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The Minimum Wage Puzzle in Less Developed Countries:
Reconciling Theory and Evidence

by Christopher Adam and Edward Buffie

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I N T E R N A T I O N A L M O N E T A R Y F U N D

IMF Working Paper

Research Department

**The Minimum Wage Puzzle in Less Developed Countries:
Reconciling Theory and Evidence***

Prepared by Christopher Adam and Edward Buffie

Authorized for distribution by Chris Papageorgiou

January 2020

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Abstract

We show that a dynamic general equilibrium model with efficiency wages and endogenous capital accumulation in both the formal and (non-agricultural) informal sectors can explain the full range of confounding stylized facts associated with minimum wage laws in less developed countries.

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Contents

| | |
|---|----|
| 1. Introduction | 2 |
| 2. Insights from a Simplified Model | 6 |
| 2.1. The Short Run | 10 |
| 2.1.1 The Impact on Real Output | 11 |
| 2.1.2 The Full General Equilibrium Solution..... | 12 |
| 2.2. The Long Run..... | 12 |
| 2.3. The Transition Path | 14 |
| 2.3.1 Welfare | 15 |
| 3. The Full Model..... | 16 |
| 3.1. Model Calibration..... | 19 |
| 3.1.1 The Problem Child: Cost Shares in the Informal Sector | 22 |
| 3.2. Solution Technique..... | 23 |
| 4. Numerical Results | 23 |
| 4.1. The Base Case and Its Variations..... | 23 |
| 4.2. The Low-Income Country Case | 25 |
| 4.3. Coherence with Empirical Estimates | 25 |
| 4.4. The Transition Path and Welfare..... | 26 |
| 5. Extensions | 27 |
| 5.1. Adding MW Increases in the Public Sector | 27 |
| 5.1.1 The Transition Path | 28 |
| 5.2. Adjustment Costs to Changing Employment | 28 |
| 6. Concluding Remarks | 29 |
| References..... | 31 |
| Appendix A..... | 38 |
| Appendix B..... | 39 |
| Appendix C..... | 41 |
| Appendix D..... | 50 |

1 Introduction

Minimum wage (MW) laws are now an important policy for combatting poverty in many LDCs, but as in developed countries, there is considerable controversy about whether they achieve their stated objective. First-generation analyses of the MW relied on the canonical Segmented Labor Markets Model (SLMM). According to SLMM, a higher MW reduces employment and output in the formal sector. Some of the workers who lose their jobs then seek employment in the informal sector (where MW laws cannot be enforced). The influx of labor to the informal sector increases employment and output but also depresses the real wage. Underemployment worsens, total output declines, and any improvement in the overall distribution of income comes at the expense of the poorest group in the country, low-paid workers in the informal sector. For standard production functions and plausible parameter values, job losses in the formal sector and the redistributive effects are large. In short, MW laws derive from good intentions but appear hard to recommend.

The data have not been kind to this narrative, however. Empirical evidence accumulated over the past twenty years casts doubt on, or strongly contradicts, every claim advanced by the SLMM. Sometimes employment increases in the informal sector; typically, however, it decreases more than employment in the formal sector (Betcherman, 2014). The real wage in the informal sector tends not to decline; reflecting the ubiquitous "lighthouse effect," it almost always increases: "No study has found that a higher minimum wage depresses wages for informal sector workers as a whole" (Gindling, 2014). Completing the rout, employment losses in the formal sector are often surprisingly small. The mean employment elasticities in the meta-analysis of Nataraj *et al.* (2014) and in surveys of the literature by Borat *et al.* (2017), the World Bank (2006), and this paper are -.08, -.011, -.20, and -.23, respectively. In some countries, the evidence suggests a *positive* impact on formal sector employment (see Appendix C).

These stylized facts represent a major challenge for theory. The lighthouse effect is consistent with employment decreasing in the informal sector. But what explains the effect in the first place? The usual explanation, that the MW serves as a norm for fairness in the informal sector, is incomplete and unconvincing.¹ Most informal sector capitalists are quite poor (La Porte and Shleifer, 2014), so the notion that they feel a social obligation to respond to an increase in the MW by paying much higher wages strains belief; evidently some unspecified change in the economic environment, causally linked to the MW, makes it profitable for firm owners to raise wages.

The finding that employment losses are often very small in the formal sector — nil is a common descriptor — is perhaps the most perplexing result uncovered by empirical studies. For a CES production function with two inputs, capital and labor, the elasticity of employment with

¹ Kristensen and Cunningham (2006) emphasize this point: ". . . enforcement mechanisms are weak, but for some reason employers, and particularly those in the informal sector who are not legally bound by the minimum, choose to adjust wages when the legally mandated wage is changed."

respect to the wage equals $-\sigma/\theta_K$, where σ is the elasticity of substitution and θ_K the cost share of capital.² When $\sigma = .5 - 1$ and $\theta_K = 0.33$, this yields an employment elasticity of $1.5 - 3$, more than ten times the mean elasticities of between $.08$ to $.20$ in the literature cited above. The true employment elasticities may be higher (in absolute value) than those reported in the literature, and certainly the variation in outcomes among LDCs deserves more attention.³ We return to these points later. If anything, however, they add to the list of unresolved empirical results. A satisfactory theory should be able to account for both high and low employment elasticities. As Bhorat et al. (2017) observe: "There is a range of potential impacts of minimum wages on employment. The heterogeneity of outcomes in LMI countries, in particular, suggests that a variety of *context-specific factors* interact with the minimum wage." (Our emphasis.)

There are pros and cons to the approach taken by the existing literature. On the credit side of the ledger, the provocative stylized facts, amassed through decades of careful empirical research, are important in their own right and exceptionally informative about the right way to model the labor market in LDCs. This will be a recurrent theme in our paper.

But the literature's strength is also its weakness. From a policy standpoint, the lopsided emphasis on empirical investigation is troubling. Absent any substantive input from theory, the stylized facts are something of a black box: a set of potentially important, policy-relevant results that we do not fully understand and therefore cannot fully trust. The black box problem was noted thirty years ago in the developed country literature by Brown (1989) and has persisted largely unchanged to the present day. In the development literature, Eyraud and Saget (2005), Lemos (2009), Betcherman (2014), and Fields (2011) have called for research to "look for the factors behind [the] weak effect" on employment (Eyraud and Saget); to develop a "coherent theoretical framework" that makes sense of the "puzzling results" in Brazil and other countries (Lemos); to help understand the long-run effects of MWs (Betcherman); and, more generally, to meet the "the need for empirically-grounded theoretical labor market models that can be used in the formulation of policy" (Fields). The appeals have yet to elicit a response. Writing in 2017, Bhorat *et al.* assert that "While work on minimum wages is fairly mature in many OECD countries, our understanding of minimum wage policy in SSA is not." To a lesser but still significant extent, the same assessment applies to Latin American and Asia.⁴

Our objective in this paper is to bridge the divide between theoretical and empirical research. Toward this end, we develop a dynamic general equilibrium model with efficiency wages (EW)

²This is the solution for the short run, where the capital stock is fixed.

³It is often asserted that failure to account for noncompliance biases estimates of the employment elasticity downward. This assertion is valid in the SLMM but dubious in our model. The informal sector in our model includes formal firms that do not comply with the MW law. But employment losses in the noncompliant sector are typically larger than in the compliant sector. Failure to account for noncompliance could therefore run in the opposite direction.

⁴Bhorat *et al.* attribute the problem to lack of data. We partly disagree. In the case of SSA, we favor a combination of Bhorat *et al.* and Fields' views: better policy analysis requires new, better theoretical models informed by better data.

and endogenous capital accumulation in both the formal and (non-agricultural) informal sectors. A large body of empirical work already attests that EW models can explain the most important characteristics of labor markets in developing countries. We show that they can also explain the full range of confounding stylized facts – those emphasized in the literature plus others that have flown below radar – associated with MWs in LDCs. In our view, this effectively settles the debate on how to model the labor market in developing countries. The evidence in support of EW theory is broad, deep, and compelling across the development spectrum. Competing theories cannot be ruled out altogether – the sheer diversity in the structure of labor markets across and within LDCs precludes that – but none come close the explanatory power of EW (See Box 1).

Outline

The main body of the paper is organized into four sections. We start out in Section 2 by deriving analytical results in a stripped-down model that assumes constant employment and output in the informal sector. The analytical results elucidate many of the key mechanisms that limit employment losses in the formal sector. In a standard setup where firms operate a CES production function and worker effort depends only on the real wage, EW effects reduce the employment elasticity from around 2 – 3 to 1. This is substantial but inadequate progress: 1 is still a long way from .1 – .3, the range that brackets the majority of empirical estimates. The solution to the problem is to strengthen fidelity to the stylized facts by incorporating two other effects: (i) the impact of the unemployment rate on work effort, as measured by wage curves estimated for LDCs; and (ii) the link between monitoring costs, effort, and the firm-size wage premium (which is much larger in LDCs than in developed countries). When these effects are added to the mix, the MW decreases the effective cost of labor, inducing firms to *increase* output and investment.⁵ The increase in output lowers the employment elasticity to .2 – .6 in the short run. Moreover, as the capital stock grows, labor demand recovers and output continues to rise. In the limiting case where the goods produced by the formal and informal sectors are perfect substitutes, labor demand recovers fully – the employment elasticity equals zero across steady states.

In Section 3 we present the full model that features EWs in both the formal and informal sectors, and discuss its calibration. In Section 4 we report the results from numerical simulations in which we explore the sensitivity of the lighthouse effect and sectoral employment and output under alternative empirically-relevant values of key parameters. The variation in the numerical results mirrors the variation in outcomes documented in empirical studies. Three "context-specific factors" condition the impact of an increase in the MW: (i) the relative size of the

⁵Investment increases provided the elasticity of substitution in consumption between the formal and informal good is not implausibly low.

formal sector; (ii) the degree of substitutability between formal and informal sector output; and (iii) the absolute and relative degree of wage flexibility embodied in the sectoral wage curves.

Section 5 extends the benchmark model in two directions. The first adapts the model to handle scenarios in which MW increases encompass the public sector. In this not uncommon scenario, a fourth context-specific factor proves important, namely the impact of the MW on the supply of public sector services/inputs complementary to private sector production. We do not assume optimizing behavior in the public sector; labor may therefore be much less productive and EW effects may be much weaker than in the private sector. Even then, however, payment of a higher MW in the public sector significantly reduces employment losses in the private sector.

The other extension treats labor as a quasi-fixed factor subject to adjustment costs. This refinement improves the fit with the empirical evidence, especially in the formal sector. Without adjustment costs, employment losses in the formal sector are two to three times larger in the short and medium run than in the long run. With adjustment costs, layoffs undershoot or slightly overshoot (in the medium run) their steady-state level. Consistent with the majority of empirical estimates, small employment elasticities of 0.2 or less predominate in the relevant parameter space at *all* time horizons.

Our paper is only a first pass at solving the MW puzzle. As such, it ignores a number of important issues. The final section discusses this and some of the topics that should be addressed in future research.

Box 1: The Case for Efficiency Wages

Efficiency wages are rarely seen in development macromodels. This needs to change. Over the past twenty years, empirical studies have amassed abundant, compelling evidence that efficiency wages operate throughout the non-agricultural sector in LDCs. Estimates of the impact of unemployment on real wages confirm the existence of wage curves in the formal and informal sectors in Argentina, Turkey, Colombia, Uruguay, Chile, South Africa, Cote d'Ivoire, Mexico, China, South Korea, and a host of other developing countries (Blanchflower and Oswald, 2005). There is also powerful, if indirect evidence supportive of efficiency wages in the stylized facts documented in microeconomic studies of LDC labor markets. Across the development spectrum, wage and employment data exhibit the same patterns: (i) firm-size wage premiums that start at very small establishment size (5+ employees) and are much larger than in developed countries; (ii) persistent, remarkably stable inter-industry wage differentials; (iii) high correlation of industry wage premiums across occupations; (iv) large wage premia for formal vs. informal sector employment and for informal non-agricultural employment vs. agricultural employment; (v) large cyclical flows into and out of unemployment in both the formal sector and the informal sector; (vi) virtually identical lists for low- and high-paying industries; (vii) large, stable wage differentials between firms in the same industry; and (viii) lower quit rates and longer job tenure in the formal sector. We do not have the space here to survey the literature in greater depth or to discuss myriad estimation issues. References and capsule summaries of the results for 50+ studies are reported in Appendix D.

None of this denies the potential relevance of other wage-setting mechanisms: depending on the country in question, wages in the formal sector may be determined by MW laws, unions, insider rent extraction, or tripartite bargaining. Some mixed model of the labor market, with EW elements entering in different sectors to different degrees, may then be appropriate. Mixed models are relevant in many countries. We would argue, for example, that optimizing union models should incorporate EW effects.

2 Insights From a Simplified Model

The full model has a lot of moving parts. It is not a black box, however. To facilitate comprehension of the model and the numerical results presented in Sections 3 and 4, we first analyze a simplified model that abstracts from most of the general equilibrium interactions between the formal and informal sectors. Variable names are familiar or at least mnemonic. K_i , L_i , Q_i , w_i , C_i and P_i refer to capital, labor, output, wages, consumption, and prices, with subscript 1 for the formal sector and 2 for the informal sector. The informal good serves as the numeraire ($P_2 = 1$).

Technology

The simplified model fixes output and employment in the informal sector in order to focus on the response of the formal sector to a higher MW. The numerous complications associated with the lighthouse effect are on hold for the time being.

Firms in the formal sector operate a linearly homogeneous CES production function. The elasticity of substitution between capital and labor is σ_1 and the supply of labor services depends on the amount of effort e_1 that workers expend:

$$Q_1 = a_f \left[a_1^{1/\sigma_1} (e_1 L_1)^{(\sigma_1-1)/\sigma_1} + (1 - a_1)^{1/\sigma_1} K_1^{(\sigma_1-1)/\sigma_1} \right]^{\sigma_1/(\sigma_1-1)}. \quad (1)$$

Factories are built by combining one unit of the informal good with f units of the formal good. The supply price of capital is thus

$$P_k = 1 + fP_1. \quad (2)$$

Preferences, Saving and Investment

All economic activity is undertaken by a single representative agent. Preferences of the agent qua consumer are given by

$$C = \left[(1 - \kappa)^{1/\epsilon} C_1^{(\epsilon-1)/\epsilon} + \kappa^{1/\epsilon} C_2^{(\epsilon-1)/\epsilon} \right]^{\epsilon/(\epsilon-1)}. \quad (3)$$

C is a CES aggregate of C_1 and C_2 , with substitution elasticity ϵ . The optimal choices for C_1 and C_2 minimize the cost of purchasing C . This yields the demand function

$$C_1 = (1 - \kappa) \left(\frac{P_1}{P} \right)^{-\epsilon} C, \quad (4)$$

and the exact consumer price index

$$P = [\kappa + (1 - \kappa)P_1^{1-\epsilon}]^{1/(1-\epsilon)}. \quad (5)$$

After choosing the best mix of C_1 and C_2 , the agent solves the problem

$$\underset{\{C_1, e_1\}}{\text{Max}} U = \left(\frac{C^{1-1/\tau}}{1 - 1/\tau} - Z \right) e^{-\rho t} dt, \quad (6)$$

subject to

$$P_k \dot{K} = P_1 Q_1 + Q_2 - PC - P_k \delta K, \quad (7)$$

where

$$Z = [e_1 - g_o - g_1 \ln(w_1/P) - g_3 u + g_4 \ln(L_1/S_1)]^2, \quad (8)$$

the unemployment rate is $u = 1 - L_1 - L_2$, and δ , ρ , τ , and S_1 denote, respectively, the depreciation rate, the pure time preference rate, the intertemporal elasticity of substitution, and the

number of managers/supervisors. The supplies of production labour and managers/supervisors are perfectly inelastic. To economize on notation, both equal unity.

Following Collard and de la Croix (2000) and Danthine and Kurmann (2004, 2010), we have introduced a term that captures the net utility loss from effort at the job. Effort *per se* does not reduce utility; what matters is whether workers perceive the firm as reciprocating their effort in a fair deal. Naturally, workers view the deal more favorably when they are paid a higher wage and when the unemployment rate is high as they feel more grateful for having a job. Utility also increases when L_1/S_1 is low so that workers receive more input from management about how to do their job properly. As will become apparent shortly, this gives rise to a firm-size wage premium. In Appendix A we show that the same wage curve emerges when $g_4 \ln(L_1/S_1)$ reflects effort enforced through monitoring worker performance. We prefer the current formulation only because it is more compact.⁶

On an optimal path,

$$e_1 = g_o + g_1 \ln(w_1/P) + g_3 u - g_4 \ln L_1 \quad (9)$$

and

$$\dot{C} = \tau C \left(\frac{r_1}{P_k} - \rho - \delta \right), \quad (10)$$

where $r_1 = P_1 \partial Q_1 / \partial K_1$ is the capital rental rate. Equation (10) is a standard Euler equation, while (9) says that effort adjusts so that $Z = 0$. (*Ex post*, current utility depends only on consumption.) In deriving (10), we assumed that P_1 enters P_k and P with the same weight.⁷

Labour Demand and the Wage Curve

Firms recognize the connection between labor productivity and the real wage. Hence they optimize over L_1 and w_1 . The profit-maximizing choices satisfy

$$\begin{aligned} P_1 \partial Q_1 / \partial L_1 &= w_1 \\ \implies L_1 &= a_1 Q_1 (a_f e_1)^{(\sigma_1 - 1)} (1 - g_4 / e_1)^{\sigma_1} \left(\frac{w_1}{P_1} \right)^{-\sigma_1} \end{aligned} \quad (11)$$

and

$$\underbrace{a_f \left(\frac{Q_1}{a_f} \right)^{1/\sigma_1} \left(\frac{e_1 L_1}{a_1} \right)^{-1/\sigma_1}}_{w_1 / P_1 (e_1 - g_4)} \frac{\partial e_1}{\partial (w_1 / P)} \frac{P_1}{P} = 1,$$

⁶In the formulation with monitoring costs, it is necessary to distinguish between two types of effort.

⁷The general form of the Euler equation is $\dot{C} = \tau C [r_1 / P_k - \rho - \delta - (\gamma - \alpha) \dot{P}_1 / P_1]$ where $\alpha \equiv f P_1 / P_k$ is the cost share of the formal good in production of the investment good and $\gamma \equiv P_1 C_1 / P C$ is the share of the formal good in aggregate consumption. Equation (10) assumes $\alpha = \gamma$.

$$\implies \underbrace{\left(\frac{\partial e_1}{\partial(w_1/P)} \frac{w_1/P}{e_1 - g_4} \right)}_{\text{Standard Solow condition}} = 1. \quad (12)$$

Equation (9) and the modified Solow condition in (12) imply⁸

$$\frac{g_1}{e_1 - g_4} = 1. \quad (13)$$

Without loss of generality, we set e_1 equal to unity at the initial equilibrium. The *wage curve* defined by (9) and (13) then reads

$$\ln(w_1/P) = \frac{1 - g_o - g_3u + g_4 \ln L_1}{1 - g_4}. \quad (14)$$

There is no "natural rate of unemployment," just a *curve* relating the equilibrium wage to the unemployment rate. Firm size shifts the wage curve in the manner shown in Figure 1. When employment rises, monitoring/managerial input per worker declines and effort decreases. The optimal response of the firm is to buy back the lost effort by paying a higher wage.

Raising the Minimum Wage (MW)

We assume the MW initially equals the EW firms pay in equation (14). In other words, firms have already optimized employment and output to all historical changes in the MW. When the government announces a new, higher MW, equations (13) and (14) are suspended and effort is determined by (9) with the real minimum wage w_m replacing w_1/P :

$$e_1 = g_o + g_1 \ln w_m + g_3u - g_4 \ln L_1. \quad (9')$$

The nominal MW is indexed to the CPI to maintain the real MW. This makes the product wage in (11) a function of the real price of the formal good:

$$L_1 = a_1 Q_1 (a_f e_1)^{(\sigma_1 - 1)} (1 - g_4/e_1)^{\sigma_1} \left(w_m \frac{P}{P_1} \right)^{-\sigma_1}. \quad (11')$$

Market-Clearing Conditions

Two market-clearing conditions close the model. Demand equals supply in the formal sector and in the rental market for capital goods when

$$Q_1 = (1 - \kappa) \left(\frac{P_1}{P} \right)^{-\epsilon} C + f(\dot{K}_1 + \delta K_1) \quad (15)$$

⁸The standard Solow condition states that the wage maximizes profits when the elasticity of effort with respect to the real wage equals unity. The condition emerges whenever the first-order condition calls for w to minimize $w/e(w)$.

$$K_1 = (1 - a_1)Q_1 a_f^{(\sigma_1 - 1)} \left(\frac{r_1}{P_1} \right)^{-\sigma_1}. \quad (16)$$

2.1 The Short Run

Getting down to business, differentiate (9') and (11'). After slight manipulation,

$$\theta_{K1} \hat{L}_1 = \theta_{K1} \hat{K}_1 + \left(\frac{\sigma_1}{1 - g_4} - \theta_{K1} \right) \hat{e}_1 - \sigma_1 \left[\hat{w}_m - (1 - \gamma) \hat{P}_1 \right], \quad (17)$$

$$\hat{e}_1 = g_1 \hat{w}_m - (g_3 L_1 + g_4) \hat{L}_1, \quad (18)$$

where θ_j is the cost share of factor j ; a hat over a variable signifies a percentage change (i.e., $\hat{x} = dx/x$); and we made use of the adding-up condition $\theta_{K1} + \theta_{L1}/(1 - g_4) = 1$.⁹ Although the capital stock is fixed in the short run, we carry it around in anticipation of future needs.

Without EW effects, the partial equilibrium solution for the employment elasticity (i.e., the solution with P_1 constant) is $\hat{L}_1/\hat{w}_m = -\sigma_1/\theta_{K1}$. With $\theta_{K1} = .33$, textbook neoclassical economics cannot explain the very small employment elasticities reported in the empirical literature unless the econometric estimates are badly wrong or the true value of σ_1 is less than .1!

EW effects reduce the employment elasticity, assuming $\sigma_1 > \theta_{K1}(1 - g_4)$. Substituting for \hat{e}_1 in (17) leads to

$$\hat{L}_1 = \frac{\theta_{K1}}{m_0} \hat{K}_1 + \frac{\sigma_1(1 - \gamma)}{m_0} \hat{P}_1 - \frac{\theta_{K1}(1 - g_4)}{m_0} \hat{w}_m, \quad (19)$$

where γ is the share of the formal good in aggregate consumption and

$$m_0 \equiv \theta_{K1}(1 - g_3 L_1 - g_4) + \sigma_1 \frac{g_3 L_1 + g_4}{1 - g_4} > 0.$$

In the expression for m_0 , the sign of $1 - g_3 L_1 - g_4$ determines whether the supply of labor services ($e_1 L_1$) rises or falls with L_1 . For empirically-plausible values of g_3 , g_4 , and the share of formal sector employment in total employment, $1 - g_3 L_1 - g_4 > 0$ is likely, but not guaranteed, to hold.¹⁰ We assume the condition always holds; none of the results in the paper depend on perverse general equilibrium effects (e.g., a downward-sloping supply curve).

⁹Write the production function as $Q = F[e(w_1/P, L_1/S_1)L_1, K_1]$. With constant returns to scale, $\lambda Q = F[e(w_1/P, L_1/S_1)\lambda L_1, \lambda K_1]$. Differentiating with respect to λ gives

$$Q = F_L e L_1 + F_K K_1$$

$$\implies Q = w_1 L_1 / P_1 (1 - g_4) + r_1 K_1 / P_1$$

$$\implies 1 = \theta_{L1} / (1 - g_4) + \theta_{K1}.$$

¹⁰ L_1 and the unemployment rate determine the formal employment share: $L_1 / (L_1 + L_2) = L_1 / (1 - u)$.

The solution in (19) is involved but easy to break down. Three distinct effects operate. All are needed to bring the employment elasticity into the general vicinity of the elasticity estimates in empirical studies. To see this, consider the outcome in an overly simple model where effort depends only on the real wage ($g_3 = g_4 = 0$). The partial equilibrium employment elasticity then reduces to $\hat{L}_1/\hat{w}_m = -1$. The intuition for the result stems from the Solow condition and is quite general. For $P_1 = 1$ and $Q = F(e_1 L_1, K_1)$, the first-order condition for employment is $F_1(e_1 L_1, K_1) = w_1/e_1$. Starting from an equilibrium where firms pay the EW, the elasticity of effort with respect to the MW equals unity, per the standard Solow condition. It follows that $e_1 L_1$ is constant in partial equilibrium and hence that $\hat{L}_1/\hat{w}_m = -\hat{e}_1/\hat{w}_m = -1$.

Return now to the solution in (19) and incorporate the firm-size wage premium ($g_4 > 0$) and the impact of higher unemployment on work effort ($g_3 > 0$). The empirical evidence discussed later in Section 3.1 places g_3 between .4 and 1.2, g_4 between .14 and .33, and L_1 between .30 and .75. For our base case model described in Section 4, where $\theta_{K1} = .40$, $\sigma_1 = .75$, $g_3 = .80$, and $g_4 = .20$, the partial equilibrium employment elasticity equals .33.¹¹ This is not the complete solution, of course (since P_1 is endogenous.), but it is clear, however, that a *fully-loaded* EW model has the potential to explain why big increases in the MW seldom result in big employment losses.

2.1.1 The Impact on Real Output

The results for employment clearly strengthen the case for raising the MW. Surprisingly, so also do the results for real output and investment. Equations (1), (18), and (19) give

$$\begin{aligned} \hat{Q}_1 = & \theta_{K1} \left[1 + \frac{\theta_{L1}}{m_0(1-g_4)}(1-g_3 L_1 - g_4) \right] \hat{K}_1 \\ & + \frac{\sigma_1(1-\gamma)}{m_0(1-g_4)}(1-g_3 L_1 - g_4) \hat{P}_1 + \frac{\sigma_1 \theta_{L1}}{m_0(1-g_4)}(g_3 L_1 + g_4) \hat{w}_m. \end{aligned} \quad (20)$$

Real output *increases* in the short run. (P_1 decreases, but only in response to output rising; while $\hat{K}_1 = 0$ in the short-run.) This strong result is inherent in the logic of the EW model. Figure 2 depicts the solution for labor services $e_1 L_1$ when there is no firm-size wage premium ($g_4 = 0$) and $P_1 = 1$. As before, $Q = F(e_1 L_1, K_1)$ and firms maximize profits by hiring labor up to the point where $F_1(e_1 L_1, K_1) = w_1/e_1$. In partial equilibrium, nothing happens: $\hat{e}_1/\hat{w}_m = 1$, so there is no change in the effective cost of labor (ECL) or the supply of labor services. In general equilibrium, however, a coordination externality comes into play: when each firm reduces employment, the increase in the unemployment rate induces workers to put out more effort. The combined effect of the higher wage and higher unemployment shifts the ECL schedule downward, increasing the supply of labor services and output.

¹¹ L_1 is backed out from the values of other parameters and variables. It equals .49 in the base case calibration.

The coordination externality is sufficient but not necessary for output to increase. The firm-size wage premium ($g_4 > 0$) also figures in the positive output response. If a larger workforce is more difficult to manage/supervise, then effort decreases with employment at the level of the firm. Thus the marginal ECL increases with employment. Turning this around, when a higher MW increases effort, the average ECL rises but the marginal ECL declines. Since $\hat{L}_1 = -\hat{e}_1$ when $g_3 = g_4 = 0$, the firm-size wage premium implies $\hat{L}_1 + \hat{e}_1 > 0$; again, the total supply of labor services increases. There are clear parallels with impact of a MW on labor demand at firms that exercise monopsony power.¹² But while both output and employment increase under monopsony, employment declines in the EW case.¹³

2.1.2 The Full General Equilibrium Solution

Finally, we bring demand-side parameters into the solution. To minimize algebraic clutter, we assume the cost share of the formal good in production of investment goods is the same as its share in aggregate consumption. Solving (15) for P_1 then yields

$$\hat{P}_1 = -\frac{\theta_{K1}}{V} \left[1 + \frac{\theta_{L1}}{m_0(1-g_4)}(1-g_3L_1-g_4) \right] \hat{K}_1 - \frac{\sigma_1\theta_{L1}}{m_0(1-g_4)V}(g_3L_1+g_4)\hat{w}_m, \quad (21)$$

where

$$V \equiv \frac{\sigma_1(1-\gamma)\theta_{L1}}{m_o(1-g_4)}(1-g_3L_1-g_4) + \epsilon \frac{C_1}{Q_1}.$$

Higher output in the formal sector depresses P_1 . Consequently, employment decreases more in the full general equilibrium solution than in the partial equilibrium solution that holds P_1 constant. Exactly how much more depends on ϵ , the elasticity of substitution between the formal and informal goods. When the two goods are (not) close substitutes ϵ is large (small) and the partial equilibrium solution is a good (poor) approximation to the general equilibrium solution. We will be more precise about what "close substitutes" means and about the value of ϵ compatible with small employment losses when we present numerical results for the full model in Section 4.

2.2 The Long Run

Across steady states,

$$r_1 = (\rho + \delta)P_k. \quad (22)$$

We rewrite (22) as

$$MPK = RCK, \quad (23)$$

¹²When MW increases, e_1 increases and the MP of e_1 decreases. This reduces the marginal ECL.

¹³Rebitzer and Taylor (1995) develop a similar idea. They demonstrate in a variant of the Shapiro and Stiglitz model (1984) that a higher MW increases employment. The result depends, however, on the strong assumption that effort is constant once the no-shirking condition is satisfied.

where MPK and $RCK = (\rho + \delta)P_k/P_1 = (\rho + \delta)(1/P_1 + f)$ are the marginal product of capital and the real cost of capital in the formal sector.

When the government raises the MW, the supply of labor services increases and P_1 falls. Both the MPK and RCK schedules shift upward, as shown in Figure 3. The relative strength of the competing effects depends on the size of the informal sector and the elasticities of substitution in consumption and production. Equations (16) and (19) - (21) deliver

$$\frac{\hat{K}_1}{\hat{w}_m} = \frac{\sigma_1 \theta_{L1} (g_3 L_1 + g_4)}{m_0 (1 - g_4) S} [\epsilon C_1 / Q_1 - \sigma_1 (1 - \gamma)] \quad (24)$$

$$\frac{\hat{L}_1}{\hat{w}_m} = \frac{\theta_{K1}}{m_o V} [\epsilon C_1 / Q_1 - \sigma_1 (1 - \gamma)] \frac{\hat{K}_1}{\hat{w}_m} - \underbrace{\left[\frac{\theta_{K1} (1 - g_4)}{m_0} + \frac{\sigma_1^2 (1 - \gamma) \theta_{L1}}{m_0^2 (1 - g_4) V} (g_3 L_1 + g_4) \right]}_{\text{Short-run outcome}} \quad (25)$$

$$\frac{\hat{Q}_1}{\hat{w}_m} = \frac{\theta_{K1} (1 + \theta_{L1} \Delta)}{V} \epsilon \frac{C_1}{Q_1} \frac{\hat{K}_1}{\hat{w}_m} + \underbrace{\frac{\sigma_1 \theta_{L1}}{m_0 (1 - g_4)} (g_3 L_1 + g_4) \epsilon \frac{C_1}{Q_1}}_{\text{Short-run outcome}} \quad (26)$$

where

$$\Delta \equiv \frac{1 - g_3 L_1 - g_4}{m_0 (1 - g_4)} > 0, \quad (27)$$

$$S \equiv \sigma_1 \left[\frac{\theta_{L1} (g_3 L_1 + g_4)}{m_0 (1 - g_4)} V + \theta_{K1} (1 - \gamma) (1 + \theta_{L1} \Delta)^2 \right] > 0. \quad (28)$$

The equilibrium capital stock increases iff

$$\epsilon > \epsilon^* = \sigma_1 (1 - \gamma) \frac{Q_1}{C_1}. \quad (29)$$

In our base case calibration, $\gamma = .65$, $\sigma_1 = .75$, and $Q_1/C_1 = 1.095$.¹⁴ For these values, ϵ^* is only .287. Sensible alternative calibrations produce higher (and lower) values for ϵ^* , but there remains a general presumption that macroeconomic life is better in the long run than in the short run. Employment always decreases less in the long run. In addition, when $\epsilon > \epsilon^*$, the capital stock and real output increase continuously on the path to the new steady state.

Could employment in the formal sector increase in the long run? This is asking too much of the current simplified model. It is possible, however, to get very close to a positive result. In the limiting case where the formal and informal goods are perfect substitutes (in consumption),

$$\left. \frac{\hat{K}_1}{\hat{w}_m} \right|_{\epsilon=\infty} = 1 - g_4,$$

¹⁴ Q_1/C_1 is backed out from other values.

$$\left. \frac{\hat{L}_1}{\hat{w}_m} \right|_{\epsilon=\infty} = 0.$$

Eventually, employment fully recovers.

The full model includes additional general equilibrium effects that reduce employment losses relative to the losses in the simplified model. These effects can flip the sign of the employment elasticity in the formal sector from negative in the short run to positive in the long run for large but believable values of ϵ .

2.3 The Transition Path

The transition path is governed by the two differential equations for C and K in (7) and (10). Linearizing these two equations around the stationary equilibrium (C^*, K_1^*) gives

$$\dot{C} = \tau C(\rho + \delta)(\hat{r}_1 - \hat{P}_k) \quad (30)$$

,

$$P_k \dot{K}_1 = P_1 Q_1 \hat{Q}_1 - P dC - P_k \delta dK_1. \quad (31)$$

Equations (2), (16), (20), and (21) link the paths of P_k , r_1 , Q_1 , and P_1 to the path of K_1 . The solutions for Q_1 , r_1 and P_k are

$$\hat{Q}_1 = \frac{\theta_{K1}(1 + \theta_{L1}\Delta)}{V} \epsilon \frac{C_1}{Q_1} \hat{K}_1, \quad (32)$$

$$\hat{r}_1 = -\frac{1}{V} \left[(1 - \gamma)\theta_{L1}\Delta + \theta_{K1}(1 + \theta_{L1}\Delta) + \epsilon \frac{C_1}{Q_1} \theta_{L1} \frac{g_3 L_1 + g_4}{m_0(1 - g_4)^2} \right], \quad (33)$$

$$\hat{P}_k = \gamma \hat{P}_1 = -\gamma \frac{\theta_{K1}}{V} \left[1 + \frac{\theta_{L1}}{m_0(1 - g_4)} (1 - g_3 L_1 - g_4) \right] \hat{K}_1. \quad (34)$$

Feeding the above solutions into (30) and (31) produces

$$\begin{bmatrix} \dot{C} \\ \dot{K}_1 \end{bmatrix} = \begin{bmatrix} 0 & u_1 \\ -1/P_k & u_2 \end{bmatrix} \begin{bmatrix} C - C^* \\ K_1 - K_1^* \end{bmatrix}, \quad (35)$$

where

$$u_1 \equiv -\tau \frac{C}{K_1} \frac{\rho + \delta}{\sigma_1 V} \left\{ \epsilon \frac{C_1}{Q_1} \frac{\sigma_1 \theta_{L1} (g_3 L_1 + g_4)}{m_0(1 - g_4)^2} + \sigma_1 (1 - \gamma) [\theta_{K1} + \theta_{L1} \Delta (1 + \theta_{K1})] \right\} < 0,$$

$$u_2 \equiv \frac{1}{V} \left\{ \epsilon \frac{C_1}{Q_1} [\rho(1 + \theta_{L1} \Delta) + \delta \theta_{L1} \Delta] - \delta \sigma_1 (1 - \gamma) \theta_{L1} \Delta \right\}$$

The stationary equilibrium is saddle-point stable. On the convergent path,

$$C - C^* = (K_{1,0} - K_1^*) \frac{u_1}{\lambda} e^{\lambda t}, \quad (36)$$

$$K_1 - K_1^* = (K_{1,0} - K_1^*) e^{\lambda t} \quad (37)$$

where

$$\lambda = \frac{u_2 - \sqrt{u_2^2 - 4u_1/P_k}}{2} < 0.$$

Figures 4 and 5 depict the transition paths for K_1 , C , and L_1 . The saddle path is positively sloped while the capital stock increases or decreases monotonically depending on whether $\epsilon \geq \epsilon^* = \sigma_1(1 - \gamma)Q_1/C_1$. In the fourth quadrant, the slope of the LL schedule takes the same sign as $\epsilon - \epsilon^*$. Thus, after decreasing at $t = 0$, employment rises continuously. From (25) and (37),

$$\frac{\dot{L}_1}{L_1} = -\frac{\theta_{K1}}{m_o V} \left[\underbrace{\epsilon C_1/Q_1 - \sigma_1(1 - \gamma)}_{\text{Sign of } K_1^* - K_{1,0}} \right] \lambda \frac{K_1^* - K_{1,0}}{K_{1,0}} e^{\lambda t} > 0. \quad (38)$$

Consumption increases in the short run when $\epsilon < \epsilon^*$ but not necessarily when $\epsilon > \epsilon^*$. Two conflicting effects operate when $\epsilon > \epsilon^*$. The increase in the equilibrium capital stock creates an incentive to temporarily reduce consumption, while the rise in output at $t = 0$ and the agent's desire for a smooth consumption path pull in the opposite direction. In the case shown in Figure 4, the intertemporal elasticity of substitution (τ) is relatively low and the consumption-smoothing motive wins out. The private sector allocates some of the increase in real income at $t = 0$ to investment and some to consumption. The counterintuitive outcome where consumption decreases in the short run obtains only when the intertemporal elasticity of substitution is implausibly large. In our base case calibration, for example, τ must exceed 1.27 (see Appendix B).

2.3.1 Welfare

Although we are primarily interested in positive analysis, we take the opportunity in passing to comment on the welfare implications of the results. The punchline is easy to guess: the MW increases welfare, subject to the caveat that a model with a representative agent ignores distributional concerns or assumes, optimistically, that the newly unemployed are compensated enough for their lost wage income. This is obvious in Figure 4, where the path of consumption is continuously higher. Other paths are possible. Consumption may be lower either in the short/medium run or in the long run (Figure 5). In every case, however, welfare improves. In

Appendix B we show that the percentage equivalent variation (EV) welfare gain is

$$EV = \frac{(1 - g_3 L_1 - g_4) \theta_{K1} \gamma}{m_0 (1 - g_4) V} (\epsilon - \epsilon^*) \underbrace{\frac{K_1^* - K_{1,0}}{K_{1,0}}}_{\text{Sign of } \epsilon - \epsilon^*} \frac{\lambda \rho}{\lambda - \rho} + \underbrace{\epsilon \gamma \sigma_1 \theta_{L1} (g_3 L_1 + g_4)}_{\text{Direct effect of } w_m \uparrow} \quad (39)$$

The welfare arithmetic in (39) is straightforward. Both employment and the real wage are suboptimal at the initial equilibrium. Raising the MW ameliorates the coordination externality. It also reduces employment. But since the total supply of labor services increases, the net welfare effect is positive. This gain is captured by the second term in (39). The first term measures the additional welfare gain generated by changes in the capital stock. Variations in K_1 have no direct effect on welfare. Indirectly, however, changes in K_1 increase welfare by increasing the supply of labor services. The sign of $\epsilon - \epsilon^*$ determines both the change in the equilibrium capital stock and the impact of increases in the capital stock on employment. Thus, regardless of whether the equilibrium capital stock rises or falls, the supply of labor services continues to increase as some of the workers laid off at $t = 0$ get rehired on the transition path to the new steady state. The MW always increases welfare because it always moves the supply of labor services closer to its social optimum.

3 The Full Model

Many elements of the full model will be familiar from the exposition of the simplified model. To save space, we present the model with minimum commentary.

Technology

CES production functions convert inputs into output. Scarce entrepreneurial talent H is a fixed factor in the informal sector:

$$Q_1 = a_f \left[a_1^{1/\sigma_1} (e_1 L_1)^{(\sigma_1-1)/\sigma_1} + (1 - a_1)^{1/\sigma_1} K_1^{(\sigma_1-1)/\sigma_1} \right]^{\sigma_1/(\sigma_1-1)}, \quad (40)$$

$$Q_2 = a_i \left[a_2^{1/\sigma_2} (e_2 L_2)^{(\sigma_2-1)/\sigma_2} + a_3^{1/\sigma_3} K_2^{(\sigma_2-1)/\sigma_2} + (1 - a_2 - a_3)^{1/\sigma_2} H^{(\sigma_2-1)/\sigma_2} \right]^{\sigma_2/(\sigma_2-1)} \quad (41)$$

The supply price of capital in both sectors is

$$P_k = 1 + f P_1. \quad (42)$$

PREFERENCES, SAVING, AND INVESTMENT

Equations (3)-(5) carry over:

$$C = \left[(1 - \kappa)^{1/\epsilon} C_1^{(\epsilon-1)/\epsilon} + \kappa^{1/\epsilon} C_2^{(\epsilon-1)/\epsilon} \right]^{\epsilon/(\epsilon-1)}, \quad (43)$$

$$C_1 = (1 - \kappa) \left(\frac{P_1}{P} \right)^{-\epsilon} C, \quad (44)$$

$$P = [\kappa + (1 - \kappa) P_1^{1-\epsilon}]^{1/(1-\epsilon)}. \quad (45)$$

The representative agent solves the more elaborate Ramsey problem

$$\underset{\{C, I_1, I_2, e_1, e_2\}}{\text{Max}} U = \int_0^{\infty} \left(\frac{C^{1-1/\tau}}{1-1/\tau} - Z_1 - Z_2 \right) e^{-\rho t} dt, \quad (46)$$

subject to

$$PC = P_1 Q_1 + Q_2 - P_k \left[I_1 + I_2 + \frac{v_1}{2} \left(\frac{I_1}{K_1} - \delta \right)^2 K_1 + \frac{v_2}{2} \left(\frac{I_2}{K_2} - \delta \right)^2 K_2 \right], \quad (47)$$

$$\dot{K}_1 = I_1 - \delta K_1, \quad (48)$$

$$\dot{K}_2 = I_2 - \delta K_2, \quad (49)$$

where

$$Z_1 = [e_1 - g_o - g_1 \ln(w_1/P) - g_3 u + g_4 \ln(L_1/S_1)]^2,$$

$$Z_2 = [e_2 - b_o - b_1 \ln(w_2/P) + b_2 \ln w_m - b_3 u]^2,$$

$$u = 1 - L_1 - L_2,$$

I_j is investment in sector j ($j = 1, 2$), and the terms $v_1(\bullet)^2 K_1/2$ and $v_2(\bullet)^2 K_2/2$ capture adjustment costs incurred in changing the capital stocks.¹⁵

Optimizing over e_1 and e_2 gives

$$e_1 = g_o + g_1 \ln w_m + g_3 u - g_4 \ln L_1, \quad (50)$$

$$e_2 = b_o + b_1 \ln(w_2/P) - b_2 \ln w_m + b_3 u. \quad (51)$$

The effort function in the informal sector differs from its counterpart in the formal sector in two ways. First, effort is independent of employment on the assumption that supervision of the small workforce at micro firms is not a problem. Second, and more importantly, the MW shifts

¹⁵Adjustment costs are required to support the assumption that capital is sector specific.

the norm for fairness among workers. When w_m increases, workers perceive their current real wage as less fair than before; disgruntled, they express their dissatisfaction with the status quo by reducing effort.

The other first-order conditions for an optimum can be compressed into two Euler equations for I_1 and I_2 . On an optimal path, investment adjusts so that the return on capital, net of adjustment costs and depreciation, continuously equals the real interest rate:

$$\frac{v_1}{K_1} \dot{I}_1 = \left[1 + v_1 \left(\frac{I_1}{K_1} - \delta \right) \right] \left[\frac{\dot{C}}{C\tau} + \rho + \delta - (\alpha - \gamma) \frac{\dot{P}_1}{P_1} \right] + \frac{v_1}{2} \left(\frac{I_1}{K_1} - \delta \right)^2 - \frac{r_1}{P_k} \quad (52)$$

$$\frac{v_2}{K_2} \dot{I}_2 = \left[1 + v_2 \left(\frac{I_2}{K_2} - \delta \right) \right] \left[\frac{\dot{C}}{C\tau} + \rho + \delta - (\alpha - \gamma) \frac{\dot{P}_1}{P_1} \right] + \frac{v_2}{2} \left(\frac{I_2}{K_2} - \delta \right)^2 - \frac{r_2}{P_k} \quad (53)$$

where $\alpha \equiv P_1 f / P_k$, is the cost share of the formal good in the production of capital.

Labor Demand and the Wage Curve in the Informal Sector

The sectoral demands for labor are

$$L_1 = a_1 Q_1 (a_f e_1)^{(\sigma_1 - 1)} (1 - g_4 / e_1)^{\sigma_1} \left(w_m \frac{P}{P_1} \right)^{-\sigma_1}, \quad (54)$$

$$L_2 = a_2 Q_2 (a_i e_2)^{(\sigma_2 - 1)} w_2^{-\sigma_2}. \quad (55)$$

Enforcement of the MW law is confined to the formal sector. In the informal sector, firms pay an EW well below w_m . Equation (51) and the Solow condition

$$\frac{\partial e_2}{\partial (w_2 / P)} \frac{w_2 / P}{e_2} = 1$$

yield

$$e_2 = b_1.$$

Conveniently, effort is constant in general equilibrium. We set e_2 equal to unity, the initial level of effort in the formal sector. The resulting wage curve is

$$\ln(w_2 / P) = 1 - b_o + b_2 \ln w_m - b_3 u. \quad (56)$$

At first glance, equation (56) delivers a lighthouse effect. This is not necessarily the case, however. Layoffs in the formal sector exert downward pressure on the real wage by increasing the unemployment rate. Moreover, estimates of wage curves find, as expected, that real wages are considerably more responsive to unemployment in the informal sector than in the formal

sector. A significant lighthouse effect requires not only b_2 sufficiently large, but also relatively small employment losses in the formal sector. The MW puzzle is multifaceted, but the three most important stylized facts – small employment losses in the formal sector, larger employment losses in the informal sector, and the lighthouse effect – are all of a piece.

Raising the MW

The policy experiment is the same as in the simplified model. Initially the MW is a penny below the EW firms pay in the formal sector. The announcement of a higher MW thus increases the wage in the formal sector dollar-for-dollar.

Market-Clearing Conditions

Three market-clearing conditions close the model. Demand equals supply in the formal sector and in the rental markets for the two capital goods when¹⁶

$$Q_1 = (1 - \kappa) \left(\frac{P_1}{P} \right)^{-\epsilon} C + f \left[I_1 + I_2 + \frac{v_1}{2} \left(\frac{I_1}{K_1} - \delta \right)^2 K_1 + \frac{v_2}{2} \left(\frac{I_2}{K_2} - \delta \right)^2 K_2 \right], \quad (57)$$

$$K_1 = (1 - a_1) Q_1 a_f^{(\sigma_1 - 1)} \left(\frac{r_1}{P_1} \right)^{-\sigma_1}, \quad (58)$$

$$K_2 = a_3 Q_2 a_i^{(\sigma_2 - 1)} r_2^{-\sigma_2}. \quad (59)$$

3.1 Model Calibration

Table 1 shows the values assigned to various deep parameters, to the formal sector wage premium, and to factor shares and expenditure shares at the initial equilibrium. We chose standard values for the depreciation rate (δ), the intertemporal elasticity of substitution (τ), the urban unemployment rate (u), and the cost share of capital in the formal sector (θ_{K_1}). With respect to the other choices (save one):

- *Pure time preference rate (ρ) and the real return on private capital.* Across steady states, the real return on private capital (net of depreciation) equals ρ . We set ρ at 10%. This is line with estimates of the return to private investment in Isham and Kaufmann (1999), Dalgaard and Hanson (2005), and Marshall (2012), and with hard data on real loan rates in LDCs.¹⁷

¹⁶We omit the market-clearing condition for H , which tracks the quasi-rent earned by entrepreneurial talent in the informal sector (a variable irrelevant to the issues under examination).

¹⁷To give a few examples, the real loan rate in 2014 was 8.9% in Colombia, 9.7% in Costa Rica, 10.3% in Guatemala, 8.4% in Kenya, and 11.1% in Tanzania (World Development Indicators, 2014). The estimates of the return on private capital cited in the text range from 12% to 16% and presumably incorporate a risk premium. Karabarbounis and Neiman (2013) assume a time preference rate of 10% in estimating a global (59-country) model of labor shares.

- *Elasticity of substitution between capital and labor services* (σ_1, σ_2). Estimates of σ in LDCs range from .5 to 1.2.¹⁸ Overall, there is more support for $\sigma < 1$ than for $\sigma \geq 1$. Since separate estimates do not exist for the informal sector, we fix both σ_1 and σ_2 at .75. The results do not change significantly when σ equals .5 or 1.
- *Adjustment costs to changing the capital stock*(v_1, v_2) *and the q-elasticity of investment spending* (Ω). The first-order condition for investment in the formal sector is

$$[1 + v_1(I_1/K_1 - \delta)] = \phi_1/\phi_2 P_k$$

where ϕ_1 and ϕ_2 are multipliers attached to the private agent's budget constraint and to the law of motion for the capital stock [the constraints in (47) and (48)]. To link the adjustment cost parameter v_1 to an observable elasticity, note that ϕ_1/ϕ_2 is the shadow price of capital measured in dollars. Thus $\phi_1/\phi_2 P_k$ is effectively Tobin's q , the ratio of the demand price to the supply price of capital. Let $\Omega_1 \equiv \hat{I}_1/\hat{q}$ denote the q-elasticity of investment spending. Evaluated at a stationary equilibrium, the first-order condition for investment then gives $v_1 = 1/\delta\Omega_1$. There are few reliable estimates of Ω for LDCs. The estimates for Egypt in Shafik (1992) and for Korea in Hong (1998) and Kim *et al.* (2015), are 2.1 – 2.6, 3.1, and 2.1 – 2.4, respectively. The assigned value is consistent with these estimates and with high-end estimates for developed countries. A sensible case can be made for both higher and lower numbers. Fortunately, the results are highly insensitive to Ω . The impulse responses presented in Sections 4 and 5 change very little when Ω equals .5 or 5.

- *Firm-size wage premium* } [$g_4/(1 - g_4)$]. Velenchik (1997), Soderbom and Teal (2004), Falco *et al.* (2009), Aigbokhan (2011), and Rand and Torm (2012) report elasticities of the real wage with respect to employment of .16 in Zimbabwe, .15 in Ghana, .38 – .50 in Tanzania, .26 in Nigeria, and .24 in Vietnam. This elasticity pins down g_4 in the formal sector wage curve in (14). Our choice for the premium of .25, which equals the average of the five estimates, implies a value of g_4 of .20.
- *Formal sector wage premium*($\psi = w_1/w_2$). The formal sector wage premium is large in LDCs. Numerous empirical studies find, after controlling for observable human capital characteristics, unobservable heterogeneity, self-selection, and workplace conditions, that workers in the formal sector earn 20 – 120% more than workers in the informal sector.¹⁹ A wage premium of 50% is representative, if slightly conservative. As explained later, the wage premium should be set jointly with the sectoral factor cost shares and the

¹⁸See Briguglio (1998), Duffy and Papageorgiou (2000), Claro (2002), Wang (2012), Shankar and Rao (2012), Martinez (2012), Shen and Whalley (2013), Goldar et al. (2014), Oberfeld and Raval (2014), and Helali and Kalai (2015).

¹⁹Summary results for 36 case studies may be found at <http://mypage.iu.edu/~ebuffie/>.

consumption share of the formal good to be consistent with the observed share of the formal sector in total employment.

- *Consumption share of the formal good* (γ). There is considerable variation in the size of the formal sector across LDCs. To accommodate this, we let γ take low, average, and high values of .45, .65, and .80. The average value equals the average output share of the formal sector in the World Bank Enterprise Surveys (La Porta and Shleifer, 2014).
- *Cost share of the formal good in production of investment goods* (α). The base case in the full model maintains the assumption of the simplified model that $\alpha = \gamma$.
- *Elasticity of substitution in consumption* (ϵ). The right value for ϵ depends on whether firms in the formal and informal sector sell in similar or distinct product markets. In the former case, values of ϵ on the order of 2 – 5 are defensible; in the latter case, .5 – 1 is the likely range.²⁰ Our prior is that formal and informal firms normally compete in the same market, producing fairly substitutable goods. The World Bank Enterprise Surveys support this view: after access to finance, the biggest "obstacle to doing business" cited by firms in the formal sector is "practices of competitors in the informal sector" (La Porta and Shleifer, 2014). We carry out runs for $\epsilon = 1, 3, 5$. In keeping with our prior, $\epsilon = 3$ is the base case.²¹
- *Lighthouse effect* (b_2). When b_2 equals unity, as in the base case, effort in the informal sector depends on the ratio of the wage to the MW. We also investigate scenarios with weaker and stronger lighthouse effects ($b_2 = .5, 1.5$). One rationale for the stronger lighthouse effect is that effort may depend on the absolute difference between the wage and the MW.
- *Real wage flexibility in the formal and informal sectors* [$g_3/(1 - g_4), b_3$]. Estimates of wage curves in LDCs suggest a modal elasticity of approximately -0.1, and that real wages are much more flexible in the informal sector than in the formal sector (see Appendix D). Thus $b_3 \gg g_3/(1 - g_4)$. But much more flexible does not always mean highly flexible. In both the formal and informal sector, the sensitivity of the real wage to the unemployment rate varies considerably across countries, time periods, and states of the economy. In our baseline we set $g_3/(1 - g_4) = 1$, but examine low and high wage flexibility scenarios ($g_3/(1 - g_4) = 0.5$ and $g_3/(1 - g_4) = 1.5$ respectively), but impose $b_3 = 2g_3/(1 - g_4)$ in all runs.

²⁰At high levels of aggregation, compensated elasticities of demand are small. In our base case, $\epsilon = .5 - 1$ implies a compensated elasticity of .175 – .35.

²¹Two other points merit comment here. First, small firms in the formal sector that do not comply with the MW law belong to the informal sector in our model. Second, in a model with only two goods, the compensated elasticity of demand for the formal sector good is much smaller than the elasticity of substitution. In our base case calibration where $\gamma = .65$, the compensated elasticity of demand is 1.05 – 1.65 for $\epsilon = 3 - 5$.

3.1.1 The Problem Child: Cost Shares in the Informal Sector

One important part of the model proved difficult to calibrate. Good, sensible data are not readily available for factor cost shares in the informal sector. National Income Accounts data are especially unreliable (Gollin, 2002). We calibrate the labor share in informal sector value added directly from the *Informal Enterprise Surveys* collected by the World Bank.²² We focus exclusively on manufactured firms and compute the labor share at the level of the firm as $\theta_{Li} = (wL/VA)_i$. The denominator, value added, is defined as $VA_i = PY_i - P_M M_i - P_E E_i$, where PY_i denotes the total value of sales, $P_M M_i$ is the cost of material inputs, and $P_E E_i$ is the cost of energy and transport. We consider two measures of the numerator, labor costs. The first is simply the firm's self-reported "labor costs" and the second ("wage bill") is computed as the product of the (reported) average wage times reported employment. As noted by Gollin (2002), in many low-income countries payments to informal labor, including family employees, are treated as residual payments to capital; to control for this, we impute the average wage for "unpaid" family members working in the firm.

Both labor measures ("labor costs" and "wage bill") are computed on a country-by-country basis where the usable sample of manufacturing firms ranges from 50 to 250 informal firms. Missing and clearly mis-reported data are endemic in the informal surveys and we therefore censor the firm-level data, excluding firms with labor shares in value added that exceed 100% or fall below 15%. Table 2 and the associated kernel densities in Figure 6 summarize the information in the 19 surveys. Mean employment in informal firms is approximately four (of which 1.75 are unpaid family/other employees) and the labor share in informal sector value added is approximately .50. Since production is less capital intensive in the informal sector than in the formal sector, we set $\theta_{K2} = .25$. This and the value of .50 for θ_{L2} imply a cost share for entrepreneurial skill of .25.

The initial distribution of non-agricultural employment between the formal and informal sectors is tied down by the formal sector's share in consumption and investment expenditure, $\gamma = \alpha$; the initial formal sector wage premium, $\psi = w_1/w_2$; and factor cost shares in the two sectors. If the values assigned to these variables are reasonable, the employment share of the formal sector should lie between .35 and .75, the range observed in the data (Terrell and Almeida, 2008; Gasparini and Tornarolli, 2009). The base case and alternative calibrations of the model satisfy this consistency check. In the baseline calibration where $\gamma = \alpha = 0.65$, formal labour accounts for 54% of total (non-agricultural) employment. Increasing the expenditure share of the formal sector to .80 implies that that formal sector labour accounts for 72% of the total, which aligns with non-agricultural employment shares in higher-income countries in Latin America,

²²Afghanistan (2008, 2009), Burkina Faso (2009), Cape Verde (2009), Cameron (2009), Cote d'Ivoire (2009), Madagascar (2009), Mauritius (2009), Angola (2010), Botswana (2010), DRC (2010, 2013), Mali (2010), Argentina (2010), Ghana (2013), Guatamala (2010), Kenya (2013), Myanmar (2014), Peru (2010) and Rwanda (2011).

while for $\gamma = \alpha = 0.45$, the share of formal employment in non-agricultural employment falls to 34%, consistent with that observed in low-income countries.

3.2 Solution Technique

There are a variety of ways to approximate the stable manifold. Given the substantial nonlinearities present in the model, we judged the method in Novales *et al.* (1999) to offer the best tradeoff between solution speed and minimization of approximation error. The method derives stability conditions from a linear approximation around the steady state, but incorporates the nonlinear structure of the model when tracking the transition path.

4 Numerical Results

Different calibrations of the model are appropriate for countries at different stages of development. To keep taxonomy to a minimum, we limit the analysis to a comparison between a middle-income / emerging market economy – our ‘baseline’ calibration – and a representative low income country calibration.

4.1 The Base Case and Its Variations

This section explores the consequences of a 10% increase in the real minimum wage in the formal private sector for a base case constructed to replicate key features of middle-income developing countries. The central settings for the unemployment semi-elasticity of wages are $g_3/(1-g_4) = 1$ in the formal sector and $b_3 = 2$ in the informal sector, against which we consider relatively flat ($g_3/(1-g_4) = .5$ and $b_3 = 1$), and relatively steep ($g_3/(1-g_4) = 1.5$ and $b_3 = 3$) wage curves.²³

We start by examining the comparative steady state (i.e. long run) effects of MW increases. The spine of Table 3 (Column [2] in Panels A, B and C) reports the results of variations in the elasticity of substitution in consumption and the formal share in expenditure; reading from left to right in each case shows variations in outcomes as the wage curves in both sectors steepen. In each panel we report the percentage change across steady states in sectoral and total employment, capital stocks, output, and the real consumption wage in the informal sector in response to a 10% increase in the formal sector minimum wage. (To compare these results with the elasticities typically reported in the empirical literature, simply divide our results through by 10.) Three results stand out. First, MW increases boost aggregate investment, GDP, and informal sector wages, and strongly favor the formal sector. The behaviour of the central base case is consistent with the intuition developed in Section 2. A higher minimum wage in the formal sector leads to a modest contraction in long-run employment, positive capital accumulation and strong output

²³The wage curve has the real wage on the vertical axis. Steeper wage curves exhibit therefore a more elastic response of the wage to the unemployment rate.

growth in the formal sector. This is combined with a larger and unambiguous contraction in employment in the informal sector accompanied by a mild contraction of the capital stock and a modest contraction in output. In aggregate, across steady states, the economy adds capital, expands output and sheds labor. Because aggregate employment losses are relatively small, the lighthouse effect raises the real wage 5% in the informal sector.

Delving deeper, and starting with Panel A, Column [2], total employment contracts by 2.6%, comprising a contraction of 1.5% in formal sector employment and a 3.9% reduction in employment in the informal sector. Total output expands by 4.1%, but also in an unbalanced fashion; formal sector output expands by 7.5% while informal sector output contracts by 2.1%. The strong growth in formal output reflects large increases in both the capital stock and labor services. Effective labor input ($e_1 L_1$) rises 8.4% as increased effort outweighs the contraction in formal sector employment. This, in turn, stimulates investment by increasing the marginal product of capital. Capital in the formal sector grows by 6.5% across steady states while that in the informal sector declines very slightly, by 0.4%.

Second, higher substitutability and a larger formal sector means higher aggregate gains favoring the formal sector. When substitutability in consumption between the output of the formal and informal sectors rises, aggregate outcomes are leveraged up and a larger share of the gains accrue to factors employed in the formal sector (compare Column [2] of Panel A and Panel B). Increasing ϵ from 3 to 5 results in the total output growth rising from 4.1% to 4.7%, while the contraction in overall employment is reduced slightly from 2.6% to 2.5%. These aggregate effects, however, conceal highly asymmetric sectoral effects: holding the initial expenditure shares constant, we actually observe a small *increase* in formal sector employment, while contraction in the informal sector increases sharply. The more asymmetric impact on employment is reflected in more asymmetric responses of sectoral capital accumulation and output. Similarly, the higher the initial share of formal sector expenditure in total expenditure (Column [2] in Panel C versus Panel A), the more the aggregate gains are leveraged, again with the employment and output gains accruing primarily to the formal sector, although in this case the 'lighthouse' effect on informal wages is significantly stronger.

Third, the steeper the wage curves, the more favorable the effects on output and employment in both sectors. Per the analysis in Section 2, the larger the unemployment semi-elasticity in the formal and informal sector wage curves, the more rising unemployment leverages effort, minimizing employment losses in both sectors. The extra boost to effective labour spurs greater capital accumulation, reducing employment losses further. Indeed, formal sector employment may *increase* in the long run (Panel B, Column [3]). This paradoxical result stems from general equilibrium interactions associated with the lighthouse effect. The simplified model of Section 2 assumes constant employment in the informal sector. Indirectly, therefore, via its impact on the unemployment rate, the lighthouse effect increases work effort, labor productivity, and labor demand in the formal sector. It follows that *for any given set of wage curves* there exists some

finite value of ϵ for which formal sector employment increases across steady states. Naturally, the critical value of ϵ is smaller the steeper the wage curves. In the runs for $\alpha = \gamma = .65$, for example, the borderline value of ϵ decreases from 7.57 when $g_3/(1 - g_4) = .5$ and $b_3 = 1$ to 3.91 for $g_3/(1 - g_4) = 1.5$ and $b_3 = 3$.

4.2 The Low-Income Country Case

The results in Table 3 sit well with the empirical evidence from middle-income developing countries studied extensively in the empirical literature (e.g., Brazil, Costa Rica, Honduras, Mexico, Indonesia, and South Africa). Low-income economies, such as those of Sub-Saharan Africa, are usually characterized by a larger informal/non-compliant sector whose output, arguably, is less substitutable with output of the formal sector. Reflecting these differences, Table 4 reports the results for $\alpha = \gamma = 0.45$ and $\epsilon = 1 - 3$. Compared to Table 3, the aggregate response and response in the formal sector are significantly attenuated. For $\epsilon = 1$ and wage curve parameters constant at their central values (Panel A, Column [2]), aggregate output growth is pegged back from around 4% to less than 1%, while the aggregate employment contraction increases from 2.6% to around 3.5% between steady states. Outcomes for the formal sector are correspondingly less favourable, with employment elasticities much closer to the high end estimates from the empirical literature (see Section 4.3). If we combine these low-income country structural characteristics with a relatively flat wage curve, as in Column [1], employment losses increase even further, to 6% for aggregate employment and over 8% for the formal sector. In this case, aggregate output contracts slightly, with the minimal output gains in the (now relatively small) formal sector failing to offset the contraction in informal sector output.

4.3 Coherence With Empirical Estimates

The benchmark estimates span the range of key results from the empirical evidence discussed in Section 2. Much the largest share of this evidence focuses on the short- to medium-run employment consequences of changes in minimum wages and, to a lesser extent, on the impact on wages in the uncovered sectors; there is much less empirical evidence on sectoral or aggregate output effects. Figure 7 presents the distribution of simulated employment elasticities against a range of estimates from the empirical literature and shows clearly how the simulated results for the "middle-income" calibration (the results from Table 2), shown in red, are concentrated within the center of the distribution of empirical estimates, while our low- and high-elasticity of substitution cases anchor the tails of the distribution.

As noted, the evidence on other variables is less complete but nonetheless our simulations are consistent with the key results emerging from the literature. The bulk of the empirical evidence suggests that wages in informal/un-covered sectors rise – or at least do not fall – following increases in the formal-sector minimum wage. Gindling and Terrell (2005) estimate an elasticity

of .15 for urban informal workers and .40 for rural informal workers (Costa Rica); Neumark et al. (2006) estimate an elasticity of .43 (Brazil); and Rani and Ranjbar's (2015) estimated elasticities vary from .45 (for India) to around .80 – .90 (for Indonesia and South Africa). See also Bhorat et al (2016), Andalon and Pages (2009), Lemos (2009), Gindling and Terrell (2007). The results from Tables 3 and 4 return uniformly positive informal wage elasticities that range from .31 to .70. Finally, while only a few empirical papers attempt to measure the impact on macro variables other than employment, those that do strongly support the predictions of our model that big positive effects on GDP, labor productivity, and investment are the norm. Rama (2001), Azam (1997), Kertesi and Kollo (2003), Bhorate et al.(2014), and Mayneris et al. (2014) report very large increases in labor productivity in Indonesia, Morocco, Hungary, South Africa, and China (Table C1, Appendix C).^{24,25} Mayneris et al. (2014), for example, estimate the elasticity of labor productivity with respect to the MW in China to be .38 for the private sector and .19 for the state sector. By way of comparison, in simulations for our base case, the mean elasticity is approximately .60 across steady states and .25 to .40 in the short/medium run.

4.4 The Transition Path and Welfare

We conclude this section by examining the transitional dynamics for employment, capital, output and consumption along with the welfare implications of raising the minimum wage. As noted in Section 2.3 and Figures 3 and 4, sectoral capital stocks, consumption, and output rise or fall monotonically towards their long-run values (after an initial jump in the case of consumption and output). These patterns are exactly replicated in Figure 8, which plots the (first 50 periods of the) transition paths for each variable for the case analysed in Table 3, Panel A Column [2]. Two features of the transition paths are worth noting. First, the difference between the short and long-run responses for output and consumption are large: for both, the impact effects are around half their long-run values. Second, and more important, formal sector employment strongly overshoots its steady-state level (which in turn causes overshooting in the unemployment rate). In the case shown in Figure 8, the short-run elasticity is more than three times as large as its long run value (.48 compared to .15) and has a half-life of more than 25 periods (years). As we show in Section 5.2, allowing for *small* adjustment costs in employment results in a much more modest degree of overshooting.

Turning to welfare considerations, recall the striking result in the simplified model that the MW *always* increases welfare. This does not generalize to the full model, but it predominates

²⁴Bhorat et al. (2014) also present evidence that an investment boom accompanied the sharp increase in the MW for agricultural workers in South Africa. Mayneris et al. (2014) do not discuss the impact on investment in China; they emphasize, however, that unit labor costs fell and firm profits held up nicely.

²⁵Kertesi and Kollo (2003) find the large increase in labor productivity "puzzling." After noting the 57% increase in the real MW was associated with a sudden increase in labor productivity, they remark: "The question of how labor productivity was raised in many hard-hit low-wage enterprises seems a hard nut to crack."

in the relevant parameter space. Welfare rises in the nine runs covered by Table 3 and the four runs for $g_3/(1 - g_4) = 1$ and 1.5 and $b_3 = 2$ and 3 in Table 4.²⁶ In all of these runs, the path of consumption is qualitatively similar to the path in Figure 8: C jumps upward at $t = 0$ and then increases monotonically, closely tracking the path of GDP . Unfortunately, the other two LIC runs dissent from the majority view. When wage curves are relatively flat (i.e., $g_3/g_1 = .5$ and $b_3 = 1$) as in Figure 9, aggregate consumption is continuously lower; ergo, the representative agent suffers a welfare loss.

5 Extensions

In this section we extend the model to incorporate MW increases in the public sector and/or adjustment costs to changing employment. Both extensions improve the fit between the model's predictions and the empirical evidence.

5.1 Adding MW Increases in the Public Sector

Private sector productivity depends on efficient delivery of a variety of public sector services. Consequently, if EW effects operate in the public sector, employment losses should be smaller in countries where MW laws cover both public and private sector employees (e.g., Costa Rica, Honduras, Brazil, China, and much of Sub-Saharan Africa).

To analyze this scenario, we assume public sector services enter the private sector production function in sector j as a shift factor with elasticity β

$$Q_1 = a_f(e_3L_3)^\beta \left[a_1^{1/\sigma_1}(e_1L_1)^{(\sigma_1-1)/\sigma_1} + (1 - a_1)^{1/\sigma_1}K_1^{(\sigma_1-1)/\sigma_1} \right]^{\sigma_1/(\sigma_1-1)}, \quad (60)$$

$$Q_2 = a_i(e_3L_3)^\beta \left[a_2^{1/\sigma_2}(e_2L_2)^{(\sigma_2-1)/\sigma_2} + a_3^{1/\sigma_2}K_2^{(\sigma_2-1)/\sigma_2} + (1 - a_2 - a_3)^{1/\sigma_2}H^{(\sigma_2-1)/\sigma_2} \right]^{\sigma_2/(\sigma_2-1)}. \quad (61)$$

Public sector employment L_3 is constant. The effort function takes the same general form as in the private sector

$$e_3 = d_0 + d_1 \ln w_m + d_3 u, \quad (62)$$

but public sector employees may be less productive and/or work less hard than private sector employees. We investigate two cases:²⁷

Case 1:

$$L_3/L_{1,0} = .5, \quad e_{3,0} = .5, \quad \eta_w = d_1/e_{3,0} = .5,$$

²⁶To repeat an earlier disclaimer, a representative agent model ignores distributional concerns. Positive results are therefore only suggestive of potential welfare gains.

²⁷Both cases assume that the semi-elasticity of effort with respect to the unemployment rate equals unity (i.e., $\eta_u \equiv d_3/e_{3,0} = 1$), the initial value in the private formal sector.

and

$$MPL_3 = \beta(Q_1 + Q_2)/L_3 = .5MPL_1$$

Case 2:

As Case 1, except $MPL_3 = MPL_1$

In Case 1, the initial level of effort, the elasticity of effort with respect to the real wage (η_w), and the marginal product of labor (MPL_3) are half as large as in the private formal sector.²⁸ In Case 2, an acute shortage of public services implies that the productive impact of *additional* services is high; hence $MPL_3 = MPL_1$ despite the disparity in work effort ($e_{3,0} = .5$ vs. $e_{1,0} = 1$).

Tables 5a and 5b report results for the long run. In Case 1, the gains relative to the benchmark model (i.e., the results in Table 3) are appreciable but modest. Case 2, by contrast, delivers large, eye-catching numbers. The employment elasticity for L_1 is positive in eight of the nine runs, varying from $-.05$ to $.25$, while GDP increases another 1.4 - 2.6 percentage points. Note also that the biggest gains occur in column 1, the pessimistic case where wage curves are relatively flat.

5.1.1 The Transition Path

Although employment losses are smaller than in the benchmark model, the desired fit with the stylized facts remains elusive. Figures 10 and 11 show impulse responses for $\epsilon = 1 - 5$, $g_3/(1 - g_4) = 1 - 1.5$, and $b_3 = 2 - 3$. The paths for $\epsilon = 1$, where the short/medium-run employment elasticity in the formal sector ranges from $.40$ to $.55$ and is larger (in absolute value) than in the informal sector, can claim empirical support, but only in a minority of cases. To account for the full diversity of outcomes documented in the empirical literature, the employment elasticity for L_1 in the rest of the parameter space should cluster between $-.1$ and $-.3$ and be continuously lower than the elasticity than the elasticity for L_2 . All of the paths for $\epsilon = 3 - 5$ fail this test, as do the runs (not shown) with $g_3/(1 - g_4) = .5$ and $b_3 = 1$. Zero for twelve is not a passing grade!

5.2 Adjustment Costs to Changing Employment

The second extension is more powerful and more general. When firms incur adjustment costs to changing employment, the representative private agent solves

$$\underset{\{C, I_1, I_2, e_1, e_2, h, g\}}{\text{Max}} \quad U = \int_0^{\infty} \left(\frac{C^{1-1/\tau}}{1 - 1/\tau} - Z_1 - Z_2 \right) e^{-\rho t} dt, \quad (63)$$

²⁸The value of β is backed out from the values assigned to the other variables. The marginal products of labor equal their marginal value products because P_1 equals unity at the initial steady state.

subject to

$$PC = P_1 Q_1 + Q_2 - P_k \left[I_1 + I_2 + \frac{v_1}{2} \left(\frac{I_1}{K_1} - \delta \right)^2 K_1 + \frac{v_2}{2} \left(\frac{I_2}{K_2} - \delta \right)^2 K_2 \right] - v_3 \frac{g^2}{2} - P_1 \frac{v_4}{2} h^2 \quad (64)$$

$$\dot{K}_1 = I_1 - \delta K_1, \quad (65)$$

$$\dot{K}_2 = I_2 - \delta K_2, \quad (66)$$

$$\dot{L}_1 = h L_1, \quad (67)$$

$$\dot{L}_2 = g L_2. \quad (68)$$

Doubtless adjustment costs for employment are much smaller than adjustment costs for the capital stock. It also seems likely that adjustment costs in the informal sector are a small fraction of adjustment costs in the formal sector. Otherwise, not much is known about how v_3 and v_4 compared with v_1 and v_2 . To calibrate the model, we assume adjustment costs for employment are 40% as large as adjustment costs for the capital stock in the formal sector and 5% as large in the informal sector (i.e., $v_4 = .4v_1$ and $v_3 = .05v_2 = .05v_1$).

At long last, the impulse responses have the right look. The paths in Figures 12 and 13 match up well with all qualitative and quantitative aspects of the empirical evidence. In keeping with the central tendency in the data, employment losses in the formal sector are often small in absolute terms and relative to employment losses in the informal sector; consistent with the variation in the data, the opposite configuration holds in some parts of the parameter space (e.g., $\epsilon \approx 1$).

6 Concluding Remarks

We have shown that a dynamic general equilibrium model with efficiency wages in both the formal and informal sectors can explain the salient features of the empirical evidence on how MW laws affect employment, wages, and output in LDCs. Variants of the model allow for small adjustment costs to changes in employment and for efficiency wage effects to operate in the public sector as well as the private sector. Calibrated to conventional values for structural parameters, to micro-level data for informal firms, and to consensus estimates of sectoral wage curves, the "fully-loaded" model has considerable leverage. Simulated under our baseline calibration, the model generates short- and long-run results that sit comfortably with the central tendency in the empirical evidence; when combined with alternative simulations that vary a small number of key parameters, the results span the full range of empirical estimates in the existing literature.

A number of important issues should be addressed in future research. Number one on our list is research on the optimal MW.²⁹ The results presented here are merely suggestive. They show only that welfare often increases when (i) the MW is slightly above the equilibrium wage in the formal sector and (ii) policy makers ignore distributional concerns. The latter qualification is obviously important. A proper analysis requires welfare comparisons in a more elaborate model with heterogeneous households.

²⁹Analysis of the macroeconomic effects of greater enforcement of the MW is another promising area for future work. See Gindling *et al.* (2015) for empirical evidence on the impact of stronger enforcement in Costa Rica and Basu *et al.* (2010) for a detailed micro-theoretic model of how enforcement and the mandated MW interact with credibility and the structure of the labor market.

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Appendix A

When firms monitor worker performance, the Z term in the private sector optimization problem is

$$Z = [e_1 - g_o - g_1 \ln(w_1/P) - g_3u]^2 - ne_s, \quad (\text{A1})$$

where

$$e_s = g_5 - g_4 \ln(L_1/S_1).$$

e_s is effort enforced through monitoring. The private agent optimizes over e_1 . Thus

$$e_1 = g_o + g_1 \ln(w_1/P) + g_3u. \quad (\text{A2})$$

Profits are

$$\pi = P_1F(eL_1, K_1) - w_1L_1, \quad (\text{A3})$$

where total effort is $e = e_1 + e_s$. The first-order conditions for L_1 and e_1 give (for $S_1 = 1$)

$$\begin{aligned} P_1F_1 \left(e - \frac{\hat{e}_s}{\hat{L}_1} e_s \right) &= w_1, \\ \implies P_1F_1 e (1 - g_4/e) &= w_1, \end{aligned} \quad (\text{A4})$$

$$\begin{aligned} P_1F_1 \frac{\partial e_1}{\partial (w_1/P)} \frac{1}{P} &= 1, \\ \implies \frac{g_1}{e - g_4} &= 1. \end{aligned} \quad (\text{A5})$$

Without loss of generality, we set $e = 1$ at the initial equilibrium. Substitute $e_1 = 1 - g_5 + g_4 \ln L_1$ into (A2) and solve for $\ln(w_1/P)$:

$$\ln(w_1/P) = \frac{1 - (g_o + g_5) - g_3u + g_4 \ln L_1}{1 - g_4}. \quad (\text{A6})$$

Apart from the irrelevant constant g_5 , the wage curve in (A6) is the same as in (14).

The only changes in the production function and labor demand are that e replaces e_1 :

$$Q_1 = a_f \left[a_1^{1/\sigma_1} (eL_1)^{(\sigma_1-1)/\sigma_1} + (1 - a_1)^{1/\sigma_1} K_1^{(\sigma_1-1)/\sigma_1} \right]^{\sigma_1/(\sigma_1-1)}, \quad (\text{A7})$$

$$L_1 = a_1 Q_1 (a_f e)^{(\sigma_1-1)} (1 - g_4/e)^{\sigma_1} \left(\frac{w_1}{P_1} \right)^{-\sigma_1}. \quad (\text{A8})$$

Repeating the steps in Sections 2.1-2.3 produces the same solutions as in the main text.

Appendix B

Since $Z = 0$ ex post,

$$U = \int_0^{\infty} \frac{C^{1-1/\tau}}{1-1/\tau} e^{-\rho t} dt, \quad (\text{B1})$$

$$\implies \frac{U - U_o}{C_o^{-1/\tau}} = \int_0^{\infty} (C - C_o) e^{-\rho t} dt. \quad (\text{B2})$$

Linearizing the private agent's budget constraint gives

$$C - C_o = \rho P_k (K_1 - K_{1,o}) - P_k \dot{K}_1 + Q_1 \theta_{L1} \left[\frac{1 - g_3 L_1 - g_4}{1 - g_4} (m_1 \hat{K}_1 + m_2 \hat{P}_1 + \frac{m_5}{\theta_{L1}} \hat{w}_m) \right], \quad (\text{B3})$$

$$m_1 = \theta_{K1} / m_o,$$

$$m_2 = \sigma_1 (1 - \gamma) / m_o,$$

$$m_5 = \frac{\sigma_1 \theta_{L1} (g_3 L_1 + g_4)}{m_o (1 - g_4)},$$

and we have chosen units so that $P_1 = P = 1$ at the initial steady state.

The envelope theorem tells us that there is no first-order impact on welfare of changes in the capital stock, holding the supply of labor services constant. Hence the sum of the first two terms in (B3) equals zero. Formally, substitute

$$K_1 - K_{1,o} = (K_1^* - K_{1,o}) (1 - e^{\lambda t}) \quad (\text{B4})$$

$$\dot{K}_1 = -\lambda (K_1^* - K_{1,o}) e^{\lambda t} \quad (\text{B5})$$

into (B3) and then substitute the resulting expression for $C - C_o$ into (B2):

$$\begin{aligned} \frac{U - U_o}{C_o^{-1/\tau}} &= P_k (K_1^* - K_{1,o}) \left[\int_0^{\infty} \rho e^{-\rho t} dt + \int_0^{\infty} (\lambda - \rho) e^{(\lambda - \rho)t} dt \right] \\ &+ \int_0^{\infty} Q_1 \theta_{L1} \left[\frac{1 - g_3 L_1 - g_4}{1 - g_4} (m_1 \hat{K}_1 + m_2 \hat{P}_1 + \frac{m_5}{\theta_{L1}} \hat{w}_m) \right] e^{-\rho t} dt. \end{aligned} \quad (\text{B6})$$

The two integrals enclosed by $[\bullet]$ sum to zero. Thus the welfare gain/loss depends only on how the MW affects the present value of the total supply of labor services ($e_1 L_1$):

$$\frac{U - U_o}{C_o^{-1/\tau}} = Q_1 \theta_{L1} \int_0^{\infty} \left[\frac{1 - g_3 L_1 - g_4}{1 - g_4} (m_1 \hat{K}_1 + m_2 \hat{P}_1 + \frac{m_5}{\theta_{L1}} \hat{w}_m) \right] e^{-\rho t} dt. \quad (\text{B7})$$

Write the solution for \hat{P}_1 in equation (21) as

$$\hat{P}_1 = -\frac{m_3}{V}\hat{K}_1 - \frac{m_5}{V}\hat{w}_m,$$

where

$$m_3 \equiv \theta_{K1} \left[1 + \frac{\theta_{L1}(1 - g_3L_1 - g_4)}{m_0(1 - g_4)} \right]$$

Substituting for P_1 in (B7) and collecting terms yields

$$\frac{U - U_o}{C_o^{-1/\tau}\hat{w}_m} = Q_1\theta_{L1} \int_0^\infty \left[\frac{1 - g_3L_1 - g_4}{1 - g_4} (m_1 - m_2m_3/V) \frac{\hat{K}_1}{\hat{w}_m} + m_5 \left(\frac{1}{\theta_{L1}} - \frac{m_2}{V} \frac{1 - g_3L_1 - g_4}{1 - g_4} \right) \right] e^{-\rho t} dt. \quad (\text{B8})$$

Now

$$m_1 - \frac{m_2m_3}{V} = \frac{m_1}{V} [\epsilon C_1/Q_1 - \sigma_1(1 - \gamma)],$$

$$m_5 \left(\frac{1}{\theta_{L1}} - \frac{m_2}{V} \frac{1 - g_3L_1 - g_4}{1 - g_4} \right) = \frac{m_5}{\theta_{L1}V} \epsilon \frac{C_1}{Q_1}.$$

So the solution in (B8) may be rewritten as

$$\frac{U - U_o}{C_o^{-1/\tau}\hat{w}_m} = \frac{Q_1\theta_{L1}(1 - g_3L_1 - g_4)}{(1 - g_4)V} \frac{m_1}{K_1} [\epsilon C_1/Q_1 - \sigma_1(1 - \gamma)] \int_0^\infty (K_1^* - K_{1,o})(1 - e^{\lambda t}) e^{-\rho t} dt$$

$$+ \frac{Q_1m_5}{V} \epsilon \frac{C_1}{Q_1} \int_0^\infty e^{-\rho t} dt. \quad (\text{B9})$$

$(U - U_o)/C_o^{-1/\tau}$ is the welfare gain measured in units of consumption. To express the equivalent variation (EV) gain as a percentage of consumption, multiply by ρ/C . Doing this and substituting for m_1 and m_5 produces the solution stated in equation (39) in the text:

$$EV = \frac{(1 - g_3L_1 - g_4)\theta_{K1}\gamma}{m_o(1 - g_4)V} (\epsilon - \epsilon^*) \underbrace{\frac{K_1^* - K_{1,o}}{K_{1,o}}}_{\text{Sign of } \epsilon - \epsilon^*}$$

Appendix C

Table C1: Minimum Wage Studies and Episodes

Notation: F = formal sector; I = informal sector; EE = employment elasticity; Overall = combined effect in formal and informal sectors; LE = lighthouse effect. Unless otherwise noted, employment elasticities are negative in the formal sector.

| Study/Country | Employment | Real Wages and Productivity |
|---|---|---|
| Bhorat et al. (2016), S. Africa. | F: Small <i>positive</i> effect in retail and taxi sectors. I: Small <i>negative</i> effect in agriculture. No significant effect in the other three sectors. Significant decrease in hours in agriculture (I), but longer hours in the taxi industry (F). Overall: No significant impact. | F: increases 11% (retail). Alternative estimate = 26.3%. I: Increases 11.3-14.6% (agriculture, domestic workers). Alternative estimate = 15.8-58.2%. LE: Yes. |
| Bell (1997). Colombia. | F: EE = .15-.33 for unskilled labor and .55-1.22 for target group (firms where the wage is ≤ 1.5 MW). | MW has a stronger impact the wage in I than F. LE: Yes. |
| Gindling and Terrell (2009). Honduras, 1990-2004. | F: EE = .46-.55 in the short run and 1.22 in the long run. I: Positive EE; range = .39-.51. ¹ Overall: Elasticity of unemployment = .96 for fixed effects, but insignificant for 2SLS and GMM. | I: No effect (small, insignificant coefficients) on wage workers or the self-employed. LE: Unclear. |

| Study/Country | Employment | Real Wages and Productivity |
|---|---|---|
| Lemos (2009). Brazil, 1982-2004. | No discernable impact on employment in F or I (including self-employment in I). | Strong evidence from kernel densities that MW affects wages in the both F and I. LE: Yes. |
| Gindling and Terrell (2007). Costa Rica, 1988-2000. | F: EE = .11-.55. (F = all salaried employees in the private sector.) | Evidence of LE in kernel densities. LE: Yes. |
| Ham (2018), Honduras, 2006-2012. | F: EE = .8-1. (F = the <i>covered</i> sector — all salaried employees in the private sector.) I: Positive EE = .5-.7 (I = only employers, self-employed workers, and unpaid family members.) Overall: -.10. | NA |
| Andalon and Pages (2008). Kenya, 1998-1999. | F: .11-.55 I: Positive EE for self-employed ranges from .27 to .59; no significant effect on employment of salaried workers. | Strong effect on average real wage in non-agriculture. Elasticity with respect to the MW equals 1.02. No separate data for F and I. LE: Probably. |
| Rama (2001). Indonesia, 1988-1994. | F: Estimates suggest <i>positive</i> impact, but effects are not statistically significant. I (small manufacturing): Average of three EE = -.96; highest EE = -1.30. Overall: Small negative, statistically insignificant effect. | 95% increase in real MW that increased average real wages 5-15% was associated with an increase in labor productivity of 14.4-22.6%. The increase in labor productivity in small-firm manufacturing was 38.7%. LE: NA. |

| Study/Country | Employment | Real Wages and Productivity |
|--|---|--|
| Kertesi and Kollo (2003). Hungary, 2001-2002. | F: No evidence of effect at large firms. I: Significant decrease in employment (3% in first year); EE with respect to the weighted average real wage ranges from -.27 to -.32. | MW enforced across all firm sizes. Widespread compliance. 57% increase in the real MW in 2001. Large increase in labor productivity in the small firm sector. No direct evidence for large firms. Aggregate labor productivity increased $\approx 7\%$. LE: Unclear (MW enforced across all firms.) |
| Jones (1998). Ghana, 1970-1991. | I: No effect of private sector MW. Total: <i>Positive</i> EE = .53; probably captures mainly the large firm response. | Kernel densities suggest a significant positive effect of MW in I. LE: Yes. |
| Azam (1997). Morocco, 1978-1991, agricultural MW. | NA | Strong positive effect on agricultural output. Elasticity on output with respect to the real MW = 1.18 for the largest crop (barley). |
| Gindling and Terrell (2005). Costa Rica, 1988-1999. | NA | Impact of MW much stronger in I than F. Reflects the bigger bite of MW in I. Elasticities of average wage with respect to MW: Urban F = .108; Urban I = .152; Rural F = .164; Rural I = .398. No impact on wages of the self-employed (not covered by MW). LE: Yes. |
| Strobl and Walsh (2003). Trinidad and Tobago, 1996-1998. | Total: Probability of job loss for those directly affected by the MW = 9% for males; statistically insignificant for females. | I (small firms): No effect for males; some evidence of a positive effect for females. LE: Unclear. |

| Study/Country | Employment | Real Wages and Productivity |
|---|---|--|
| Dinkelmann and Ranchbod (2012). S. Africa, 2001-2004, domestic workers | No statistically significant effect on employment on either the intensive or extensive margins. | Despite no monitoring or active enforcement of the MW in this part of the informal sector, wages increased 19-22%. Work conditions also improved. LE: Yes. |
| Bosch and Manacorda (2010). Mexico, 1989-2001. | NA | F: Kernel densities show pronounced spikes at exact multiples and fractions of the MW. I: No discernible impact. LE: Yes for the formal sector; No for the informal sector. |
| Neumark et al. (2006). Brazil, 1996-2001, six largest metropolitan areas. | Total: EE = .07 for total employment (not the target group most directly affected by the MW). | I: Strong evidence that the MW serves as a reference wage; 11.4% of informal workers paid exactly the MW. Total: One dollar increase in MW increases the average wage 43 cents. LE: Yes. |
| Suryahadi et al. (2003). Indonesia, 1988-1999. | Urban F: EE = .112 for all workers, .196 for less-educated workers, and .140 for blue-collar workers. | NA |
| Comola and De Mello (2011). Indonesia, 1996-2004. | F: Employment decreases. I: Employment increases. Total: Employment increases. | I: Positive significant effect on earnings. LE: Yes. |

| Study/Country | Employment | Real Wages and Productivity |
|---|---|---|
| Murray and van Walbeek (2007). S. Africa, 2005, large sugar farms | F: No layoffs but total work hours reduced 10.1% in response to the MW introduced in agriculture in 2002. | NA |
| Khamis (2013). Argentina, 1993 and 2004. | NA | MW bites more and increases wages more in I than in F. Strong statistically significant effect. LE: Yes. |
| Conradie (2004). S. Africa, grape industry. | EE = .3-.6 | NA |
| Kristensen and Cunningham (2006). 19 countries in Latin America and the Caribbean, 1998-2012. | NA | Kernel densities indicate the MW affects the wage distribution in I in 14 of 19 countries. Impact is stronger than in F. Numeraire effect suggested by spikes at exact multiples of the MW. LE: Yes. |
| Feliciano (1998). Mexico, 1970-1990. | Large decreases in the real MW had no effect on male employment, including employment of young males. EE = .43-1.25 for female employment, but the effect disappears when four states (out of 32) are excluded. | NA |
| Wang and Gunderson (2011). China, 2000-2007, rural migrants (target group). | Considerable heterogeneity. No effect in the eastern or western provinces. EE = -1.02 in central provinces. (Results for non-state enterprises.) | NA |

| Study/Country | Employment | Real Wages and Productivity |
|--|--|---|
| Rani and Ranjbar (2015). Brazil, India, Indonesia, S. Africa, and Mexico, 2005-2010. | NA | MW strong affects the wage distribution in all countries. In Brazil, Mexico, and India the impact in I is stronger than in F. Elasticity of real wage in I with respect to the effective MW (log of Katz index) at 20 th and 40 th quantiles in 2009-2010: .79, .64 in Brazil; .46, .48 in India; .82, .92 in Indonesia; .74, .66 in Mexico; .82, .82 in S. Africa. LE: Yes. |
| Nataraj et al. (2014). Survey and meta-regression analysis of nine studies of the impact of MW on F and I sector employment. | F: Meta-regression concludes that $EE = .078$. Based on only four studies. I: Share of total employment increases (four studies). Impact on self-employed is uncertain. | NA |
| Fang and Lin (2013). China, 2004-2009, | Young adults: $EE = .156-.244$. At-risk group: $EE = .265-.553$. Total: $EE = .073-.086$. | NA |
| Lemos (2004). Brazil, 1982-2000, six largest metropolitan areas, total public and private employment. | No significant effect on employment. If anything, the estimates suggest a positive effect (Table 40. Maximum possible $EE = .16$ in the short run and $.05$ in the long run. Adjustment occurs almost entirely through hours worked. | Strong evidence in kernel densities and estimated wage equations that the MW strongly compresses the wage distribution. Impact on F-sector wage is 2-3 times larger than the impact on I-sector wage. LE: Yes. |

| Study/Country | Employment | Real Wages and Productivity |
|---|--|--|
| FainzyIber (2001). Brazil, 1982-1997, panel data for six major metropolitan areas. | F: EE = .16 for workers earning < 90% of the MW and .09 for those earning 90-110% of the MW. I, salaried: .35 for workers earning <90% of the MW and .25 for those earning 90-110% of the MW. Self-employed: .34 for workers earning <90% of the MW and .29 for those earning 90-110% of the MW. | Strong effects of the MW on the entire wage distribution in both the F and I sectors. Elasticity of real wage with respect to the real MW in F = 1.43 for workers earning 50-90% of the MW; 1.08 for workers earning 90-110% of the MW; and .89 for workers earning 110-150% of the MW. Corresponding elasticities = 1.18, 1.03, and .82 I-salaried and 1.18, 1.32, and .77 for self-employed workers. LE: Yes. |
| Hertz (2005). S. Africa, 2001-2004, domestic workers. | Significant disemployment effects. Average (median) EE = .42 (.46) for women and .48 (.33) for men. Employment losses greater in areas with a larger fraction of workers initially earning less than the new MW | Despite high levels of noncompliance, real wages increase 6.6-12.3% for men and 19-21.5%, for women. Larger increases seen in areas with a larger fraction of workers initially earning less than the new MW. |
| Alatas and Cameron (2003). Indonesia, 1990-1996, production workers in clothing, textiles, footwear and leather industries (formal sector). | No impact on employment at large firms. Baseline difference-in-differences estimate gives an average EE = .163 for small firms in 1990-1991. But estimates are statistically insignificant in sensitivity tests that utilize alternative control groups. | NA |
| SMERU (2001). Indonesia, urban formal sector, 1988-1999. | Baseline estimate of EE = .11 for all workers, .31 for youths and women, .20 for less-educated workers, and .14 for blue-collar workers. EE remain negative in sensitivity tests, but most become statistically insignificant. | NA |

| Study/Country | Employment | Real Wages and Productivity |
|--|---|---|
| Islam and Nazara (2000). Indonesia, 1990-1998. | With no controls, <i>positive</i> EEs of .136 for all employment and .395 for paid workers. With pre-crisis and area dummies and regional GDP as controls, EE = -.097. | NA |
| Del Carpio et al. (2013). Vietnam, 2006-2010. | F: <i>Positive</i> EE = .737. I: EE = 1.055. Total: EE = -.293. | Strong positive effect on the average real wage in combined F and I sectors; elasticity of real wage with respect to the real MW = .482. LE: Unclear (no separate estimate for I). |
| Mayneris et al. (2014). China, 2003-2005, industrial sector: | No impact. | Higher MW was binding. Elasticity of the average real wage with respect to the real MW = .361. Large increases in labor productivity. Elasticity of labor productivity with respect to the MW = .38 for private firms; elasticity of labor productivity was half as large in the state sector. Labor productivity increased 8.4% vs. average real wage increase of 7.9%. No decrease in firm profitability. |
| Gindling et al. (2015). Costa Rica, 2011-2012. | No evidence that an effective campaign to enforce the MW at small and medium-sized firms reduced full-time employment. Some weak evidence that part-time employment decreased. | No evidence of adverse an effect on GDP growth. |

| Study/Country | Employment | Real Wages and Productivity |
|--|---|---|
| Alaniz et al., 2011. Nicaragua, 1998-2006. | EE for large firms = -.615 (-1.197 for workers earning within 20% of the MW) Total EE = -.310 (-.522 for workers earning within 20% of the MW). No significant effect on employment at small firms. | Significant positive effect on wages at both large and small firms. (Compliance much lower at small firms.) For workers earning within 20% of the MW, the wage increase at small firms was 55-68% as large as at large firms. LE: Yes. |
| Bhorat et al., 2014. S. Africa, agricultural sector, 2001-2007. | MW increased farm workers 30% relative to control group. Employment decreased 13-18%, with an increase in full-time employment and large losses in part-time employment. ² | Large increase in labor productivity: output growth stays on trend despite the large decrease in employment. Investment boom following the introduction of the agricultural MW. |

¹ The authors caution that, due to the limitations of the data and a weak identification strategy, the positive EE is not strong evidence of an indirect of the MW on employment in the informal sector.

² The 13-18% figure is our estimate based on data in the paper.

Appendix D

Wage Curves and Unemployment

| Study, Country | Elasticity of the real wage with respect to the unemployment rate |
|---|--|
| Berg and Contreras, 2004. Chile, Greater Santiago Area, 1974-1996. | Aggregate = -.076 University education = -.08, No university education = -.11 Formal Sector = -.08, Informal Sector = -.03 (insignificant) Estimate for informal sector is restricted to self-employed workers. |
| Balagi et al., 2012. Turkey, 2005-2009. Excludes agricultural workers. | Aggregate = -.105 Formal Sector = -.069, Informal Sector = -.264 High Education: Aggregate = -.089, Formal = -.073, Informal = -.139 Low Education: Aggregate = -.098, Formal = -.037, Informal = -.288 |
| Ramos et al., 2009. Colombia, 2002-2006. Covers 13 metropolitan areas and 45.2% of the population. | Aggregate = -.071 Formal Sector = -.060 (insignificant), Informal Sector = -.179 |
| Bucheli and Gonzalez, 2007. Uruguay, 1991-2005. National sample, but under-coverage of agriculture. | Aggregate (private sector) = -.132 Formal Sector = -.058, Informal Sector = -.241 Education: 0-8 years = -.158; 9-11 = -.144; 12+ = -.04 (insignificant) |
| Lugo, 2006. Chile, 1992-2002. Sample covers only the urban sector. | Aggregate (private sector): Male = -.039, Female = -.048 Formal: Male = -.031, Female = -.040 Informal: Male = -.040, Female = -.057 High Education: Male = 0, -.045, Female = insignificant |

| Study, Country | Elasticity of the real wage with respect to the unemployment rate |
|---|---|
| Alcaraz, 2009. Mexico, 1995-2001 | Formal Sector = 0, Informal Sector = -.138. |
| Arango et al., 2010. Colombia, different data sources for 2001-2009. | Local (city) unemployment rate: -.009 to -.167; cell mean = 0 to -.103 Group unemployment rate: -.086 to -.108; cell mean = -.044 to -.097 |
| Garcia and Granados, 2005. Chile? Three different national surveys for 1990-2003. | Regional unemployment rate: -.04 Sectoral unemployment rate: -.13. |
| Hoddinott, 1996. Cote d'Ivoire, 1985-1987, males in urban areas. | Aggregate = -.119 Education: Primary or less = -.215, More than primary = -.111 Professional = 0, Non-professional = -.127 |
| Ilkaracan et al., 2003 Turkey, firms with 10+ employees. | Aggregate private sector = -.135 Primary education or less = -.23 Primary-high school education = -.172 College graduates = -.029 (insignificant) Non-agricultural labor = -.15-.43 |
| Kingdon and Knight, 1999 S. Africa, 1993 | All workers = -.108 Private sector = -.141 at mean unemployment rate; Urban = -.135 (evaluated at mean unemployment rate) Rural = -.122 (evaluated at mean unemployment rate) |

| Study, Country | Elasticity of the real wage with respect to the unemployment rate | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|--------------------------|--|------|--------|----------------|-------|-------|---------------|-------|-------|-----------------|-------|-------|----------------|-------|-------|---------------|--------------|--------------|----------|--------------|--------------|-----------|--------------|--------------|
| Mudiriza and Edwards, 2018 S. Africa, 2011 | Aggregate = -.091 Private sector = -.052 | | | | | | | | | | | | | | | | | | | | | | | | | |
| Von Fintel (2015) S. Africa, 2000-2004 | Close to zero in the short run and approximately -.1 in the long run | | | | | | | | | | | | | | | | | | | | | | | | | |
| Balagi et al., 2017 Brazil, 2002-2009 | Aggregate = -.107 Formal = -.039 (insig), Informal = -.246 Age 15-29 = -.154, Age 30-44 = -.030 (insig) Tenure < 1 = -.180, 1 < Tenure < 5 = -.137, Tenure > 5 = insig Low skill = -.048 (insig), Medium skill = -.110 | | | | | | | | | | | | | | | | | | | | | | | | | |
| Castro (2006) Mexico, urban workers, 1993-2002 | <table border="0"> <tr> <td colspan="2">Aggregate = -.037, -.039</td> </tr> <tr> <td>Male</td> <td>Female</td> </tr> <tr> <td>Private Sector</td> <td>-.039</td> <td>-.048</td> </tr> <tr> <td>Formal Sector</td> <td>-.031</td> <td>-.040</td> </tr> <tr> <td>Informal Sector</td> <td>-.058</td> <td>-.057</td> </tr> <tr> <td>High education</td> <td>-.045</td> <td>insig</td> </tr> <tr> <td>Low education</td> <td>-.044, -.065</td> <td>-.041, -.057</td> </tr> <tr> <td>Age < 26</td> <td>-.063, -.068</td> <td>-.063, -.070</td> </tr> <tr> <td>Age 26-45</td> <td>-.017, -.067</td> <td>-.023, -.031</td> </tr> </table> | Aggregate = -.037, -.039 | | Male | Female | Private Sector | -.039 | -.048 | Formal Sector | -.031 | -.040 | Informal Sector | -.058 | -.057 | High education | -.045 | insig | Low education | -.044, -.065 | -.041, -.057 | Age < 26 | -.063, -.068 | -.063, -.070 | Age 26-45 | -.017, -.067 | -.023, -.031 |
| Aggregate = -.037, -.039 | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Male | Female | | | | | | | | | | | | | | | | | | | | | | | | | |
| Private Sector | -.039 | -.048 | | | | | | | | | | | | | | | | | | | | | | | | |
| Formal Sector | -.031 | -.040 | | | | | | | | | | | | | | | | | | | | | | | | |
| Informal Sector | -.058 | -.057 | | | | | | | | | | | | | | | | | | | | | | | | |
| High education | -.045 | insig | | | | | | | | | | | | | | | | | | | | | | | | |
| Low education | -.044, -.065 | -.041, -.057 | | | | | | | | | | | | | | | | | | | | | | | | |
| Age < 26 | -.063, -.068 | -.063, -.070 | | | | | | | | | | | | | | | | | | | | | | | | |
| Age 26-45 | -.017, -.067 | -.023, -.031 | | | | | | | | | | | | | | | | | | | | | | | | |

| Study, Country | Elasticity of the real wage with respect to the unemployment rate |
|---|---|
| Lopez and Mendoza (2017) Mexico, manufacturing sector, 2007-2015 | Aggregate = -.023, -.045 Period of greater stability (07/2009 – 12/2015) = -.092, -.131 |
| Galvani (1999) Argentina, urban areas, 1990-1997 | Aggregate: short run = -.04, long run = -.082 Males: short run = -.057, long run = -.119 |
| Galindo and Catalan (2010) Mexico, 1989-2008 | Aggregate = -.032 |
| Balagi et al. (2012) Turkey, 2005-2008 | Aggregate = -.099 Young (< 34) = -.108 (-.243), Old = -.081 (-.177) Male = -.069, Female = -.237 Urban = -.101, Rural = -.105 (insig) Tenure: Low (< 6.9 years) = -.175 (-.309), High = -.011 (insig) Education: Low (< 8 years) = -.086 (-.632), High = -.067 (-.088, insig) Note: Figures in parentheses are for women. |
| Wu (2004) China, urban youth, 1989-1999 | Aggregate = -.224 |
| Park and Shin (2008) Korea, 1998-2002 | Aggregate = -.075, -.098 |

Table 1: Calibration of the Model.

| Parameter/Variable | Value in Base Case |
|--|--------------------|
| Depreciation rate (δ) | .05 |
| Intertemporal elasticity of substitution (τ) | .5 |
| Urban unemployment rate (u) | .10 |
| Cost share of capital in the formal sector (θ_{K1}) | .40 |
| Cost share of production labor in the formal sector (θ_{L1}) ¹ | .48 |
| Pure time preference rate (ρ) | |
| Elasticity of substitution between capital and labor (σ_1, σ_2) | .75 |
| q-elasticity of investment spending (Ω) | 2 |
| Firm-size wage premium [$g_4/(1 - g_4)$] | .25 |
| Formal sector wage premium ($\psi = w_1/w_2$) | .5 |
| Consumption share of the formal good (γ) | .65 |
| Elasticity of substitution in consumption (ε) | 3 |
| Cost share of the formal good in production of investment goods (α) | .65 |
| Ratio of user fees to recurrent costs (f) | .5 |
| Lighthouse effect (b_2) | 1 |
| Real wage flexibility in the formal sector [$g_3/(1 - g_4)$] | 1 |
| Real wage flexibility in the informal sector (b_3) | 2 |
| Cost share of capital in the informal sector (θ_{K2}) | .25 |
| Cost share of labor in the informal sector (θ_{L2}) | .50 |
| Cost share of entrepreneurial talent in the informal sector (θ_H) | .25 |

¹ The cost shares for capital and production labor satisfy the adding-up constraint $\theta_{K1} + \theta_{L1}/(1 - g_4) = 1$. This and the value of g_4 backed out from the size premium imply $\theta_{L1} = .48$ and a cost share of .12 for managerial/supervisory labor.

Table 2

| Labour Share in value added ^[c] | | | | | |
|--|-------|--------|----------|-------|-------|
| | Mean | Median | Std. dev | Max | Min |
| (Reported labour costs) / VA ^[a] | 0.238 | 0.244 | 0.129 | 0.548 | 0.081 |
| (Avg.wage*emp)/VA ^[b] | 0.426 | 0.118 | 0.118 | 0.702 | 0.119 |
| Labour Share in value added – trimmed ^[c] | | | | | |
| | Mean | Median | Std. dev | Max | Min |
| (Reported labour costs) / VA | 0.494 | 0.482 | 0.093 | 0.706 | 0.354 |
| (Avg.wage*emp)/VA | 0.488 | 0.481 | 0.087 | 0.702 | 0.363 |

| Employment | | | | | |
|------------------|------|--------|----------|------|-----|
| | Mean | Median | Std. dev | Max | Min |
| Paid Employees | 2.8 | 2.0 | 2.3 | 10.5 | 0.7 |
| Unpaid Employees | 0.7 | 0.7 | 0.5 | 1.6 | 0.0 |
| Family | 0.8 | 0.8 | 0.4 | 2.2 | 0.4 |

Source: Cross sectional means from 19 World Bank Informal Firms Survey (see Note 23)

Notes: [a] Total self-reported labour costs as a share of derived value added; [b] Average reported wage time total employment (including family members); [c] Labour share initially calculated excluding firms where measured labour share in excess of 1; trimmed calculation excludes firms with calculated labour share less than 0.15.

Table 3: Long-run outcome when the MW increases in the private sector.

| | PANEL A ($\varepsilon = 3, \alpha = \gamma = .65$) | | |
|-------------|--|------------------------|--------------------------|
| | [1] | [2] | [3] |
| | $g_3/g_1 = .5, b_3 = 1$ | $g_3/g_1 = 1, b_3 = 2$ | $g_3/g_1 = 1.5, b_3 = 3$ |
| w_2/P | 5.83 | 4.95 | 4.57 |
| L_1 | -2.99 | -1.52 | -0.87 |
| L_2 | -5.83 | -3.91 | -3.07 |
| $L_1 + L_2$ | -4.29 | -2.61 | -1.87 |
| K_1 | 4.87 | 6.47 | 7.17 |
| K_2 | -1.77 | -0.39 | 0.21 |
| Q_1 | 5.84 | 7.47 | 8.18 |
| Q_2 | -3.41 | -2.08 | -1.50 |
| GDP | 2.60 | 4.13 | 4.80 |
| | PANEL B ($\varepsilon = 5, \alpha = \gamma = .65$) | | |
| | [1] | [2] | [3] |
| | $g_3/g_1 = .5, b_3 = 1$ | $g_3/g_1 = 1, b_3 = 2$ | $g_3/g_1 = 1.5, b_3 = 3$ |
| w_2/P | 5.98 | 5.13 | 4.77 |
| L_1 | -1.32 | 0.16 | 0.81 |
| L_2 | -7.49 | -5.69 | -4.91 |
| $L_1 + L_2$ | -4.14 | -2.51 | -1.80 |
| K_1 | 6.66 | 8.25 | 8.95 |
| K_2 | -3.40 | -2.12 | -1.50 |
| Q_1 | 7.42 | 9.05 | 9.76 |
| Q_2 | -4.66 | -3.42 | -2.87 |
| GDP | 3.19 | 4.68 | 5.33 |
| | PANEL C ($\varepsilon = 3, \alpha = \gamma = .80$) | | |
| | [1] | [2] | [3] |
| | $g_3/g_1 = .5, b_3 = 1$ | $g_3/g_1 = 1, b_3 = 2$ | $g_3/g_1 = 1.5, b_3 = 3$ |
| w_2/P | 7.02 | 6.39 | 6.11 |
| L_1 | -1.87 | -0.76 | -0.28 |
| L_2 | -6.05 | -4.65 | -4.03 |
| $L_1 + L_2$ | -3.05 | -1.86 | -1.33 |
| K_1 | 6.04 | 7.23 | 7.76 |
| K_2 | -1.17 | -0.14 | 0.31 |
| Q_1 | 6.65 | 7.86 | 8.39 |
| Q_2 | -3.37 | -2.40 | -1.97 |
| GDP | 4.64 | 5.81 | 6.32 |

Table 4: Long-run outcome when the MW increases in the private sector (LIC calibration).

| | PANEL A ($\varepsilon = 1, \alpha = \gamma = .45$) | | |
|-------------|--|------------------------|--------------------------|
| | [1] | [2] | [3] |
| | $g_3/g_1 = .5, b_3 = 1$ | $g_3/g_1 = 1, b_3 = 2$ | $g_3/g_1 = 1.5, b_3 = 3$ |
| w_2/P | 4.52 | 3.38 | 2.89 |
| L_1 | -8.19 | -6.53 | -5.81 |
| L_2 | -4.37 | -1.84 | -0.73 |
| $L_1 + L_2$ | -5.68 | -3.45 | -2.47 |
| K_1 | -0.61 | 1.20 | 1.98 |
| K_2 | -1.16 | 0.62 | 1.40 |
| Q_1 | 1.01 | 2.97 | 3.79 |
| Q_2 | -2.50 | -0.77 | -0.21 |
| GDP | -0.88 | 0.91 | 1.69 |

| | PANEL B ($\varepsilon = 3, \alpha = \gamma = .45$) | | |
|-------------|--|------------------------|--------------------------|
| | [1] | [2] | [3] |
| | $g_3/g_1 = .5, b_3 = 1$ | $g_3/g_1 = 1, b_3 = 2$ | $g_3/g_1 = 1.5, b_3 = 3$ |
| w_2/P | 4.71 | 3.61 | 3.14 |
| L_1 | -4.52 | -2.76 | -2.00 |
| L_2 | -5.98 | -3.62 | -2.58 |
| $L_1 + L_2$ | -5.48 | -3.32 | -2.38 |
| K_1 | 3.29 | 5.19 | 6.02 |
| K_2 | -2.71 | -1.05 | -0.32 |
| Q_1 | 4.62 | 6.58 | 7.43 |
| Q_2 | -3.71 | -2.09 | -1.38 |
| GDP | 0.04 | 1.81 | 2.58 |

Table 5a: Long-run outcome when the MW increases in both the private and public sectors (Case 1).

| | $\varepsilon = 3$ and $\alpha = \gamma = .65$ | | |
|-------------|---|------------------------|--------------------------|
| | $g_3/g_1 = .5, b_3 = 1$ | $g_3/g_1 = 1, b_3 = 2$ | $g_3/g_1 = 1.5, b_3 = 3$ |
| w_2/P | 7.1 | 6.2 | 5.8 |
| L_1 | -1.9 | -.8 | -.3 |
| L_2 | -6.1 | -4.5 | -3.7 |
| $L_1 + L_2$ | -3.8 | -2.5 | -1.9 |
| K_1 | 6.0 | 7.2 | 7.8 |
| K_2 | -1.3 | -.2 | .3 |
| Q_1 | 7.2 | 8.4 | 9.0 |
| Q_2 | -2.9 | -1.9 | -1.4 |
| GDP | 3.7 | 4.8 | 5.4 |
| | $\varepsilon = 5$ and $\alpha = \gamma = .65$ | | |
| | $g_3/g_1 = .5, b_3 = 1$ | $g_3/g_1 = 1, b_3 = 2$ | $g_3/g_1 = 1.5, b_3 = 3$ |
| w_2/P | 7.2 | 6.4 | 6.0 |
| L_1 | -.1 | 1.0 | 1.5 |
| L_2 | -7.9 | -6.4 | -5.7 |
| $L_1 + L_2$ | -3.7 | -2.4 | -1.8 |
| K_1 | 7.9 | 9.1 | 9.7 |
| K_2 | -3.0 | -2.0 | -1.5 |
| Q_1 | 8.9 | 10.1 | 10.7 |
| Q_2 | -4.3 | -3.3 | -2.8 |
| GDP | 4.3 | 5.4 | 5.9 |
| | $\varepsilon = 3$ and $\alpha = \gamma = .80$ | | |
| | $g_3/g_1 = .5, b_3 = 1$ | $g_3/g_1 = 1, b_3 = 2$ | $g_3/g_1 = 1.5, b_3 = 3$ |
| w_2/P | 8.4 | 7.9 | 7.6 |
| L_1 | -.6 | 0 | .4 |
| L_2 | -6.4 | -5.4 | -4.9 |
| $L_1 + L_2$ | -2.2 | -1.5 | -1.1 |
| K_1 | 7.3 | 8.0 | 8.4 |
| K_2 | -.6 | .1 | .4 |
| Q_1 | 8.2 | 8.9 | 9.2 |
| Q_2 | -2.8 | -2.2 | -1.9 |
| GDP | 6.0 | 6.7 | 7.0 |

Table 5b: Long-run outcome when the MW increases in both the private and public sectors (Case 2).

| | $\varepsilon = 3$ and $\alpha = \gamma = .65$ | | |
|-------------|---|------------------------|--------------------------|
| | $g_3/g_1 = .5, b_3 = 1$ | $g_3/g_1 = 1, b_3 = 2$ | $g_3/g_1 = 1.5, b_3 = 3$ |
| w_2/P | 7.8 | 7.0 | 6.6 |
| L_1 | -.5 | .3 | .6 |
| L_2 | -5.8 | -4.6 | -3.9 |
| $L_1 + L_2$ | -2.9 | -1.9 | -1.5 |
| K_1 | 7.5 | 8.3 | 8.7 |
| K_2 | -.4 | .4 | .8 |
| Q_1 | 8.9 | 9.7 | 10.1 |
| Q_2 | -2.0 | -1.3 | -.9 |
| GDP | 5.1 | 5.8 | 6.2 |
| | $\varepsilon = 5$ and $\alpha = \gamma = .65$ | | |
| | $g_3/g_1 = .5, b_3 = 1$ | $g_3/g_1 = 1, b_3 = 2$ | $g_3/g_1 = 1.5, b_3 = 3$ |
| w_2/P | 7.9 | 7.2 | 6.8 |
| L_1 | 1.5 | 2.2 | 2.5 |
| L_2 | -7.7 | -6.6 | -6.0 |
| $L_1 + L_2$ | -2.7 | -1.8 | -1.4 |
| K_1 | 9.6 | 10.3 | 10.7 |
| K_2 | -2.3 | -1.6 | -1.2 |
| Q_1 | 10.8 | 11.5 | 11.8 |
| Q_2 | -3.5 | -2.8 | -2.5 |
| GDP | 5.8 | 6.5 | 6.8 |
| | $\varepsilon = 3$ and $\alpha = \gamma = .80$ | | |
| | $g_3/g_1 = .5, b_3 = 1$ | $g_3/g_1 = 1, b_3 = 2$ | $g_3/g_1 = 1.5, b_3 = 3$ |
| w_2/P | 9.2 | 8.9 | 8.7 |
| L_1 | .8 | 1.1 | 1.3 |
| L_2 | -6.1 | -5.6 | -5.3 |
| $L_1 + L_2$ | -1.1 | -.8 | -.6 |
| K_1 | 8.8 | 9.1 | 9.3 |
| K_2 | .3 | .6 | .8 |
| Q_1 | 9.8 | 10.1 | 10.3 |
| Q_2 | -2.0 | -1.7 | -1.6 |
| GDP | 7.5 | 7.8 | 7.9 |

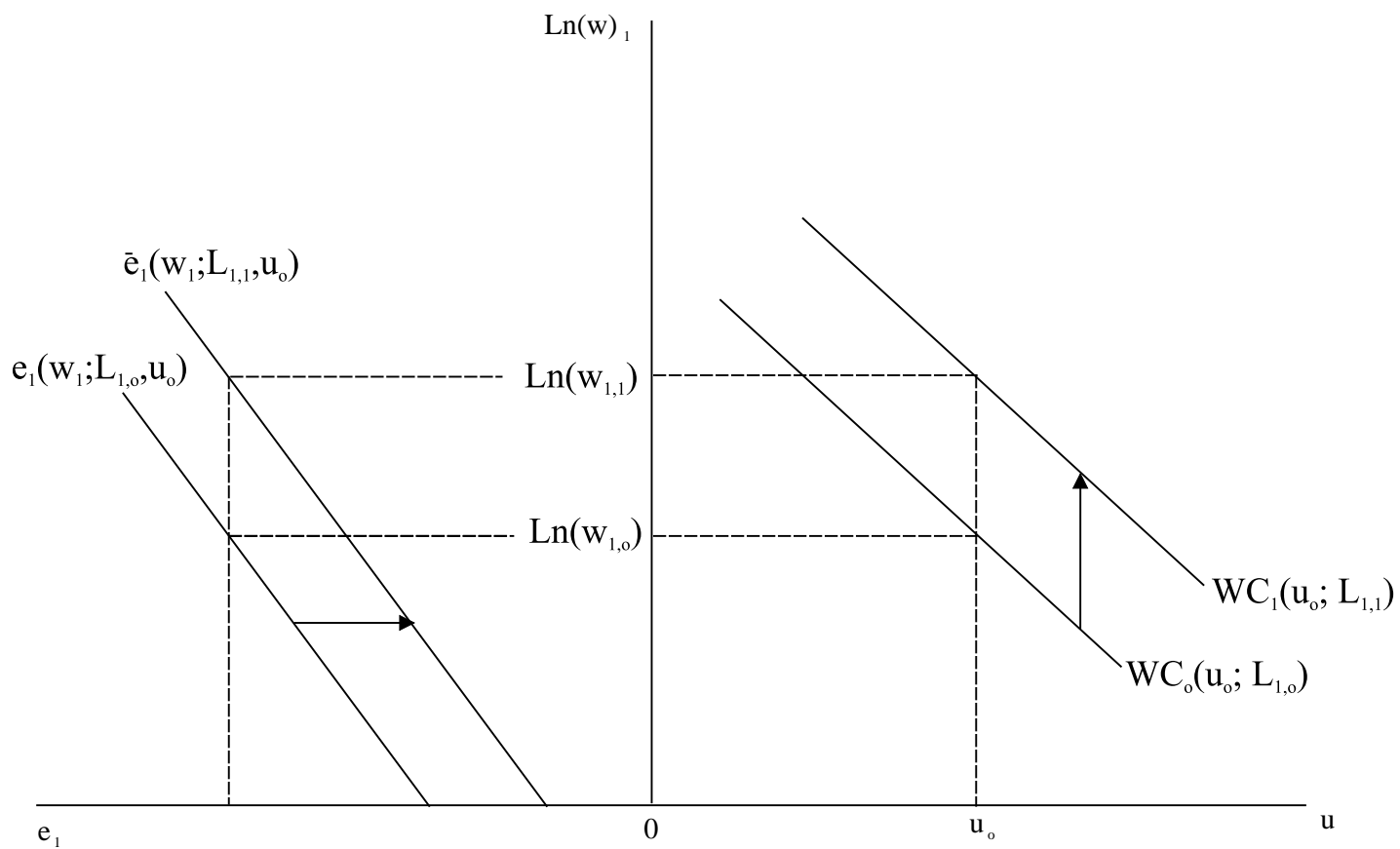


Figure 1: Impact of higher employment on the wage curve.
 ($P_1 = P = 1$, so w_1 equals the real wage.)

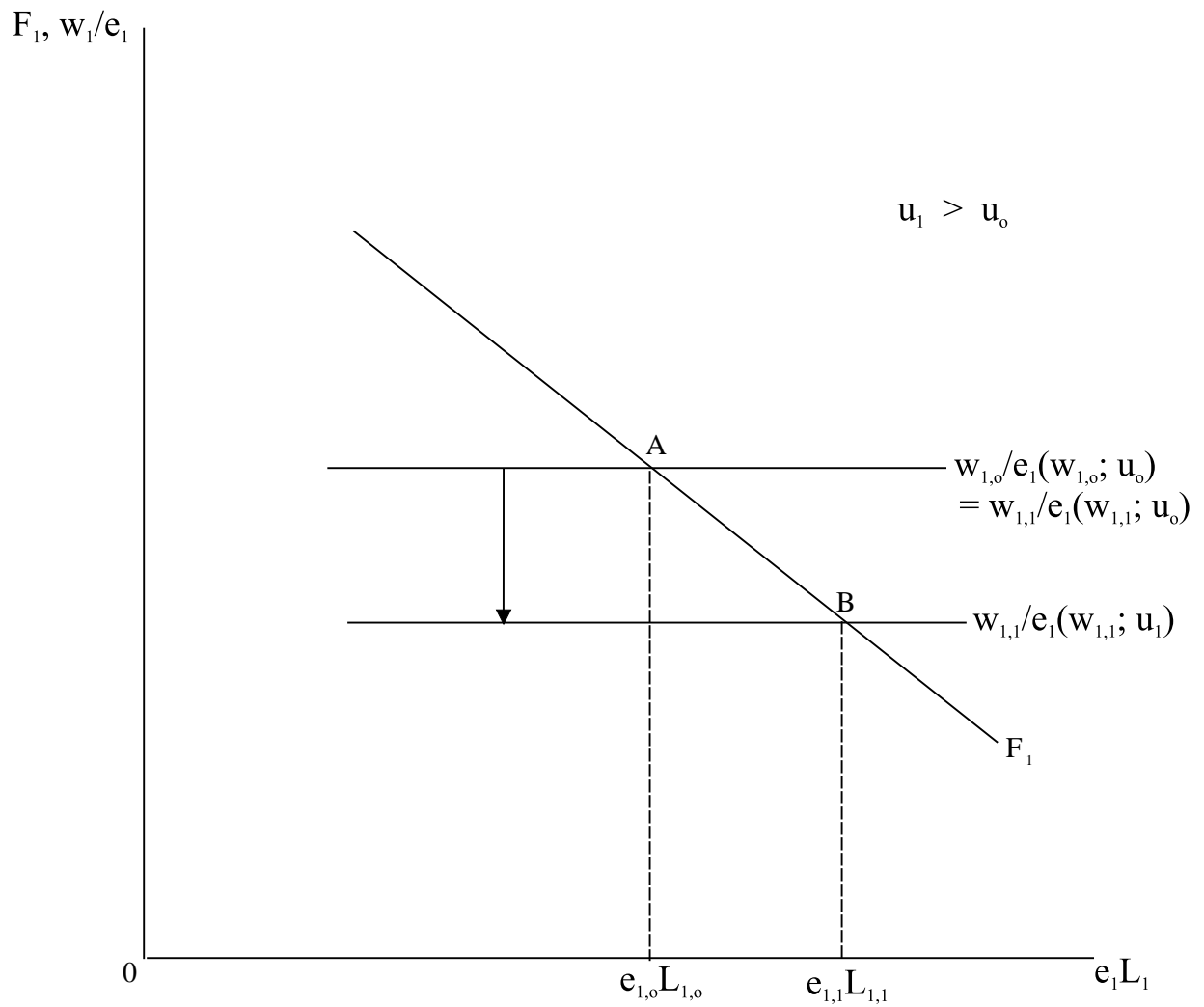


Figure 2: Impact on the total supply of labor services when the minimum wage increases the real wage in the formal sector from $w_{1,0}$ to $w_{1,1}$ ($P_1 = P = 1$).

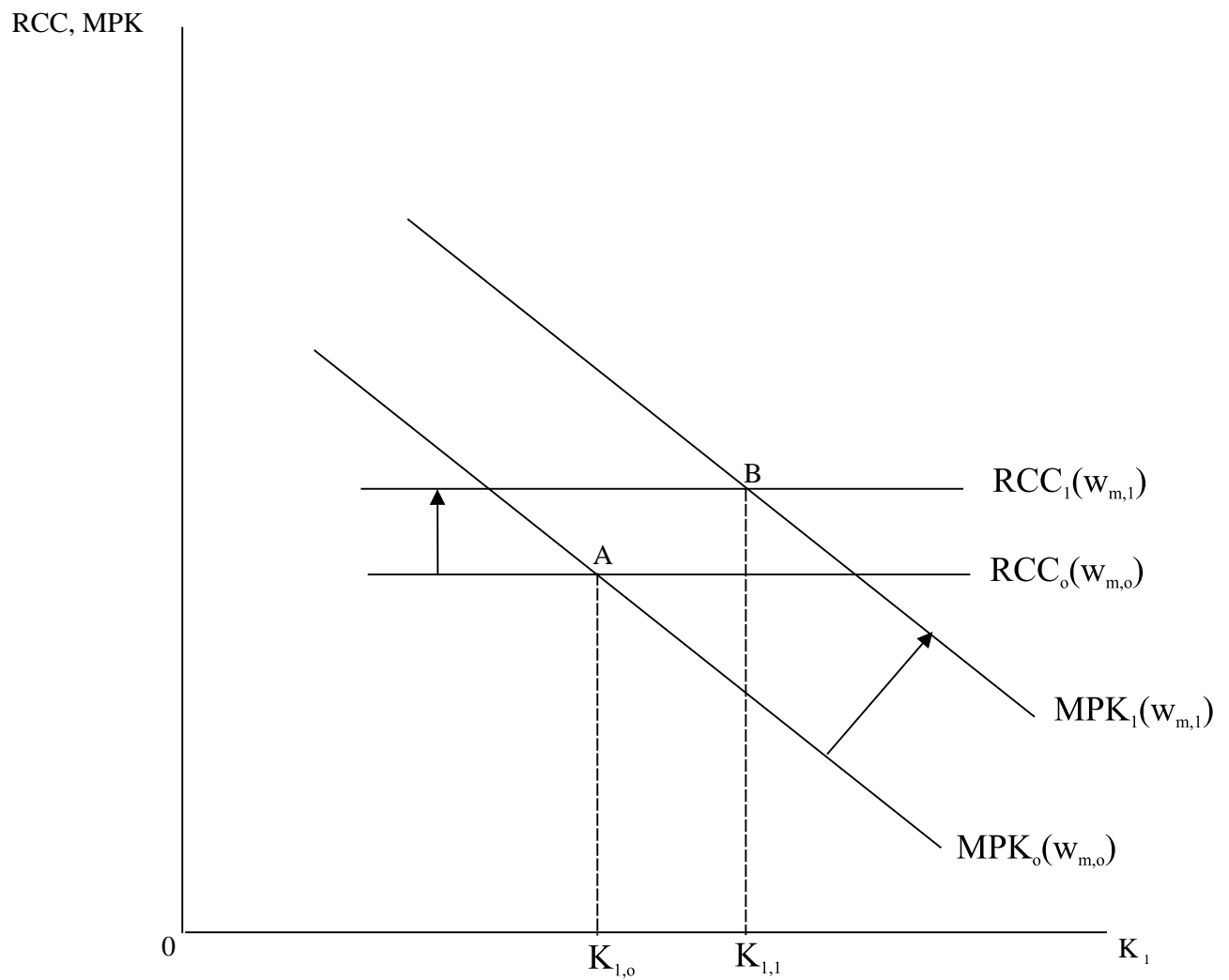


Figure 3: Impact on the capital stock in the formal sector when the real minimum wage increases and $\epsilon > \epsilon^*$.

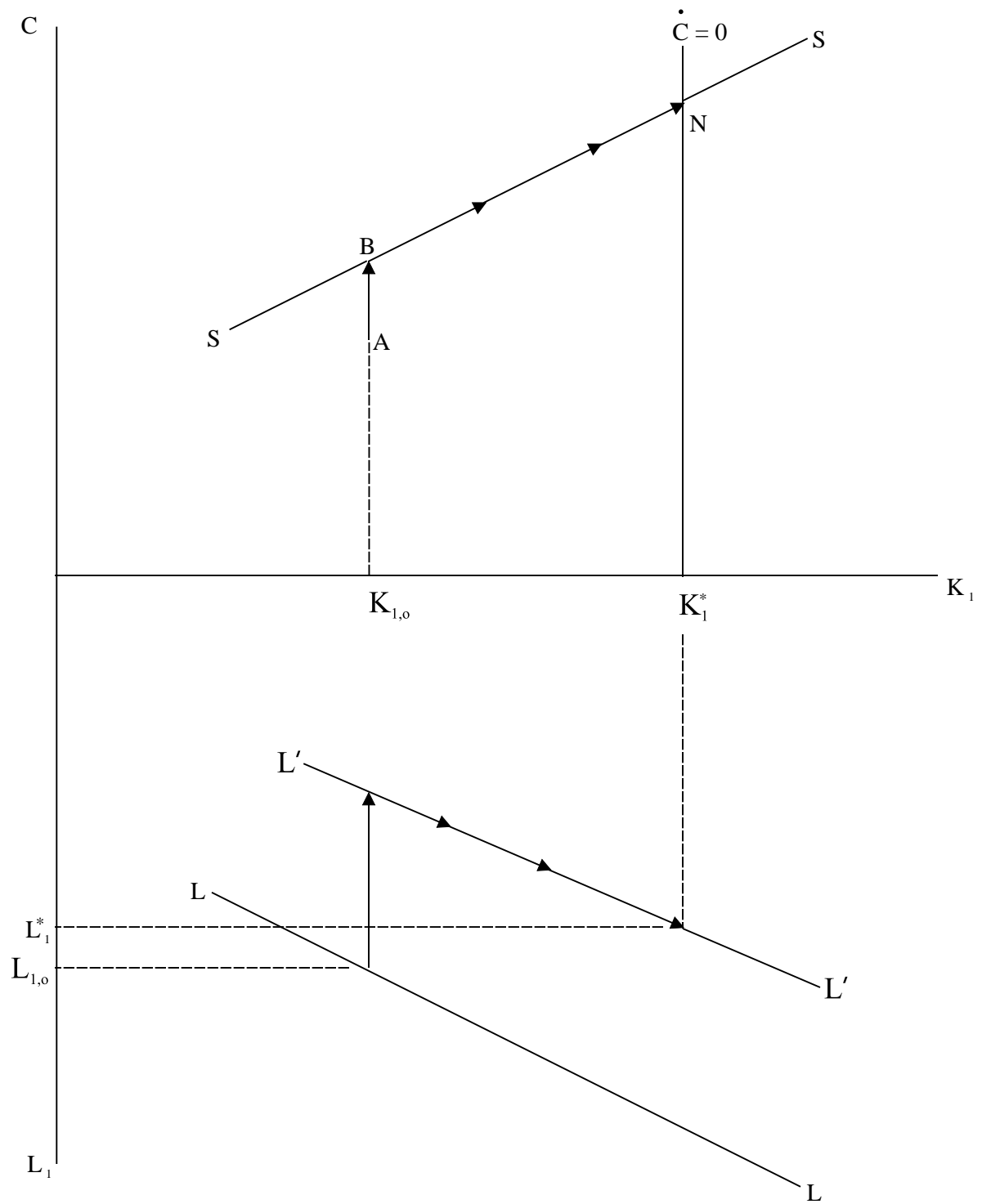


Figure 4: Transition path when K increases.

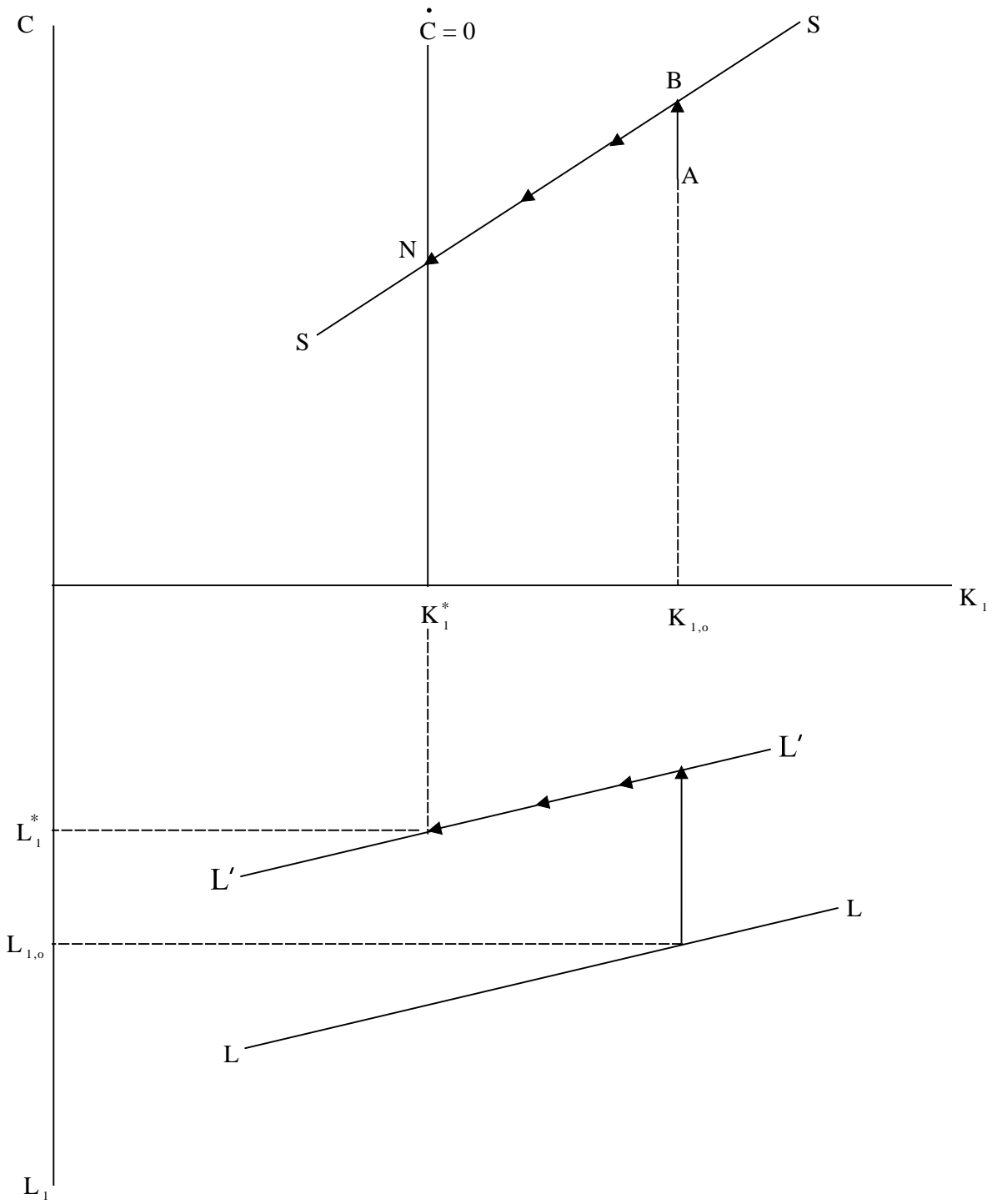


Figure 5: Transition path when K decreases.

Figure 6

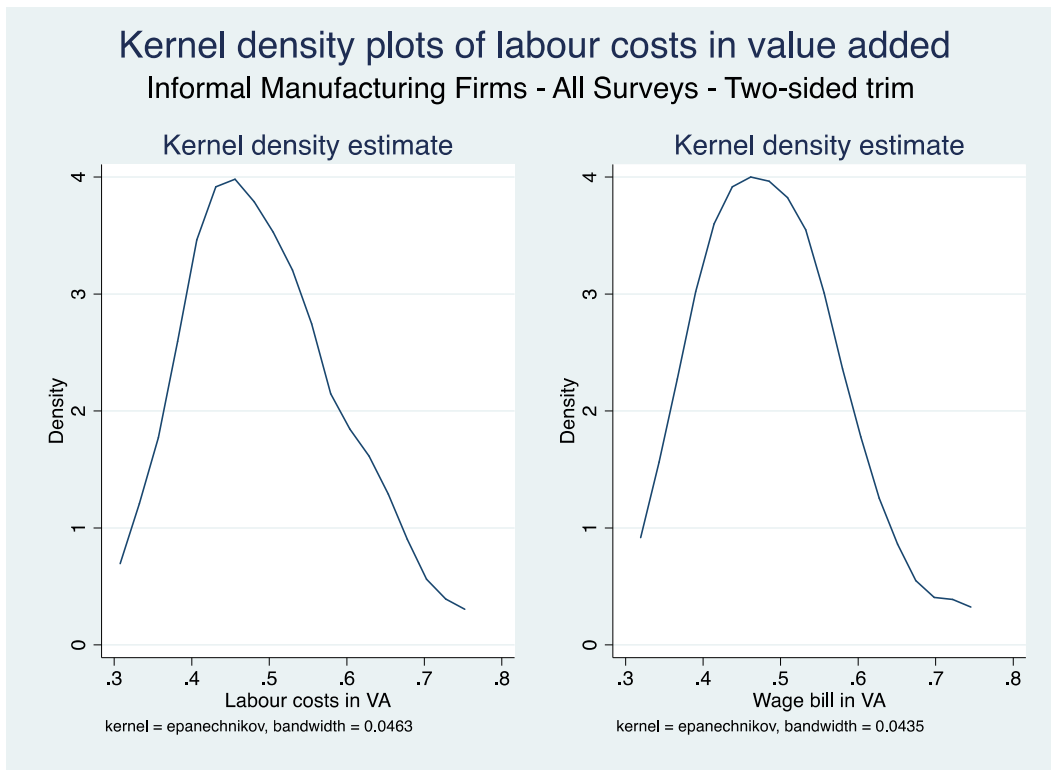
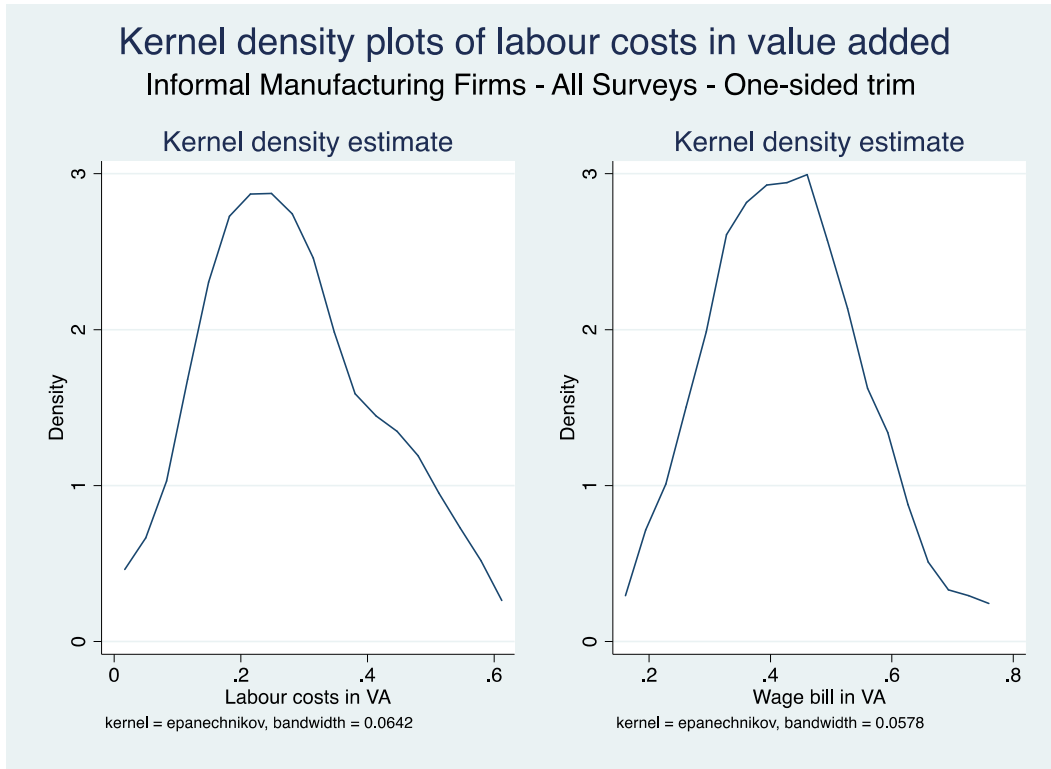
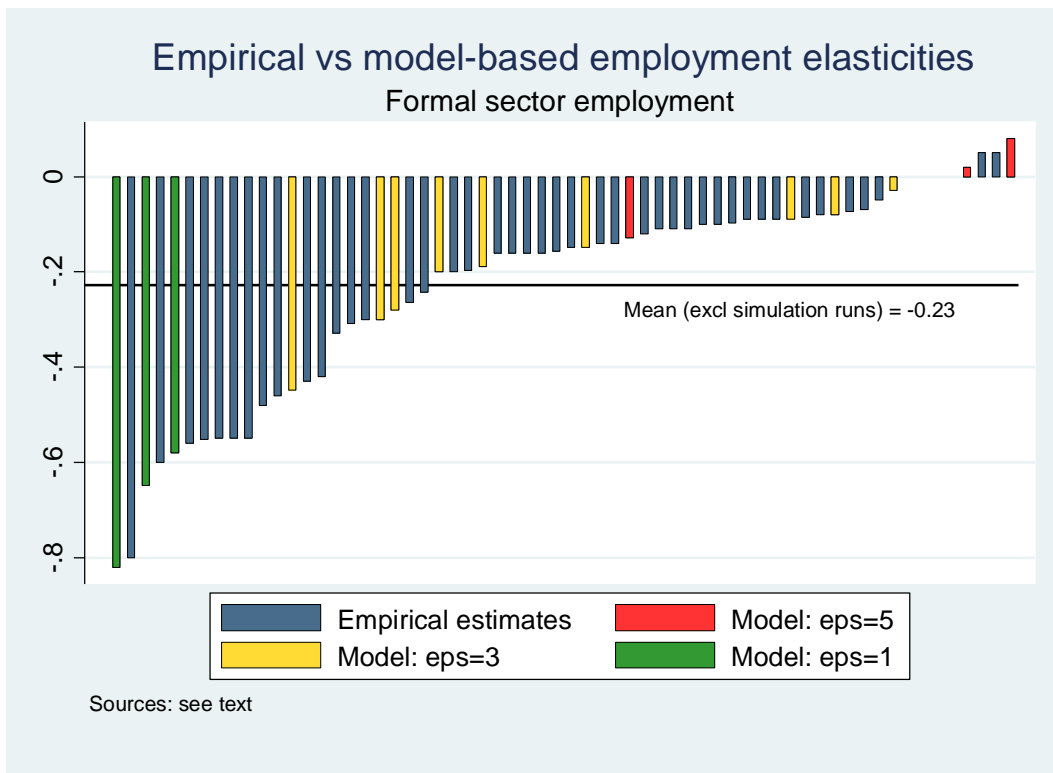


Figure 7



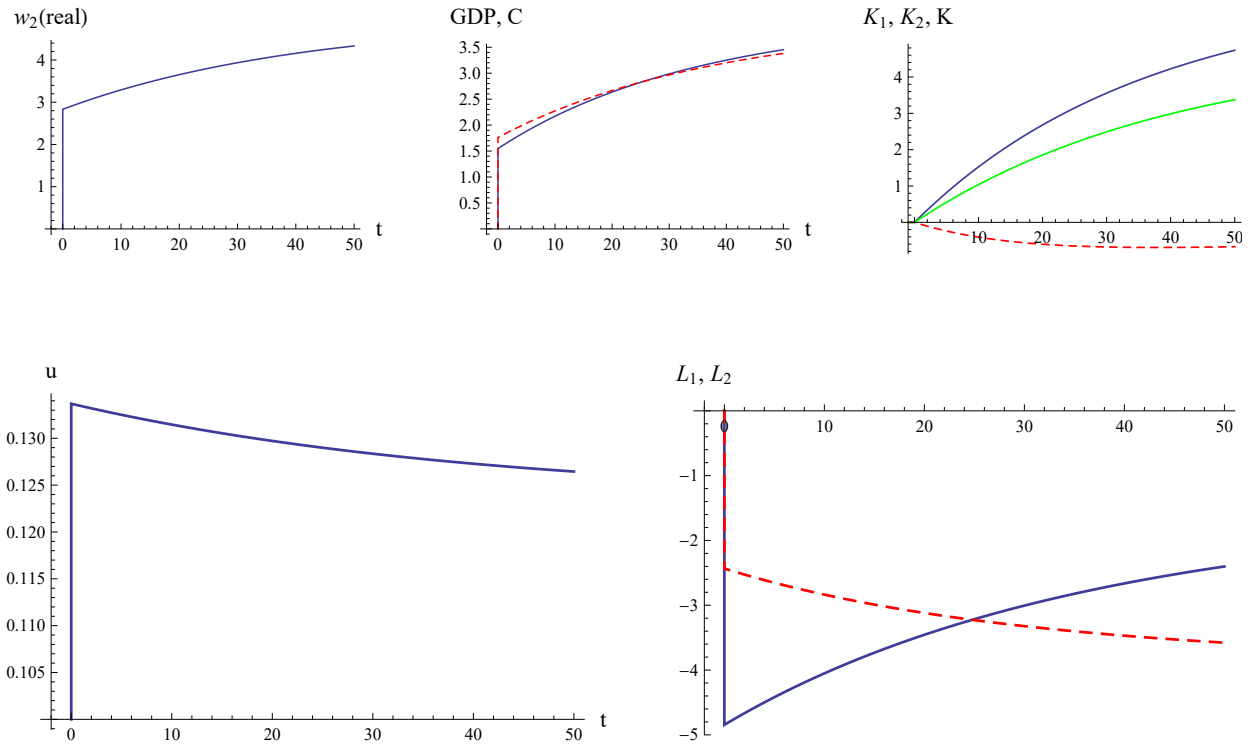


Figure 8: Transition path in the base case. Dashed lines refer to the paths of consumption, the capital stock in the informal sector, and informal sector employment. The green line in the plot for the capital stocks tracks the path of the aggregate capital stock.

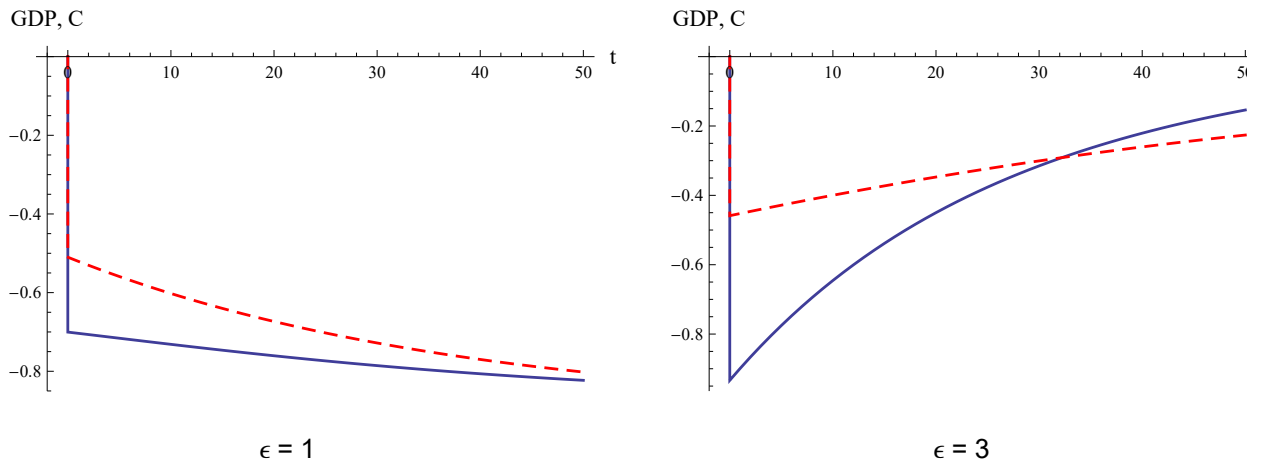


Figure 9: Paths of GDP (solid) and consumption (dashed) in the calibration for LICs ($\alpha = \gamma = .45$).

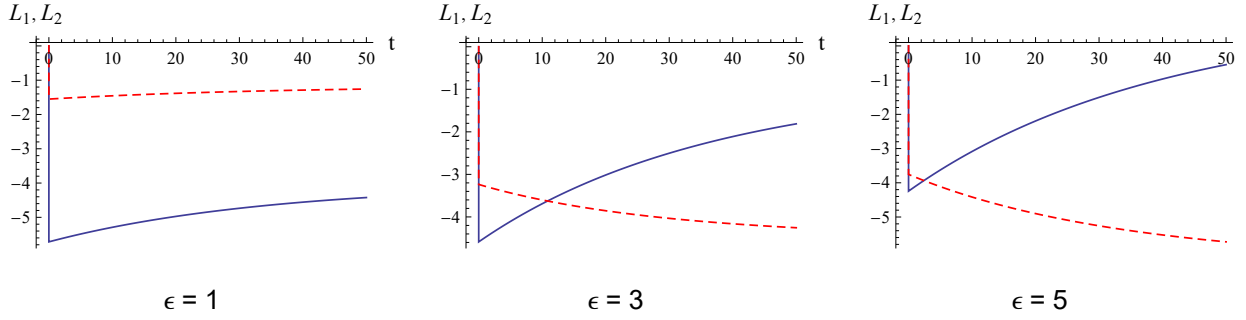


Figure 10a: Paths for formal (solid) and informal sector (dashed) employment when the MW increases in both the public and private sectors, Case 1 with $g_3/g_1 = 1$ and $b_3 = 2$.

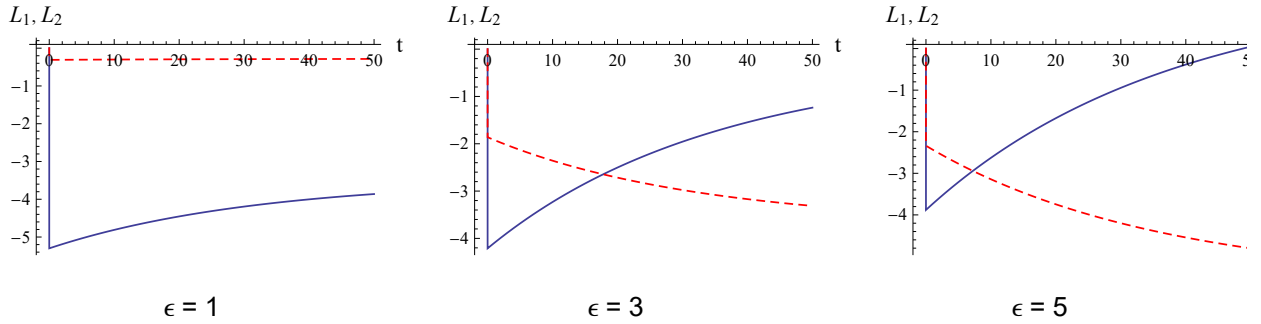


Figure 10b: Paths for formal (solid) and informal sector (dashed) employment when the MW increases in both the public and private sectors, Case 1 with $g_3/g_1 = 1.5$ and $b_3 = 3$.

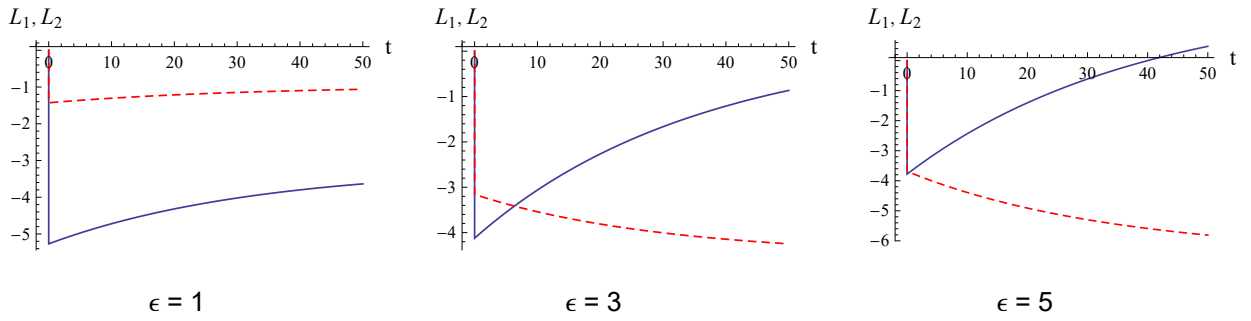


Figure 11a: Paths for formal (solid) and informal sector (dashed) employment when the MW increases in both the public and private sectors, Case 2 with $g_3/g_1 = 1$ and $b_3 = 2$.

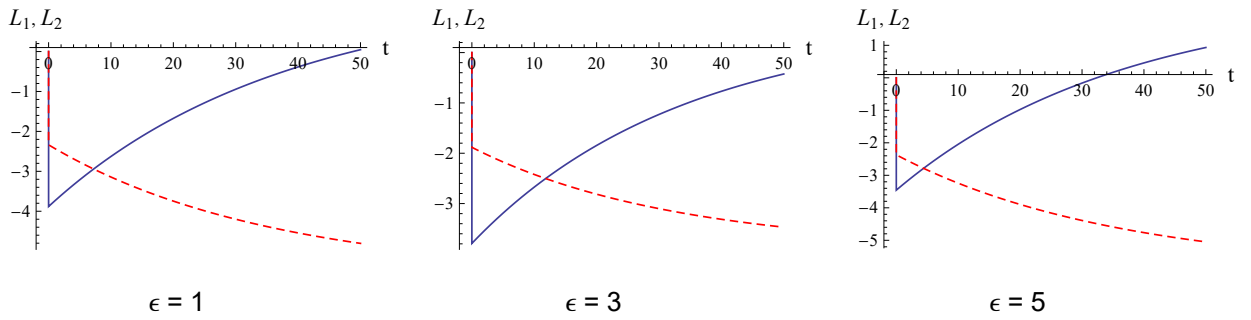


Figure 11b: Paths for formal (solid) and informal sector (dashed) employment when the MW increases in both the public and private sectors, Case 2 with $g_3/g_1 = 1.5$ and $b_3 = 3$.

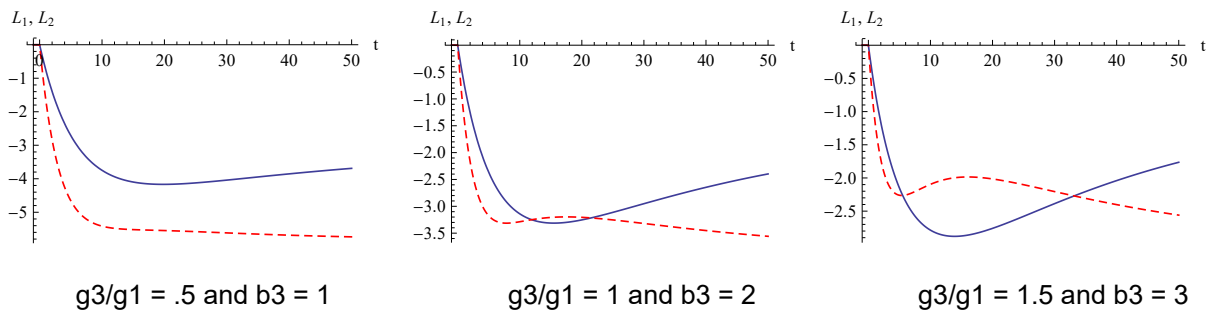


Figure 12a: Paths for formal (solid) and informal sector (dashed) employment when $\epsilon = 3$, the MW increases only in the private sector, and there are adjustment costs to changing employment.

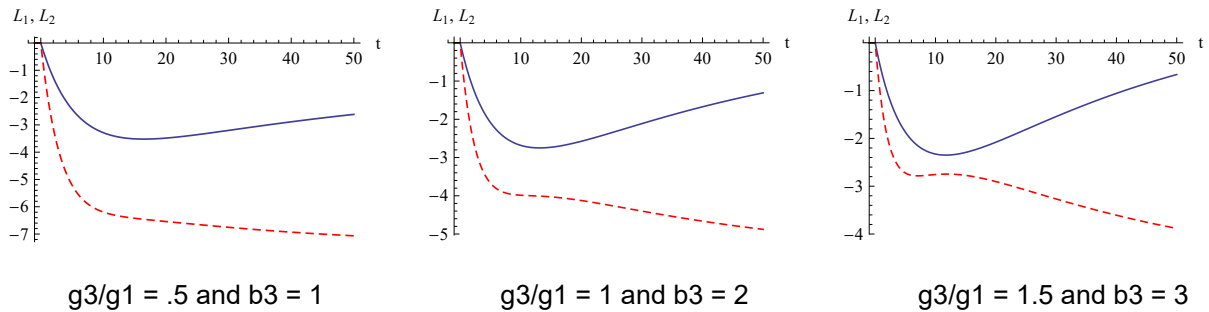


Figure 12b: Paths for formal (solid) and informal sector (dashed) employment when $\epsilon = 5$, the MW increases only in the private sector, and there are adjustment costs to changing employment.

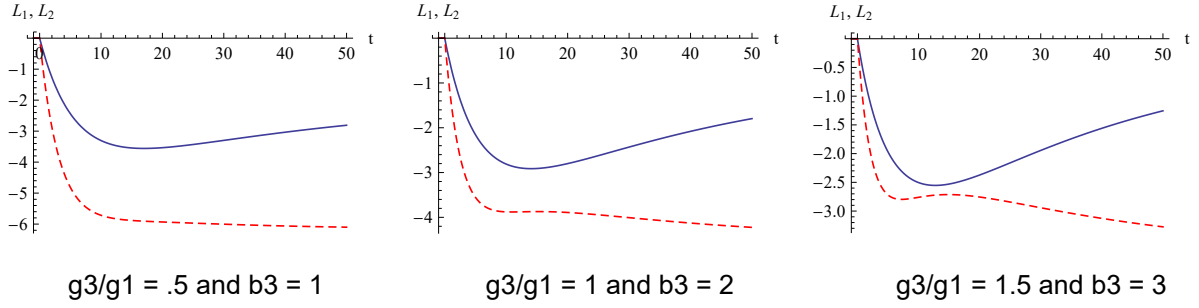


Figure 13a: Paths of formal (solid) and informal sector (dashed) employment when $\epsilon = 3$, the MW increases in both the public and private sectors, and there are adjustment costs to changing employment. The public sector is highly inefficient (Case 1).

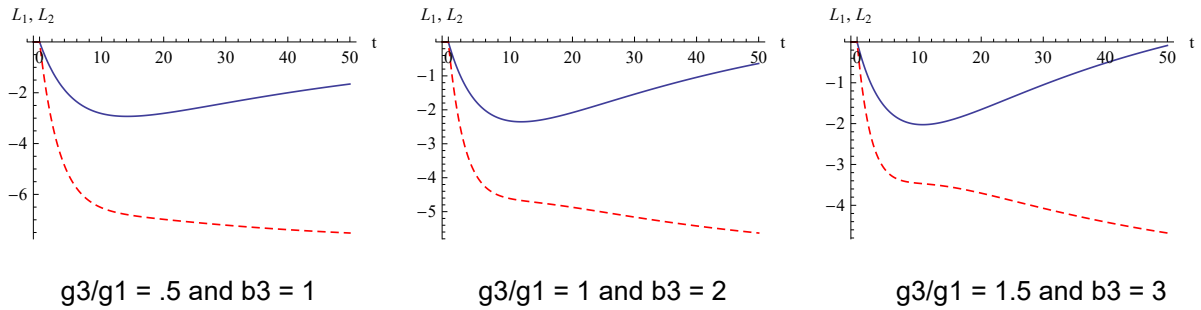


Figure 13b: Paths of formal (solid) and informal sector (dashed) employment when $\epsilon = 5$, the MW increases in both the public and private sectors, and there are adjustment costs to changing employment. The public sector is highly inefficient (Case 1).