‘COST OF PRODUCTION’ AND THE THEORY OF THE RATE OF PROFIT*

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Adam Smith set economists an examination question: what determines long-run normal prices (he called them ‘natural prices’) and the associated rate of profit. In neoclassical theory, equilibrium prices of produced commodities are equal to their costs of production. But to avoid being trapped in a circular argument this requires that no commodity is used in its own production. The definition of commodities in intertemporal general equilibrium theory ensures that this is the case, since commodities at different times are defined as different commodities and production takes time. A simple intertemporal general equilibrium model is used to demonstrate that the consequent characterisation of equilibrium eliminates the possibility of answering Smith’s question – it is not possible to determine the rate of profit and associated normal prices. The definition of equilibrium has been changed and a different question answered. Given the well-known failings of attempts to determine the rate of profit by the marginal product of ‘capital’, the lack of a theory of profit in Debreu’s Theory of Value, the locus classicus of neoclassical theory, leads to the conclusion that there is no neoclassical theory of the rate of profit.

JEL Classifications: B12; B13; B16; C62; D24; D33; D46; D50

INTRODUCTION

‘in the case of a basic product the prices of its means of production depend on its own price no less than the latter depends on them. A less one-sided description than cost of production seems therefore required.’ (Sraffa, Production of Commodities by Means of Commodities, p. 9).

‘A good at a certain date and the same good at a different date are different economic objects, …. A commodity is therefore defined by a specification of all its physical characteristics, of its availability date, and of its availability location. As soon as one of these three factors changes, a different commodity results.’ (Debreu, Theory of Value: An axiomatic analysis of economic equilibrium, pp. 29–30).

*None of the results in this paper is particularly novel. However, I hope that by setting out a number of related propositions in simple form I may provide useful insights into a complex debate. I am very grateful to the referees for valuable observations. I regret I have not been able to accommodate all of them.

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In chapter 7 of Book 1 of the *Wealth of Nations* Adam Smith sets the examination question that economists have attempted to answer over the succeeding 240 years: what determines prices in a market economy?

In order to pose the question precisely Smith began by providing a definition of ‘price’. In doing so he rejected simple empiricism, declaring ‘I have no great faith in political arithmetick’ (Smith, 1776, IV.v.69). Instead, he introduced a characterisation of price that embodied the systematic working of a competitive market economy.

The object of the analysis of price determination should be what he called the ‘natural price’ (Smith, 1776, Book 1, chpt. 7). The natural price derives from the persistent tendency of competitive behaviour to establish a single price for each commodity and for the services of (the various varieties of) land and of labour. Competition also tends to establish a uniform rate of profit on the value of capital invested in the production of commodities. Competitive behaviour\(^1\) provides the economy with a systematic impetus by the consequential market adjustment of the composition of output to the composition of demand. This adjustment and the establishment of natural prices are two sides of the same coin. Accordingly, competition ensures that the natural price is the ‘center of gravitation’ for actual prices, or, as Smith labels them ‘market prices’ (Smith, 1776, p. 65). Market prices are what would today be called disequilibrium prices, reflecting the ‘noise’ in markets. They are not associated with a uniform rate of profit, or indeed with uniformity of input prices of, say, land or labour, or even with a single price for each commodity. A little over 100 years later Alfred Marshall re-labelled natural prices as ‘long-run normal prices’ and Smith’s market prices as ‘the day-to-day oscillations’ that follow no consistent pattern\(^2\) (Marshall, 1890, pp. 378-9). The essential concepts remained the same. So did the task: to explain how natural prices are determined\(^3\).

The characteristics of Smith’s definition are:

- The object of analysis – natural prices and the associated rate of profit – is defined *independently* of any theory that may be used to explain the determination of the magnitude of those unknowns. Competitive behaviour ensures that for each commodity there is a single price. It does not determine what that price might be. Competition ensures that the rate of profit is uniform. It does not determine the magnitude of that rate. It is therefore not surprising that there have been different theories of price determination. The theory of the rate of profit and of the associated prices proposed in the first half of the nineteenth

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1. For Smith the concept of a competitive economy is a statement about behaviour. The later neoclassical concept of ‘perfect’ competition is a statement about market structure.

2. Marshall used the term ‘market price’ for a very different concept: the equilibrium price in a market in the extreme short run in which no change in output is possible.

3. There has been an extensive debate over Smith’s concept of ‘gravitation’, see for example the Special Issue on ‘Convergence to Long-Period Positions’ of the journal *Political Economy: Studies in the Surplus Approach* (1990); or more recently Bellino and Serrano (2017). The central question posed by Smith ‘what determines natural/long-run equilibrium prices?’ is considered in this paper. The nature of gravitation is a separate question, not considered here. However, it should be noted that any model necessarily abstracts from forces deemed less relevant or ‘noise’, and hence the issue of stability is important in the evaluation of all models.
century by classical economists such as Ricardo is quite different from the theory of the same phenomena advanced later by neoclassical economists such as Marshall and Walras.4

- The depiction of the natural price as a centre of gravitation and market prices as ‘day-to-day oscillations’ or ‘noise’ derives from the view that the adjustment of output to demand proceeds more rapidly than change in the technical conditions of production. So even if natural prices – Marshall’s long-run prices – are changing through time (the centre of gravitation is moving), nonetheless the movement can be usefully analysed by means of comparisons of natural prices and the associated rate of profit at different times - in neoclassical terms, by comparisons of long-run equilibria.5

Within the corpus of neoclassical theory the prices of producible commodities are, in equilibrium, equal to the sum of the factor rentals expended in their production – their costs of production.6 This requires that no commodity should enter its own production. If it did, as in Sraffa’s argument cited above, the ‘cost of production’ could not be determined independently of the price of that commodity itself. Put simply, the supply curve could not be drawn independently of the demand curve. Debreu’s definition of a commodity, also cited above, ensures that no commodity can enter its own production, since physically identical commodities available at different times are different commodities. However, it will be demonstrated in what follows that a consequence of the definitions adopted in Debreu’s Theory of Value (the definitive statement of the neoclassical theory of value) is that the concept of long-run equilibrium as a centre of competitive gravitation is abandoned and the question posed by Smith, ‘What determines natural prices and the general rate of profit?’ is left unanswered.7

4 In neoclassical theory the fact that long-run normal equilibrium is defined by the adjustment of output to demand, whilst natural prices are to be determined by the balance of supply and demand functions, can be a source of confusion: a confusion between definition and determination. Supply and demand functions appear to embody a market process – moving along the curves toward equilibrium. However, this appearance is false. The functions are loci of potential equilibria, not representations of the disequilibrium upon which competitive market forces are brought to bear. Moreover, the description of long-run normal equilibrium as a condition in which the composition of productive capacity is adjusted to demand, there are single prices for each good and service, and there is a uniform rate of profit on the value of capital invested, does not require any knowledge of demand and supply functions, or, indeed, any of the apparatus of neoclassical theory (or, indeed, of any other theory).

5 Marshall (1890, p. 315) distinguished the ‘normal relations of wages, profits, prices etc. for rather long periods’ from ‘the very gradual or Secular movements of normal price caused by the gradual growth of knowledge, of population and of capital’.

6 The price of a produced commodity may, of course, be less than its cost of production (the equality replaced by an inequality), in which case the level of output of that commodity is zero. This refinement, necessary for the proof of the existence of general equilibrium, is ignored in this note, since it has no bearing on the generality of the argument. Similarly ignored (other than in footnote 13) is the condition that excess demand for an element of the endowment may be negative even at zero price (that element of the endowment is a free good).

7 There have been numerous attempts since 1870s to develop a neoclassical theory of the rate of profit. Many of these have contained logical flaws associated with definition of the ‘quantity of capital’ with which the economy is endowed (see Garegnani, 1987). A different approach was taken by Walras (1874–7).
NON-REPRODUCIBLE MEANS OF PRODUCTION

Consider an economy in which production is solely by means of non-reproducible means of production (apples, $a$, and bread, $b$, are produced by land, $\lambda$, and labour, $l$). The technology is quasi-concave, with constant returns to scale\(^8\).

The prices of the produced commodities will, in a perfectly competitive market, be equal to their cost of production\(^9\). Suppose that the competitive technique is represented by a matrix of unit input coefficients, $N$, then

$$x = N'c \quad \text{and} \quad Np_j = p_i \quad i = a, b; \quad j = \lambda, l$$

where $x$ is the vector of endowments, $N'$ is the transpose of $N$, $c$ the vector of outputs (consumption), and $p_j$ with relevant subscript, represents the vectors of input prices $j$ and output prices $i$. The markets for land and labour are cleared at prices $p_\lambda$ and $p_l$ respectively. The prices of apples and bread, $p_a$ and $p_b$, are equal to their costs of production; i.e. the sum of the factor rentals expended in their production.

PRODUCTION OVER TIME

Consider next an economy in which a set of goods are produced over a sequence of discrete time periods, $0, 1, \ldots, T$, by means of a set of physically identical goods\(^{10}\). Following Debreu, physically identical goods available at different times are defined as different commodities\(^{11}\). The endowment of goods at time 0 is $x_0$ and the vector of prices for the endowment is $p_0$. The output of goods at time $t$ is $y_t$, and $p_t$ the prices of goods at time 0 for delivery at time $t$.

The competitive technique for the production of goods at time $t$ from inputs at time $t-1$ is $A^t$. The unit input coefficients $a_{ij}$ refer to the input of good $i$ into the production of good $j$. The time dimension is recognised in the superscript $t$ in $A^t$. It is assumed that all $A$ are square, non-negative and connected (all commodities are necessary as inputs for the production of all outputs) and display simple production expressed the endowment of capital as list of quantities (a vector). Nonetheless, Walras’s theory of the determination of what he called the ‘rate of net income’ is also logically flawed (Garegnani, 1960 second part, chpts. 2 and 3; Eatwell, 1987b).

\(^8\) The assumption of constant returns to scale is that which is compatible with the assumption of perfect competition. (Eatwell, 1987a; Sraffa, 1926).

\(^9\) Constant returns to scale are defined by a production function that is homogeneous of degree 1, and hence the cost minimising condition that rent and the wage are equal to the marginal value products of land and labour is, by Euler’s theorem on homogeneous functions, equivalent to the condition that price is equal to cost of production.

\(^{10}\) Markets exist for all commodities at all times, and all transactions at all times will be honoured (there is certainty) or the probability that they will be honoured is known; Debreu (1959) chapter 7. The analysis presented in this paper is a (very) simplified exposition of that developed by Debreu.

\(^{11}\) Debreu attributes this definition of commodities, and hence of the nature of the equilibrium he determines, to the works of Lindahl (1929) and Hicks (1939). But the essential idea of prices determined by the relationship of production and demand over time derives from Fisher (1907; 1930), See also the papers by Arrow and Debreu (1954) and McKenzie (1959).
(all inputs are fully used up in one production cycle from $t-1$ to $t$). Hence the relationship between the production of goods at time 1 and the inputs into production is

$$x_0 = A^\top_1 y_1$$

where $A^\top_1$ is the transpose of $A^1$.

The *price* paid for the delivery of a good at time 1 is equal to its cost of production:

$$A^1 p_0 = p_1$$

where $p_0$ are the market clearing prices for the endowment $x_0$. The cost of production does not include any interest charge\(^{12}\), even though goods are produced over time, since the $p_1$ are prices paid at time 0 for delivery at time 1, and hence may be interpreted as prices discounted *at an as yet unknown interest rate*.

Of the goods produced at time 1, some may be consumed at time 1, $c_1$, and others used as means of production at time 1, $m_1$, for goods at time 2:

$$x_0 = A^\top_1 [c_1 + m_1]$$

Since $m_1 = A^2 y_2$, for the production of goods at time 2

$$x_0 = A^\top_1 c_1 + A^\top_1 A^2 y_2$$

and prices at time 0 for goods to be delivered at time 2 are equal to their cost of production:

$$A^2 A^1 p_0 = p_2$$

For time $T$, when $y_T = c_T$

$$x_0 = A^\top_1 c_1 + A^\top_1 A^2 c_2 + \ldots + A^\top_1 A^{T-1} c_T$$

and

$$A^T A^{T-1} \ldots A^1 p_0 = p_T$$

It is clear that the definitions of quantity and price relationships over time are formally identical to the quantity and price relationships in the case of non-reproducible means of production. The demand for the factors that make up the endowment is derived from the demand for consumption goods. Physically identical goods at

\(^{12}\) There is no distinction between the terms ‘rate of interest’ and ‘rate of profit’ in the argument of this paper, which contains no reference to money.
different times have different prices, and indeed they are produced differently. In the simple example above the technique for producing goods at time 1, \( A_1' \), is different from the technique for producing physically identical goods at time 2, \( A_1'A_2' \). No good is ‘reproduced’. The prices of all produced commodities are equal to their costs of production, and for no commodity do ‘the prices of its means of production depend on its own price no less than the latter depends on them’. The concept of cost of production is well defined.

The interpretation of \( p_t \) as the prices paid at time 0 arises from the condition that price is equal to cost of production without any charge for the ‘cost of time’, i.e. interest on the value of the elements of the endowment invested in production. The prices of the endowment are determined by the condition of market clearing, by the balance of supply and demand. In effect, the model of production over time takes the land and labour, apples and bread model, and re-labels land and labour as ‘goods at time 0’ and apples and bread as ‘goods at time 1’.

If the contemporaneous price paid at time \( t \) for commodity \( i \) delivered at time \( t \) is \( P_{it} \), the price \( p_{it} \) may be interpreted as \( P_{it} \) discounted to time 0.

By Walras Law there is a degree freedom in the determination of prices. This is used to define the numéraire, for example \( \Sigma_i \Sigma_t p_{it} = 1 \).

Suppose that over and above this standard fixing of a numéraire, an additional, intertemporal, normalisation is imposed, say \( P_{it} = p_{i0} \) for a particular \( i \), all \( t \). Then \( P_{it} \) is an ‘intertemporal numéraire’. Taking, for example, the unitary production period 0 to 1, \( p_{i0} = P_{i1} \). Hence

\[
p_{i0} = P_{i1} = p_{i1}(1 + r_{i1}) \quad \text{and} \quad r_{i1} = p_{i0}/p_{i1} - 1
\]

where \( r_{i1} \) is the discount rate (own-rate of return) specific to commodity \( i \) over the period 0 to 1. Discount rates may be calculated in this manner between all pairs of commodities \( i \) at different time periods (Debreu, 1959, pp. 33–35).

The contemporaneous prices of all goods other than that chosen as intertemporal numéraire may be determined by their relationship to that specific intertemporal normalisation:

\[
P_{ij} = p_{ij}(1 + r_{i1})
\]

Note that there is no reason to expect the discount rate between any pair of commodities to be the same, indeed in general \( r_{it} \neq r_{jt} \). The discount rate will depend on the choice of the (arbitrary) intertemporal normalisation. Indeed, in a model which includes many non-reproducible means of production (say labour performed at different times) as well as physically reproducible commodities, the discount rate could be defined as the ratio of prices of a non-reproducible good at different times.
REPRODUCTION

Suppose that the proposition that commodities at different times are different commodities is abandoned, and instead physically identical commodities are defined as the same commodity – i.e. commodities are reproduced. Then the contemporaneous prices of physically identical commodities must be the same, so that for all commodities \( p_{j0} = P_{j1} \). Given that the contemporaneous price of good \( j \)

\[
P_{j1} = p_{j1} (1 + r_{j1}), \text{ and since } p_{j0} = P_{j1} \text{ then } p_{j0}/p_{j1} = (1 + r_{j1}) = (1 + r_{j1}).
\]

That is, the discount rate for the period 0 to 1 is now independent of whichever commodity is chosen for intertemporal normalisation. The discount rate is the general rate of profit, \( r \), over the production period 0 to 1. Accordingly, given that \( p_0 = P_1 = p_1 (1 + r) \), the price relations

\[
A^t p_0 = p_1
\]

can, by multiplying each side by \((1 + r)\), be re-written as

\[
A^t p_0 (1 + r) = p_0
\]

Since \( A^t \) is square and non-negative, then, by the Perron-Frobenius theorems, there is a unique solution for economically meaningful \( p_0 \) and \( r \) (i.e. for which \( p_0 \) is non-negative) and, of course, for \( p_1 \) (see Debreu and Herstein, 1953). There is no reason to suppose that the \( p_0 \) so determined will be equal to the prices that, determined by the balance of supply and demand, would clear the markets for the vector of endowments \( x_0 \). In other words, the only possible solution for prices in this case is incompatible with an arbitrary endowment. Moreover, for every commodity ‘the prices of its means of production depend on its own price no less than the latter depends on them’. The term ‘cost of production’ has no meaning independent of the solution for \( p_0 \).

This is the ‘Walrasian’ version of the impossibility of determining the general rate of profit by supply and demand, or, more generally, by competitive utility maximisation subject to constraints. The phenomenon of re-switching is another

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13 Indeed whilst \( p_0 \) is all positive, it may be that the markets for one or more items of the endowment clear only at zero price (they are free goods), or it may be the case that at \( p_0 \) the demand for a good exceeds the arbitrary quantity in the endowment. The point is that \( p_0 \), the only vector of prices compatible with the solution for \( r \), is not determined by the condition of market clearing but the condition that there is a definable rate of profit.

14 It has been argued that, under a specific set of conditions, notably linear Engel curves and stationary demand (relative utilities are the same in each period) the intertemporal sequence will converge in the manner of turnpike theorems to display normal prices and a determinate uniform rate of profit (Schefold, 1997, chpt. 18). Leaving aside consideration of the special conditions, this seems to be an unfortunate interpretation of Debreu’s characterisation of equilibrium, presenting it as an amalgam of the Marshallian short-run and the long-run. But in Debreu’s formulation the equilibrium set of prices, \( p_x \) is the equilibrium. There is no sense in which, say, \( p_{t0} \) prices at time \( t \) where \( t \) is ‘large’, have any different significance than \( p_{t1} \), prices
manifestation of this same impossibility in models in which techniques of production are characterised in terms of quantities of non-reproducible means of production and the value of reproducible means of production, i.e. in terms of ‘capital-labour ratios’ (Garegnani, 1970).

Neoclassical theory ‘works’ as a relationship between the composition of demand and the composition of the fixed endowment. In models of production the two are linked by the condition that cost of production equals price. Other than in a one commodity world, this requires that commodities are differentiated by their location in time. Otherwise, as noted already, it is not possible to define the cost of production of reproducible inputs independently of their own price.

The rate of profit is a pure number, the ratio of the value of the share of net product accruing to profits to the value of the produced means of production (‘capital goods’) employed. If numerator and denominator are evaluated in the same relative prices, the ratio is independent of the choice of numéraire. If numerator and denominator are evaluated using different relative prices, then the value of the ratio will depend on the choice of the ‘extra’ numéraire, as is the case in Debreu’s construction.

FUTURES MARKETS

Debreu (1959, p. 32–3) argues that the concept of price ‘as the amount paid now’ for a good delivered at time \( t \), is ‘very closely related to ‘price’ as understood on a futures market’. Whilst, noting that in a futures market ‘this price shall be paid at the delivery date … This difference from the price concept which will be used here is inessential’.

The relationship between current (spot) and forward price (price as understood on a futures market) was analysed by Sraffa (1932) in his critique of Hayek (1931). What Sraffa referred to as the ‘own-rate of interest’ or the ‘natural rate’ was defined as the spot price of a commodity divided by the future price of the commodity discounted at the going general rate of interest, \( r \), less one:

\[
r_{j1} = \frac{P_{j0}}{P_{j1}/(1 + r)} - 1
\]

Note that the discount rate is presumed equal to the general rate of interest/profit, as is made clear in the discussion of divergence between ‘natural’ rates:

In equilibrium the spot and forward price coincide, …. and all the ‘natural’ or commodity rates are equal to one another, and to the money rate. But if, for any reason, the supply and the demand for a commodity are not in equilibrium, …. its spot and forward prices diverge, and the ‘natural’ rate of interest on the commodity diverges from the ‘natural’ rates on other commodities. (Sraffa, 1932, p. 50).

at time 1. They are all part of the same equilibrium, an equilibrium that is specified in quite different terms from the natural price/market price characterisation proposed by Smith and adopted subsequently by Marshall.

15 The ‘price paid at the delivery date’ is the same concept referred to above as the contemporaneous price.
The difference between the forward price on a futures market and the concept as defined by Debreu is not ‘inessential’. It is the difference between the identification of equilibrium with, on the one hand, Smith’s concept of natural price and Marshall’s equivalent long-period equilibrium, both characterised by a uniform rate of profit in the production of reproducible commodities, and on the other hand, with a sequence of time-specific prices for dated commodities. The definition of equilibrium in each case is quite different (Garegnani, 1976).

THE OBJECT OF ANALYSIS

Debreu has not answered the question posed by Smith – what determines natural price and the associated general rate of profit. He has answered a quite different question associated with a different definition of equilibrium. Does it matter that the definition of equilibrium in each case is different?

Debreu (following Lindahl and Hicks) defines the object of analysis as the prices of dated commodities that have the characteristic of balancing supply and demand for those commodities in competitive markets – circumstances described as ‘an equilibrium’. It is notable that these concepts cannot be defined other than in terms of the theory that will be used to determine the unknowns. As Debreu (1959, p. 35) puts it: ‘…. it remains to sum up the formulation of all the above concepts in the language of the theory …. All [characterisation of commodities and prices] that precedes this statement is irrelevant for the logical development of the theory. Its aim is to provide a possible interpretation of the latter’.

The characteristics of this concept are therefore:

- The object of analysis is not defined independently of the theory, indeed it is an interpretation of the theory. In Debreu’s analysis perfectly competitive markets result in a single price for each commodity, and prices are market-clearing. But market-clearing is defined in the context of a fixed endowment, i.e. part of the specification of the theory. The term ‘equilibrium’ is a synonym for ‘the solution of the model’.
- No consideration is given to the question of whether the equilibrium so defined is a centre of gravitation. Given that the endowment of producible means of production is arbitrary the equilibrium will typically be sub-optimal in relation to the composition of demand and hence ‘unstable’\(^{16}\). It cannot be argued that there will be a competitive tendency toward the equilibrium.

\(^{16}\) Suppose that \(x_0\) is drawn from a set of possible endowments \(K = R(x_0)\) where \(K = x_0'p_0\). There will be a composition of endowment \(x_0^*\) that is utility maximising. Given that an arbitrary endowment will only conform to \(x_0^*\) by a fluke, any chance deviation (‘noise’) will tend away from the equilibrium defined with respect to \(x_0\) toward that defined by \(x_0^*\) (Eatwell and Milgate, 1999). The term ‘unstable’ is in quotation marks, since any formal analysis of instability involves defining behaviour at dis-equilibrium prices, see the survey of the literature by Franklin Fisher (1983, chpt. 2). As is now well known, the Mantel-Sonnenschein-Debreu theorem demonstrates that nothing can be said about the stability of general equilibrium (see Rizvi, 2006).
SUMMING-UP

The neo-classical theory of value is incapable of answering the challenge posed by Smith: the determination of natural prices and the associated rate of profit. The fundamental difficulty is that the natural (long-run normal) prices of reproducible means of production must satisfy two conditions at the same time: they must clear the markets for the endowment of reproducible goods, and must be equal to the (unique set of) prices associated with the reproduction of those goods.

In the determination of normal prices in an economy with reproducible means of production, the term ‘cost of production’ has no meaning. The concept of cost of production is valid in an economy in which goods are defined by their physical essence and their location in time, when ‘discounted’ prices are determined without reference to a rate of interest. But the interpretation of such prices as ‘embodying’ a general rate of interest lacks any coherent foundation, as was made clear by Malinvaud (1965, p. 233):

…the relationships studied here are meaningful only under precise normalisation rules for undiscounted prices. Only discounted prices are determined by competitive equilibrium, or by the price system associated with an optimum. In the absence of a normalisation rule the interest rates are not defined and can assume any value greater than \(-1\).

There is no neo-classical theory of the rate of profit.

REFERENCES


The conclusion of the ‘re-switching debate’ was that attempts to determine the rate of profit by the marginal product of capital are logically flawed (Symposium, 1966). However, it has been argued that if the endowment of capital is expressed as a vector of capital goods, each in their own physical units, rather than as a ‘quantity of capital’ then a logically sound theory will result. See for example, Samuelson, 1962, p. 193: ‘Repeatedly in writings and lectures I have insisted that capital theory can be rigorously developed without using any Clark-like concept of aggregate ‘capital’, instead relying upon a complete analysis of a great variety of heterogeneous physical capital goods and processes through time’. The argument above suggests that such reliance is misconceived.


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