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CAPITAL EXPANSION, RATE OF GROWTH, AND EMPLOYMENT[1]

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I. INTRODUCTION

This paper deals with a problem that is both old and new—the relation between capital accumulation and employment. In economic literature it has been discussed a number of times, the most notable contribution belonging to Marx. More recently, it was brought forth by Keynes and his followers.

A thorough analysis of economic aspects of capital accumulation is a tremendous job. The only way in which the problem can be examined at all in a short paper like this is by isolating it from the general economic structure and introducing a number of simplifying assumptions. Some of them are not entirely necessary and, as the argument progresses, the reader will see how they can be modified or removed.

The following assumptions and definitions should be noted at the outset: (a) there is a constant general price level; (b) no lags are present; (c) savings and investment refer to the income of the same period; (d) both are net, i.e., over and above depreciation; (e) depreciation is measured not in respect to historical costs, but to the cost of replacement of the depreciated asset by another one *of the same productive capacity;*[2] (f) productive capacity of an asset or of the whole economy is a measurable concept.

The last assumption, on which (e) also depends, is not entirely safe. Whether a certain piece of capital equipment or the whole economy is considered, their productive capacities depend not only on physical and technical factors, but on the whole interplay of economic and institutional forces, such as distribution of income, consumers' preferences,

[1] This is a summary of a paper presented before a joint session of the Econometric Society and the American Statistical Association in Cleveland on January 24, 1946. It contains the logical essence of the argument with relatively little economic detail. I hope to develop the latter in a separate paper to be published in one of the other economic journals.

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[2] If the original machine worth $1,000 and producing 100 units is replaced by another one worth also $1,000, but producing 120 units, only $833.33 will be regarded as replacement, and the remaining $166.67 as new investment. A similar correction is made when the new machine costs more or less than the original one. The treatment of depreciation, particularly when accompanied by sharp technological and price changes, presents an extremely difficult problem. It is quite possible that our approach, while convenient for present purposes, may give rise to serious difficulties in the future.

wage rates, relative prices, structure of industry, and so on, many of which are in turn affected by the behavior of the variables analyzed here. We shall nevertheless assume all these conditions as given and shall mean by the productive capacity of an economy (or an asset) its total output when all productive factors are fully employed under these conditions.[3]

The economy will be said to be in equilibrium when its productive capacity P equals its national income Y. Our first task is to discover the conditions under which this equilibrium can be maintained, or more precisely, the rate of growth at which the economy must expand in order to remain in a continuous state of full employment.

II. THE PROBLEM OF GROWTH

The idea that the preservation of full employment in a capitalist economy requires a growing income goes back (in one form or another) at least to Marx. It has been fully recognized in numerous studies (recently made in Washington and elsewhere) of the magnitude of gross national product needed to maintain full employment. But though the various authors come to different numerical results, they all approach their problem from the point of view of the size of the labor force. The labor force (man-hours worked) and its productivity are supposed to increase according to one formula or another, and if full employment is to be maintained, national income must grow at the combined rate. For practical relatively short-run purposes this is a good method, but its analytical merits are not high, because it presents a theoretically incomplete system: since an increase in labor force or in its productivity only raises productive capacity and does not by itself generate income (similar to that produced by investment), the demand side of the equation is missing. Nor is the difficulty disposed of by Mr. Kalecki's method according to which capital should increase proportionally to the increase in labor force and its productivity.[4] As Mrs. Robinson well remarked, "The rate of increase in productivity of labor is not something given by Nature."[5] Labor productivity is not a function of technological progress in the abstract, but technological progress embodied in capital goods, and the amount of capital goods in

[3] It should undoubtedly be possible to work out a more precise definition of productive capacity, but I prefer to leave the matter open, because a more precise definition is not entirely necessary in this paper and can be worked out as and when needed.

[4] See his essay, "Three Ways to Full Employment" in *The Economics of Full Employment*, Oxford, 1944, p. 47, and also his "Full Employment by Stimulating Private Investment?" in *Oxford Economic Papers*, March, 1945, pp. 83–92.

[5] See her review of *The Economics of Full Employment*, *Economic Journal*, Vol. 55, April, 1945, p. 79.

general. Even without technological progress, capital accumulation increases labor productivity, at least to a certain point, both because more capital is used per workman in each industry and because there is a shift of labor to industries that use more capital and can afford to pay a higher wage. So if labor productivity is affected by capital accumulation, the formula that the latter should proceed at the same rate as the former (and as the increase in labor force) is not as helpful as it appears.

The standard Keynesian system does not provide us with any tools for deriving the equilibrium rate of growth. The problem of growth is entirely absent from it because of the explicit assumption that employment is a function of national income. This assumption can be justified only over short periods of time; it will result in serious errors over a period of a few years. Clearly, a full-employment level of income of five years ago would create considerable unemployment today. *We shall assume instead that employment is a function of the ratio of national income to productive capacity.* While this approach seems to me to be superior to that of Keynes, it should be looked upon as a second approximation rather than a final solution: it does not allow us to separate unused capacity into idle machines and idle men; depending upon various circumstances, the same ratio of income to capacity may yield different fractions of labor force employed.

Because investment in the Keynesian system is merely an instrument for generating income, the system does not take into account the extremely essential, elementary, and well-known fact that investment also increases productive capacity.[6] This *dual* character of the investment process makes the approach to the equilibrium rate of growth from the investment (capital) point of view more promising: if investment both increases productive capacity and generates income, it provides us with *both* sides of the equation the solution of which may yield the required rate of growth.

Let investment proceed at the rate I per year, and let the ratio of the potential net value added (after depreciation), i.e., of the productive capacity of the new projects to capital invested in them, i.e., to I, be indicated by s.[7] The net annual potential output of these projects will then be equal to Is. But the productive capacity of the whole econ-

[6] Whether every dollar invested increases productive capacity is essentially a matter of definition. It can safely be said that investment taken as a whole certainly does. To make this statement hold in regard to residential housing, imputed rent should be included in the national income. See also note 19.

[7] The use of the word "project" does not imply that investment is done by the government, or that it is always made in new undertakings. I am using "project" (in the absence of a better term) because investment can mean the act of investing and the result of the act.

omy may increase by a smaller amount, because the operation of these new projects may involve a transfer of labor (and other factors) from other plants, whose productive capacity is therefore reduced.[8] We shall define σ, the *potential social average investment productivity* as

$$\sigma = \frac{\frac{dP}{dt}}{I}. \tag{1}$$

The following characteristics of σ should be noted:

1. Its use does not imply that other factors of production and technology remain constant. On the contrary, its magnitude depends to a very great extent on technological progress. It would be more correct to say that σ refers to an increase in capacity which accompanies rather than one which is caused by investment.

2. σ refers to the increase in *potential* capacity. Whether or not this potential increase results in a larger income depends on the behavior of money expenditures.

3. σ is concerned with the increase in productive capacity of the whole society, and not with the rate of return derived or expected from investment. Therefore σ is not affected directly by changes in distribution of income.

4. s is the maximum that σ can attain. The difference between them will depend on the magnitude of the rate of investment on the one hand, and the growth of other factors, such as labor, natural resources, and technological progress on the other. A misdirection of investment will also produce a difference between s and σ.

We shall make the heroic assumption that s and σ are constant.

From (1) it follows that

$$\frac{dP}{dt} = I\sigma. \tag{2}$$

It is important to note that, with a given σ, dP/dt is a function of I, and not of dI/dt. Whether dI/dt is positive or negative, dP/dt is always positive so long as σ and I are positive.

Expression (2) showing the increase in productive capacity is essentially the supply side of our system. On the demand side we have the multiplier theory, too familiar to need any comment, except for an emphasis on the obvious but often forgotten fact that with any given marginal propensity to save, *dY/dt is a function not of I, but of dI/dt.* Indicating the marginal propensity to save by α, and assuming it to be constant,[9] we have the simple relationship that

[8] I am disregarding the external economies and diseconomies of the older plants due to the operation of the new projects.

[9] Over the period 1879–1941 the average propensity to save (ratio of net capital

$$\frac{dY}{dt} = \frac{dI}{dt}\frac{1}{\alpha}. \tag{3}$$

Let the economy be in an equilibrium position so that[10]

$$P_0 = Y_0. \tag{4}$$

To retain the equilibrium position, we must have

$$\frac{dP}{dt} = \frac{dY}{dt}. \tag{5}$$

Substituting (2) and (3) into (5) we obtain our fundamental equation

$$I\sigma = \frac{dI}{dt}\frac{1}{\alpha}, \tag{6}$$

the solution of which gives

$$I = I_0 e^{\alpha\sigma t}. \tag{7}$$

$\alpha\sigma$ is the equilibrium rate of growth. So long as it remains constant, *the maintenance of full employment requires investment to grow at a constant compound-interest rate.*

If, as a crude estimate, α is taken at 12 per cent and σ at some 30 per cent, the equilibrium rate of growth will be some 3.6 per cent per year.[10a]

The reader will now see that the assumption of constant α and σ is not entirely necessary, and that the whole problem can be worked out with variable α and σ.

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formation to national income) was fairly constant and approximately equal to some 12 per cent. See Simon Kuznets, *National Product Since 1869*, National Bureau of Economic Research (mimeographed, 1945) p. II–89 and the *Survey of Current Business*, Vol. 22, May, 1942, and Vol. 24, April, 1944. In a problem of cyclical character, an assumption of a constant propensity to save would be very bad. Since we are interested here in a secular problem of continuous full employment, this assumption is not too dangerous.

[10] The problem can be also worked out for the case when $P_0 > Y_0$.

[10a] After this paper was sent to the printer, I found a very interesting article → E. H. Stern, "Capital Requirements in Progressive Economies," *Economica*, Vol. 12, August, 1945, pp. 163–171, in which the relation between capital and output in the U. S. during 1879–1929 is expressed (in billions of dollars) as *capital* $= 3.274$ *income* $- 3.55$. My estimates gave roughly similar results. This would place s around 30 per cent, though this figure should be raised to account for the underutilization of capital during a part of that period. It is also not clear how the junking process (see p. 144) was reflected in these figures.

The average rate of growth of real national income over the period 1879–1941 was some 3.3 per cent. See Table V, p. 818, and Appendix B, pp. 826–827, in my paper, "The 'Burden' of the Debt and the National Income," *American Economic Review*, Vol. 34, December, 1944.

III. THE EFFECTS OF GROWTH

Our next problem is to explore what happens when investment does grow at some constant percentage rate r, which, however, is not necessarily equal to the equilibrium rate $\alpha\sigma$. It will be necessary to introduce two additional concepts: average propensity to save I/Y and the average ratio of productive capacity to capital P/K. To simplify the problem, we shall assume that

1. $I/Y=\alpha$, so that average propensity to save is equal to marginal.
2. $P/K=s$, i.e., the ratio of productive capacity to capital for the whole economy is equal to that of the new investment projects.

We shall consider first the special simple case $\sigma=s$, and then the more general case when $\sigma<s$.[11]

Case 1: $\sigma=s$. Since $I=I_0e^{rt}$, capital, being the sum of all net investments, equals

$$K=K_0+I_0\int_0^t e^{rt}dt=K_0+\frac{I_0}{r}(e^{rt}-1). \tag{8}$$

As t becomes large, K will approach the expression

$$\frac{I_0}{r}e^{rt}, \tag{9}$$

so that capital will also grow at a rate approaching r.

As $Y=(1/\alpha)I_0e^{rt}$, the ratio of income to capital is

$$\frac{Y}{K}=\frac{\frac{1}{\alpha}I_0e^{rt}}{K_0+\frac{I_0}{r}(e^{rt}-1)}, \tag{10}$$

and

$$\lim_{t\to\infty}\frac{Y}{K}=\frac{r}{\alpha}. \tag{11}$$

Thus so long as r and α remain constant (or change in the same proportion) no "deepening" of capital takes place. This, roughly speaking, was the situation in the United States over the last seventy years or so prior to this war.

[11] It is also possible that, owing to capital-saving inventions in existing plants, $\sigma>s$. Formally this case can be excluded by falling back on the definition of depreciation given in note 2. This, however, is not a very happy solution, but the approach used in this paper will hardly offer a better one. I think, however, that α in our society is sufficiently high to make $\sigma>s$ in a continuous state of full employment more an exception than a rule.

Substituting $K=P/s$ into (11) we obtain

$$\lim_{t\to\infty}\frac{Y}{P}=\frac{r}{\alpha s}. \tag{12}$$

Since in the present case $\sigma=s$,

$$\lim_{t\to\infty}\frac{Y}{P}=\frac{r}{\alpha\sigma}. \tag{13}$$

The expression

$$\theta=\frac{r}{\alpha\sigma} \tag{14}$$

may be called the *coefficient* of *utilization*. When the economy grows at the equilibrium rate, so that $r=\alpha\sigma$, $\theta=100$ per cent and productive capacity is fully utilized. But as r falls below $\alpha\sigma$, a fraction of capacity $(1-\theta)$ is gradually left unused.[12] *Thus the failure of the economy to grow at the required rate creates unused capacity and unemployment.*

Case 2: $\sigma<s$. As investment proceeds at the rate I, new projects with a productive capacity of Is are built. Since the productive capacity of the whole economy increases only by $I\sigma$, it follows that somewhere in the economy (not excluding the new projects) productive capacity is reduced by $I(s-\sigma)$. Therefore every year an amount of capital equal to $I(s-\sigma)/s$ becomes useless.

The problem can now be approached from two points of view. The amounts $I(s-\sigma)/s$, can be looked upon as capital losses, which are not taken into account in calculating income and investment.[13] In this case, I still indicates the rate of net investment, and all other symbols retain their old meaning, except that capital has to be redefined as the integral of investment *minus* capital losses: every year chunks of capital (over and above depreciation) are written off and junked. The annual addition to capital will then be

$$\frac{dK}{dt}=I-\frac{I(s-\sigma)}{s}=I\frac{\sigma}{s}, \tag{15}$$

and

$$K=K_0+I_0\frac{\sigma}{s}\int_0^t e^{rt}dt=K_0+I_0\frac{\sigma}{sr}(e^{rt}-1). \tag{16}$$

[12] It should be noted that if r, α, and σ are constant, θ is also a constant. Even though the economy fails to grow at the required rate, the relative disparity between its capacity and income does not become wider, because its capital also grows not at the $\alpha\sigma$ but at the r rate.

[13] These losses are not necessarily losses in the accounting sense. See note 14.

Also,

$$\lim_{t\to\infty}\frac{Y}{K}=\frac{r}{\alpha}\cdot\frac{s}{\sigma}, \tag{17}$$

and

$$\lim_{t\to\infty}\frac{Y}{P}=\frac{r}{\alpha\sigma}, \tag{18}$$

which is exactly the same result we had in (13).

The second approach consists in treating the amounts $I(s-\sigma)/s$ not as capital losses but as a special allowance for obsolescence. Net investment would then have to be defined not as I, but as $I\sigma/s$. Other symbols would have to be redefined accordingly, and the whole problem could then be reworked out in the same way as on pp. 142–143.

In a sense the choice between these two methods is a matter of bookkeeping; depending upon the character of the problem in hand, one or the other can be used, though I suspect that the second method can easily become misleading. The nature of the process will be the same whichever method is used. The fact is that, owing to a difference between s and σ, the construction of new investment projects makes certain assets (not excluding the new projects themselves) useless, because under the new conditions brought about by changes in demand, or a rise in the wage rates, or both, the products of these assets cannot be sold.[14] As stated on p. 140 the difference between s and σ is created either by misdirection of investment or by the lack of balance between the propensity to save on the one hand, and the growth of labor, discovery of natural resources, and technological progress on the other. So long as mistakes are made or this lack of balance exists, the junking process is inevitable.

From a social point of view, the junking process is not necessarily undesirable. In this country, where saving involves little hardship, it may be perfectly justified. But it may present a serious obstacle to the achievement of full employment, because the owners of capital assets headed for the junk pile will try to avoid the losses. So long as they confine themselves to changes in their accounting practices, no special consequences will follow. But it is more likely that they will try to accumulate larger reserves either by reducing their own con-

[14] To be strictly true, the statement in the text would require considerable divisibility of capital assets. In the absence of such divisibility, the expression "junking" should not be taken too literally.

The fact that these assets may still be operated to some extent or that their products are sold at lower prices or that both these conditions exist, does not invalidate our argument, because σ, being expressed in real terms, will be higher than it would be if the assets were left completely unused.

sumption or by charging higher prices (or paying lower wages). As a result, the total propensity to save may rise. This will be exactly the opposite measure from what is needed to avoid the junking process, and will of course lead to greater trouble, though I am not prepared to say to what extent capital owners will succeed in passing on these losses.

In so far as they are able to control new investment, they will try to avoid losses by postponing it. Consequently, the rate of growth may well be depressed below the required $\alpha\sigma$, and unused capacity will develop. Our present model does not allow us to separate unused capacity into idle capital and idle men, though most likely both will be present.[15] Because of humanitarian considerations, we are more concerned with unemployed men. But *unemployed capital is extremely important, because its presence inhibits new investment.*[16] It presents a grave danger to a full-employment equilibrium in a capitalist society.

IV. GUARANTEED GROWTH OF INCOME

In the preceding sections it was shown that a state of full employment can be maintained if investment and income grow at an annual rate $\alpha\sigma$. The question now arises as to what extent the argument can be reversed: suppose income is guaranteed to grow at the $\alpha\sigma$ rate; will that call forth sufficient investment to generate the needed income?

We are concerned here with a situation where spontaneous investment (i.e., investment made in response to changes in technique, shifts in consumers' preferences, discovery of new resources, etc.) is not sufficient, and therefore a certain amount of induced investment (made in response to a rise in income) is also required.[17] To simplify the argument, let us assume that spontaneous investment is absent altogether. It should also be made clear that the problem is treated from a theoretical point of view, without considering the numerous practical questions that the income guarantee would raise.

If an economy starts from an equilibrium position, an expected rise in income of $Y\alpha\sigma$ will require an investment equal to $Y\alpha\sigma/s$. As before, two cases have to be considered.

[15] The presence of unemployed men may be obscured by inefficient utilization of labor, as in agriculture.

[16] It is true that a given capital owner may often have a hard time distinguishing between capital idle because of $\sigma < s$, and capital idle because of $r < \alpha\sigma$. The first kind of idleness, however, is relatively permanent, and cannot be corrected by greater expenditures, while the second is temporary (it is hoped) and is due to poor fiscal and monetary policies.

[17] Cf. Alvin H. Hansen, *Fiscal Policy and Business Cycles*, New York, 1944, Part Three, and particularly p. 297.

1. If σ is equal or reasonably close to s, the resulting amount of investment of $Y\alpha$ will equal the volume of savings that will be made at that level of income, and equilibrium will be maintained.[18] *Thus a mere guarantee of a rise in income* (if taken seriously by the investors) *will actually generate enough investment and income to make the guarantee good without necessarily resorting to a government deficit.*

2. If σ is appreciably below s, investment will probably fall short of savings and equilibrium will be destroyed. The difficulty arises because a full-employment rate of investment in the face of a $\sigma<s$ makes the junking process (discussed on pp. 143–145) inevitable, while a mere guarantee of a rise in income, as a general rule, lacks the instrument to force the capital owners to discard their equipment. They will simply invest $Y\alpha\sigma/s$ instead of $Y\alpha$. Only if in the economy as a whole there is a considerable number of products the demand for which is highly elastic with respect to income, and therefore a good number of others the demand for which is negatively elastic with respect to income, will a larger amount than $Y\alpha\sigma/s$ be invested and a corresponding amount of capital junked. Of course, if the rise in income is accompanied by shifts in consumers' preferences, the appearance of new products, aggressive competition, and other changes, the junking process will be speeded up, but if these changes do take place they may give rise to spontaneous investment of their own and the guaranteed *rise* in income will not be important. Still, the assurance of a high and rising income is undoubtedly one of the best methods for encouraging investment.

As explained before, a substantial difference between s and σ simply indicates that with the available labor force and the current progress of technology, the maintenance of full employment under a given α requires the accumulation of capital at a faster rate than it can be used. As a general rule, this applies equally well to both private and public investment, though there may be special cases when, owing to the development of particular consumers' preferences (e.g., for vacations), or to technological reasons (e.g., need for power), or to institutional conditions (as in urban redevelopment), considerable need for public investment still exists.[19]

[18] There is a slight error in the magnitudes in the text because of the use of discontinuous functions.

[19] As soon as the government enters the picture we find ourselves in a maze of definitional problems. From the point of view of this paper, saving and investment should be understood in reference to the whole economy, including the government, and not to its private sector only. But which government expenditures should be regarded as investment? The difficulty is present in the private sector as well, except that there we can take refuge in formal definitions, which cannot be well applied to government. I leave the question open. Certainly, investment need not be limited to inventories, steel, and concrete.

I am not prepared to say whether we already are or shall soon be faced with a serious difference between s and σ, though I doubt that it was an important problem in the past, except perhaps for the short boom years. My own guess is that we shall be more concerned with the disparity between $\alpha\sigma$ and r, that is with the failure of income to grow at the required rate.

If, however, the difference between σ and s becomes serious and inhibits investment, or if the junking process proceeds at a faster rate than is deemed socially desirable, the society will have at its disposal two methods not mutually exclusive: (1) the reduction of the propensity to save, or (2) the speeding up of technological progress. I hope that the main emphasis will be placed on the latter.

This paper attempted to analyze the relation between investment, rate of growth, and employment. The analysis was carried out on a very abstract and simplified level—a procedure which may be justified at the beginning of an investigation, but which must be corrected later on. In general, there is no such a thing as an absolutely good or bad assumption: what may be safe in one kind of a problem can become fatal in another. Of the several assumptions made here, that regarding depreciation is likely to cause the greatest difficulties, but it is by no means the only one. I hope to develop the whole subject further at a later date.

The central theme of the paper was *the rate of growth*, a concept which has been little used in economic theory, and in which I put much faith as an extremely useful instrument of economic analysis. One does not have to be a Keynesian to believe that employment is somehow dependent on national income, and that national income has something to do with investment. But as soon as investment comes in, growth cannot be left out, because for an individual firm investment may mean *more* capital and *less* labor, but for the economy as a whole (as a general case) investment means *more* capital and *not less* labor. If both are to be profitably employed, a *growth* of income must take place.

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