there is no reason for there to be any such condition if we are only interested in characterising equilibrium states and not in explaining how those states might be achieved.

17.7 FINANCIAL MARKETS

Financial markets and their role as providers of capital have to a large extent been left on one side in this set of contributions. This is not quite fair to Morishima who deals at some length with the relationship between Japanese firms and their banks. However, where capital has appeared in the Classical contrada, it has been more of the physical aggregate sort.

Nevertheless there is a new contrada which has devoted a great deal of attention to the importance of financial markets, and this is what is known as the GEI approach. As can be seen from the contributions of Geanakopulos, Magill and Quinzii this has added quite a lot to our appreciation of the role of these markets in making up for the lack of certain markets. It is clear that with the highly specific definition of goods in the Arrow–Debreu model it would be ridiculous to assume that all markets exist. Yet, many questions remain to be answered and the most important one is perhaps why are certain markets missing.

A good point made by Geanakopulos is that default has to be included as a normal phenomenon in a risky environment and that an equilibrium default rate could be established. This is quite far removed from Arrow–Debreu.

However, once we start to discuss financial markets we also have to talk about firms, and we also have to allow for the fact that firms behave strategically which leads me to visit the next contrada.

17.8 GAME THEORY

This approach has found little space in this conference with the exception of the contributions by Gintis and Sabourian. An early paper of Debreu (1952) showed how general equilibrium could be interpreted as a Nash equilibrium. This is a line that Hamid Sabourian pursues. A typical argument that has been widely used in general equilibrium theory since the fundamental contribution by Aumann (1964) is that to be logically consistent one needs an infinite number of economic agents. One of the most important results of Aumann's contribution was that one was able to show how continuum economies could be regarded as the limits of a sequence of large finite economies. This is not the case, for example, with Arrow's impossibility theorem which does not hold in the continuum case but holds in all large finite economies (see Kirman and Sondermann, 1972).

In this volume, Sabourian looks at the problem of the difference between the equilibria of large but finite games and those with a continuum of players. In general equilibrium theory, the argument advanced for using a continuum of agents is that this makes each individual genuinely negligible. What Sabourian explains is that when there are only a finite number of players, the players can choose history-dependent strategies. Having elucidated this problem, he shows how, by using noise and limiting the rationality of the players, one can obtain competitive behaviour in dynamic models with a finite number of agents. The only question that arises here is as to whether such a clever reformulation of the problem is really worthwhile. Some of us would suspect that game theory and strategic thinking are not appropriate for large economic models.

Another clever use of game theory is that by Gintis who analyses how certain incentive effects can be provided by an appropriately specified game. He shows us how certain preconceptions about *homo economicus* are undermined in an experimental game theoretic framework. He develops the idea of a more altruistic individual, *Homo reciprocans*. Here I have to make a remark in passing: One of the striking features of this collection of papers is the extent to which the authors have changed their original contradas and yet in a certain sense remained faithful to their origins. Let me take as an example Gintis' delightful analysis of the motivations of individuals. Not so very long ago he would have been in a contrada along with a number of other New Radical Economists. We now find him backing a horse which is at least as radical as his old steed, yet it has very different genes.

Gintis' position is that we have to find some sort of general equilibrium model, but if it is to be useful and to explain the actual behaviour of actors in the economy it will not be like the Walrasian model. Actors will learn to make contracts which are not explicit and optimal but which can be, but may never actually be, reinforced by punishments. Where we expect to see rational egoistical behaviour and observe apparent altruism we should not just dismiss the phenomena as due to irrationality but should try to see how interactions can reinforce such behaviour. He suggests that this is the route to explaining experimental observations in ultimatum or public goods games. A number of authors have suggested that people learn to punish free riders and potential free riders learn that they will be punished. Thus endogenous contracts emerge which do not correspond to Walrasian outcomes. However, it is enough to set up the model carefully for strong reciprocators to be able to invade a population of defectors. This is encouraging and fits well with Peyton Young's work on the evolution of conventions, even though I think Herb Gintis would regard Young as belonging to another contrada.

The lesson to be learned from Gintis' contribution is that even in strategic situations we should not necessarily attribute the classical non-cooperative behaviour of homo economicus to individuals. This is one reason for not adopting the radical view that all economics should be seen from the strategic standpoint and that all the actors should be assumed to adopt classical and extremely sophisticated reasoning.

Indeed it seems unreasonable to attribute so much rational calculation to agents in making all their decisions. In particular, one imposes upon them very strong beliefs about the rationality of the other agents. One has to know what players expect other players to do. But this brings me to another important contrada which was here largely occupied by Mordecai Kurz.

17.9 BELIEFS AND EXPECTATIONS

Beliefs and expectations enter into our considerations in two ways, one of which is discussed at length in this book with a new approach by Mordecai Kurz. In any intertemporal situation with uncertainty, agents must form expectations or at least have beliefs. The standard way of closing general equilibrium models in the setting is to assume that we only consider situations in which individuals have rational expectations, in the sense that their probability distribution over events in the future coincides with the underlying distribution from which those events are drawn. This implies that individuals in such a situation should have the same beliefs and that trade takes place because individuals have different characteristics and not because of any divergence in their beliefs. This does not seem realistic when one is looking, for example, at financial markets. It seems clear that most of the trades that take place in such markets are due to the different anticipations of the traders as to the evolution of prices in the future.

Mordecai Kurz's contribution shows that even different beliefs about the same future may, by his criterion, be rational. His approach is therefore much more general than that of rational expectation equilibria. The advantage of Kurz's approach is that he is able to reproduce many of the phenomena found in financial time series.

It is perhaps worth pointing out here that approaches in which individuals hard on different expectations can also produce some of the stylised facts associated with financial time series (see Banerjee (1992), Bikhchandani *et al*. (1992), Kirman (1993, 2000)).

Another remark which should be made here is that we often assume that agents are fully aware of the world within which they operate. In the standard general equilibrium model this is not very troubling since all individuals have to know and observe are the central price signals. However, as soon as we introduce more complicated situations it may well be that there is a discrepancy between what agents believe and reality. As Woodford (1990) and others have shown, individuals may well come to converge on situations in which their incorrect beliefs are confirmed and in which they have no way of refuting those beliefs. This was the essence of the famous "sunspots" models, and similar phenomena can be exhibited in a game theoretic setting (see Brousseau and Kirman (1993)).

A last word on this subject is that there are many situations in which expectations will change constantly and never settle to any fixed state. In such a case, with switching regimes for example, an alternative equilibrium notion is needed. This is particularly the case in which agents are trying to form their expectations from their experience and they all, by their learning, affect the variables they are learning about. In Frank Hahn's terms there is a feed-back from the micro-behaviour to the macro-variables which in turn are used as signals by the agents. Thus learning has to play an important part in any reasonable description of economic reality and it is this that leads me to the next subject.

17.9.1 Learning, adaptation and evolution

Learning and adaptation are particularly significant from two points of view. First, individuals may, by learning, reduce the uncertainty with which they are faced and this is related to the previous topic. Second, though individuals may not be clever they may learn to do clever things. This is the argument that underlies Lucas' justification for the use of optimisation. His way of saving optimising behaviour from its critics is made explicit, when he says

> In general we view, or model, an individual as a collection of decision rules (rules that dictate the action to be taken in given situations) and a set of preferences used to evaluate the outcomes arising from particular situation–action combinations. These decision rules are continuously under review and revision: new decisions are tried and tested against experience, and rules that produce desirable outcomes supplant those that do not. I use the term "adaptive" to refer to this trial-and-error process through which our modes of behaviour are determined.

However, Lucas then goes on to argue that we can safely ignore the dynamics of this process because, "Technically, I think of economics as studying decision rules that are steady states of some adaptive process, decision rules that are found to work over a range of situations and hence are no longer revised appreciably as more experience accumulates."

De Vroey has described Lucas's point of view as one of "benign neglect". However, it is more than that. His argument is that the evolution of the economic environment is very much slower than the speed at which agents adjust to that evolution. Thus the two processes can safely be separated. Surprisingly you will find, in Petri's contribution, the same sort of justification for a convergence to a long-term state. We are justified in worrying about these long-term states because the economic environment changes very slowly in relation to the speed at which economic actors learn. Thus, Petri finds himself backing the same horse as Lucas, though neither of them would, I believe, claim for a moment to share the same ideology. Yet, the position on which they agree seems to me to incorporate a potentially fatal error. The environment in many economic situations is composed of the other economic actors, and since the latter are also learning, the convergence of such an interactive system is far from being obvious.

If we put this problem on one side for a moment, one stream of thought wants to show how optimal behaviour may arise from a process of learning. Yet, the pitfalls of the evolutionary approach as a justification for optimal behaviour are well known. The delightful idea that, had the famous apple fallen on Darwin's head rather than on that of Newton, we would have a very different explanation of why apples fall downwards is a nice illustration. Clearly, Darwin would have assumed that apples originally fell in all directions, but only those that fell downwards were capable of sowing their seeds and reproducing themselves. Hence, natural selection soon led to only these apples surviving and it is this and not gravity that is the explanation for the fact, that apples fall to earth. In the same vein, Steven Gould after hearing a long and technical talk on the optimal adaptation of sunflowers remarked that it would have been much better if sunflowers had simply developed feet and they could then have walked to wherever they needed to be in relation to the sun.

Game theory evolution, at least as a metaphor, has become very important and the use of learning took second place as it was largely replaced by some sort of evolutionary argument (e.g. Gintis). Of course, evolution as presented in evolutionary game theory can always be replaced by a learning interpretation. Nevertheless, the aim of game theorists was to justify certain equilibrium notions and to select among them. Thus, their purpose was not to show how individual behaviour might evolve to some sort of optimum but how the aggregate outcome might do so. The basic idea is that equilibrium results from natural selection among strategies and that individuals are identified with strategies.

The difficulties with such an approach do not imply that we can afford to do without some account of how different behaviours and social situations evolve and emerge but do suggest that we should be very careful in using the evolutionary paradigm. Contrary to many of my colleagues, I share Mirowski's (2001) position in suggesting that if we are to talk about evolution we have to specify carefully what we mean by their various terms such as genes, reproduction and fitness.

Learning is a very different story and one which we surely cannot overlook. One definition of intelligence is that it is the capacity to profit from previous experience. The purely evolutionary approach leaves no place for such intelligence whilst most economic models probably attribute too much intelligence to the participants. The sort of learning that is widely used in the game theoretic literature is of the Bayesian type and this implies considerable intelligence and sophistication of the actors. However, the problem is that, unlike the sort of situation portrayed in the two-armed bandit literature, for example, what agents are learning about is, in general in economics, the behaviour of other agents and the latter are also learning and are often aware that all agents are involved in the same process of reflection. This leads to an infinite regress problem which is similar to that posed by the assumption of "common knowledge" in game theory.

Yet other learning procedures, for example the "classifier systems" of John Holland can produce striking results while only requiring that indivduals have a tendency to use most often those strategies that have been successful in the past. In such a framework individuals may well learn that it pays to cooperate without making any sophisticated calculations as to the intentions of those with whom they interact. Indeed this sort of approach has been formalised by Easley and Rustichini (1999) and they show that under certain assumptions a group of individuals may achieve a socially efficient outcome without holding any beliefs at all.

I think, many people would agree that even if we do not go as far as Easley and Rustichini, we have to incorporate some sort of dynamic which would allow for adjustment as people learn. As I have said, my feeling is that learning should be portrayed as a selection among rules. Although many economists are reluctant to abandon rationality as embodied in the standard assumptions on preferences in general equilibrium theory, a process of trial and error seems a much better description of how people arrive at most decisions. Of course, this casts a shadow on the whole of the standard model since we then have to specify carefully the learning process and the associated evolution of aggregate outcomes.

17.10 THE WAY FORWARD

I agree with Frank Hahn on the need for a second look at the nature of macroeconomics. To look at the details of every individual's situation at every point in time is not appropriate for most economic analysis. We need statistical descriptions of the states of an economy, the evolution of which will be generated by, but not similar to, the evolution of the individuals' states. This sort of statistical approach is related to that proposed by Foley in this volume and is also linked to the work on the distribution of agents characteristics by Werner Hildenbrand (1994). Perhaps we should go much further and ask ourselves whether we should look at "equilibrium paths" of states in the standard sense or whether we should look at a more general class of dynamic paths. Suppose that we take Frank Hahn seriously when he says, "Everything is changing in the long run". In this case, we have to accept the idea that the economy will have periods of relatively stable behaviour, periodically interrupted by movements as the space of the states itself changes. This is a view which will be very familiar to evolutionary biologists but one which may be difficult to accept for economists.

This brings us back to a question I have raised earlier. What is important is the relative speed of adaptation and evolution. If the evolution of the environment, in this case, the economic environment, is rather slow and the behaviour of economic agents rather fast then one would see precisely the sort of evolution that I have just mentioned. I have to repeat that it is not at all clear that in a situation where the environment is made up of the other individuals that such a picture is appropriate. It seems to me that there is a great deal of work to be done before we have an adequate picture of how a complex adaptive system such as an economy organises itself and evolves over time. Indeed, in this conference, little or no attention has been paid to the organisation of the economy.

For example, the networks in which individuals operate are important; they govern transactions, they govern opinions, preferences and expectations, and they govern the transmission of information and the spread of technologies. Although economists have made some effort to take the influence of networks into account, very little has been said about where they come from and their evolution over time. Their effect may be "stabilising" in the sense that they permit a free flow of information and facilitate arbitrage or "de-stabilising" in the sense that they facilitate the development of epidemics and bubbles particularly on financial markets.

We will probably also have to devote more attention to the emergence of institutions and, in particular, the structure of markets. With the possible exception of the GEI contributions the chapters in this volume have little to say about the actual structure of markets and a great deal to say about the abstract and rarefied "market mechanism" which underlies general equilibrium theory. This is something which economists interested in real phenomena should be more concerned about.

17.11 CONCLUSION

What then are we to make of the result of this Palio? From the outset it was clear that no horse would win since most of them were competing in different races and not really against each other. What did emerge was a great deal of clarification of the competing views of the world and perhaps optimistically more understanding of each others viewpoints. Whilst old and long-standing battles were re-fought in a rather more gentlemanly way than in the real Palio, other new competitors were entering the fray. If we look in to the future it would seem that there are two rather different points of view. There are those who believe that by modifying and enlarging the scope of standard general equilibrium models we will come to some better explanation of economic phenomena. On the other hand, there are those like myself who believe that the whole paradigm is in need of a radical shake-up. I was happy to find that I was not alone but of course did not find any intellectual horse that was up and ready to run and to beat the existing paradigm. If the general equilibrium theory contrada is in a difficult period this is due to weaknesses which developed and were made explicit from the inside rather from the outside. The nearly fatal Sonnenschein, Mantel, Debreu syndrome could not have been generated by more respectable members of that contrada. Indeed as Frank Hahn has pointed out, Debreu's own project has turned out to be immune to the disease. The problem that many of us have is that the project does not seem to be the right basis for an empirically verifiable model. The model is intact but now looks almost irrelevant to the understanding of real economic phenomena. Yet the fact that general equilibrium theory in its standard form is in a weakened state is a small comfort to those who are looking for a rigorous framework within which to think about these economic phenomena. The Palio will continue to happen and the General Equilibrium contrada will continue to run a horse, albeit a rather different one from that described by Arrow and Debreu. The question is how well it will compete with its other, often rather undisciplined rivals.

In fact, I believe that the action will be in other contradas but not in those which would like to throw analysis away and concentrate on description. My suspicion, which the 10 years to come will put to the test, is that we will probably borrow another framework from physics, that of statistical mechanics, to make our next step. This does not mean that we should necessarily adopt, as Walras and his contemporaries did, a physical framework without question. We should rather make use of the insights that such an approach provides in terms of notions of equilibrium and the relationship between aggregate and micro phenomena to improve our capacity to explain and perhaps even forecast real economic-phenomena, something which we have been rather poor at in the past.

NOTE

1 It is only fair to say that recent contributions by Billette de Villemeur (1999, 2001), Hildenbrand (1998), Hildenbrand and Kneip (2000) and Giraud and Maret (2001) have shown that the idea that diversity of characteristics will lead to uniqueness and stability is illusory and depends on a very special and limited form of diversity. The hope of curing, in this way, the horse representing the general equilibrium contrada of the Sonnenschein Mantel Debreu syndrome may well prove to have been in vain.

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18 General equilibrium: problems, prospects and alternatives

Final discussion

Editorial note: The last day of the Workshop was dedicated to a general discussion, chaired by Fabio Petri; it was opened by Prof. Kirman's lecture, which explains why many interventions refer to his talk. The following is a transcript of the discussion which followed. Some interventions have been edited by the respective authors, but the arguments have remained fundamentally unchanged.

Foley

I think Prof. Kirman rose admirably to a very difficult task. I would like to comment on some observations he made in the first part of his talk. We are in the midst of a period where science in general and the science of economics in particular is coming to an understanding that there is a much broader range of ways in which systems can order themselves and exhibit regularities than were recognized up to 20 or 30 years ago. Typically in the past some notion of equilibrium was almost the only notion of the way that you could have regularities and could study them in a complicated system. In the last 20 or 30 years, developing the work of Poincaré, we have begun to understand that there are ways in which a system can have regularities that are not necessarily describable as equilibrium, and we should think about that in terms of economics. Let me mention a couple of examples. In dynamic economics one pattern arises in chaotic systems where there is no tendency to settle down to an equilibrium and yet there can be strong statistical regularities. But perhaps more relevant to economics are complex systems, which are not as unstable as chaotic systems, but are more unstable than equilibrium systems, are on the boundary between chaotic and stable systems, and often show certain kinds of self-organization. Complex systems do not exhibit a tendency to equilibrium, but despite the fact that they can amplify shocks, they don't explode. So, for example, Per Bak, in a book called "How nature works", has advanced a modest proposal for understanding all systems in terms of self-organization, using a model he calls "self-organized criticality". These systems sustain themselves in states that are far from equilibrium in the traditional sense. In economics it seems to me that the challenge of the future is to sort out the appropriate time scales for the different aspects of economic life in order to be able to use each of these different concepts of self-organization to model them. I myself believe that there is strong evidence for something like thermodynamic statistical equilibrium on some very short time scale, reflected in the emergence of prices which give us the quantitative data we can work with. On the other hand, on a somewhat longer time scale it seems to me quite likely that the economy is not stationary, and is not going to have classical equilibrium properties, though it may have other self-regulating properties, self-organizing properties, which are what we need to study.

Kirman

I basically subscribe to this point of view; and I think that everybody who spends some time reflecting on these issues will tend to do the same. Perhaps when someone writes books with titles as ambitious as "How nature works", this tends to have a counter-productive effect because, by exaggeration, it tends to make the thing look less serious than it is; I think it is an extremely serious project.

Garegnani

I believe that the Debreu, Mantel, Sonnenschein's "syndrome", as Prof. Kirman calls it, is less well diagnosed and more serious than many here believe. But let me come first to two points Kirman has just raised about "classical" theory. On consumers (Kirman, p. 468) I can be brief. Consumers are not ignored in classical theory: the notion of a subsistence wage and of its socio-historical determinants provide a good example of that. The difference from neoclassical theory lies in the *kind* of analysis conducted, not in the absence of it. Kirman connects however the absence he sees of a classical consumption theory with a lack of interest for "Pareto's optimality"; but the point here, rather than lack of interest, is that such an optimality is *a priori* excluded in a theory where – witness Ricardo's chapter on Machinery in the *Principles* – labour unemployment and a social product below what is technically feasible are seen to be compatible with competitive prices and the natural wage.

Kirman's second point concerns the "stability" of the competitive normal positions of the economy to which Smith, Ricardo or Sraffa refer their classical prices. The question has certainly not been overlooked in the present revival of classical theory. Discussion is proceeding (e.g. this university held a symposium on the subject, edited in 1990 by our chairman of today): but I would dare say, and have so argued at the time, that Adam Smith's conclusions on the subject in Chapter VII, Book I of the *Wealth of Nations*, "Of Natural and Market Prices of Commodities", still stand solidly after more than two centuries. The key to the stability of the normal prices – whether the classical ones of Smith and Sraffa, or the neoclassical ones of Marshall, J.-B. Clark or Wicksell – can be seen to lie in the stability of the *real wage*. Given a stable wage, the competitive tendency to uniform prices for productive services may be shown to be broadly sufficient to warrant a tendency to the normal prices: but whereas in classical theory the stability of the wage is ensured by the socio-institutional factors determining it, in neoclassical theory it depends on the shapes of labour demand and supply functions and it is there that the stability of normal prices meets the obstacles of "income effects" or, more radically, reverse capital deepening.

Returning to the neoclassical sickness, Arrow and Hahn (1971) reminded us of the exclusive interest of "sensible equilibria", since

> Certainly ... we should not be much interested in an equilibrium with a zero real wage (pp. 354–5)

But we saw in my paper yesterday that reswitching and reverse capital deepening may indeed entail just *that* kind of "non-sensible" intertemporal equilibrium.

This runs counter to what Hahn wrote several years ago:

> I have said that neither Sraffa nor his followers have made anything of reswitching. By this I meant that they have continued to believe that it is damaging to neo-classical *equilibrium theory*, which is not, and have neglected various neo-classical *adjustment theories* which are certainly at risk.
>
> (Hahn, 1982: p. 373, our italics)

Surely Hahn would agree now that what causes an equilibrium *not* to be "sensible" has to do with "equilibrium theory"?

As for the "adjustment theories" of the same passage, Prof. Hahn continues:

> the famous Solow parable in which all equilibrium paths seek the steady state depends on just those possibilities of aggregation which reswitching examples show not to be available. Indeed [...] professor Robinson was right in arguing that capital aggregation [...] has had the consequence that no agent needs to have correct expectations concerning [...] future prices

Yesterday we also saw some of the difficulties which reverse capital deepening can cause for the adjustments to, and uniqueness of, intertemporal equilibria – a strict counterpart, as should perhaps have been expected, of those that had been pointed out long ago for the traditional equilibria (e.g. Garegnani, 1970). In his passage, Hahn seems therefore to only discern the tip of the iceberg, i.e. the difficulties for the trajectory of the equilibrium path, and not its body which regards the properties of each of the dated equilibria that should make up the "path". Indeed Hahn's reference to "correct expectations" indicates that he may simply be referring to the "problem" named after him, which seems in effect to show how difficult it would be for expectations out of line with objective factors ever to be realised in a longer period and, therefore for an economy, to correspondingly suffer from those causes of instability, rather than from objective ones like those we saw yesterday. In any case the "Hahn problem" is altogether independent of the reswitching to which Hahn seems to connect instead the "risk" it raises for "adjustment theories": reswitching and reverse capital deepening are compatible with the complete "futures" markets which would void the "Hahn problem". So, when Hahn concludes (ibid.):

> it was left to neo-classical economists to attempt to study the precise pathology of the price mechanism which may result when heterogeneous inputs are modelled explicitly

it comes natural to reflect that perhaps the neoclassical study of reswitching as a "pathology of the price mechanism" (or is it only the pathology of a particular *theory* of the price system?) has not gone further in "adjustment theories" than it has in "equilibrium theory", when alleging the latter's immunity from such a "pathology".

All this may also make it clear that if the problem of capital is, as Kirman says, one of aggregation (p. 473), it certainly is not one having to do only, or even mainly, with "aggregate production functions". It has to do with the questions it raises for the validity of the neoclassical – or, if we like, of the general equilibrium – attempt to explain distribution and prices in terms of the substitutability between "factors" and the resulting demand and supply functions.[1] Thus, I have some difficulty in following Prof. Kirman when he says that

> capital goods as such make no sense in [intertemporal] general equilibrium theory any more than a single rate of interest does (p. 473)

The fact that the expenditure side of budgets drawn over the entire duration of the economy can be written in terms of consumption only (when terminal capital is zero) does not contradict the other fact that production occurs in definite "years" (t) with outputs which include capital goods. The intertemporally aggregated all-consumption expenditure may therefore be seen to result from the summation of *yearly* budget equations dealing with the gross income coming from those yearly outputs: in those equations the gross "savings" corresponding to the capital goods produced in (t) appear on the expenditure side of budgets for (t), but then they cancel out with the amount which in ($t + 1$) will result on the income side from those savings. And with such savings there come the problems of "capital" and of its employment with labour in proportions dictated by the techniques and the outputs whose relative profitability changes as the intertemporal prices change.

As for the role of "the" rate of interest also mentioned by Kirman, I remarked (my paper, par. 22–23) that it is misleading to argue, as e.g. Arrow (1989) appears to have done, that adjustments in *intertemporal* consumption outputs raise no more problems than adjustments in *contemporary* consumption. In the latter case resources can be shifted between the two or more productions in *direct* response to the change in their relative (contemporary) price plausibly resulting from the excess demands, but in the former no such direct shift is ever possible. Excess savings – i.e. excess supply in t of "corn" (imagine it to be the only consumption good), and corresponding excess demand for it in ($t + 1$) – can only be remedied if the fall of the intertemporal relative price of "corn" (i.e. the "corn" interest rate in t) induces entrepreneurs to increase the application of "capital goods" to assist the fully employed labour of t. That, and only that would, on the one hand, increase the demand of capital goods in (t), thus absorbing the resources of ($t - 1$) freed by eliminating the excess "corn" supply of t – and, on the other, raise the productivity of the fully employed labour of t so as to satisfy the excess "corn" demand of ($t + 1$). This process, Arrow and Kirman will agree, is quite different from that of adjusting contemporary consumptions: indeed it is essentially the process traditionally described as the adjustment of investment to savings by means of the interest rate: it suffers therefore from all the shortcomings we *now* know to attend the response to interest rates of the ratio between labour and means of production. The point raised by Kirman, that there is no single rate of interest but as many as there are commodities, complicates the matter but makes no essential difference in the nature and difficulties of the adjustment. There are, incidentally, reasons to think that such rates would generally tend to move in the same direction (par. [9], in Appendix I to my paper) so that we might often talk as if there were a single interest rate.

It is then ironical, I noted in my paper, that Keynes should have written:

> If savings consisted not merely in abstaining from present consumption but in placing simultaneously specific orders for future consumption, the effect might be easily different; for in that case the resources released from preparing for present consumption could be turned over to preparing for future consumption
>
> (Keynes, 1936: 210–211; cf. also n. 52 in my paper)

Keynes was granting too much here, and it appears that even within a correct analysis based on the received premises the conclusions of the *General theory* would be stronger than he felt when he relied only on uncertainty and incorrect expectations to counter the received conclusions. But that only means that such a logically correct analysis throws into question the entire approach to distribution in terms of demand-and-supply for "factors". Indeed the fault of orthodoxy may be seen to lie in the *theory* rather than in the *method:* outside neoclassical theory the possibility of deficiencies of aggregate demand seems inescapable. (We may also see, therefore, how misleading is the view of some authors on an affinity, between the current resumption of the theory of the old classical economists, and some resurgences of pre-Keynesian neoclassicism, because of a similarity in their long-period method of analysis. Far from overlooking the Keynesian critique, the classical resumption can however be seen to draw the full implications of it by *abandoning* the theory, rather than attempting *to save* it by abandoning the method by which theory and observation have been traditionally related in economics.)

It may also be clear now why, unlike what Kirman attributes to authors of the classical "contrada", I do not think the problem with general equilibrium theory is its failure to answer the right questions. The ultimate questions are there the same as in classical theory: what makes the distribution of the social product, or the prices, or the level of aggregate output, etc. be what they are: the neoclassical failure lies in the answers.

As I see it, there are two levels in that failure. At a first level contemporary general equilibrium falls between two stools. As the significant theory of the economy which it was trusted to rigorously be at the times of the Marshalls, Wicksells, or Pigous, and which is still *applied* today as an alleged approximation, neoclassical theory is generally recognised to be inconsistent because of its reliance on a "quantity of capital". The attempt to remove the inconsistency has, on the other hand, robbed the theory of much of its explanatory capacity and turned it into the "mock up plane that will not fly" recalled by Kirman.

At a second, deeper level, however, neoclassical theory fails because, as is now beginning to come into the open, the conception of "capital" as a factor of production is indissoluble from the neoclassical attempt to replace with a generalization of classical rent theory, and the resulting demand-and-supply forces, the explanation of the division of the product between profits and wages which the old classical economists gave, essentially, in terms of socio-historical forces. And the first-level failure is but an expression of this second, more deep-seated failure.

Thus the question about neoclassical theory is about the theory itself, rather than on the problems it deals with: i.e. the question is, what to make of a theoretical approach which, if not reined in by ad hoc assumptions, can bring to what, in Arrow and Hahn's words, are "non sensible" results. The answer that there is nothing new about the present results on capital, since the possibility of non-uniqueness and instability of the equilibria has been generally admitted, is tempting but not adequate.

The systems from which such possibilities had previously emerged were systems of either pure exchange, or of atemporal production also in effect excluding capital. Negative results could therefore be only due to "income effects", with which the theory has somehow managed for long to coexist. And the conviction that the difficulties – however dramatized by the Debreu, Mantel and Sonnenschein's formalizations – could only result from such income effects is, I believe, what ultimately explains the persisting trust in the theory from a majority of the profession and, therefore, its peculiar continuance on the basis as of two "modes" complementing each other: one of "rigour", and one of actual practice in the profession. In the first "mode" one really finds only one general, non-negative result: the existence of a solution of the general equilibrium equations, often misleadingly described as "the existence of an equilibrium".[2] But then in the second "mode" the significant traditional propositions are applied, thus in effect using the mode of "rigour" as an entirely unwarranted confirmation of the essential validity of the neoclassical approach, and therefore, of the propositions traditionally drawn from it.

In fact, the negative results in question here, due to the impossibility of consistently conceiving capital as a single quantity, have a significance essentially different from those due to income effects. Far from being compatible as the latter "effects" are, with a more exactly defined "substitution effect", the former undermine the substitution principle in the full generality of its decisive application to "factors of production", whether through alternative techniques or through alternative consumption goods.

Moreover, those negative results do not point to an isolated difficulty of the notion of capital in neoclassical theory: they rather constitute only one aspect, however central, of a whole set of difficulties which beset that notion. Another aspect is what has prevented the keeping of the questions of the uniqueness and stability of the demand-and-supply equilibria in the transparent terms which the founders of the theory believed would follow from the twin principles of decreasing marginal utility and decreasing marginal productivity. By the time of Hick's *Value and Capital*, the failure of the attempt to arrive at a consistent concept of capital as a single factor along the lines of the average period of production had begun to enforce in mainstream neoclassicism a recognition that consistency on capital left really no way out but the Walrasian one of treating each kind of capital good as a distinct factor in the endowment. That entailed proceeding to the intertemporal setting imposed by the impermanence of the resulting equilibria, with not only the difficulties of that impermanence for relating theory and observation, but also the lack of transparence and the enormous complexities which that attempted treatment necessarily entails.

This second aspect of the difficulties has indeed been alone sufficient to cause Christopher Bliss – in a (1975) book paradoxically intended to deny or decry the difficulties relating to neoclassical capital – to choose as the "more sensible" notion of equilibrium (why this returning need to rein the theory in from non "sensible" conclusions or notions?) not that of a state to which the economy tends, but that of a state which is simply "assumed", and thus

> regard the object of our investigations not as 'the economy' but as 'economic equilibrium' (1975, p. 28)

independently, that is, of the capacity of the "equilibrium" to represent the economy, a capacity which, we are told, "one might eventually hope" to argue only when a "full specification of [the] disequilibrium dynamics [of the economy]" will be achieved, a "Herculean programme" Bliss admits. The author does not seem to ask himself why exactly a "programme" of explaining *the economy*, which seemed accomplishable, and indeed largely accomplished already, in the times of Walras or Marshall more than a century ago, has in the meantime become an impossibly Herculean one.

Kirman

I did not spend much time on classical theory simply because of my ignorance. As to the "syndrome", I think you are absolutely right, the syndrome is a very important one. But let me make one comment on demand: I agree that there is an asymmetry in the use of the word in classical and neoclassical authors; but I think that really one cannot write down equations where one takes quantities demanded as a given vector; the notion of a subsistence bundle of goods fixed in its contents may have been applicable in the past but definitely in the modern world demand has become more and more important, and the pattern of demand will be determined by tastes.

Garegnani

There are two aspects, in demand and tastes, which we should distinguish: the *content* of tastes and the *formal property* of decreasing marginal rates of substitution between goods consumed. Now, "subsistence" is no more than an example of the classical focus on that "content" of tastes which is also what would seem to mainly matter for Kirman's "pattern of demand" (thought not for supporting the Pareto optimality I referred to in my intervention). The neoclassical focus is instead on the formal property rather than the content of tastes.

This exclusive neoclassical focus is of course only natural, because the formal property of tastes provides, alongside the alternative to techniques of production, the basis of factor substitution, and hence of the specific demand-and-supply apparatus assumed to determine everything from distribution down to prices and outputs. But it is equally natural that the formal property be of little interest to the old classical economists since their theory of distribution between wages and profits, based on broader social conditions, neither required, nor allowed for a determination of prices and outputs by means of demand and supply *functions* of the neoclassical kind. As the "non substitution" theorem reminds us, relative prices are determined independently of demand, once the real wage is given – and once increasing returns from the division of labour, or decreasing returns from scarce natural resources are left for consideration in the context of accumulation and growth. And besides, the absence of demand and supply of "factors" and of their equality would have deprived, even in principle, any demand functions for products of their basis in simultaneously determined incomes.

But without demand functions, the outputs are *in fact* left for *separate consideration*, i.e. are taken as "intermediate data" in determining prices where what should be stressed is the word "intermediate". However, Kirman says that we cannot write *equations* where the quantities demanded are a given vector, by which I believe he means that we cannot have a *theory* which is not concerned with determining the quantities demanded. As follows from what I just said about content of tastes, I entirely agree with him there. But the determination of the quantities demanded may however be separate from (though not necessarily independent of) that of prices, and therefore does not consist of equations to be solved simultaneously with those determining prices: indeed the general determination of outputs may in part or all not run in terms of equations at all. That is what lies behind the quantities demanded being given in the equations determining prices.

Determination we may recall need not take the form of equations: a *dependence* of quantities demanded and produced on prices (as on the several other circumstances, which they are likely to depend on) does not suffice for us to usefully postulate functional relations between them. For that we should be able to attribute to those relations, properties universal and definite enough to render a mathematical treatment of them helpful at a level of general theory (as distinct from "models" which are a different matter): otherwise such relations are, as Edgeworth once aptly put it, "arbitrary functions representing [only] ignorance" (1881, p. 4). Now, neoclassical theorists believed that such fruitful functional relations exist, consisting essentially of generally decreasing demand functions derivable from tastes, techniques, endowments and that they so interact with other elements as to determine prices and outputs. Hence their determination of outputs simultaneously with prices. Smith and Ricardo did not see any such functions and determinations, which would indeed have been unnecessary for, or in conflict with, their vision of a distribution between wages and profits determined essentially by demographic and socio-institutional circumstances. Hence their different treatment of outputs, representable by taking outputs as (intermediate) data, once their price determination is expressed in equations.

Kirman

I still think that the demand side has to be treated by theory; maybe the neoclassical way is the wrong way of treating demand, but certainly demand equations have a very important part to play in the determination of outputs. As to the second part of your opening comments, given these instability problems I can see perfectly well that the economy could settle itself on a path going toward zero prices, and in particular, zero wages, but of course a zero wage does not make sense, there are many forces which are going to prevent this, so we can point to a separation between forces which may put the economy on a path tending toward zero, and forces which prevent the economy from actually getting to zero. That may be a reason why we get unemployment in concrete economies but not zero wages. It may well be the case that for Keynesian reasons the economy starts spiralling away from an equilibrium in the labour market, but then these other forces stop the spiralling for all sorts of reasons, this is an issue which we should perhaps discuss some other time. A last remark. Capital for me has always been a mysterious notion, and I still find it very difficult to understand; I have this feeling, and it is a purely intuitive feeling, that the Cambridge U.K. side won the debate with Cambridge U.S., but somehow afterwards all the difficulties with aggregate capital and its use in an aggregate production function were just swept under the rug by modern macroeconomists as Fabio Petri has pointed out.

Chichilnisky

First of all thanks to Prof. Kirman for a very nice summary. I would like to pick up some threads of what Prof. Foley and others said. We are now in a peculiar time in history where the coupling of human and physical systems is becoming much more intense. Because of economic growth, now it is the first time in history where humans have the ability to change the composition of the atmosphere of the planet and the distribution and composition of species. This coupling emphasizes the importance of interdisciplinary work (economics and environmental science). It also leads to the realization that human activities can undermine important ecosystems on earth. Our knowledge about these situations becomes increasingly important in many ways. We are, I would say, in the midst of a *knowledge revolution*, in which knowledge is replacing the role of capital much like capital replaced the role of land after the industrial revolution. General equilibrium is about markets, but now it must be about markets which are coupled with nature, in which natural resources play a very special role, and also markets in which knowledge plays a very special role as well.

It is perhaps a coincidence that both knowledge and environmental assets (like the atmosphere of the planet or the composition of species) are "privately produced public goods". These are public goods in which the decisions about production or consumption are taken on private grounds. For example, the carbon dioxide composition of the planet's atmosphere is determined by our decisions, purely private decisions about using cars or burning oil to heat homes. Knowledge is also a privately produced public good. Markets with privately produced public goods are therefore going to be some of the most important markets in the future. They include markets for environmental assets and markets for knowledge. If that many of the topics, which were mentioned at this conference, fit this observation well indeed, markets with privately produced public goods exhibit a strong connection between distribution and efficiency issues, a connection, which does not exist in the markets for purely private goods. Markets require a different form of organization from markets for private goods. We thus get close to Herb Gintis' concerns for the types of solutions which were presented in his paper, examples of how a coordination of property right policies distribution and the introduction of new markets can help solve problems with strong externalities as in the case of knowledge and in the case of environmental assets. Since markets with knowledge and environmental goods are indeed markets with public goods, concerns that the connection between efficiency and distribution are going to become increasingly important. Furthermore, as in the case of knowledge there are reasons to believe that due to strong increasing returns to scale, in those markets efficiency and distributional concerns are likely to be important connected efficiency in markets with increasing returns also calls for property right policies to move the economy closer to the Pareto frontier. It seems to me that the connection between distribution and efficiency is likely to become one of the most important economic issues in this new century. There is also the connection to the complexity of the system. We are considering complex, coupled systems and the fact that generically resource-economic dynamical systems are often chaotic, with several "attractor" regions. Typically we will be faced with economies, which are jumping between "chaotic attractor" regions, and we should study what forces are responsible for jumps and how frequently these occur.

Kirman

I could not possibly respond to all the points Prof. Chichilnisky raised here. I will just make two very quick remarks. It is commonly held that we are now in a situation relative to environmental problems which is fundamentally new and different from what went on in the past since, as Prof. Chichilnisky says, we now modify our physical environment. I would like here to make a quick reference to some very interesting work. Recently someone decided to start to sell spring water and since all spring water is polluted so he started collecting water from deeper strata of ice in Greenland and Iceland so it comes from centuries back. Parallel with this enterprise someone else found that in some deep strata of ice in Iceland there are lots of lead. And where does this lead come from? From the Roman Empire; there is even a story going around that the Roman Empire collapsed because of lead pollution; anyway there was so much lead pollution that lead even arrived in Greenland and is there in ice strata. So there have been moments in the past when pollution was very important. In any event, this was only an observation in passing.

On complex systems, one person who has tried to do work along the lines you mentioned is Jean-Pierre Aubin, in Paris, whose idea is basically to look at sustainable development, forgetting about equilibrium, one just looks for which set of paths is feasible. He does not study how to move from one path to another.

Gintis

In regard to Prof. Kirman's talk, I want to make two points. First, I want to defend evolutionary theory. Evolutionary theory does not argue that the results of an evolutionary process are optimal. This representation is quite incorrect, as can be gleaned from even an elementary textbook on the subject. Stephen Jay Gould has made the non-optimality of biological organisms a centerpiece of his argument against creationism, for instance. Evolution is always subject to developmental constraints, and hence gives results that persist through time not because they are optimal but because they happen to be better than other things given the circumstances.

Second, Prof. Kirman's simile with the Palio was ingenious, but to me it is very sad that this is a reasonable way to represent economics. I used to be in physics and mathematics and I feel happier about the way physics and biology treat their subject – as a common enterprise. Only on the edge of new knowledge there is much dispute, not about the fundamentals, about which there is virtually unanimous agreement.

I think we should firmly chastise those who treat doing economics as a tournament for which winning is the only goal.

Kirman

My point about adaptation was precisely the one you made, to observe that something is persistent does not mean that it is optimal; and yet one hears so often this kind of inference, that if something is observed and persistent then it is the optimal way. As to the Palio, in some sense you are right, we need much more mutual respect, but as far as the "anything is permitted" aspect of the Palio, the very nasty actions during the race, this happens in other disciplines too.

Gintis

But on the edge of the discipline, not on fundamentals.

Hahn

I do very much disagree with the picture of the loving scientists in the natural sciences given by Prof. Gintis. It is totally and completely wrong. I agree with him that perhaps in economics the fight is on a slightly greater scale, but I had an experience of being asked by Oxford University to consult various mathematics professors in Cambridge to collect opinions about an appointment on which in Oxford they were unable to agree, and you should have heard them – I can assure you that there is as much disrespect, and as much disagreement about the relative importance of research directions, in mathematics as in economics. Game theory, by the way, was abhorred. I think all the subjects have this kind of disagreements. But other subjects are slightly different because we have rather solid groupings in our subject, and I experienced that in Cambridge where it was very difficult to reach agreements on the Faculty Board.

End of the morning discussion

AFTERNOON SESSION

Sabourian

Professor Kirman's talk I think was wonderful; but the gist of what came out, I take it as not being representative of what our profession is about. I have no comments on the classical side; but on the rest of the profession, I do not think that there is this kind of division, there is no division between game theorists and evolutionary game or general equilibrium and contract theory, there is no situation like "we each have our own research agenda"; these things are actually or can be quite complementary. A second point: I think that what Prof. Foley has done is very interesting work, but it is very difficult in order to have predictive power to avoid the question of the rational actor, of some actors who have some range of choice and power to affect things but may be in a strategic situation; game theorists have all kinds of problems, but the issues the game theorists are concerned with would appear one way or another in all the other models with some kind of actors. One last thing very quickly: I am always surprised when I hear general equilibrium people or macro people, when you are dealing with large economies the complexity of decision-making is really important, and somehow we avoid that, we kind of always say something like "bounded rationality is difficult", but we should look at this, it is as important as the question of beliefs that Prof. Mordecai Kurz raised in his talk.

Foley

This question of the voluntary action of agents in the statistical equilibrium model is very important. The theory represents everything as quite passive: you just come into the market with an offer set and you do not have any control over where you land. I suppose in the back of my mind is an argument along the lines: "Sure, everybody is going out in the market and they are struggling like hell for the surplus, trying to find good deals there, scurrying around, looking for bargains and so forth. But the problem is that they are all trying to do that, and there is no reason to think that one of them in particular will be more successful than any other. The theory effectively assumes that these efforts just compensate each other statistically, through the process that is impounded in the black box of the trading process".

Gintis

My feeling is a little bit different from Dr Sabourian's: there *is* tension between the general equilibrium theory and at least game theory in the United States; and general equilibrium theorists are unhappy because they have been pushed, from the center of the research that's being done, to specialized journals; I think Profs. Magill and Quinzii were complaining about that in one of their lectures; there is also some tension between contract theorists and game theorists, although it never comes out in the form "let's fight about it".

Fisher

I wish to discuss a point that I regard as not very important but which has the property that the issue of its importance will raise considerable controversy. I wish to argue that two issues should be separated:

1. The question of the existence of an aggregate capital stock or of aggregate production functions; and
2. The question of whether there are things that are unsatisfactory with the state of general equilibrium theory.

While I believe the state of general equilibrium theory to be partly unsatisfactory, I also believe this to have nothing whatever to do with the existence of aggregate capital, however important that subject may have been to various people in the past. My argument goes as follows:

There is nearly universal agreement that aggregate capital considered as a factor in an aggregate production function exists only under extraordinarily special circumstances. This is a technical result long ago proved by myself, Gorman, and others. Nevertheless, the implications of that result do not always seem to have been fully realized. One of those implications, in particular, is that if one attempts to use aggregate capital as though it were a factor in an aggregate production function, there will be a paradox somewhere, something will go wrong. One cannot depend on any intuition that comes from considering aggregate relationships as production functions.

Now, Garegnani in his paper demonstrates that in a very simple general equilibrium model there are some things which appear to be problems. If one defines aggregate capital in terms of total savings, one gets reswitching and reverse capital deepening. He concludes from this that aggregate capital is an important ingredient of neoclassical general equilibrium theory and indeed, that there are problems with the theory itself. But neither conclusion follows. Neoclassical general equilibrium theory does not require the existence of aggregate capital as a factor in an aggregate production function. The fact that it is impossible to define aggregate capital does not change this. The general non-existence of aggregate production functions means that phenomena such as reswitching or reverse capital deepening that violate the properties that we would expect to observe if an aggregate production functions existed and aggregate capital were a factor in it can have no bearing on the evaluation of general equilibrium theory.

In short, such problems lie with aggregation and not with general equilibrium theory itself. Those who argued that aggregate production functions and aggregate capital stock as a factor of production do not generally exist were correct (although often not for the right reasons). But that very fact means that one cannot conclude that the strange behaviour of such constructs implies a failure of a theory that is built without them.

Garegnani

I am glad for Prof. Fisher's question which allows me to render explicit something I clearly took too much for granted in my paper yesterday. The essence of the answer to his question is simple: if capital could be measured independently of distribution, so that a given set of capital goods is the same "quantity of capital" whichever the wage and interest rates happen to be, *then* reswitching and reverse capital deepening would *not* be possible (see e.g. in my paper par. 21). *Then* the zero solutions and/or the multiplicity and/or instability of equilibria which we saw yesterday to result from those phenomena would also *not* be possible. The connection between the impossibility of consistently measuring capital as a single magnitude and those particular "unsatisfactory things" in general equilibrium, as Fisher just put it, could hardly be any stricter than this cause to effect relation.

In particular, the question is not, as Fisher seems to say, that if one *defines* aggregate capital in terms of savings, then one gets reswitching and reverse capital deepening. The point is that one *gets* reswitching, a result independent of our subjective definitions; similarly one *gets* the possibility emerging from reverse capital deepening of results like non-uniqueness, instability – and this, again, is so independent of whether or how savings are defined in the equations. Savings enter there merely as what seems the most natural way of showing the existence of those phenomena and of explaining them – a rather useful way, I think, if one is to judge from the fact that *that* cause of "unsatisfactory things" in general equilibrium had not been seen, or not seen with any clarity, by other means before.

Fisher

That just says, if capital could be aggregated the world would be a much simpler place.

Garegnani

Simplicity of the world is also a question of the theory we have about it, and of how good that is. I expect that with Kepler and Galileo the universe looked simpler than it had progressively come to look with the epicycles and eccentrics of the Ptolomeans. Certainly, the economy seen along Smith's and Ricardo's lines, where profits are essentially the difference between the outputs one can get from workers, and what demography, history and present social power have allowed real wages to be, does not look like being troubled by reswitching or reverse capital deepening.

Let me however notice now some expressions in Fisher's earlier question to me, and thereby return on an important point raised in Kirman's summing up this morning. There is an ambiguity when Fisher refers to "aggregate capital". One thing is the use of that quantity in the *aggregate production function* to which Fisher seems to mainly refer (a representation, that is, of the production of the whole economy in terms of a single production function): a question which, whatever its importance for practical applications, is hardly relevant in pure theory. A quite different thing is the single quantity of capital which the founders of neoclassical theory – Jevons, Marshall, Böhm-Bawerk, Clark, and everybody else really – used before recent decades for their *multi-sectoral* (general-equilibrium) pure theory. That single "quantity of capital" was needed for two basic reasons quite independent of any interest in aggregate production function. The first is that one can reasonably suppose alternative techniques or outputs to give rise to substitutability of "factors" only if big tractors and small tractors or looms are seen as different homogeneous *quantities* of "capital", and *not* as distinct physical "factors of production". And, of course, that substitutability of "factors" was what gave confidence in the uniqueness and stability of the equilibrium, and made the theory acceptable at the time of those founders.

The second reason is the strictly logical one that a conception of the capital endowment as susceptible of assuming the physical composition adjusted to the equilibrium to be determined is entailed in the uniformity of the *effective* rate of return on the supply prices of the capital goods (not to be confused with the uniformity of the *own* commodity rates: cf. Appendix II [A] to my paper). And that uniform rate had traditionally been the basis for relating theory to observable variables via the gravitation *around* what could then be argued to be the sufficiently persistent position determined by the theory. Given the perfect substitutability between capital goods in the eyes of the savers – for whom they are homogeneous bearers of future income – persistence of the forces determining the equilibrium requires such a uniform rate, just as it requires a uniform price and its equality with the supply price for any commodity (my paper, par. 25–26).

Incidentally that perfect substitutability of capital goods for savers explains why, contrary to what is still often held (e.g. Kirman, this volume, p. 473; Hahn, 1982: 354; Bliss, 1975: 144–145), heterogeneity of capital goods qualitatively differs from that of labour or land, where of course no similar tendency to some uniform price can be supposed to exist over the relevant period of time.

Foley

I am in a somewhat peculiar position because due to certain idiosyncrasies of my education I never learned "neoclassical economics". The economic theory that I learned was from Herbert Scarf and it was couched entirely in terms of the abstract general equilibrium model with *n* commodities, abstract production sets, and so forth. Someone has defined economic intuition as what you learn in your first course in economics, and since I did not learn the same things as many other people, my intuition is not the same. My economic intuition was always that there were no theorems of the following kind: suppose you increase the supply of labour, as a result the equilibrium real wage will fall. I knew there were no such theorems available in general equilibrium theory. I also knew, and maybe this became clearer because of Scarf's mathematical point of view, that once you took the step to a simultaneous general equilibrium vision, you had to give up hope of sustaining some traditional ideas. For one thing, you lost any sense of causality. In a general equilibrium model there is no real sense in which one factor causes another: everything is determined simultaneously by the whole collection of relationships. Now, Prof. Garegnani has made some strong methodological criticisms of that way of thinking about the world, arguing that the world is actually deeper, that it has got layers, and that there are senses in which some things are causal and other forces are less causal; but if you think in general equilibrium terms you just cannot see what he is getting at. And I never thought there was any hope of proving general stability theorems, because Herbert Scarf had showed that you could not prove stability, at least tâtonnement stability, without unacceptably strong hypotheses on aggregate demand functions. So these things are not counterintuitive to me, these are just facts of the matter. I see them as inherent defects of the general equilibrium point of view. I tend to read this as a case where neoclassical economics, in an attempt to purify itself logically, leached out all of the substance, the concrete substance and content, of what it had to say about the world. And it raises the question in my mind whether it is really fair to talk about general equilibrium theory as if it were neoclassical economics, precisely because it does not have that content of substitutability, well-behaved stability properties and well-behaved comparative statics properties. In fact, when I was at MIT it became apparent to me that for, say, Solow and Samuelson, who are perhaps closest to being true believers in the old neoclassical rules, general equilibrium theory was just as much a threat as the Cambridge controversy to their point of view.

So from all this what lessons do I draw? First, I have personally learned a lot from this discussion, and I think there are some real issues at stake; second, I think we have reached a point where there is really not much disagreement about how the different models work, either the long-period equilibrium model, or the general equilibrium model. I think most of the neoclassically trained economists here can figure out long-period equilibrium prices after this week, and Profs. Garegnani, Schefold, Salvadori, Heinz Kurz, and Petri have shown that they can do general equilibrium theory if they want to, and in some cases make very ingenious suggestions which go beyond the published general equilibrium results. Indeed, in some ways it is the classicals, such as Prof. Schefold, whose results go beyond received theory, who are developing general equilibrium theory. I think that what is at issue here is really more methodological and more what Schumpeter called "vision"; and perhaps it would help, if this debate continues, if we tried to put it more on that plane.

We have to discuss questions like: what are the advantages and disadvantages of forcing all theory into the Procrustean bed of a simultaneous set of equations with exogenous and endogenous variables? I think that the long-period and classical point of view is that simultaneous methodology has limitations, that there are other ways of doing coherent theory, which have their own virtues, and that those ways should be explored. That's a point which needs to be discussed in more depth. Perhaps, people brought up in the general equilibrium tradition are unjustifiably impatient with ideas that aren't couched as a well-determined set of simultaneous equations, and maybe they need to be more explicit about the methodological advantages of expressing everything as a well-determined model with exogenous and endogenous variables. I suggest that we make more explicit the methodological and philosophical level of this discussion, which I think in the end will be more fruitful.

Schefold

I wish to reply to a number of observations by Prof. Kirman. First, Prof. Kirman has said that real economic problems seemed to him to be the least concern of the participants in this Conference. I presume that everybody present here would like to protest. I rather believe that we have been talking about real economic problems although with different views about what the real economic problem is and how to approach it; and that everybody has a way of relating e.g. comparisons between the economic problems of the United States and Europe to their own theory. So I think that, each in his own way, we all are trying to be serious and also to learn from each other. I for instance did not know much, very little indeed, about the stock exchange and yet found Magill's presentation very stimulating and learned from it that the modern stock exchange is subject to an evolutionary process, and that there were not only new institutions created and transformed in the fourteenth and fifteenth century when bills of exchange were invented and developed, but that also the modern stock exchange is undergoing transformations, for reasons capable of theoretical explanation.

Second, regarding my own presentation, I wish to say that I think the agreement reached by Fisher and Garegnani at the end of their exchange about the importance of the concept of aggregate capital is only apparent, because there is a real difficulty in agreeing as to what it means to discuss the concept of aggregate capital if one knows that it does not exist. I have great respect for the work of Prof. Garegnani and for what Garegnani has tried to do in his lecture, but I think it is quite difficult to make it clear to others what Garegnani was doing, and I therefore have used here an intertemporal model which allows me to make important points related to capital theory without using the concept of aggregate capital explicitly. In reply to Alan Kirman's questions, I wish to stress that the point of the model was to consider on the one hand paths of accumulation and on the other hand, a utility maximizing agent. The paths of accumulation could be, among other things, such that the system was in a stationary state, then the amount of labour available to the economy increased, and a technique could emerge which allowed to absorb that amount of labour while at the same time the wage rate was lowered. However, it also was possible that there was a technique which allowed to absorb that amount of labour while the wage rate paradoxically was raised. In both cases one could find a consumer who liked to be on that path and whose intertemporal preferences were such that that path was realized. Now only one of those two can be stable, I believe, and therefore there is a problem with intertemporal equilibrium. Alan Kirman has asked, why only one agent? The reason of course is that with only one agent the equilibrium is unique and the model becomes simpler. The instability can then be traced unambiguously to the conditions of production. With several agents, additional equilibria, besides the one that I wanted to construct, inadvertently might come up and that would have made the construction less transparent. (I am of course fully aware of the fact that neoclassical theory is interesting because it deals with a multiplicity of agents.) The reswitching problem here appears as a problem of stability. Stability usually is discussed in pure exchange economies; it then depends on the utility functions. Restrictions have to be imposed in order to rule out instability. In consequence, it is a modern trend in neoclassical theory to reduce the generality originally postulated regarding the characters of agents, and to say "well, on average they may be such that gross substitution holds" or something of that kind. Hildenbrand in Germany is doing such research. The point of the approach presented by me is to show that instability problems come up even if the consumers are perfectly well behaved, because the cause of instability resides in production, and this is a new contribution.

Having only one more minute at my disposal, I wish to add, on the supposed lack of concern for consumers mentioned by Kirman, that it is the point of the theory of joint production that consumers are taken into account, although in a way rather different from the one adopted in neoclassical theory: I have described the vector of given needs as a ring between two chains of reasoning: one chain of reasoning is of course the classical theory as we now know it, dealing with matrices and all that; the other chain of reasoning is to discuss how the needs themselves change and more specifically how the commodities and the means to satisfy those needs change over time in a – I should say – historical theory.

My last point – only an assertion, really – is this: we, the classical economists, can agree on what neoclassical theory is because we define it as that part of modern economic theory where supply-and-demand determines distribution. So we can define neoclassical theory by what we regard as possibly its central mistake. This is a very provocative way of putting it. It is quite another question whether there is one classical theory or one correct theory. Perhaps, truth is many and only that mistake is one. The manifold theories built around the "core" of classical theory (to use Garegnani's expression) can lead in different directions and I agree with those who say that ultimately the attraction of the approach stands or falls with the attraction of those manifold approaches.

Heinz D. Kurz

I want to refer to some of the points made by Prof. Kirman in his excellent talk. Professor Kirman openly admits that his acquaintance with classical economic theory is less than optimal, what explains the emphasis in his talk on neoclassical theory with which he is familiar. So let me say a few things on classical theory. First, *gravitation*: do natural prices act as centres of gravitation which make themselves felt fairly quickly? "Quickly", I think, is the crucial word. Now, asking for convergence of market prices to natural prices, as is done in many studies on gravitation, is probably asking for too much, and the classical economists, Smith and the others, were not that demanding: in their view gravitation involved, in Smith's words, market prices "oscillating" around natural prices, which may perhaps be translated as meaning that market prices must never move too far away from natural prices. I need not tell you how intrinsically complex the issue of gravitation is. Indeed, there is no presumption that we shall ever be possessed of a *general* theory of gravitation. While in some analyses the problem of gravitation is seen in an overly simplistic way, there is the opposite danger of burdening it with demands which are hard to meet. It should then be clear that there can be no fear that the issue of gravitation will be settled in the foreseeable future. In this state of affairs the observation that in competitive conditions the rates of profit never seem to deviate too much from one another may prompt one to start from the stylized fact – and it is only a stylized fact – of a uniform rate of profit. This is equivalent to adopting a *long-period method*, which brings me to my second point. As regards the "vision" of the economic system, to which Prof. Foley referred, it seems that many economists – the classical economists but also many neoclassical economists, both early and contemporary ones – share that vision, that is, assume that there is gravitation. From what we heard these days, for example in the talks of Prof. Hahn or Prof. Mordecai Kurz, this vision is still around. Let me recall a statement by Prof. Mordecai Kurz in this volume, p. 248: "Despite the fact that the economy may undergo structural changes yielding non-stationarity, the economic universe is stable in the sense that . . . [i]n such a system the concept of normal patterns makes empirical sense and provides useful knowledge. It is represented by the long-term averages of economic variables". You could find similar statements using almost the same terminology in Smith and Ricardo and Marx, and many other authors. This is decidedly indicative of the application of a long-period point of view or method which differs substantially from the short-period point of view or method of the Arrow–Debreu model. In other words, several neoclassical models presented during this conference belong firmly to traditional long-period neoclassical theory.

We now come to the crux of the matter: differences in the *content* of alternative theories. The analytical procedure generally adopted is the following. We distinguish between the "unknowns", or dependent variables, of the problem at hand, on the one hand, and the "data", or independent variables, in terms of which the former are to be explained, on the other. Different choices in this regard involve different theories and potentially radically different theories, as Neri Salvadori and I tried to make clear in our paper. Indeed, classical theory and traditional neoclassical theory provide fundamentally different explanations of normal income distribution and normal prices in competitive conditions. The two theories see different factors at work in the economy deciding the sharing out of the product amongst workers and capitalists. Put in a nutshell, in the classical economists the emphasis in the explanation of the rate of profits and relative prices, in terms of a *given* real wage rate, is on the relative strength of the parties, "whose interests are by no means the same", in the "dispute" over the distribution of income, as Smith (*Wealth of Nations*, I.viii.11) kept stressing. The given real wage rate was meant to express the balance in that dispute in a given time and place. In contradistinction, for the neoclassical economists, the emphasis is on the interplay between preferences and endowments. However, with their concept of "capital" as a magnitude that can be determined prior to, and independently of, relative prices, traditional neoclassical authors became entangled in insoluble theoretical difficulties showing that their explanation of normal income distribution in terms of relative scarcities of factors of production cannot be sustained. This criticism applies to all kinds of long-period neoclassical theories, including those presented during these days.

While each of the two theories starts from some givens, or data (output levels, technical alternatives, and the real wage rate in the case of classical theory; preferences, technical alternatives, and initial endowments, including "capital", in the marginalist case), this does not mean, of course, that what are considered as data are meant to reflect historical constants. Any serious scholar would allow interactions among the sets of data themselves, and also between the unknowns and the data. For example, earlier neoclassical economists stressed the endogeneity of preferences, certainly in the long run, but also in the short run. Friedrich von Wieser, for example, was opposed to the idea of taking preferences as given independently of the state of affairs in the system as a whole. He argued that in an economy experiencing a famine, preferences would be bound to change, one would have to learn to make use of, say, edible roots or grass and whatever one can find in order to survive. Similarly in classical theory, where interactions between prices, output levels and income distribution are, of course, not denied. However, as Ricardo stressed: "No law can be laid down respecting quantity." Therefore, in the theory of value and distribution he started from given output proportions. His unwillingness to take into account interactions between quantities and prices in that theory simply means that there is no reason to presume that the theorist can expect to find general laws expressing their interrelations, as they are postulated in neoclassical demand functions. The economist simply cannot avoid studying the historical particulars of an economic change – whether it is predominantly due to the introduction of a new method of production and whether this affects the system of production as a whole or is confined to a single industry only; or whether it is due to the introduction of an entirely new kind of commodity; or whether it is due to the exhaustion of some natural resources; etc. Increasing returns to scale that turn out to be external to each industry or group of industries, as in Smith's discussion of the division of labour, for example, are a case highlighting Ricardo's "no law respecting quantities" dictum: how could one ascertain *a priori* the evolution of quantities and prices? There are simply no demand functions that could be known by the theorist.

This draws attention to the fact that the classical authors, for very good reasons, it seems, distinguished between different spheres of economic analysis necessitating the employment of different methods. While one sphere is suited to the application of deductive reasoning – this relates to the investigation of the relations between the distributive variables and relative prices, given the system of production in use – the other sphere requires more inductive lines of reasoning and research – this relates to an investigation of the sources and consequences of economic change, in particular technological progress, economic growth, changing consumption patterns, the exhaustion of natural resources etc. Contemporary neoclassical theory (see, for example, the so-called "new" growth models) typically presupposes given and unchanging "preferences" in deriving demand functions; however, preferences will certainly not remain unaffected by these changes.

I think what is at issue is an important methodological question, as was pointed out by Prof. Foley and also Prof. Garegnani; it is a question about what economic theory and especially mathematical model building can accomplish and what it cannot, and what we think the driving forces of the economic system are.

Just two further remarks. What our neoclassical colleagues presented during these days were more or less partial equilibrium models, not general equilibrium models. Thus the model in Prof. Mordecai Kurz's paper is clearly a very partial model: he takes the rate of growth as given from outside. More generally, I find the term "General Equilibrium Theory" misleading and at any rate somewhat grandiloquent. If you allow "data" to be endogenised, then you are in an entirely different world and things become much more complex.

One final remark about prediction vs. understanding. We have heard quite often that economic theory ought to be concerned with predicting. I wish to remind the audience of a statement by Ronald Coase: "Faced with a choice between a theory which predicts well but gives us little insight into how the system works, and one which predicts badly but gives us this insight, I would choose the latter and I am inclined to think that most economists would do the same." Listening, for example, to Prof. Hahn during this seminar, I felt that there are advocates of Coase's point of view among our neoclassical colleagues. Understanding, *Verstehen* in the Weberian sense, plays an important role and one must not believe that if the predictions are not as good as they seem to be, e.g. in Prof. Mordecai Kurz's paper, then the model has no explanatory value, and, vice versa, if the fit is good the model by definition gives us an insight into how the system works.

Gintis

I wish to pose a question to Prof. Schefold: I am not clear as to which economic events Prof. Schefold would explain differently from me or from other people. Since these differences about the real world have not been sufficiently spoken about in the Conference, some clarification would be useful. I used to think that the classicals believed that class struggle, as opposed to market clearing, determined the wage rate and the profit rate; but I have not heard this at all at this Conference except briefly from Prof. Petri in his lecture. Is there a set of beliefs about the nature of the world that the classicals and those they criticize differ on?

Schefold

My reply will obviously have to be very brief. First – and I think this would be shared by most people in this paradigm – I do not believe in a general automatic tendency to full employment, and I believe that to a large extent it is left to the participation rate, to migration and so on to adapt the supply of labour to the state of effective demand, if growth is not sufficiently fast ultimately to exhaust labour reserves as in Kaldor's model. Regarding distribution, as I have tried to indicate briefly during my lecture, I believe that the forces which predominate change over time, so that, while it seems to me most relevant to speak about distribution as mainly demand-determined in the sense of the Cambridge theory or Kaleckian theory for the decades immediately after World War Two on the Continent, I think that afterwards there was a period of slower growth in which the idea that the rate of interest determines distribution is more plausible. Yet another theory would apply to the period when a subsistence wage theory seems applicable. One might make similar examples regarding output and consumption: the dialectic of necessaries and luxuries obviously has to be different in different phases of accumulation if what I have said about distribution is right. So one does arrive at different conclusions. Moreover, I see an important methodological divide. Many aspire to make predictive theories. Others are content to understand processes which are going on. Prediction presupposes that people act according to predictable motives. Understanding (Verstehen), as Prof. Kurz has reminded us, is concerned with changing forms of motivation. An understanding of historical transformations, such as occur between different phases of accumulation, is a natural complement of classical theory.

Petri

I would not concede to neoclassical theory that income effects do not question its plausibility. Professor Garegnani said that neoclassical theory has managed to live with income effects; but he did not enter into the question, whether neoclassical theory *had the right* to disregard them in the "mode of actual practice". I think that Prof. Kirman, for one, would answer in the negative; and I would concur. For example, I do not think that the current work on heterogeneous agents will be able to justify the common assumption in macroeconomics that the labour supply curve is not "backward-bending". A long series of esteemed neoclassical economists of the past, starting from Walras, admitted that it might easily be "backward-bending", and it is unclear how the reasons they gave might be dismissed. Now in neoclassical theory, even conceding downward-sloping factor demand curves, "backward-bending" factor supply curves may cause multiple equilibria (possibly very close to one another), or a near coincidence of supply-and-demand for a factor for an ample interval of values of its rental; in either case the forces of supply-and-demand would leave the factor rental indeterminate to all practical purposes, suggesting that the theory does not grasp the forces which in actual economies make income distribution well-determined and generally quite resistant to drastic changes. The assumptions usually made in mainstream macroeconomic analysis to exclude these cases appear motivated only by a readiness to assume what is necessary to make the theory yield non-implausible results.

Still, I agree with Prof. Garegnani that the difficulties connected with capital are more fundamental, and I think there is a simple way to see why. The conception that the several capital goods are only embodiments of a single factor "capital" had not only the (demand-side) role, on which Prof. Garegnani concentrated in his lecture, of making it possible to believe in a capital/labour substitution process making the rate of interest capable of bringing investment into equality with full-employment savings; it also had the (supply-side) role of leaving the *composition* of the capital endowment to be determined endogenously by the (long-period) equilibrium. It was thus possible, when discussing the stability of equilibrium, to admit time-consuming disequilibria involving out-of-equilibrium exchanges and productions; which is what all founders of marginalism found natural and necessary (including Walras, who in the first three editions of his *Eléments* described the tâtonnement as time-consuming and involving actual disequilibrium productions, without perceiving the contradiction between this description and his specification of the data relative to the capital endowment).

It may legitimately be doubted that the marginalist/neoclassical approach would have been able to impose itself, if it had presented itself from the beginning in its neo-Walrasian versions, with their enormously problematical connection with the explanation of real events. Let me remember that the impermanence problem is not the only new problem making that connection problematical for the neo-Walrasian versions. Another one is the dilemma (arising out of the possible quick changes of relative prices, what obliges one to take into account the subsequent evolution of relative prices in order to determine the first-period decisions) between (a) assuming complete futures markets (or perfect foresight), with a clear lack of realism; or (b) including exogenous unobservable subjective expectations among the data, with a danger of arbitrariness and of indeterminateness of the sequence of equilibria. Still another one, stressed by Prof. Garegnani a few minutes ago, is the insufficient substitutability between inputs when capital goods are not treated like embodiments of a single factor "capital" of variable "form". These problems too did not arise in the long-period equilibria based on "capital"; in these equilibria, the composition of capital being adapted to demand, the equilibrium relative prices could be taken to be sufficiently close to constant as to allow the neglect of their changes over time; and the possibility to change the "form" of capital when changing the capital–labour ratio made the assumption of a relevant substitutability between "capital" and labour more plausible. (The exercises by profs. Garegnani and Schefold, which accept for the sake of argument the neo-Walrasian framework, should not make one forget that the equilibria thus determined would anyway have unclear significance even if unique and "stable" in a tâtonnement sense.) It is therefore important to remember that Wicksell in his early work had not yet had doubts about the average period of production and the subsistence-fund conception of capital and therefore believed that a long-period equilibrium was determinable without problems; that Walras too, in the first three editions of his treatise, believed he had satisfactorily determined a *long-period* equilibrium with complete uniformity of rates of return on supply price; that Wicksell's and Walras' later retreats and hesitations went largely unnoticed; and that they had been the only ones among the founders of marginalism to tackle the problem of introducing capital into the system of general equilibrium equations. So neoclassical theory was born and became dominant in the mistaken belief that it was capable of determining *long-period* equilibria, i.e. sufficiently persistent centres of a time-consuming trial-and-error gravitation of market prices and quantities. The shift to neo-Walrasian versions only came after the theory had thoroughly impregnated the economists' minds in *that* form.

This "supply-side" role of the conception of "capital" as ultimately a single factor makes it possible to see a simple reason why the difficulties due to heterogeneous capital are more fundamental: they give rise to problems *logically prior* to the possibility of discussing the impact of income effects on stability. If one accepts that adjustments take time, and that the role of equilibrium must be that of determining the situation the economy gravitates towards through time-consuming disequilibria, then the equilibrium cannot include among its data a given vector of endowments of the several capital goods, it can only be a long-period equilibrium, but then the non-validity of the notion of aggregate "capital" means that its endowment cannot be determined, so the equilibrium *cannot even be determined*: so one cannot even *start* to discuss its stability, nor the relevance of income effects to that end.

In the light of all this, it is understandable that Prof. Foley should raise the question of the connection between "neoclassical economics" and (neo-Walrasian) general equilibrium analyses. My view, as I have argued in my lecture, is that in order to see that the latter cannot justify the former it is sufficient to notice the latter's inability to admit time-consuming adjustments, what renders it a theory barren of implications for the tendencies of economies where adjustments take time. Professor Kirman appears to concur with this view when he admits on p. 474 that "one cannot extract tendencies towards any particular state from Walrasian equilibrium theory". But I wonder how widespread a clear consciousness of the implications of this fact is, even among the general equilibrium specialists who, like Hahn, declare general equilibrium theory not to aim at describing the working of actual economies. Without an ultimate acceptance of the thesis that the tendency towards an equilibrium between supply and demand determines distribution and outputs, it is unclear why so many energies should still be dedicated to trying to extend general equilibrium theory to new fields (e.g. infinite-horizon models). The supply-and-demand approach is instilled into the students' minds from their very first readings, so (unless one encounters fairly early on a massive dose of non-conventional teaching) it is extremely difficult to avoid accepting the supply-and-demand forces (e.g. the tendency of labour demand to increase if real wages decrease) as obviously present in reality, independently of the difficulties of the theory in demonstrating their existence. On the contrary, it should be admitted that without a consistent theory behind them, those forces cannot be assumed to exist, and one should look for different forces, for an alternative approach.

But then I would tend to disagree with Prof. Foley – if I have understood him correctly – on whether a central problem now is the *methodological* one concerning "the advantages and disadvantages of forcing all theory into the Procrustean bed of a simultaneous set of equations with exogenous and endogenous variables". The method – what can be formalized, what can be determined endogenously, what should be treated as given – derives from the *theory*. For example, in neoclassical theory, treating tastes as data derives from an argument that they are not so dependent on the variables that the equilibrium tries to determine, as to make it illegitimate to treat them as given (a much shakier argument than generally admitted, in view e.g. of the pervasiveness of advertising, but I cannot enter this issue now).

Let me give one example: the consequences of discarding the labour demand curve and more generally of abandoning the thesis that a decrease of real wages increases the demand for labour. Imagine that there is labour unemployment and that the unemployed workers offer themselves at a lower wage than the prevailing one. The employed workers, in order not to be fired, will themselves accept the lower wage. Sooner or later the unemployed workers will learn that the lower wage does not increase the demand for labour, and that, since there are some, however small, firing and hiring costs, they are not obtaining any benefit by asking for a lower wage, because they remain unemployed, and accomplish only a reduction of the wage of their employed relatives and neighbours. Notice how much easier it becomes to explain downward wage rigidity: isn't it now natural to assume that this collective learning process will produce social conventions and customs, which will exclude wage undercutting from the normal, acceptable social behaviours?

If one accepts this picture (which I think is very realistic), the question then remains, of explaining the level of the real wage which the unemployed workers do not bid down. Suppose one follows the classical authors and believes that this level results from very complex processes, and that it changes most of the time only slowly and for reasons which are also largely political, so that it is not influenced by unemployment in a univocal and predictable way; then it will be natural to take it as given when determining output (and "given" will *not* mean unexplained, but only explained in another part of the overall analysis); and it will also be natural to determine output separately from it, since one has given up the neoclassical idea of a necessary connection between real wage and level of employment. If on the contrary, one believes in some other stable and univocal influence – expressible through a function of known qualitative shape – of unemployment on the real wage (e.g. efficiency wages), then the real wage will be determined endogenously once outputs (and hence unemployment) are determined; if one also believes that aggregate output depends in a fairly stable and univocal way on the real wage, e.g. because investment depends on income distribution, then one will be justified in attempting a simultaneous endogenous determination of both the real wage and output. So the answer to the "methodological" question, of how far one can go with stable functional relations specified at least in their qualitative shape, depends on the *theoretical* beliefs, and the important questions are about theory, i.e. about the forces at work. So the central problem now is the theoretical one of supplying a convincing theory of income distribution, and of outputs (and employment and growth).

Mordecai Kurz

Whenever I work with graduate students on a dissertation or research, I don't let them get anywhere before they write to me what the empirical facts are that this dissertation or research wishes to explain, and somewhere along the line I also want them to write what are the empirical facts that would falsify their theory and would induce them to abandon that line of research. So let me ask you guys, you have been lecturing to us for a week now on what is wrong with neoclassical theory, let me ask you a simple question, is there a body of empirical facts that you believe the theory you have been working with explains? And, can you give me a set of five empirical facts that would induce you to abandon your line of research?

An intervention from the audience

Professor Mordecai Kurz, I don't understand why you believe so firmly in falsifying theories through empirical facts. You are very Popperian. There are other ways to justify or falsify theories. Feyerabend would not agree with you.

Mordecai Kurz

That does not answer my question.

Gintis

I agree with Prof. Kurz, of course, that we ought to have falsifiable theories, and I think that Feyerabend is wrong. He presents a radically incorrect interpretation of the history of the natural sciences, for which falsification is absolutely central to the development of knowledge. The fact is that there is a lag between the time at which an anomaly occurs in a discipline and the time when the anomaly leads to a rejection of the dominant theory, because people will only replace a theory which is defective with a better theory. So for instance the Michelson–Morley experiments, which showed that the speed of light is independent of ether, took place several years before Einstein developed the special theory of relativity. Perhaps he was not aware of that anomaly, but definitely he was aware of a contradiction in Maxwell's theory, solved in a purely arbitrary manner by the Lorenz contraction. It always takes time for people to move from an anomaly to an alternative: that is not to say that facts don't matter. To do science you must believe that the world out there is real, and that you are trying to explain the way the world out there works. Philosophers like Feyerabend have this idea that they are like priests. They like to tell scientists how to conduct their lives correctly. I believe philosophy is wonderful just as ballet is wonderful, but neither helps us to do science. If you want to be an economist, you explain how the world works.

Garegnani

On the question raised by Prof. Kurz, I would stress that the interaction between theory and empirical evidence is a very complicated matter when it concerns a general theoretical approach. Keynes' *General Theory* is perhaps the only example in which empirical evidence played an *immediate* role – though, significantly enough, not a lasting one – in the acceptance of a theoretical approach.

For the sake of an example, I would like Kurz to list the five empirical facts that would falsify neoclassical theory and induce him to abandon it. In the meantime I will do my part and list five empirical facts which I think would have falsified neoclassical theory, and supported classical theory if, on the validity of alternative theoretical approaches, things could ever be as simple as Prof. Kurz professes to think. By inverting the sign of those facts he can get five facts which could have induced me to form different views on the relative merits of the two theories. I will be sketchy but, given time, the point could be made more stringently. The five facts are:

1 The generalized presence of persistent involuntary labour unemployment.
2 Alternative methods of production generally use different *kinds* of capital goods and not different *proportions* of the same capital goods, as would be required for any high substitutability between factors to be postulated independently of the admittedly inconsistent reduction of heterogeneous capital goods to quantities of homogeneous capital.
3 Different consumption goods similarly require different *kinds* of capital goods for their production and not different proportions of them, with the same implication as under 2 for the substitutability between factors postulated by the theory.
4 The levels of capital accumulation do not appear to depend upon autonomous individual choices to save out of the incomes of normal utilisation of productive capacity (no sudden burst of such saving propensities was noticed by economic historians at the time e.g. of the English industrial revolution, or of the post-war reconstruction of Germany, or of the Korean growth 1970–1990, etc.).
5 As Marshall had to admit, it is an empirical fact that "in economics every event causes permanent alterations in the conditions under which future events occur", contrary to the reversibility of economic changes on which the demand-and-supply functions making up neoclassical theory essentially rest.

Now, I have no doubt that Kurz and many people here will have what they think are good arguments to counter these five empirical facts. But this is just the point I wanted to make: that things are less simple than Prof. Kurz has professed to believe in his intervention about classical theory. As the list I have drawn might perhaps show, that is rather unfortunate – but for the classical, not the neoclassical, theory.

Foley

People seem to have risen to the methodological and philosophical level I was advocating but there is perhaps space for refining the discussion a bit, there is some space between Feyerabend and Popper. Even the strong empiricists among us would probably agree that there is the level of theory which Lakatos calls "core theoretical beliefs". These notions, general ideas, visions, are not directly falsifiable by evidence, but they play the heuristic role of posing questions, of making a framework for hypotheses which can be falsified, or do have explanatory content. I think that both sides to this debate, the classical and the neoclassical, responded to the Cambridge controversy by retreating back into their respective core beliefs, and that this is why the debate has been so difficult to settle. The neoclassicals did it by retreating to the Arrow–Debreu type of model, which has no falsifiable content – that is really the Debreu–Mantel–Sonnenschein result. I gather that's what bothers Alan Kirman, and perhaps Michael Mandler has something to say about this as well, because I know he has thought a lot about it. But I think that the classical and long-period equilibrium side also needs to present more of a penumbra of explanatory models that have some content and could be falsified; that does not mean you give up the whole approach because one model turns out to be wrong, but I think that would make for a more vigorous classical economics.

Geanakoplos

I have a short question for Prof. Foley. You said something which I found intriguing and I may have misheard it. You said earlier that Arrow–Debreu theorists think in terms of exogenous and endogenous variables when they think of a model, and that one of the things which we have learnt at this Conference which we should spend more time discussing is what a different approach to theorizing could be, one that presumably does not use the distinction between exogenous and endogenous – and you left it there, and I thought that you perhaps might elaborate on it with another sentence or two.

Foley

I think that the distinction between exogenous and endogenous variables is a good habit, and I spend a lot of time with my graduate students trying to get them to put ideas into that form. But clearly there are limitations to it. In somewhat Hegelian language, it tends to compress all of the 3D levels through which we could understand reality into two dimensions: it flattens everything out. For example, dynamical forces in economies like the equalization of the rate of profit through intersectoral flows of capital, and other dynamical forces like the responses of households to changes in prices are presented in a simultaneous system on exactly the same level. That has some great logical advantages but maybe we also lose something in terms of what Heinz Kurz calls Verstehen and in terms of a layeredness of the theory by insisting on that. Perhaps, there is something in between exogenous and endogenous variables, if you see what I mean: factors which are sometimes pretty constant and you want to take them as given, but are subject to longer-terms forces and evolution.

Petri

If one really wants to respect the empirical evidence, as Prof. Mordecai Kurz insists we should, then the introductory economics we teach our students should start from two facts. First, all econometric enquiries into the behaviour of aggregate investment, according to the *Journal of Economic Literature* recent survey (1993: Chirinko) as well as according to earlier surveys, confirm that investment depends above all on “quantity” variations (i.e. on demand variations) and not on “price” variations (i.e. not on the rate of interest); so we should have investment, in our textbooks, depend on demand (on the accelerator, fundamentally) and not on the rate of interest: the IS–LM model should not be present in our textbooks except as a historical curiosity. Second, the labour market: I rely on Marshall, Solow, and other studies by non-Sraffian economists (e.g. recently Truman Bewley), for the thesis that empirical evidence shows that wage labourers behave according to ideas of fair wages which change slowly through time, and when unemployed are not ready (except in very special circumstances where their different behaviour is usually easily explainable) to compete with the employed workers by offering themselves at lower wages. So we should start with models which take investment as given and then explain its variations through the accelerator, and which take the real wage as given and determined by historical circumstances and changing slowly. Empirical evidence means that this is how we should teach economics. Classical economists are much closer to teaching economics this way than mainstream economists, so who has greater respect for the empirical evidence?

An intervention from the audience

A quick comment about investment being a function of output. Fazzari and Petersen have done extensive investigation of investment by firms, and their morale is that investment is financially constrained, so it is true that output is correlated with investment, but the reason is that firms are financially constrained and they actually look at the rate of interest for all their possible opportunities. So you may get a correlation between output and investment, but if one just specifies behavioral functions then it is hard to understand why output drives investment.

Petri

Clearly if firms do not invest more the reason is that they think they would incur losses, or those who should lend them the money think so, so you can always put it that demand generates a financial constraint on firms. I disagree though with what I think you mean when you say that firms "look at the rate of interest for their possible opportunities". One cannot treat the rate of return on investment as being independent of the rate of interest: the rate of interest, being a cost, influences prices, and if competition is operative a lower rate of interest will also lower prices relative to money wages and to other costs such as imports, and thus will also lower the rate of return; so even if firms are financially constrained in a way which is not simply a reflection of demand conditions, one cannot derive from this that a lower rate of interest stimulates investment (in an open economy, it might on the contrary induce capital to emigrate to countries where the rate of interest, and hence the rate of return, is higher).

Mandler

I want to add a different perspective on the now decades-long debate between Sraffian and neoclassical economists. It has been suggested several times that the two camps do not share enough common ground to resolve their analytical differences; the primitive assumptions of Sraffian and neoclassical theory simply lie too far apart. But if this explanation were correct, the controversy would not be so pointed. The intense, even angry, dispute persists because one of the theories advances internal arguments against the other: Sraffian criticisms are meant to be valid if one begins from a neoclassical, supply-and-demand starting point. Only this fact, I think, can explain both the acerbity of debate and frequent puzzlement of the participants. Consider the two schools' explanations of the multiplicity of equilibria in supply-and-demand models. As

Prof. Garegnani has rightly emphasized, the Sraffian and neoclassical accounts differ dramatically. Neoclassical economists attribute the multiplicity problem – and paradoxical comparative statics as well – to the heterogeneity of agents and specifically to the differences in income effects across agents. Sraffians in contrast have claimed that the absence of the aggregate factor capital can be the source of the problem. Mainstream economists find this disagreement perplexing; the multiplicity problem is after all a long-standing subject of neoclassical research. Aggregate capital is neither necessary nor sufficient for there to be multiple supply-and-demand equilibria, while the heterogeneity of agents, in contrast, is necessary. So the two schools come to markedly different conclusions despite the fact – when Sraffians are in critique mode at least – that they begin from the same assumptions. It would seem, therefore, that either there is room for analytical progress or some participants must conclude that their positions are in error. Schools of thought that start from truly incommensurate positions cannot argue at great length; they lack the analytically tractable differences that reproduce controversy. Only the common ground that accompanies internal criticisms can generate heat, and create opportunity for consensus.

Garegnani

I agree with Prof. Mandler that the "classical" criticism on capital is "internal" to neoclassical theory and is therefore conducted on grounds strictly shared with neoclassical theory. I believe however that the conflict *in logic* between the two sides which he deduces from that, does not really exist.

Thus, nobody from the "classical" side disputes that the multiplicity of equilibria may follow also from "income effects": no conflict there with the "classical" claim that the impossibility of measuring capital as a magnitude independent of distribution, entails the possibility of multiple and unstable equilibria. Nor does that "classical" claim contradict the neoclassical position that "aggregate capital" is neither necessary nor sufficient for multiple supply-and-demand equilibria. Indeed (a) the presence of produced means of production is not necessary for multiple equilibria since, as just said, the latter could result from income effects without any production, or with production without capital; (b) the presence of produced means of production is not sufficient either, since of course it is compatible with a unique equilibrium (e.g. in the case of an economy where only corn is produced, with labour and corn).

Where there would be disagreement in logic is if neoclassical theorists were to claim that income effects are *necessary* for multiplicity of equilibria. But are neoclassical economists really claiming that, or in any case will they claim that in the future, when present day results will be acknowledged and the distinction between production with and without capital, clear in Walras and Wicksell's times, will spread to contemporary general equilibrium theory?

Mordecai Kurz

I regret that I did not get any answer to my question, so let me re-propose the question. One of the propositions that has been advanced many times this week was the uniform rate of return principle of the Sraffian system. So let us look at the facts. Over the last two hundred years in the United States of America the average rate of return on risky capital has been about 8 per cent. The standard deviation of the distribution of the means of the rates of return across different industries is about 18 per cent. Which means that there is an enormous distribution of rates of return depending upon the use of capital and depending upon the industry in which it is employed. That does not sound to me like a uniform rate of return. Certainly it should have been your position that that proposition is empirically testable. It seems to me that if you have a proposition which is central to your thinking, you should have been able to test it. The statistics that I know suggest that there is not even proximate uniformity of rates of return, because in modern times by far the most dominant factor in the return to capital is risk, which is exactly what I was talking about, because the riskless rate of return is about 1 per cent, with a small standard deviation of about 5 per cent; due to fluctuations in business cycles, sometimes there is a negative rate of return which is riskless, and sometimes it is positive, due to inflation as well. But risky returns differ widely because there are different levels of risk. Furthermore, the fundamental force in modern technology is the emergence of knowledge as a capitalized item, a firm like Microsoft has virtually no assets and its capital is worth 500 billion dollars. You have not talked this week at all about risk, nor about capitalized knowledge; without these, the uniform rate of profit has decisive empirical evidence against it.

Garegnani

Yes, Prof. Kurz is quite right, these are very important things, but if we started now to theorize about general prices and distribution by considering for example Microsoft, we would not quickly end. Basically that was done over the last two centuries and a half. Kurz's criticism of the uniform rate of return would not be a criticism of Sraffa's theory in particular: it would be a criticism of much of economic theory since Adam Smith and his "natural prices", including that of all neoclassical authors until comparatively recent decades.

All those authors, Smith included, of course knew and wrote that profit rates would *permanently* differ between industries because of degrees of risk and other net disadvantages, but, as Prof. Kurz will agree, if these different long-run rates of return tend to move in parallel, they can be treated as one, which is just what those authors did and Sraffa obviously implies. Those authors also knew and wrote that profit rates at any given time differ between firms or industries because of "causes whose action is fitful and short lived" (Marshall). Thus they may differ because of innovations introduced in advance relative to competitors, the resulting extra-profits being capitalized in the stock exchange as for Microsoft. But those authors, like us, were not mainly interested in what profit rates are firm by firm, or industry by industry (supposing such *ex-post* rates to have meaning). Only averages over a sufficient period of time, broadly indicating what is to be expected on investment or, more exactly, the *changes* in such averages, are of interest in a general theory of distribution and relative prices. Indeed, in this respect there seems to be nothing qualitatively different to the profit rates: the same is broadly true with regard to both uniformity and deviations for any other price which competition leads us to take as uniform in our equations.

In fact, as I have argued in my paper, what has caused the pure theory of last few decades to abandon the condition of a "uniform rate of profit", i.e. more exactly, a uniform rate of return on the supply prices or production prices of the capital goods (not to be confused of course with a uniformity of own rates of interest: cf. Appendix II [A] to my paper) was certainly not the desire for a closer approximation to reality: I am sure that Prof. Kurz is under no illusion about the ease of tracing in the reality of firms the several quasi rents, or the several own rates of return, resulting from the equations of contemporary general equilibrium. He was instead able to trace the "riskless rate" he has just referred to – which, whatever the level attributed to it, appears to be the very uniform rate he criticizes classical people for theorizing about.

Sabourian

I would like to change the direction of the discussion slightly. I was an undergraduate and a graduate in Cambridge and I have heard it all, classicals against neoclassicals, many times before. When I was a first-year undergraduate, we had a King's Economics party, and Joan Robinson came to me and asked me what I thought of marginal productivity theory. I went "Huh, I think it is a very good theory", I had absolutely no idea about it. She walked away and never talked to me again for 3 years. I think I was very lucky because by the time I became a graduate student I stopped hearing these things in Cambridge. Now I want to ask the classical economists here a history-of-modern-economic-thought question, not at all in an aggressive mood: why is it, in your opinion, that you have been saying these things and – at least this is my perception – you are becoming more and more marginalized in the profession? Do you think that there is a conspiracy, or that we the rest of the profession are stupid or we don't understand the argument? Even in Cambridge we don't hear about these things any more.

Heinz D. Kurz

First, on Prof. Mordecai Kurz's question on differentials. Clearly, there exist profit rate differentials, and there is a theory about this. Adam Smith has a very interesting chapter on it; there was a remark on this in the text of the lecture by Prof. Salvadori and myself, but, given the time constraint, we could not dwell on this issue in our talk; let me refer you to Chapter 11 in our 1995 book. So definitely there is a theory; you cannot accuse us of not seeing that there might be permanent reasons in favour of differentials in profit (and wage) rates; the assumption of a uniform rate of profits is simply a device to focus attention on the ideal case of free competition (which assumes these reasons to be absent) in order to discuss what determines the *level* of that uniform rate.

Second, on the issue of what would make one change one's theory, I remember events which shook my confidence in neoclassical economic analysis. But let me comment on your view that a good fit with the data is fundamental. This I think is rather simplistic. I give as an example an article I came across recently by Hansen and Prescott, "A neoclassical theory of the industrial revolution". The argument there is that you have two production functions, one which is called a Malthusian production function, which has labour, capital and land as inputs, and then there is a second production function which, miraculously, has only labour and capital. The problem is reduced to a choice of technique problem. For a long time, the Malthusian production function is said to have dominated, and therefore the system was stuck: there was no substantial growth because of diminishing returns to land. This hypothesis is then "tested" in terms of an aggregate Cobb–Douglas production function, and it is contended that the fit with the empirical data is very good. Next, it is assumed that for some reason there is an improvement in the other production function, which then becomes dominant. As a consequence scarce land plays no longer any role whatsoever and the system begins to develop and grow. Here we have a case where irrespective of the fit with the data the analysis, I think, is just rubbish. There is no presumption that a good fit is the litmus test of a good theory. A good theory might, however, very well be reflected in a good fit.

On Dr Sabourian's question. Are we really marginalized? If you look around in our subject there are lots of analyses which are classical in spirit without carrying that name. The situation is not as bleak as it might look at first sight and, moreover, what matters is the quality of an argument. At the same time I cannot deny that I have the feeling that a relative of Gresham's Law is active in economic theory.

Garegnani

Professor Sabourian of course knows that what matters in science is not necessarily the opinion of the majority. Science would otherwise never move. By definition any novelty is initially held by a minority, and the "initially" can be long when it regards a non-experimental science, as disturbed by practical interests as economics is. However, in these days in Siena, Prof. Sabourian might have noticed already how a claim he evidently heeded in his early Cambridge time has in the meantime turned out to be incorrect. In Prof. Hahn's (1982) view reswitching and reverse capital deepening had nothing to do with "equilibrium theory". Those phenomena have however turned out now to cause "unsatisfactory things in general equilibrium" as was admitted a short while ago. So things are moving, and Prof. Sabourian might be interested in following them.

An intervention from the audience

During my studies of econometrics I never came across a regression which had a 100 per cent fit. If you had a 100 per cent fit it would mean you have a perfect theory, you have found all the explanatory variables. But you never get a 100 per cent fit, so your theory is always flawed. So if there are errors, you cannot find a perfect theory, and statistics cannot improve it; as long as you have errors, you cannot say that you have a better theory because you have a better fit, this is standard econometrics.

Petri

One cannot simply start from facts, and do without theory. For instance, let us accept that real wages are determined by prevalent ideas as to fair wages. That does not settle the following question: if, in the presence of unemployment, workers did accept a lower real wage, would that improve the employment situation? These kinds of questions require a logical structure of the theory, a complicated deductive reasoning. In traditional marginalist theory this deductive reasoning was performed, it was admitted that in the short run there were financial and monetary problems, rigidities of fixed plants, fluctuations etc., but it was thought possible to conclude that, yes, on the average over the cycle a lower wage would increase employment because of technological and psychological substitution, and this was described through a labour demand curve which was downward-sloping and which showed what would happen. An empirical observation that in certain cases an increase in employment had not followed a fall in real wages might have been explained in many ways, e.g. mistakes of monetary policy, changes in the international situation, temporarily pessimistic expectations, a wave of labour-saving technical innovations, etc. Now, I have argued in my lecture that the labour demand curve cannot even be drawn, because we do not know what to keep fixed when we change labour employment or the real wage: we cannot keep a vector of capital goods' endowments fixed because disequilibrium takes time, nor can we keep an aggregate capital endowment, measured as an amount of value, fixed in the face of changes in relative prices and quantities produced. So this is a case where reflection on the logical structure of a theory becomes very important: a certain notion, can we use it or not? A certain chain of logical deductions which concludes to the existence of a certain force or tendency, is it correct or not? If not, the tendency or force cannot be presumed to exist: the theoretical criticism may make it possible to cut through the uncertainties deriving from the seldom decisive role of the non-experimental evidence only available to economists.

Gintis

I have a lot of sympathy with how Prof. Petri would start an economics course. I like zoology and if I were to teach zoology the first thing I would do is to show people the animals and how they move and how they eat and scratch and mate. I think that is what one should do in economics, including presenting empirical regularities long before trying to explain them. We should teach people what happens in economies, differences between different types of economies, which ones are successful and which not. I totally agree with you on that, but I do not think that is a debate between paradigms.

On another point, the argument about the downward-sloping demand for labour curve is incorrect; if you maintain an artificially high wage by legislating it, or by taxing employment as they do in Europe, the presumption is that if you take away those taxes and the cost of labour to firms goes down then employment will go up; it is not because you believe the curve has a particular shape. Rather, it is because you believe that the government is keeping the wage at a certain level, it is likely therefore to go down if you take away that constraint.

But let me make now a more general point. I am confused as to why we teach economics the way we do. I do not believe in general equilibrium theory. I have the same view as Prof. Foley. To me general equilibrium theory only says, all markets clear in equilibrium, and I do not believe that. I do not know why people place so much faith in general equilibrium theory as to make it the basis for their undergraduate training, when in fact they don't do research on it, mostly. I think Prof. Quinzii was right, most people have moved on to do other things, because the marginal rate of return, so to speak, from research in general equilibrium theory is quite low, but they continue to teach that theory. I don't understand it.

When I teach intermediate microeconomics, I start with IBM vs. Toshiba who are trying to introduce a standard for Betamax, and they immediately learn about coordination games, cooperation vs. competition, and all sorts of neat stuff you can get from game theory. My colleagues who start out by trying to get people to understand the idea that the core, as the number of people goes to infinity, reduces to the competitive allocation, are nothing but irrelevant ideologues. No one understands this unless having had years of training in economics, and then having had that much training, they know it is not a reasonable general theory.

It is very frustrating to see this theory that nobody really accepts – economists know it is a formalism – and yet they use it as the basis for all their teaching. By the way, I do not think they use it as their basis for their economic advice. The economic advice that the economists that I know give is based on experience – they see what works and they see what does not work, they look at a lot of different countries and they tentatively give some advice which is probably pretty sane compared to the possible alternative advices that could be given; it is not based on this grand theory, it is not based on the shape of the labour curve, it is just based on, "well you know, if you do what Jimmy Carter did you are going to get a lot of inflation, so maybe you should run the Federal Reserve in a different way".

Petri

Our time is up. I thank you all for the controversies, for the contributions, for the attention, for the participation. I hope you are as glad as I am about the outcome of this Summer School.

NOTES

1 The view of the neoclassical problem of capital as one of "aggregation" is discussed further in an intervention below.

2 The phrase "existence of an equilibrium" rather than "existence of solutions of the equations etc." does not seem to be properly chosen in that it may be interpreted as if the question only concerned the equilibrium of the demand and supply forces represented in the equations, the existence of the forces themselves being somehow taken for granted, whether or not an equilibrium between them exists. However, clearly the forces are as much in question in the equations as their equilibrium is. Behaviour in exchange may conceivably be represented by other equally consistent relations having nothing whatsoever to do with the general equilibrium equations, like e.g. the classical effectual demand, and the dichotomy between market and natural prices.

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