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The Economic Ideas of Marx's Capital

Steps towards post-Keynesian economics

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The Economic Ideas of Marx's Capital

Nearly two hundred years have passed since the birth of Karl Marx and continuing to this day the influence of his economic views, insights and theories can still be felt. However, since the publication of *Das Kapital*, the scientific community has not been sitting idle – it is time to evaluate Marx as an economist and explore what he can bring to modern economic thinking, particularly post-Keynesian economics.

Starting with Marx's schemes of reproduction, which, it is shown, are the basis of the linear model of production as used since the 1960s by Piero Sraffa, Michio Morishima and others, the book reviews and assesses Marx's major economic theses. These include: the labour theory of value; accumulation and technical change and its impact on labour; the concept of unproductive labour; the tendential falling rate of profits; the evolution and determinants of the share of wages in national income; as well as short-run and long-run economic dynamics. *The Economic Ideas of Marx's Capital* updates the theses of the labour theory of value and the conditions for balanced growth using the recent scholarly literature, and also further develops issues related to Marx's concept of productive labour. Moreover, the book analyses the intellectual relationship of Marx's economic theory with post-Keynesian neo-Marxism, particularly in the writings of Michał Kalecki, Joan Robinson and others. By doing so, the book shows the need and possibilities of integrating major insights of Marxist and post-Keynesian theory.

This volume will be of interest to those who wish to explore Marx's economic theories through a non-ideological approach, as well as students of Marxist economics, post-Keynesian economics and the history of economic thought.

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Foreword

I feel honoured and privileged to write a Foreword to Ludo Cuyvers' remarkable book. Ludo has long been a careful scholar of the classical political economists, Marx, and on mainstream and on heterodox economic writings. He has taken a special interest in the contributions of Joan Robinson and Piero Sraffa.

Opening the Sraffa archives at Trinity College, Cambridge, has allowed scholars to establish the tremendous admiration that Piero Sraffa had for Marx's system which he thought dominated all others. He thus saw his own positive contributions as having sorted out some unfinished or incoherent problems within the whole corpus of Marx's theory of the laws of motion of capitalism, by reviving the surplus approach in a rigorous manner. Ludo conceived the grand project of using Sraffa's approach and methods to work through all the issues with which Marx's analysis was concerned in his lifetime, and further issues in capitalism which have arisen since Marx's death. The result is a monograph not only of great originality and relevance, but also of great pedagogical value for the serious student who works through Ludo's careful and illuminating examples in each of his chapters. Ludo blends this careful analysis with astute marshalling of evidence from Marx's own writings, and of the writings of other leading economists concerned with the same issues.

Ludo Cuyvers' book is a splendid example of how a deep understanding of the history of our subject may be combined with careful analysis to make sense of current happenings. I cannot recommend it too highly.

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Introduction

> Let one hundred schools of thought blossom, but let them state their assumptions clearly.
>
> Joan Robinson

In 2018, 200 years will have passed since Karl Marx was born. His economic views, insights and theories still have influence, but they also have many opponents – often very rabid opponents. Moreover, since the publication of *Das Kapital*, the scientific community has not been sitting idle. It is therefore time to conduct an evaluation of Marx as an economist.

Starting with Marx and Rosa Luxemburg, Michał Kalecki discovered and developed (independently of John Maynard Keynes) the "New Economics". During the 1950s and 1960s, economists such as Joan Robinson, Josef Steindl, Paul Baran and Paul Sweezy made impressive strides in building economic models of capitalist development by integrating key Marxian and Keynesian insights. However, in spite of being heavily indebted to Marx, they were critical of his "law of value". With the publication in 1960 of *Production of Commodities by Means of Commodities*, Piero Sraffa, who from Cambridge in the UK over time had a major influence on many people, added his linear production model to these post-Keynesian neo-Marxist theories. This then set in motion a powerful critique of mainstream neo-classical economics. Looking back over the years, it would seem that post-Keynesian economists have diverged into a number of sub-currents according to their particular source of inspiration: Kaleckian, Robinsonian, Kaldorian, Sraffian, . . . , to which other powerful heterodox insights have also been added. In this book, we wish to present the Marxian views of *Das Kapital* and the early post-Keynesian insights right up to the views of present-day post-Keynesian economists, many of whom have strayed from, or have forgotten, the earlier contributions.

Many Marxists, in turn, being highly critical of "vulgar bourgeois economics", have arrived at theoretical insights that are often lacking in academic rigour and/or have focused, using an apparent "dialectic methodology", on theoretical debates about Marx's theory of value, while attacking all non-Marxist (or each other's) ideas. Too few Marxist economists in the "Western world" have contributed to the academic and scientific development of Marxist economics. We feel that it is high time to give back to the Marxist political economy the basic ideas emanating from the early post-Keynesian neo-Marxists of the 1950s and 1960s. Evidently, times have changed since then and it will not suffice to once again merely review Marxist theory in the light of these ideas. Nevertheless, they can be of great help in reconstructing and rethinking Marxist economic theory as the "scientific socialism" (to use Friedrich Engels' terminology) of today. All of this creates a massive agenda for future research, for which this book hopefully lays a modest foundation.

The Marxist economic model is a coherent set of theses, which in turn are rooted in a strictly defined methodology. Marx sets out to show, starting with the classical economists' labour theory of value, that profits in the capitalist system stem from the exploitation of the working class. He also asserts that the accumulation of these profits leads to the expansion of the capitalist economy but that there are limits to such expansion. Following on from these limits are, among other things, a theory of the economic cycle and a theory of how the exploitation of the working class changes during this cycle. In addition, according to Marx, labour power is systematically replaced by machines, which, for various reasons that we will discuss in due course, leads to a tendency of the rate of profits to fall and economic stagnation to take root in the long run. The methodology that Marx uses is one of a builder of a theoretical economic model that adjusts the analysis step by step as new elements are introduced in the model. His value theory provides him with the so-called "law of value", which indicates how changes (increases) in labour productivity will alter (reduce) the value of the goods produced, such that the continuous introduction of new technology has to be considered as a formidable weapon in competition and is the basis of capitalist dynamics. This then brings us back to the replacement of labour by machines.

In the pages that follow, Marx's model of the capitalist economy is not looked at through non-Marxist glasses. Rather, it is investigated within its own logical framework, based on its axioms, assumptions and implications. Insights and starting points for further research are given at regular intervals from the perspective of present-day economic thinking, particularly post-Keynesian economic theory. This book is targeted at a broad public that is interested in penetrating Marx's economic theories through a "no nonsense" approach, and provides insights into how Marx's legacy is linked to economic ideas of today relating to the workings of the capitalist economy. Some of our readers will be economists or future economists. Most of them will be educated in neo-classical economics. For them, the starting points of Marx's analysis, as well as his methodology, theoretical reasoning and results, will be mostly – if not completely – new. Another section of our audience, we hope, will be non-economists, who want to get to the bottom of Marx's theories based on a "no nonsense" approach and/or want to gain insight into how Marx's economics legacy is (or can be) associated with certain elements of contemporary economic thinking.

Marx's economic model is taken from *Das Kapital*. We consciously opted not to consult the other writings of Marx since we did not want to indulge in text exegesis or delve into the origins of his ideas. The first volume of *Das Kapital* was prepared for publication by Marx himself and the next two volumes were the work of Friedrich Engels, based on the notes that Marx left behind after his death. The unfinished manuscript of Marx's *Grundrisse der Kritik der Politischen Ökonomie* dates back to 1858 (before volume 1 of *Das Kapital*) while *Theorien über den Mehrwert*, which is often called volume 4 of *Das Kapital*, comprises notes compiled by Marx during the period 1861 to 1863 during his reading of the work of other economists. The *Theorien* is a "sequel" to *Kritik der Politischen Ökonomie* which was published in 1859 and for the most part can be found in *Das Kapital*. However, we do not consider it appropriate to tire the reader with discussions on parts of earlier versions of *Das Kapital* or the unfinished writings of Marx, with the obvious exception being Friedrich Engels's posthumous publication of volumes 2 and 3 of *Das Kapital*.[1]

Throughout this book we will use a linear production model of the input-output type, or the so-called Marx-Leontief model, which is based on the pioneering work of Piero Sraffa (1960), András Bródy (1970), Michio Morishima (1973) and others. These pioneers published their work some 40 years ago, and the model is still the best suited for analysing Marx's economic views and acquiring a good understanding of his logic. The approach is today known as "analytical Marxism" (Roemer, 1981), a somewhat misleading term. The major advantage of using a linear production model is that it mathematically "translates" Marx's, as well as our, arguments, thus eliminating ambiguous answers and throwing light on the importance of explicit and implicit assumptions. However, the model also has some disadvantages in that it is ill-suited for an integration and analysis of social-economic interactions and endogenous economic dynamics. For this, we rather consulted the godfathers of neo-Marxist economic theory of the 1940s, 1950s and 1960s, i.e. Michał Kalecki, Josef Steindl, Joan Robinson, Paul Sweezy and Paul Baran. Where appropriate, we also refer to more recent literature, but we have kept these passages to a minimum to prevent the book becoming unnecessarily heavy and tedious. These neo-Marxist authors, without our having to refer to or comment on the more recent developments, offer important insights and advance many nuances, which should enable the reader to better appreciate how to react to Marx's thinking and theories.

It should be clear that we never intended to develop yet another Marxist or neo-Marxist economic theory, not even when looking at the recent economic crisis. When various Marxist and neo-Marxist views are discussed, we simply endeavour to show how Marx's economics is carried over into the later classes of economic ideas. This is more than sufficient for our assessment. Yet we do make an exception with the linear production model as it is the golden thread running through our analysis.

We will in due course deal with a number of basic concepts and, in the interests of a better understanding or where theoretically justified, we will illustrate these concepts with numerical examples.

In Chapter 1 we deal with the economic circular process, using the reproduction schemes of *Das Kapital*, and look at the relationship with present-day concepts.

In this regard, Marx's schemes of reproduction have assumed a unique place in the history of economic thought as an instrument that allows an investigation into the conditions for the normal reproduction of a capitalist economy.

Chapter 2 discusses the labour theory of value, which leads to important conclusions about the origin of capitalist profits and the definition of the average rate of profits in the capitalist economy. The labour theory of value is a cornerstone of Marx's economic theory, not least because it creates an analytical and historical link with his theory of the exploitation of the working class.

Chapter 3 focuses on long-term economic development, the accumulation of capital and economic growth, concepts that are aligned with what these days is called "effective demand" but in Marxist jargon is the "surplus value realisation problem". In Chapter 4, we introduce the often misunderstood concept of unproductive labour and we critically analyse its theoretical implications. This fourth chapter might be "a bridge too far" for some readers due to the technicalities in the discussion, in which case we would advise that they simply skip it.

As Chapter 3 shows, the linear production model also allows the integration of economic reproduction, accumulation and economic growth. The Marxist theory of long-term capitalist economic development strongly stresses the relationship between the accumulation of capital and its limits and the introduction of technical change in the production processes.[2]

The relationship between accumulation of capital and technical change is elaborated on in Chapter 5, where the immediate impact of such technical change on profits is also analysed.

Closely related to the labour-saving character of technical change are its implications for the average rate of profits in the capitalist economy. Marx's theory of the tendency of the rate of profits to fall is assessed in Chapter 6, where we also deal with the impact of a higher degree of mechanisation in the production processes.

In Chapter 7, the forces that influence the rate of surplus value or, stated more simply, the ratio of profits to wages, are investigated. This ratio changes continuously, both in the short and the long term. We address how and why this happens. In the chapter we also deal in detail with the neo-Marxist views on how the share of profits in national income is determined by the price-setting behaviour of oligopolistic enterprises in "monopoly capitalism" (or "late capitalism").

Chapter 8 reviews Marx's theory of the economic cycle. Although this theory cannot be separated from his theory of long-term economic development, it is interesting to compare it with the contemporary theory of the business cycle, which is largely of Keynesian inspiration and descent. We deal similarly with the monetary theory that *Das Kapital* presents. Although considered unfinished by today's standards, Marx's views on money clearly illustrate his open and innovative mind as he endeavours to integrate money and credit into his economic theory.

Chapter 9 looks at the role of the rudimentary underconsumption theory, which is expressed in *Das Kapital*, and how this theory has been further developed by the neo-Marxist economists. In the chapter we also encounter the theories of long waves in accumulation and the role of technological innovations. The problem here, though, is that on the basis of the periodicity of such wavelike movement in economic activity, we should already be experiencing a new upswing of this long wave, but this does not appear to be happening. Instead, what has been witnessed over the past 30 years is the spectacular rise of the financial sector, which obligates us to discuss the neo-Marxist interpretation of this "financialisation" and its importance.

In Chapter 10, the final chapter, we present the provisional balance of our investigation – provisional, since we hope that our thoughts and assessments will stimulate further discussion, and we are naturally open to all input. We make a plea in the chapter for the integration of the many contemporary insights and theoretical views – if indeed reconcilable – into the Marxist economic model. However, first and foremost, we consider (although not exclusively) the post-Keynesian contribution to be of relevance here.

What ultimately concern us are the questions: have we, in the end and with reference to Marxist economic theory, learned something that we somehow did not know before? And: Marxist economic theory offers a coherent model, but can it still be reconciled with the economic realities of today and can its building blocks, if necessary, be replaced? Because we do not consider ourselves to be sufficiently qualified to assess the philosophical and sociological core of Marxism, including historical and dialectical materialism, class society and the definition of the social classes, we do not discuss it. Hence, we make the assumption, while assessing Marxist economic theory, that there are two antagonistic economic classes in capitalist society – the working class and the capitalist class. The same holds for the economic determination of social development, i.e. the relationship between the so-called "base" and the social, political and cultural "superstructure" of society. No doubt this neglect impoverishes our analysis and does not do justice to the holistic view that Marxism offers. However, others will have to take up these philosophical and sociological challenges.

Some readers may well consider concepts such as "workers" and "capitalists" as largely, if not completely, outdated. And perhaps they are. However, the naming as such is irrelevant. Most employees today are no longer industrial workers, let alone proletarians whose only "ownership" extends to their children. However, their labour collectively produces what we call the national product. And where are the capitalists of yesteryear? They are the ones who decide what will be produced and how it will be produced in order to maximise their profits. True, many enterprises are still owned by private capital owners, in which case it is clear who the capitalists are. But in most large companies the share capital is spread around and only a few people are pulling the strings: some rich families or the CEO, possibly with members of the management committee. They receive breath-taking bonuses or share options on top of their generous salaries. Let us therefore defer to Marx and his economics and accept that there are workers and capitalists, at least for the sake of the argument.

In a nutshell, the core of Marx's economic theory, which will be investigated and assessed in this book, is as follows:

1 The workers produce, using the means of production that are owned by the capitalists.
2 All value is produced by labour, and hence by the workers.
3 The workers produce more value – the surplus value – than they receive as compensation for their labour, according to a given social norm – the wages.
4 What workers produce above their wages is a surplus, which will go to the capitalists after what is needed for (for example) the state apparatus is deducted.

Of course, reality is much more complex. The above statements flow from a model which, by definition, is based on assumptions and simplifies reality. In his time, though, Marx made an impressive attempt to make his model a dynamic one, as follows:

1 The capitalists, both individually and collectively, have an unbridled hunger for profits and accumulation.
2 The capitalist hunger for profits is appeased by maximising the surplus value extracted from the workers.
3 The squeezing of wages is the most important instrument for maximising the surplus value.
4 The squeezing of wages in capitalism generates a tendency for underconsumption and overproduction, which in contemporary economics jargon is known as insufficient effective demand.
5 By squeezing wages and by replacing labour with capital/means of production (the mechanisation of production), the surplus value produced will decrease in the long run.
6 As a result of the proportionate decline in surplus value and profits, there will be a tendency for the rate of profits to fall.

The reader will no doubt notice that the assumptions, logic and predictions of Marx's economic model are quite subversive, although they are built on the theories of the classical economists Adam Smith (1723–1790) and, to an even greater extent, David Ricardo (1772–1823). It will come as no surprise that the economists after Marx, mostly from the upper or middle classes of society, spare no pain or expense in lashing out at the Marxist model. The "neo-classical" economists invented ingenious alternatives to Marx's theory of value and the origin of profits. Whereas, according to Ricardo and Marx, the means of production should be considered "crystallised labour", i.e. produced and combined with "living labour", the "neo-classicists" regard "capital" as a separate factor of production, the marginal product of which is the remuneration of the entrepreneur for committing his capital. This is analogous to the wage rate which is due to the production factor of labour. Neo-classical economics has increasingly received academic and scientific status as a result of its thorough use of mathematical analysis. Marxist economics has also been portrayed as a mathematically developed model. This is the linear production model which, at first hesitatingly but afterwards with increasing success, was developed during the 1960s and subsequent decades and which we have adopted in this book.

A mathematical approach, as demonstrated in the following pages, allows rigorous argumentation and proof; without such an approach, achieving the desired outcomes would be difficult, if not impossible. However, the other side of the coin is abstract reasoning. We therefore endeavour to illustrate the mathematical reasoning with numerical examples. The reader is asked to work through these examples as carefully as possible, with sharpened pencils and a notebook within easy reach!

Finally, we need to issue a small warning. While we investigate the core of Marxist economic theory, as outlined above, it is simply that – Marx's economic theory. We have not felt called to also review and evaluate the economic theories of his many followers. Thus we are neglecting a lot of theoretical developments that took place after Marx.[3]

It is no doubt rapidly becoming clear to the reader that we devote little attention to money, except in Chapter 8. The reason for this is easy and straightforward: the basics of Marx's economic theory can best be explained by neglecting the money phenomenon. Although we show that the monetary insights of Marx look very contemporary – even Keynesian – there is a deep gulf between Marxist and post-Keynesian economic theory, which needs to be bridged.

It is for a different reason that we do not dwell on the monopolisation process of capitalism. Although Marx devotes attention to what he calls concentration and centralisation of capital, most of this is researched by the Marxists after Marx. The same holds for the Marxist and neo-Marxist-inspired theories of imperialism and underdevelopment. Moreover, dealing with these theories is at odds with the economic model that we have adopted. This neglect invariably amounts to a further "reductionism" which might disappoint some readers. Alas . . . The linear production model adopted in this book is one of a closed economy. We are not considering exports or imports of goods and services, nor international capital mobility. Marx devotes little attention to the implications of international capital mobility in his model of value and growth, and he ignores international labour mobility completely. Some of the implications of the workings of the international economy for Marx's economic theory have been determined only recently and are still the subject of much debate. This evidently forces us once again to simply concentrate on Marx's views and how his theory works within the linear production model. If there is merit in exploring the implications of the linear production model for a Marxist theory of international political economy, this should be on the agenda for future research, and should not be elaborated on here.[4]

We also refer regularly, at least where relevant, to our previous research and publications. This is not to browbeat the reader with alleged erudition, but rather to indicate in what sense and to what degree some of our previous publications agree with or contradict the rethinking of Marx's economics. We apologise for these personal road markings, which are only intended to guide us on our way.

While writing and re-writing the manuscript, we benefited from the generous help of some good colleagues and friends. Geert Molenberghs introduced us to

the mathematical software package R, which saved us much time and calculation work and solved the many systems of linear equations that our numerical examples required. Although in the beginning we were confronted by some pitfalls, we could always count on Geert to help us onto our feet again. Geoff Harcourt, David Laibman, Glenn Rayp, Michel Dumont and Henry van Maasakker offered their comments and directed us towards (or provided) relevant literature which we then integrated into the text. We are most grateful for their insight and support, which have significantly enhanced the quality of the book. Responsibility for all remaining mistakes and confusion lies entirely with us. We also benefited greatly from comments and observations by Hugo Buyssens on Chapter 2, by Simon Mohun, Gary Mongiovi and Daniel E. Saros on Chapter 4, and from André Mommen's review of the previous Dutch edition of this book, but they cannot be held responsible for the views expressed. We are also immensely thankful to Mrs Alice Parry, who painstakingly revised the language in all the chapters, which often came back to us with more red than black. We finally wish to thank *New Left Review* for permission to use copyright material of Karl Marx, *Capital*, Volume 1, translated by Ben Fowkes, Harmondsworth, Penguin, 1976; Karl Marx, *Capital*, Volume 2, translated by David Fernbach, London, 1978; Karl Marx, *Capital*, Volume 3, translated by David Fernbach, London, 1981.

Notes

1 There is no need to repeat excellent work. The reader is referred to Mandel (1971).
2 We use the terms "technical change" and "technological change" interchangeably. Technical change, however, is not necessarily technological, i.e. not necessarily the result of new inventions. Technical change can also be the result of changes in the organisation of production, changes in input and output prices, etc.
3 A monumental and almost exhaustive overview of the development of Marxist economic thought since Marx is Howard and King (1989, 1992).
4 For an overview, see Howard and King (1992, chs. 9–10). For recent work on production and distribution as a worldwide process, as well as a critical review of the contributions of some Marxist scholars, see Carchedi (1991, chs. 6–7).

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1 Economic reproduction

In every economic system, goods and services are produced by employing means of production and labour power. For this production to be repeated, the economic system has to reproduce itself. This holds true for all types of economic system: slave economy, feudal economy and full-blown capitalist economy. How this reproduction takes place, however, differs from system to system, with the economic classes in the system – workers, peasants, craftsmen, commercial intermediaries, etc. – all playing their own specific role. The reproduction of an economic system relates to the workings of the so-called economic circuit, which was first conceptualised and coherently analysed by the French Physiocrat, François Quesnay (1694–1774), in his *Tableau économique*. The economic circuit concept was further developed and applied by Marx in Volume 2 of *Das Kapital*, where he uses his famous schemes of reproduction.[1]

In this chapter we begin our assessment of the economic theory of Karl Marx. It will become evident that Marx's concept of the reproduction process leads logically to his theory of value, which we consider in Chapter 2. We will therefore portray this process as a set of relationships between spheres of production, which evolve on the basis of given proportions and can be analysed according to a reproduction scheme.

1.1 Production and reproduction

In order to analyse the economic circuit and *a fortiori* the way in which an economic system reproduces itself, we will consider – for the sake of simplicity – an economy in which only two goods are produced: iron and wheat. It is assumed that iron ore is up for grabs, i.e. a free gift of nature, but is mined using an iron tool (e.g. a shovel).[2]

The iron ore is then transformed into a plough, using other iron tools (e.g. an oven, a hammer and an anvil). In this example, the iron ore, being a free gift of nature, has no value. Therefore, it can be stated that 50 kg of iron is needed to produce a plough of 150 kg, as well as 50 hours of labour time (used in the mining of the ore, the production of the tools and the forging of the plough), as follows:

50 kg iron + 50 hours labour → 150 kg iron + 50 kg iron
(tools) (1 plough) (tools)

One can say that in this production process, the tools are produced together with the plough.

The plough is used in the wheat farming sector. At this point in our argument, we assume that the plough is used entirely for wheat cultivation, i.e. used up only during one production period. With inputs of 50 hours of labour time and 40 kg of wheat (used for sowing the seed), the sector produces 100 kg of wheat:

150 kg iron + 40 kg wheat + 50 hours labour → 100 kg wheat
(1 plough) (sowing seed)

In this imaginary economy, a total of 200 kg of iron means of production are produced, which serve to replace the 200 kg of iron means of production that are used up in the production of both means of production and wheat. The next production period will thus start at the same level as before. Apart from this, 100 kg of wheat are cultivated, of which 40 kg can be used in the next production period to sow the seed. What remains – 60 kg wheat – is the *net product* (the term "le produit net" goes back to the Physiocrats) and is available, in terms of the above example, as consumer goods. It might well be that this quantity of wheat exceeds what is needed to reproduce the labour power, i.e. to support the workers and their families, such that in the next production period 100 hours of labour can be extracted. This surplus of wheat will allow an increase in consumption by the workers and their families, or alternatively, it will be consumed by those who are not directly related to the two production processes but are providing other services (artists, priests, warriors . . .). The "surplus approach" in economic theory goes back to François Quesnay and was very prevalent among the classical economists (Dobb, 1973, pp. 62ff.; also Garegnani, 1984, pp. 291–325).[3]

In summary, the following scheme can be written:

50 kg iron	+ 50 hours labour →	150 kg iron + 50 kg iron
150 kg iron + 40 kg wheat	+ 50 hours labour →	100 kg wheat
200 kg iron + 40 kg wheat		

leaving a net product of 60 kg of wheat. Let us now assume that each working hour is compensated by supplying 0.5 kg of wheat to the workers. In the means of production sector, 50 hours of labour were spent, with the result that the workers receive 25 kg of wheat. Similarly, in the consumer goods sector, 50 kg in total (2 × 25 kg) of the net product of 60 kg of wheat is used to reproduce the labour power, which will allow its use in the next production period. Therefore, a surplus of 10 kg of wheat remains (60 kg of wheat as net product, minus 50 kg of wheat consumed by the workers). Marx calls this the *surplus product*, which is available for additional workers' consumption and/or as compensation for the above-mentioned services.

One thing is clear, however: in our imaginary economy, no expansion is possible. With the wheat surplus, nothing can be put aside to produce a second plough or additional tools in the next production period since a surplus of iron is lacking. Hence, only *simple reproduction* is possible.

This can be summarised in the following scheme:

$$\begin{array}{lll}
50\,\text{kg iron} & +\ 25\,\text{kg wheat} & \rightarrow 150\,\text{kg iron} + 50\,\text{kg iron} \\
150\,\text{kg iron} & +\ 65\,\text{kg wheat} & \rightarrow 100\,\text{kg wheat} \\
\hline
200\,\text{kg iron} & \quad 90\,\text{kg wheat} &
\end{array}$$

leaving a surplus product of 10 kg of wheat.

1.2 The Marxist schemes of reproduction: introduction

In reality, the economic system in evidence today is much more complex than our imaginary economy as with the former, a multitude of goods and services is produced. However, if we can value and aggregate the many goods and services used in production, then the economic system can be represented by two aggregated sectors: a sector that produces means of production and a sector that manufactures consumer goods. This is, in fact, the process followed by Marx in Volume 2 of *Das Kapital*. For the valuation of the means of production, the respective prices (which reflect their respective values) can be used. Where these prices come from is the subject of Chapter 2.

The value of the means of production, thus aggregated, which are used in the production of means of production and of consumer goods, is called constant capital by Marx, whereas the value of the consumer goods that serve the reproduction needs of labour power is called variable capital.[4]

In the sector where the means of production are produced (sector 1), these are C_1 and V_1, respectively, while in the consumer goods sector (sector 2), these are C_2 and V_2. The value of this constant and variable capital is found at the end of the production period, in the value of the output produced in both sectors: W_1 and W_2. The difference between W_1 and the used-up constant and variable capital $C_1 + V_1$ is called the surplus value M_1, while the difference between W_2 and the used-up constant and variable capital $C_2 + V_2$ is called the surplus value M_2.

In value terms, the following scheme can be presented:

$$\begin{array}{ccccccl}
C_1 & + & V_1 & + & M_1 & & = W_1 \text{ (sector 1)} \\
C_2 & + & V_2 & + & M_2 & & = W_2 \text{ (sector 2)} \\
\hline
C_1 + C_2 & & V_1 + V_2 & & M_1 + M_2 & &
\end{array}$$

This is a reproduction scheme, as developed and used by Marx in his analysis of the conditions of reproduction of any arbitrarily chosen economic system. From such a reproduction scheme, the following conditions of simple reproduction can be derived:

$C_1 + C_2 = W_1$	or: the value of the output of means of production is equal to the value of the means of production used up in the economy
$V_1 + V_2 + M_1 + M_2 = W_2$	or: the value of the output of consumer goods is equal to the value of the consumer goods consumed in the economy

Since the first condition is $C_1 + V_1 + M_1 = W_1$, it follows that:

$C_1 + C_2 = C_1 + V_1 + M_1$, or $C_2 = V_1 + M_1$	or else: the value added of sector 1 is equal to the value of the means of production used in sector 2

And since $C_2 + V_2 + M_2 = W_2$, the second condition can be rewritten as: $V_1 + V_2 + M_1 + M_2 = C_2 + V_2 + M_2$, or $V_1 + M_1 = C_2$, which produces the same result.

It should be very evident to the reader that at the start of the next production period, producers will be able to start producing the same quantities as before. In other words, if the above condition is met, then the economic circuit is closed. Once again, sector 1 has C_1 at its disposal, as well as the labour power that can function for another 50 hours. Similarly, C_2 and the required labour power are back in sector 2. At the end of the second production period, the reproduction scheme will show exactly the same values as at the end of the first production period. For this reason, this type of reproduction process is called simple reproduction.

Simple reproduction implies that the full surplus value is consumed. What remains of the value of the net product after workers and their families have consumed what is needed for the reproduction of the labour power, will be consumed by other classes and strata of the population who are not related to the production processes. In a feudal society, for instance, this surplus value would be appropriated by the feudal lords, their vassals and armies, the priests, the artists, the cathedral builders, etc.

Simple reproduction stands in contrast to *expanded reproduction*, which takes place under capitalist conditions. In fact, the economy has to be sufficiently advanced technologically (in Marx's words: "the forces of production have to be developed sufficiently") to allow expanded reproduction. In our previous numerical example, the surplus product consisted of consumer goods only, whereas additional means of production are required as well if the economy is to operate during the next production period on an expanded scale.

This development of the forces of production is assumed in the following example:

$$\begin{array}{llll}
50\,\text{kg iron} & & +\ 50\ \text{hours labour} \rightarrow & 170\,\text{kg iron} + 50\,\text{kg iron} \\
150\,\text{kg iron} + & 40\,\text{kg wheat} & +\ 50\ \text{hours labour} \rightarrow & 100\,\text{kg wheat} \\
\cline{1-2}
200\,\text{kg iron} & 40\,\text{kg wheat} & &
\end{array}$$

The net product now consists of 20 kg of iron and 60 kg of wheat.[5]

Clearly, a larger output of ploughs and iron tools is produced using the same input of labour, which implies that labour has become more productive as a result of better technological capabilities or more specialisation. However, in our example, productivity in the consumer goods sector has remained the same.

If we assume that, arising out of social necessity, the consumption of the workers remains at 0.5 kg of wheat per hour of labour spent, it follows that:

$$\begin{array}{lll}
50\,\text{kg iron} & +\ 25\,\text{kg wheat} \rightarrow & 170\,\text{kg iron} + 50\,\text{kg iron} \\
150\,\text{kg iron} & +\ 65\,\text{kg wheat} \rightarrow & 100\,\text{kg wheat} \\
\cline{1-2}
200\,\text{kg iron} & 90\,\text{kg wheat} &
\end{array}$$

Based on this numerical example, the surplus product consists of 20 kg of iron and 10 kg of wheat, and therefore also contains means of production. The example was chosen such that it allows an extension to the scale of production of means of production in the next production period, as follows:

$$\begin{array}{llll}
70\,\text{kg iron} & & +\ 70\ \text{hours labour} \rightarrow & 238\,\text{kg iron} + 70\,\text{kg iron} \\
150\,\text{kg iron} + & 40\,\text{kg wheat} & +\ 50\ \text{hours labour} \rightarrow & 100\,\text{kg wheat} \\
\cline{1-2}
220\,\text{kg iron} & 40\,\text{kg wheat} & &
\end{array}$$

After the surplus product of the first production period has been entirely "ploughed back" into sector 1, there is a net product of 88 kg of iron and 60 kg of wheat at the end of the second production period. Taking into account the socially necessary consumption of workers of 0.5 kg of wheat per hour of labour spent, the following scheme emerges:

$$\begin{array}{lll}
70\,\text{kg iron} + & 35\,\text{kg wheat} \rightarrow & 238\,\text{kg iron} + 70\,\text{kg iron} \\
150\,\text{kg iron} + & 65\,\text{kg wheat} \rightarrow & 100\,\text{kg wheat} \\
\cline{1-2}
220\,\text{kg iron} & 100\,\text{kg wheat} &
\end{array}$$

Clearly, we are dealing here with a situation of expanded reproduction, which is characteristic of the capitalist production system.[6]

Using our previous notation of constant capital, variable capital and surplus value, the necessary conditions for expanded reproduction[7] in the second production period in the above example can be written as follows:

$C_1 + C_2 < W_1$	or: the value of the output of means of production is *higher* than the value of the means of production used in the economy
$V_1 + V_2 + M_1 + M_2 < W_2$	or: the value of the output of consumer goods is higher than the value added of both sectors taken together

Stated differently: for expanded reproduction, it is required that a surplus remains in *both* sectors on top of what is needed for simple reproduction.

1.3 The workings of the Marxist schemes of reproduction under capitalism

In order to produce and therefore reap the surplus value, so Marx explains, the individual capitalist must first purchase raw materials, machines and labour power by spending an amount of money, G. This amount of money G is the monetary expression of the value of the means of production and labour power purchased by the capitalist. It is hoped that the output will later be sold in its entirety and that the capitalist will receive his investment of G, but increased by the surplus value, ΔG. The cycle can then start over again, either on the same or on an expanded scale.

Marx represents this process as follows:

	raw material				
G →	machines	→	P	→	G' = G + ΔG
	labour power		(production process)		(money after sales)

The process follows an economic circuit and holds for each individual capitalist. For this circuit to be closed at the *macro-economic level* – i.e. for it to end well – some conditions have to be fulfilled. Which ones? This is a question that was first studied from a macro-economic perspective by François Quesnay in his *Tableau économique*, from which Marx derived his reproduction schemes. Marx's real rationale was to lay bare the conditions that have to be met if the capitalist production system as a whole is to make steady progress in *the long term* (apart from fluctuations due to the business cycle), in spite of the growing division of labour. It would appear from his analysis that the circuit is disturbed – if not interrupted – by overproduction and over-savings, and that a sufficient supply of additional labour is, furthermore, required for the expansion of the capitalist economy. With Marx, we are a very long way indeed from the harmony of the "natural order" that the classical economists, together with many later economists, postulated.[8] We will return to this later.

Simple reproduction, as we have found, takes place when no surplus value is accumulated, but rather when the surplus product is fully consumed by the capitalists. We will use the following numerical example to illustrate constant capital, variable capital and surplus value:

Sector 1 (means of production): $4000(C_1) + 1000(V_1) + 1000(M_1) = 6000$

Sector 2 (consumer goods): $2000(C_2) + 500(V_2) + 500(M_2) = 3000$

The output value of consumer goods is equal to the total purchases of workers and capitalists of consumer goods:

$$2000(C_2) + 500(V_2) + 500(M_2) = 1000(V_1) + 1000(M_1) + 500(V_2) + 500(M_2)$$

Or, after deleting $500(V_2) + 500(M_2)$ from both sides of the equation:

$2000(C_2) = 1000(V_1) + 1000(M_1)$ as the condition for simple reproduction.

How this comes about can be explained as follows:

The capitalists of sector 1 use $4000(C_1)$ of the output value of means of production to replace the means of production used up. Of the output value of consumer goods (3000), $500(V_2)$ goes to the workers and $500(M_2)$ goes to the capitalists of sector 2. In order to replace the C_2 used up, the capitalists of sector 2 advance 2000 units of money and buy $2000(C_2)$ in sector 1. With these 2000 units of money, the capitalists of sector 1 pay the wages of their workers ($1000(V_1)$) and use the rest ($1000(M_1)$) for personal consumption. Through the purchases of consumer goods by the workers and the capitalists of sector 1, the 2000 units of money return to the pockets of the capitalists of sector 2 who originally advanced the money.[9]

It should be noticed that simple reproduction can hardly be considered a typical feature of a capitalist economy. The capitalists can be expected not to squander their surplus value, but rather to accumulate as much as possible. This evidently leads to a situation of expanded reproduction. In other words, the above numerical example can only serve a pedagogical purpose, since a capitalist economy is driven by a hunger for profits (i.e. a need for surplus value, accumulation of capital and expansion in order to survive in a competitive environment) and therefore expanded reproduction. This is illustrated in the following reproduction scheme:

$$\begin{array}{ll} \text{Sector 1:} & 4000(C_1) + 1000(V_1) + 1000(M_1) = 6000 \\ \text{Sector 2:} & \underline{1500(C_2)} + 750(V_2) + 750(M_2) = 3000 \\ & 5500(C) \end{array}$$

As in the previous example, it is assumed here that all intra-sectoral transactions take place without money. Money is only required for inter-sectoral transactions. Therefore, the $4000(C_1)$ is replaced by sector 1 in kind. To pay their workers, the capitalists of sector 1 advance 1000 units of money or $1000(V_1)$. With this money, the workers of sector 1 buy consumer goods in sector 2 for the same amount. To replace the C_2 in sector 2, the capitalists of that sector use the 1000 units of money thus received and advance 500 units of money. With this amount of money they buy means of production in sector 1, representing 1500(C_2). In this way, 1500 units of money are channelled to sector 1, of which 1000 units return to the pockets of the capitalists who previously advanced 1000(V_1). The remaining 500 units of money are used by the capitalists of sector 1 as consumption expenditure. The capitalists of sector 1 then buy, in sector 2, consumer goods for 500(M'), which will return the money advanced by sector 2 to that sector. The remainder of the surplus value in sector 1 amounts to 1000(M) – 500(M') = 500. Means of production to an amount of 400 is accumulated and the next production period will commence with a constant capital equal to:

$$4000(C_1) + 400(\Delta C_1) = 4400$$ [10]

For each 4 units of money value of additional means of production, 1 unit of money has to be spent on labour power. Therefore, the capitalists of sector 1 once again advance money: 100(ΔV_1). The additional workers, who are thus employed, buy consumer goods for that amount in sector 2. The capitalists of sector 2, in turn, then buy additional means of production in sector 1, as well as additional labour power, to which the remaining consumer goods will go for a monetary value of 50 after the capitalists have consumed 600(M'_2). The complete mechanism that is assumed is depicted schematically in Figure 1.1. The proof that the total output value of sector 1 and sector 2 has been sold is as follows:

$$\begin{aligned}
\text{Sector 1:}\quad & 4000(C_1) + 1000(V_1) + 1000(M_1) = \\
& 4000(C_1) + 400(\Delta C_1) + 1000(V_1) + 100(\Delta V_1) + 500(M'_1) = 6000 \\
\text{Sector 2:}\quad & 1500(C_2) + 750(V_2) + 750(M_2) = \\
& 1500(C_2) + 100(\Delta C_2) + 750(V_2) + 50(\Delta V_2) + 600(M'_2) = 3000
\end{aligned}$$

In fact, both sectors are expanding unevenly. If macro-economic equilibrium is to be maintained, the capitalists of sector 2 have to consume exactly what they fail to invest.

At this point in the argument, the difference between *surplus value produced* and *surplus value realised* should be clear, but will be explored at length in the next chapter. From our scheme of reproduction outlined above it can be deduced that in the production period being considered, a surplus value of $M_1 + M_2$ is produced. This surplus value produced will be realised only to the extent that the total output value of means of production and consumer goods is sold. When there is a shortfall in demand for additional means of production and labour power because of a temporary lack of interest in additional investment (accumulation of capital), part of the surplus value produced will remain unsold and will therefore not be realised.

To generalise and neglecting capitalist consumption, with expanded reproduction, M_1 is accumulated as capital, i.e. is spent on additional means of production and additional labour power in sector 1, with respective values of ΔC_1 and ΔV_1.

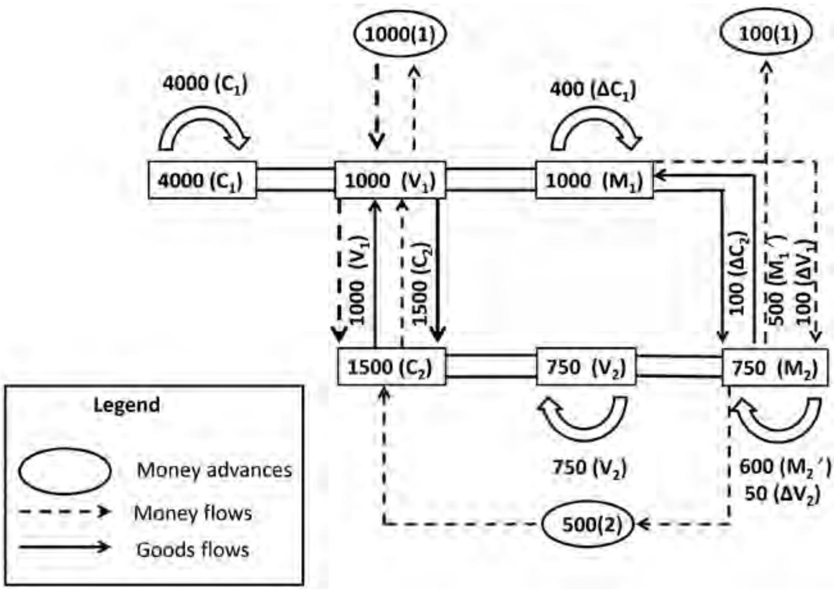

Figure 1.1 Inter-sectoral flows in the case of expanded reproduction

Also, M_2 is accumulated by procuring additional means of production and additional labour power in sector 2, with respective values of ΔC_2 and ΔV_2. Neglecting capitalist consumption, the equilibrium condition for expanded reproduction can be written as follows:

$$W_1 = C_1 + V_1 + M_1 = C_1 + C_2 + \Delta C_1 + \Delta C_2$$

$$W_2 = C_2 + V_2 + M_2 = V_1 + \Delta V_1 + V_2 + \Delta V_2$$

Or:

- The output value of means of production must be equal to what is required for the replacement of the used-up means of production, as well as for *additional* means of production in both sectors.
- The output value of consumer goods must be equal to what is required for the replacement of the originally invested variable capital, as well as for *additional* variable capital in both sectors.

If accumulation is insufficient, the produced surplus value M_1 and M_2 will not be fully *realised*. If, for instance, there is no additional investment in constant capital, $\Delta C_1 + \Delta C_2 = 0$. As a result, in the above condition for expanded reproduction:

$$W_1 = C_1 + V_1 + M_1 = C_1 + C_2 + \Delta C_1 + \Delta C_2$$

$\Delta C_1 + \Delta C_2 = 0$. It follows that the output value of what sector 1 produced will only go to the replacement of $C_1 + C_2$, the constant capital of sectors 1 and 2. Although surplus value is produced, it will not be realised in the process of reproduction due to its insufficient accumulation (i.e. insufficient capitalist spending).

The above conditions for expanded reproduction can be further simplified by dropping from both sides of the equations the corresponding terms. As a result we get:

$$V_1 + M_1 = C_2 + \Delta C_1 + \Delta C_2$$
$$C_2 + M_2 = V_1 + \Delta V_1 + \Delta V_2$$

Expressed plainly, these conditions mean:

- The value added of the capital goods[11] sector must be equal to the value of the constant capital that must be replaced in the consumer goods sector, plus the additional spending of surplus value on constant capital in both sectors.
- The replacement of the variable capital of the capital goods sector, plus the additional spending of surplus value on variable capital in both sectors, must be equal to the surplus value and the constant capital that must be replaced in the consumer goods sector – i.e. the output value of consumer goods on top of the variable capital required to produce it.

It should be noticed from all this that the spending of surplus value on capitalist consumption has not been taken into account. It should be sufficiently clear, however, that based on the above conditions, expanded reproduction is a complicated process which goes in fits and starts in an economic system where decision making in respect of production and consumption is atomised.

1.4 What can be learned from Piero Sraffa's "standard system"?

It was shown above that the full or partial investment of the surplus value leads to expanded reproduction. In physical terms, such investment implies the utilisation of the surplus product, or surplus for short. Such full utilisation in the process of reproduction can only be reconciled with sustained expanded reproduction (or balanced expanded reproduction) if each sector produces a quantity of output that leaves an output surplus, after having supplied to all sectors what is needed for simple reproduction, which is in the same proportion as the total output in each sector. This can easily be explained by assuming that the economy is a pure "wheat economy". This was the approach followed by David Ricardo in 1814–1815.[12]

In such an economy, wheat is both a means of production in its own production process (seeds for sowing) and necessary consumption for the farmers and their families and their employees. Assume, for example, that in such an economy, 100 kg of wheat is required on an annual basis as seed to produce 500 kg of wheat and the producers need 300 kg of wheat for their livelihood. This economy thus produces a surplus of wheat of 100 kg, i.e. 500 – 100 – 300. In this wheat economy, the surplus represents 25% of what is needed as seed and as necessary consumption. Put in another way, the ratio of the wheat output to the total wheat input is 500 kg wheat/(100 kg wheat + 300 kg wheat) or 1.25.

This wheat surplus allows expanded reproduction at a 25% rate of growth of inputs and outputs. A quarter of the wheat surplus is then used as seed and the remaining three-quarters is put aside as necessary consumption in the next production period.[13] The following year, 125 kg of wheat will be used as seed and 375 kg as necessary consumption, while 625 kg of wheat will be produced. In other words, expanded reproduction will have taken place, with the economy growing at 25%. The ratio of wheat output to total wheat input will remain at 1.25: 625 kg wheat/(125 kg wheat + 375 kg wheat).

In our imaginary economy where both iron and wheat are produced, things become more complicated. We once again assume that 150 kg of iron and 100 kg of wheat are produced. Taking into account the necessary consumption by the workers in the respective sectors, such consumption could be produced by the allocation of clearly specified quantities of iron and wheat as inputs, as follows:

					Total production
	50 kg iron	+	25 kg wheat	→	150 kg iron
	50 kg iron	+	65 kg wheat	→	100 kg wheat
Total supplies	100 kg iron	+	90 kg wheat		

The proportion of production to supplies in the iron sector is thus: 150/100 = 1.50, while in the wheat sector it is: 100/90 = 1.11. This means that if the complete surplus product of iron is used for expanded reproduction, it would permit an economic growth rate of 50%. The problem is, however, that this rate of growth cannot be attained in the wheat sector, since the surplus product of wheat is not in sufficient proportion to the wheat supplies required for simple reproduction. The production of iron and wheat should be increased or reduced such that in both sectors the same proportion of outputs and input supplies is realised. The question, therefore, is: what are the required outputs of iron and wheat that would lead to balanced expanded reproduction? It should be stressed, though, that such balanced expanded reproduction is not to be expected in a capitalist economy as the proportions between the sectors in the economy are, for a variety of reasons, regularly disturbed.

Piero Sraffa answered this question in his slender but ground-breaking 1960 book, *Production of Commodities by Means of Commodities* (Sraffa, 1960). However, in contrast to the procedure we adopt here, he did not consider the necessary consumption to be a physical input in the production process.[14]

To construct Sraffa's standard system, we must enlarge or reduce the scale of production of the sectors until the proportions between quantities of output and total supplies are equal between all sectors. First, we must transform the above production system of our imaginary economy such that it relates to unit outputs:

$$\frac{50}{150}\text{ kg iron} + \frac{25}{150}\text{ kg wheat} \rightarrow 1\text{ kg iron}$$

$$\frac{50}{100}\text{ kg iron} + \frac{65}{100}\text{ kg wheat} \rightarrow 1\text{ kg wheat}$$

The output quantities of iron and wheat to be determined are represented by q_1 and q_2, respectively. For the production of q_1 kg of iron and q_2 kg of wheat, the iron sector will supply $50/150 \times q_1$ kg of iron and $50/100 \times q_2$ kg of iron to the wheat sector, while the wheat sector will supply $25/150 \times q_1$ of wheat to the iron sector and require $65/100 \times q_2$ kg of wheat for itself. The proportions q_1 and q_2, which are such that in both sectors the same (still unknown) proportion $(1 + R)$ is realised, can be found by solving the following system of equations:

$$\left(\frac{50}{150}q_1 + \frac{50}{100}q_2\right)\text{kg iron} \times (1 + R) = q_1\text{ kg iron}$$

$$\left(\frac{25}{150}q_1 + \frac{65}{100}q_2\right)\text{kg wheat} \times (1 + R) = q_2\text{ kg wheat}$$

To solve such equations, matrix algebra is used. A matrix is a table of elements ordered in rows and columns, such as data on necessary inputs. A matrix can be square (equal number of rows and columns) or rectangular (unequal number of rows and columns). A special type of "matrix" is the vector, which consists of only one row or column. Matrices, if they have the appropriate dimensions, can be added up, subtracted or multiplied, as follows:

Matrix **A** + Matrix **B**: the elements that correspond per row and column are added up (the same holds for the deduction):

$$\begin{bmatrix} 12 & 5 \\ 0 & 7 \end{bmatrix} + \begin{bmatrix} 3 & 8 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 12+3 & 5+8 \\ 0+4 & 7+5 \end{bmatrix}$$

Matrix **A** × Matrix **B**: row-by-row multiplication of the row elements by the corresponding column elements, after which the results are summated:

$$\begin{bmatrix} 12 & 5 \\ 0 & 7 \end{bmatrix} \times \begin{bmatrix} 3 & 8 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} (12\times 3)+(5\times 4) & (12\times 8)+(5\times 5) \\ (0\times 3)+(7\times 4) & (0\times 8)+(7\times 5) \end{bmatrix}$$

Vector **A** × Matrix **B**: multiplication of the row elements of the vector by the column elements (column-by-column) of the matrix, after which the results are summated:

$$[12\ 5] \times \begin{bmatrix} 3 & 8 \\ 4 & 5 \end{bmatrix} = \left[(12\times3)+(5\times4)\ (12\times8)+(5\times5)\right]$$

The matrix form of the standard system of our imaginary economy is:

$$\left(q_1\, q_2\right)\times\begin{bmatrix} \frac{50}{150} & \frac{25}{150} \\ \frac{50}{100} & \frac{65}{100} \end{bmatrix} \times (1+R) = \left(q_1\ q_2\right)$$

or, in matrix notation:

$$\mathbf{q}\,\mathbf{B}\,(1+R) = \mathbf{q}$$

with $\mathbf{q} = \left(q_1\ q_2\right)$, the row vector of output quantities,

$\mathbf{B} = \begin{bmatrix} \frac{50}{150} & \frac{25}{150} \\ \frac{50}{100} & \frac{65}{100} \end{bmatrix}$, the square matrix of necessary input coefficients of means of production and necessary consumption.[15]

This system of equations can be solved. However, if we want outputs that can be compared with the 150 kg of iron and the 100 kg of wheat in our imaginary economy, an additional equation has to be added as shown below.

The 150 kg of iron and the 100 kg of wheat were produced through the spending of direct labour time of 100 hours, or 50 hours in sector 1 and 50 hours in sector 2. In order to solve the standard system for other output quantities of iron and wheat, the 100 hours of direct labour also have to be allocated differently. This re-allocation has to satisfy the condition that:

$$\left(\frac{50}{150}\text{ hours} \times q_1\text{ kg iron}\right) + \left(\frac{50}{100}\text{ hours} \times q_2\text{ kg wheat}\right) = 100\text{ hours.}$$

In matrix form, this is written as:

$$\left(q_1\, q_2\right)\begin{pmatrix} \frac{50}{150} \\ \frac{50}{100} \end{pmatrix} = 100$$

or, in matrix notation:

$$\mathbf{q}\,\mathbf{l} = 100,$$

with $\mathbf{l} = \begin{pmatrix} \frac{50}{150} \\ \frac{50}{100} \end{pmatrix}$ the column vector of labour input coefficients per unit of output.

Solving the system of equations of the standard system gives:

$$R = 21.8\%$$

$$q_1 = 123.8\,\text{kg iron (instead of 150 kg iron)}$$

$$q_2 = 118.5\,\text{kg wheat (instead of 100 kg wheat)}$$

It is thus found that if 123.8 kg of iron and 118.5 kg of wheat are produced, then both sectors will produce 21.8% more output than is required as input supplies for replacement. If the surplus is totally spent on additional means of production and necessary consumption for the additional labour force, both sectors and the economy at large will expand at a rate of 21.8%, which means there is balanced expanded reproduction.

There are only two sectors in the above example. However, a diversified economy with a large number of sectors actually makes no difference. The application of Sraffa's procedure (at least, when necessary consumption is considered to be an input in the production process) allows us to calculate for a given economy and, based on the necessary input proportions, the output quantities required for balanced expanded reproduction. It should be stressed that it was not Sraffa's intention to determine the conditions for such expanded reproduction, but rather to construct an "invariable measure of value" – the Fata Morgana of David Ricardo. Moreover, it is one thing to calculate the outputs that allow balanced expanded reproduction, but it is another to reflect on the economic realities of capitalism, with expanded reproduction proceeding in a stop-start fashion. A standard system à la Sraffa, as a representation of balanced expanded reproduction and balanced growth, is not reality but rather a thought construct which not only allows further analysis but can be used as an analytical tool.

1.5 The significance of the Marxist schemes of reproduction

Generations of economists have been inspired by the Marxist schemes of reproduction.

First, from the conditions of simple reproduction it can be deduced that the economic expansion of a productive system is only possible if advances in technological knowledge have increased productivity sufficiently to allow the production of an economic surplus or, in Marx's terminology, a surplus product. Marx's schemes of reproduction, as an analytical tool, contribute to a better understanding of this process, both conceptually and historically.

Next, the conditions of expanded reproduction, which Marx derived from his schemes of reproduction, point to the role that the accumulation of the surplus product plays and how precarious this process of economic expansion is. The proportions between the economic sectors are continuously upset by changes in the production processes and, therefore, in the precise economic input needs. Technological changes can be both labour saving and capital saving (see above), and to the extent that output increases in some sectors, the unit production costs (and thus the required unit inputs) will fall because of so-called economies of scale. Rosa Luxemburg stressed that for the study of the conditions of capitalist reproduction, only average intra- and inter-sectoral input flows are relevant and that Marx's schemes of reproduction should be viewed in this way (Luxemburg, 1951, pp. 36ff.). She therefore considered these schemes as useless for explaining the real processes involved in capitalist development. Marx's schemes of reproduction, according to Luxemburg, assume an identity between the surplus value produced and the surplus value realised, and therefore relate to the *average* reproduction needs (Marx, 1978, p. 159). The reader should remember this point when we apply these schemes further as a linear model of production equations.

It is interesting to linger for a short while and contemplate Rosa Luxemburg's thoughts on this issue. For accumulation to take place, she stated, additional labour power as well as additional capital goods and consumer goods – moreover, in the right proportions – must be available. However, the capitalist hunger for profits and the resultant leaning towards boundless accumulation will ensure that capitalist expansion repeatedly brushes up against its limits. Hence, steady and balanced, long-run growth is out of the question (Luxemburg, 1951, pp. 36–37, 48–49).

In contrast, the contradiction between the growing capacity to produce and the limited possibilities to consume gives rise to the problem of selling the ever-increasing output, and thus the surplus value realisation problem. This contradiction can only be resolved temporarily to the extent that the capitalist system finds new and/or external markets, i.e. both sales outlets and sources of supply of both raw materials and labour power in the non-capitalist "entourage" of the capitalist system (Luxemburg, 1951, pp. 351–352, 366). A special type of "external market" is found in armaments spending of the imperialist state. We return to this issue in Chapter 9, §9.3.

Rosa Luxemburg reproached Marx on the basis that his reproduction schemes assumed "balanced growth" of the economic sectors, or at least appeared to see it as a possibility – a situation that did not reflect reality in any way and which she disliked tremendously (Luxemburg, 1951, p. 417). Luxemburg not only touched a raw nerve here, but she also indicated that the conditions whereby expanded reproduction would guarantee steady and stable economic growth are mostly not met. That such steady and stable growth can be derived mathematically from the reproduction schemes illustrates, in fact, its impossibility under capitalist conditions. These reflections later inspired the growth theorists of the so-called Cambridge School, such as Roy Harrod and Joan Robinson, and the many neo-Marxist and post-Keynesian authors. We will return to this in Chapter 3.

Luxemburg's criticism of Marx seems largely based on a misunderstanding. The situation of balanced growth, which Marx's reproduction schemes indicate, is a thought construct. Joan Robinson, who also used a reproduction scheme à la Marx, defined a "Golden Age" as a situation in which technological changes are evolving steadily and are neutral, population growth is constant, and the rate of accumulation is sufficiently high (to avoid unemployment) and is constant. In such a situation, at least in a competitive economy, real wages will proportionately change in line with labour productivity. The rate of profits, and the share of profits and wages in total value added, will then remain unchanged during the process of accumulation – provided, though, that no political or economic disturbances are expected and that the capitalists have faith in the future (Robinson, 1956, p. 99; Robinson, 1962, p. 52). The aim of this theoretical situation that Robinson sketched is to elucidate the many factors that prevent such balanced or equilibrium growth. By focusing on the inherent instability of capitalism, Robinson's "Golden Age" entails a severe critique of the many theories of economic growth that assume equilibrium growth. Here the critique runs similarly to that of Marx on the classical theory of economic growth – i.e. that such theories forget that they model the dynamic process of capitalist development, which by its own nature is unstable.[16]

In Marx's reproduction schemes, the business cycle – or fluctuations in prices, profits, etc. – are disregarded. A balanced scheme of reproduction shows average inter-sectoral flows of goods and services (Marx, 1978, p. 159). Marx assumes a "normal course of reproduction" (1978, pp. 571, 186–187). The outputs and prices that allow this normal reproduction can be denoted as "equilibrium outputs" and "equilibrium prices", respectively, but do not refer to the equilibrium concept as used in neo-classical economic theory. In fact, "normal outputs" or "natural outputs" and "normal prices", "natural prices" or "prices of production" would be better descriptions. However, such terms are rarely encountered in the literature on the linear production model.

At this point it will also be interesting to consider the linear production model that Joan Robinson developed and used in her *The Accumulation of Capital* (1956) and on which her theory of economic growth is built. She elaborates on a reproduction scheme in which a sector definition is used that differs from that of Marx. As we have seen, the sectors in Marx's scheme of reproduction are aggregated (using long-term prices/labour values) and produce means of production and consumer goods, respectively. Robinson, however, considers the two sectors as integrated with the respective production of the equipment, raw materials and other means of production (Robinson, 1956, p. 17; Robinson, 1962, pp. 126–127),[17] such that all the required intermediate supplies take place within sector 1 and sector 2 thus integrated. Of course, the value of these intermediate supplies is part of the output value of the respective sectors, but this can be neglected in the further analysis. Robinson took this approach from Michał Kalecki. Next, she considers only the surplus of the consumer goods sector in determining economic expansion. This surplus, in turn, consists of the equivalent of the wages paid out in the capital goods sector, i.e. the consumer goods that are acquired by the workers in the capital goods sector.

We denote the output value of the capital goods sector and consumer goods sector, respectively, as X_1 and X_2.[18] This output value consists of the wages

L_1 and L_2, and what Robinson calls the quasi-rent Q_1 and Q_2 of both sectors.[19] These quasi-rents are the sum of profits, depreciation and rent payments of the sectors. In the absence of capitalist consumption and workers' savings, the following equalities will hold:

Sector 1 (capital goods): $X_1 = L_1 + Q_1 = Q_1 + Q_2$

Sector 2 (consumer goods): $X_2 = L_2 + Q_2 = L_1 + L_2$

The first equality shows that the total quasi-rents are spent on capital goods since it is assumed that capitalist consumption is absent. The second equality indicates that the total wages are spent on consumer goods. From these equalities it follows that:

$$Q_2 = L_1$$

This is not new. In terms of our previous numerical example, simple reproduction and absent capitalist consumption, a Marxist scheme of reproduction will lead to $1000(C_2) = 1000(V_1)$.[20] The importance of Robinson's approach is, however, twofold: (1) If the capitalists consume $c(Q_1 + Q_2)$ (with c the average consumption quote from quasi-rents, and $0 \leq c < 1$), the above equality becomes:

$$Q_2 = L_1 + c(Q_1 + Q_2),$$

and after rearrangement:

$$Q_2 = \frac{L_1 + cQ_1}{(1 - c)}$$

(Robinson, 1956, pp. 48, 75, 77, 142–143, 198, 206, 255)

This formula indicates a simple ex post linear relationship between the quasi-rents of sector 2 and the workers' and capitalists' consumption of sector 1.[21] (2) The equality, Robinson argues, can be interpreted in physical terms, with the surplus of consumer goods of sector 2 (above what the workers of sector 2 consume) being what allows the sector 1 workers' consumption (and that of the sector 1 capitalists, if positive). At this stage of her argument, Robinson thus avoids aggregation, in which prices or labour values would have to be used (as was Marx's approach).[22]

Although leading to essentially analogous conditions for expanded reproduction, as in Marx's reproduction schemes, Robinson's differing sector definition is partly motivated by her attempt to avoid aggregation of inputs and outputs using prices or labour values (Cuyvers, 1977, pp. 210ff.).[23] Moreover, following Rosa Luxemburg, Robinson derives from her model the importance of "profits realisation" and the development of a theory of economic growth that in many ways can be considered as neo-Marxist (Cuyvers, 1979, pp. 326–348). The disadvantage of the Robinsonian approach, as compared to that of Marx, is that it leaves hidden an essential condition of expanded reproduction, i.e. a positive surplus of capital goods (means of production) on top of the constant capital that needs to be replaced.[24] We will come back to this point in a later chapter.

The Marxist schemes of reproduction have significance, not just for the economic theory of long-term development and growth, but for the theory and analysis of the short term. It has already been stressed how, from the reproduction schemes, it can be deduced how the capitalists' spending of the surplus value enables the realisation of the surplus value. As is sufficiently well known, the theory of effective demand was developed largely in *The General Theory of Employment, Interest and Money* (1936) by John Maynard Keynes (1883–1946) during the same period that Michał Kalecki (at the time an obscure Polish and Marxist-inspired accountant and would-be civil engineer) developed basically the same theory, but independently of Keynes.[25] Kalecki started with Marx's schemes of reproduction. Joan Robinson, who was closely involved in the discussions in "The Circus"[26] in Cambridge in 1930–31 (on *A Treatise on Money*, published by Keynes in 1930) and subsequently in the later 1930s with Kalecki, wrote:

> Kalecki had one great advantage over Keynes – he had never learned orthodox economics (. . .). The only economics he had studied was in Marx. Keynes could never make head or tail of Marx (. . .). But starting from Marx would have saved him a lot of trouble. Kahn, at the "Circus" where we discussed the *Treatise* in 1931, explained the problem of saving and investment by imagining a cordon round the capital-goods industries and then studying the trade between them and the consumption-goods industries; he was struggling to rediscover Marx's scheme. Kalecki began at that point.
>
> (Robinson, 1966a, p. 338. See also Robinson, 1966b, p. x)[27]

Apart from this version of the reproduction schemes, Kalecki also elaborated on one with three sectors: means of production (sector 1), workers' consumption (sector 2) and capitalists' consumption (sector 3), allowing equality between the profits of sector 3 and the wage bill in the other sectors (variable capital). Again, both sectors are assumed to be vertically integrated with their suppliers of intermediate goods. If we denote the gross profits of sectors 1 and 2 by P_1 and P_2, the wage bills of these sectors by L_1 and L_2, the respective value of output by X_1 and X_2, as well as workers' consumption by C_w, capitalists' consumption by C_c, and investments by the capitalists by I, then the following equalities can be written:

Sector 1 (means of production):	$X_1 = P_1 + L_1 = I$
Sector 2 (capitalists' consumption):	$X_2 = P_2 + L_2 = C_c$
Sector 3 (workers' consumption):	$X_1 = P_3 + L_3 = C_w$

The consumption expenses C_w by the workers are equal to the total wage bill $L_1 + L_2 + L_3$ (it is assumed that workers are not saving), such that:

$$L_1 + L_2 + L_3 = P_3 + L_3 = C_w$$

from which it follows that:

$$L_1 + L_2 = P_3$$

The wage bills of sector 1 and sector 2 are thus equal to the profits of sector 3.

We also know that the capitalists' consumption spending C_c and investment I are equal to the total gross profits $P_1 + P_2 + P_3$:

$$C_c + I = P_1 + P_2 + P_3 = P_1 + L_1 + P_2 + L_2$$

which, after some rearranging, leads to the same result as above.[28] The crucial question is then whether gross profits determine C_c + I, or the other way round. Kalecki answers as follows: "the capitalists can decide how much they will invest and consume next year, but they cannot decide how much they shall sell and profit" (Kalecki, 1968, p. 461. See also Trigg, 2002, pp. 104–114.)

The big advantage of Marx's schemes of reproduction is that they lay bare the conditions that have to be fulfilled if simple or expanded reproduction is to take place. We have already indicated that these conditions can be reduced to a set of equalities between inter-sectoral spending and reproduction needs, and to certain well-balanced transactions between the capital goods and the consumer goods sectors. Such inter-sectoral relations are analysed in detail using input-output analyses which, like the reproduction schemes, are in essence based on technologically determined proportions between required inputs and outputs. However, input-output tables are not 2 × 2 or 3 × 3 tables of highly aggregated sectors, but rather show the inter-sectoral flows at the lowest level of sectoral disaggregation. Input-output analysis was developed by Wassily Leontief, who received the Nobel Prize for Economics in 1973 in recognition of his contribution to the field. In the 1920s, Leontief was involved in the construction of the first "material tables" of inter-sectoral flows which would serve in the formulation of the first economic plans in the Soviet Union. In spite of Leontief acknowledging Marx's influence on his work, this is often given insufficient attention (see, for example, Clark, 1984). Leontief went on to construct the first input-output tables for the United States in the 1930s (Leontief, 1941).

Today, the relationship between the input-output approach and Marx's reproduction schemes is widely acknowledged and documented. An important first step was taken by Oskar Lange, who mathematically showed how Marx's two-sector schemes could be transformed into an input-output table with a large number of industrial sectors. Whereas Marx's schemes indicate the importance of balanced proportions between his two large sectors for the steady expansion of the economic system, input-output analysis generalises these conditions for an economy across a large number of sectors (Lange, 1964). At the time, Lange's arguments came under heavy attack from some orthodox Marxist-Soviet economists. In retrospect, these critics were fighting a rear-guard battle.

Since the 1960s, the theoretical elaboration of input-output analysis has led to the impressive mathematical development of Marxist economic theory. Since this is particularly relevant for the analysis of Marx's "law of value", the main results of this "analytical Marxism" will be discussed at length in subsequent chapters.

Notes

1 Marx made a thorough study of Quesnay's *Tableau économique* in the early 1860s, during which time he also made adaptions and corrections. In addition, he wrote a chapter on the subject in Friedrich Engels' *Herrn Eugen Dührings Umwälzung der Wissenschaft* of 1878, levelling severe criticism – in his typical caustic and mocking way – at those authors who had found fault with the *Tableau*. For an in-depth comparison of Quesnay's *Tableau économique* and Marx's reproduction schemes, we refer to Tsuru (1942, pp. 365ff.), which after all these years remains highly relevant.

2 This assumption is made only in the interests of simplicity. If iron ore has to be mined, not only is labour required but also all kinds of means of production which have to be produced. This takes place in a separate sphere of production "mining", which is different from the "iron" sector. Dividing sector 1 where means of production are produced into a subsector 1b which supplies only the consumer goods sector 2, and the subsector 1a which only supplies itself and the 1b sector has important and illuminating implications for the further results. The reader is referred to Lowe (1976) and the mathematical appendix by Nell (1976).

3 However, putting Marx in the "surplus tradition" of the classical school has been criticised by a number of Marxists, who emphasise that Marx in an important way broke away from the classical economists and that he severely criticised their fetishistic approach. See Howard and King (1992, pp. 295–297).

4 At this point in the text, such terminology still has no relevance. The reasons why these capital components are called constant and variable capital are revealed in Chapter 2.

5 Total output of 170 + 50 kg iron − 200 kg iron to replace the iron used up in sector 1 and sector 2. Total output of 100 kg wheat − 40 kg wheat to replace the wheat used up in sector 2.

6 We can say that no expanded reproduction is possible in the third production period, as no surplus of consumer goods was produced in the second production period. Economic growth, which our numerical example generates, will stop and be unbalanced. However, this issue is of no relevance to our current discussion and we will return to it later.

7 These are necessary but not sufficient conditions. The exact composition of the surplus product is also a necessary condition.

8 Marx was not interested in studying balanced growth, as he regarded this under capitalist conditions highly improbably (see Sardoni, 1981).

9 In Chapter 3 and later chapters we will see that these capitalists' spending is essential for the full realisation of the surplus value in the circulation process. If they spend less, the surplus value will not be fully realised.

10 The symbol Δ stands for "additional". ΔC_1 is thus the additional constant capital in sector 1.

11 From now on we will use "capital goods" and "means of production" interchangeably.

12 As Piero Sraffa and Maurice Dobb pointed out in their introduction to the monumental *Works and Correspondence* of Ricardo, this argument goes back to Ricardo's "Essay on the Influence of a Low Price of Corn on the Profits of Stock" (1815) (see Sraffa and Dobb, 1951, pp. xxx–xxxi). It is important to stress that a single-good economy which leads to the classical and Marxist surplus approach, is a useful abstraction in the analysis of the dynamics of the capitalist system as Laibman (1997) shows.

13 We assume that additional labour is available to cultivate the additional wheat.

14 In fact, Sraffa's model is not even about an expanding economic system as he assumes a completely stationary economy for the sake of the argument against the marginalist, neo-classical theory of distribution. Geoffrey Harcourt has commented that Sraffa has a snapshot at a moment of time, regardless of how the economy is changing over time. Investigating an expanding economy using a linear production model goes back to Von Neumann (1945–6).

15 We will further define the **B** matrix in Chapter 3.

16 For further elaboration on the parallelism between Marx with Robinson, we refer to Cuyvers (1979, pp. 326–348).

17 Robinson's sector definition goes back to the discussions in Cambridge in the early 1930s on Keynes's *Treatise on Money*. Later, it transpired that it was also Kalecki's sector definition.

18 We are using symbols other than W_1 and W_2 since Robinson defines a sector as vertically integrated with its suppliers, therefore differing from Marx's sector definition.

19 In Kalecki's terminology, these are gross profits.

20 In our earlier example, $2000(C_2) = 1000(V_1) + 1000(M_1)$, with $1000(M_1)$ consumed by the capitalists. If capitalist consumption is zero, we evidently have $1000(C_2) = 1000(V_1)$.

21 This is a profits realisation mechanism which we will revisit in Chapter 3.

22 Most neo-Marxist economists are either reluctant to refer to Marx's labour theory of value or they attempt to avoid it in their arguments. It seems to us that they consider Marx's methodology to be too fundamental to be dependent on a price theory, which in their eyes is unimportant. See Chapter 2, § 2.7.

23 For instance, Robinson stresses that with simple reproduction, it follows from her equalities that $Q_2 = L_1$, which leads to $Q_2/L_2 = L_1/L_2$. In other words, with equal wage rates, the ratio of the gross profits (quasi-rents) to the wages in sector 2 will be equal to the ratio of labour used in sector 1 and sector 2. This, in turn, is important to her analysis of neutral technological progress.

24 A linear production model should ultimately lead to positive outputs in all sectors. This is only possible if the capital goods sector also produces a surplus. In the relevant literature this condition is known as the Hawkins-Simon condition (see Hawkins and Simon, 1949).

25 A number of interesting papers have been published recently in the *liber amicorum* for Tadeusz Kowalik, on Kalecki's treatment of Marx's reproduction schemes and Rosa Luxemburg's analysis (see e.g. Harcourt and Kriesler, 2013; Levy-Orlik, 2013; Bellofiore, 2013).

26 "The Circus" was a group of young collaborators of Keynes comprising Richard Kahn, James Meade, Joan Robinson, Austin Robinson and Piero Sraffa. During the academic year 1930–1931, they discussed Keynes's *Treatise on Money*, after which Keynes was informed of the outcome of the discussions. Although no notes of these meetings were kept, it is generally assumed that the discussions contributed heavily to Keynes's *General Theory on Employment, Interest and Money* of 1936 (see Kahn, 1985; Robinson, 1985). After "The Circus" came to an end (March 1931) discussions continued, however, with Richard Kahn, Piero Sraffa and Joan Robinson, who closely followed the process in which Keynes's *General Theory* was created (Robinson, 1985, p. 53).

27 Interestingly it has been argued that “Keynes’s First Fundamental Equation in the *Treatise* (which gives the prices of consumer goods) derives from an intersectoral balance condition, not much different from the one in Marx’s schemes of reproduction” (Erturk, 2006, p.456).

28 $C_c + I = P_1 + P_2 + P_3$ is an ex post equality.

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2 The (mostly quantitative) labour theory of value today

One of the most contested Marxist theoretical constructions is, no doubt, the labour theory of value. In fact, Marx took that theory from the classical economists, and David Ricardo, in particular. However, he reformulated and refined it so that it became the theoretical foundation of many of his further insights into the workings of a capitalist economy and the forces that he believed shaped prices and profits in the long term.

In the first section, we examine the main propositions of Marx's theory of value. Thereafter, we construct a bridge that stretches toward an understanding of long-term prices – Marx's so-called prices of production. These are prices which, under capitalist conditions of production and in the long run, apply on average and allow the individual capitalist producers to receive a (tendentially) uniform rate of profits, or average rate of profits. Marx's transformation of his labour values into these production prices and the problems he encountered in doing so, are dealt with in the second section.

As the chapter unfolds, the relevant contributions from the literature that attempt to correct Marx's incorrect solution to the "transformation problem" are discussed. These constitute mainly formal corrections and mathematical solutions, which we illustrate through numerical examples of a simple linear production model of the input-output type. We also delve into the relationship between wages and profits, based on Piero Sraffa's "standard system", and indicate that this system represents a special kind of average sector in which all changes in wages lead – directly proportionately – to changes in profits. A standard system is also highly relevant in the subsequent chapters. Finally, we revisit the theory that we developed in the past that labour values and production prices are both prices which, on average, will apply if either the capitalist logic of profits rate equalisation or the workers' logic of equal "rates of exploitation" holds.

2.1 What is the labour theory of value saying?

In every society that has implemented a system of division of labour (farmer, blacksmith, potter, etc.) and where the commodities produced are exchanged, average exchange relations will be established between the producers. The fact that money will act as a means of exchange does not in any way change the fact that each individual selling his product, as a producer, also has to buy. For example, the farmer will sell his wheat to the miller, but will buy a plough from the blacksmith. The blacksmith, in turn, will buy his daily bread from the baker who has procured the flour from the miller, and so on. As a result of repeated exchanges, the following scenario could present itself:

1 plough = 600 kg wheat = 400 kg flour = 500 loaves = . . .

The question that the labour theory of value attempts to answer is: What exactly is equated between, for example, 1 plough and 600 kg of wheat? Marx states that in the process of exchanging such qualitatively different commodities, what is equated is the amount of labour that is directly and indirectly required to produce the respective commodities. Or, to be more precise: the labour time that is required on average under a given level of development in terms of what Marx calls the "forces of production", or else: the "average socially necessary labour time". In the above example, the labour time required to produce a plough is equal to the labour time that, on average, is spent on the cultivation and harvesting of 600 kg of wheat, etc. Moreover, not only is the labour time that is directly spent relevant, but so too is the indirect labour time of the suppliers, as well as the time consumed in the used means of production. The direct and indirect labour time that on average, in given social conditions of production, is needed to produce one unit of a product, we will call the labour value – or value, for short – of that product.

How high will the value of a plough or a kilogram of wheat be? This can be calculated if we know the average conditions for producing a plough or a kilogram of wheat. Starting from the numerical example introduced in Chapter 1:

$$\underset{\text{(tools)}}{50\,\text{kg iron}} + 50 \text{ hours of labour} \rightarrow \underset{\text{(1 plough)}}{150\,\text{kg iron}}$$

Denoting the value of 1 kg of iron by λ_1, we can now write:

$$50\,\lambda_1 + 50 \text{ hours of labour} = 150\,\lambda_1 \text{ (= value of one plough)}$$

and λ_1 can easily be found by rearranging this equation[1] such that:

$$50 \text{ hours of labour} = 150\,\lambda_1 - 50\,\lambda_1 = 100\,\lambda_1$$

$$\text{or } \lambda_1 = \frac{50}{100} \text{ hours of labour time = half an hour.}$$

The value of one plough is thus $150\,\lambda_1$ = 75 hours of labour time.

It can be shown that λ_1 equals the direct and indirect labour time necessary to produce 1 kg of iron, as follows:

In order to produce 150 kg of iron as output, 50 hours of direct labour time is needed, as well as 50 kg of iron as input. Or, per kilogram of iron output: 1/3 hour direct labour time and 1/3 kg iron input. In order to produce this 1/3 kg of iron input, 1/3 × 1/3 hours of labour time is needed, together with 1/3 × 1/3 kg of iron (= 1/9 kg of iron), etc.

By adding up all the labour time spent, the following picture emerges:

$$\frac{1}{3} \text{ hour of direct labour time} + [(\frac{1}{3} \times \frac{1}{3}) + (\frac{1}{3} \times \frac{1}{9}) + \ldots] \text{ hours of indirect labour time}$$

$$= \frac{1}{2} \text{ hour of total labour time}$$

$$= \text{labour value of 1 kg of iron}$$

$$= \lambda_1$$

Based on the average production conditions for wheat, the labour value of 1 kg of wheat can be calculated as well. If a plough can be used over, say, a three-year period and therefore transfers one-third of its value each year to the value of wheat produced, then one-third of $150\,\lambda_1 = 50\,\lambda_1 = 25$ hours of "coagulated" labour time.[2]

The farmer can cultivate 100 kg of wheat if he can also devote 50 hours of direct labour time to ploughing, planting seeds, mowing, threshing, etc., and can use 40 kg of seeding material (wheat) in the process. Denoting the labour value of 1 kg of wheat by λ_2, the value equation is:

	$50\,\lambda_1$ +	$40\,\lambda_2$ +	50 hours =	$100\,\lambda_2$
	Depreciation of the plough	value of the seeding material	direct labour time	value of the output of wheat
or:	25 +	$40\,\lambda_2$ +	50 =	$100\,\lambda_2$

or else: 75 hours of labour = $60\,\lambda_2$, such that $\lambda_2 = \frac{75}{60} = \frac{5}{4}$ hours.

In addition, it can be shown here that $\lambda_2 = 5/4$ hours of labour time is exactly equal to the sum of all direct and indirect labour necessary to produce 1 kg of wheat.

At this point, it is necessary to stress that we are assuming that one hour of the labour time needed to produce iron and one hour of the labour time needed to cultivate wheat are equal or, stated differently, that labour is homogeneous. In the real world, however, this is evidently not the case. Marx therefore assumes that concrete labour in the physical production process becomes abstract labour in the value creation process. He states very early in Volume 1 of *Das Kapital*:

> As use-values, commodities differ above all in quality, while as exchange-values they can only differ in quantity, and therefore do not contain an atom of use-value.
>
> If then we disregard the use-value of commodities, only one property remains, that of being products of labour. But even the product of labour

> has already been transformed in our hands. If we make abstraction from its use-value, we abstract also from the material constituents and forms which make it a use-value. It is no longer a table, a house, a piece of yarn or any other useful thing. All its sensuous characteristics are extinguished. Nor is it any longer the product of the labour of the joiner, the mason or the spinner, or of any other particular kind of productive labour. With the disappearance of the useful character of the products of labour, the useful character of the kinds of labour embodied in them also disappears; this in turn entails the disappearance of the different concrete forms of labour. They can no longer be distinguished, but are all together reduced to the same kind of labour, human labour in the abstract.
>
> (Marx, 1976, p. 128)

A couple of pages further on in Volume 1, Marx writes:

> Use-values cannot confront each other as commodities unless the useful labour contained in them is qualitatively different in each case. In a society whose products generally assume the form of commodities, i.e. in a society of commodity producers, this qualitative difference between the useful forms of labour which are carried on independently and privately by individual producers develops into a complex system, a social division of labour.
>
> (Marx, 1976, p. 133)

In addition, skilled labour is thus reduced to abstract labour:

> More complex labour counts only as intensified, or rather multiplied simple labour, so that a smaller quantity of complex labour is considered equal to a larger quantity of simple labour. Experience shows that this reduction is constantly being made. A commodity may be the outcome of the most complicated labour, but through its value it is posited as equal to the product of simple labour, hence it represents only a specific quantity of simple labour. The various proportions in which different kinds of labour are reduced to simple labour as their unit of measurement are established by a social process that goes on behind the backs of the producers; these proportions therefore appear to the producers to have been handed down by tradition.
>
> (Marx, 1976, p. 135)[3]

For an economist, it nevertheless remains an enigma that concrete labour can become abstract labour in the value creation process. How can an hour of labour of a blacksmith become comparable to an hour of labour of a farmer? From the perspective of the economic model builder, it is not sufficient to say that this homogenisation is "established by a social process that goes on behind the backs of the producers".[4] We will avoid this discussion by simply assuming throughout that all labour is qualitatively the same, i.e. homogeneous.

Mathematically, the labour values of wheat and iron in our numerical example are the solutions in a system of linear equations:[5]

$$50\,\lambda_1 \qquad\qquad + 50 = 150\,\lambda_1$$

$$50\,\lambda_1 + 40\,\lambda_2 + 50 = 100\,\lambda_2$$

This is the linear production model that we will use in this and the following chapters, although sometimes with other numerical examples. This model will allow us not only to "translate" many of Marx's theses into mathematical "language", but to investigate them in more depth. Moreover, it is important to stress that this model represents a mini-scheme of reproduction where, under capitalist conditions of production, $50\,\lambda_1$ is the constant capital C_1 of sector 1 (iron production). Marx calls it constant capital since its value is transferred as such, without any additional value, to the value of the output of the sector being considered. Also, in the above example, 50 represents the added value in the same sector, expressed in labour time. We will see that this added value, expressed in hours of labour time, consists of variable capital V_1 and surplus value M_1. Marx's variable capital (again expressed in labour time) is what the capitalist producers invest in wages. It is called variable since, as we will see, the investment allows the capitalists to extract surplus value – profits – from labour.

The same holds for sector 2 (wheat production): $(50\,\lambda_1 + 40\,\lambda_2) = C_2$ is the constant capital deployed in sector 2, and $50 = V_2 + M_2$ is the value added in the sector. *Exchange of labour values is an exchange of equivalents and allows a balanced reproduction of the production process.*

There is more. A change in the ratio of V to M (the wages to the surplus value/profits) in a given production process – Marx calls this ratio the rate of exploitation or the rate of surplus value – has no effect whatsoever on the labour value of what is produced. Whether the 50 hours of direct labour time – which in sector 2 is added to the value of the constant capital used – consists of 10 hours of "necessary labour" (the total necessary consumption of the workers expressed in labour value = V_2) and 40 hours of surplus labour (or surplus value = M_2), or whether it consists of 30 hours of necessary labour and 20 hours of surplus labour is irrelevant for the determination of the labour value of the output. Wheat output based on the spending of 50 hours of direct labour remains 100 kg of wheat and has a unit labour value of $\lambda_2 = 5/4$ hours. *Hence, labour values are not influenced by the distribution of the value added between wages and profits. They are "distribution free".*

The above linear production model can be written as a scheme of reproduction, the methodological innovation of which Marx introduced in Volume 2 of *Das Kapital* to analyse the conditions of capitalist reproduction (see Chapter 1), as follows:

$$C_1 + V_1 + M_1 = 150\,\lambda_1 = W_1$$

$$C_2 + V_2 + M_2 = 100\,\lambda_2 = W_2$$

Or per unit of output:

$$c_1 + v_1 + m_1 = \lambda_1$$
$$c_2 + v_2 + m_2 = \lambda_2$$

with $c_1 = C_1/150$ and $c_2 = C_2/100$, $v_1 = V_1/150$ and $v_2 = V_2/100$, and $m_1 = M_1/150$ and $m_2 = M_2/100$.

If we assume for simplicity's sake that all labour is equal in quality and leads to the same necessary consumption per unit of labour, then uniform rates of surplus value will apply over the sectors. We take up the example presented in Chapter 1, where on average 0.5 kg of wheat per hour of labour time spent is needed to maintain the worker and his family according to the usual standard of living. For 50 hours of labour, the worker must receive 25 kg of wheat. The value of 0.5 kg of wheat is: $0.5\,\lambda_2 = 5/8$ hours, which Marx calls the value of labour power. In the iron production sector, the necessary labour time is:

$$50 \text{ hours} \times 0.5\,\lambda_2 = 31.25 = V_1$$

and the surplus labour time is:

$$50 \text{ hours (value added)} - 31.25 \text{ hours } (V_1) = 18.75 = M_1$$

The rate of surplus value is then $M_1/V_1 = 18.75/31.25 = 0.6$. This rate is identical to that in the wheat production sector. We therefore denote this uniform rate by σ. Thus:

$$C_1 + (1 + \sigma)\,V_1 = 150\,\lambda_1$$
$$C_2 + (1 + \sigma)\,V_2 = 100\,\lambda_2$$

These equations can be rewritten per unit of output as:

$$c_1 + (1 + \sigma)\,v_1 = \lambda_1$$
$$c_2 + (1 + \sigma)\,v_2 = \lambda_2$$

In both sectors, the surplus value is what remains after the capitalists have replaced the value of their used-up constant capital and paid their workers the value of the necessary consumption. The capitalists appropriate this surplus value as owners of the means of production and *not* as direct participants in the production process. The 50 hours of direct labour time are performed by the workers, not by the capitalists. However, the labour value of the necessary consumption (= necessary labour time) is less than 50 hours. The difference is the surplus labour which, under capitalist conditions of production, is "due" to the capitalists. Thus, the labour theory of value not only explains how equivalent values are exchanged, but also that unpaid labour – ergo exploitation – is the source of the wealth of the ruling classes, both today and in the distant past (capitalists, feudal lords and slave owners). In capitalism, the surplus value is the source of the profits, received by the capitalists, whereas wage earners earn a wage.[6]

2.2 The transformation problem

In this section, we investigate the relationship between labour values and long-period normal prices of production. First, we discuss Marx's views on how to transform labour values into production prices.[7] We show that Marx's transformation procedure is incorrect and then compare his solution with what is mathematically correct.

2.2.1 Marxian prices of production

If the exchange of goods or services is at labour values, the surplus value in each sector will be in the same proportion (σ) to the variable capital that is used in production (the wage bill). This proportion is, however, of no interest to the capitalists, who are rather interested in the return on the total invested capital, i.e. the ratio of the surplus value to the constant and variable capital. If the capitalists in sector 1 obtain a higher return than those in sector 2, the latter will start to invest capital in iron production. When the reproduction needs (the demand for iron and wheat) remain the same, the price of iron will drop compared to the price of wheat. The capital inflow from sector 2 into sector 1 will result in the production of more means of production than are needed and the price of these means of production will drop.

Conversely, the price of consumer goods that are produced in sector 2 will rise since less capital is used, thus leading to a smaller output of consumer goods against an unchanged level of demand. The capital inflows in sector 1 and the resulting price adjustments will continue until both sectors yield the same return on invested capital. This return r is the general (or average) rate of profits. Marx calls the corresponding p_1 and p_2 the prices of production.

It thus holds that:

$$\mathrm{C}_1 + \mathrm{V}_1 + r(\mathrm{C}_1 + \mathrm{V}_1) = 150\, p_1$$
$$\mathrm{C}_2 + \mathrm{V}_2 + r(\mathrm{C}_2 + \mathrm{V}_2) = 100\, p_2$$

Since, according to Marx, total profits are equal to the total surplus value:

$$r(\mathrm{C}_1 + \mathrm{V}_1) + r(\mathrm{C}_2 + \mathrm{V}_2) = \mathrm{M}_1 + \mathrm{M}_2$$

it follows that:

$$r = \frac{\mathrm{M}_1 + \mathrm{M}_2}{\mathrm{C}_1 + \mathrm{V}_1 + \mathrm{C}_2 + \mathrm{V}_2}$$

It is important to realise that the equality of total surplus value and total profits does *not* hold at the sector level. In other words, $r(C_1 + V_1)$ is not equal to M_1; nor is $r(C_2 + V_2)$ equal to M_2. In the normal case where the sectoral proportions C_1/V_1 and C_2/V_2 differ, the prices of production will bring about a difference between sectoral profits and sectoral surplus value. The total surplus value $(M_1 + M_2)$ is thus redistributed over the two sectors.

In terms of the above example:

$C_1 = 50\,\lambda_1 = 25$ $\qquad$ $C_2 = 50\,\lambda_1 + 40\,\lambda_2 = 75$

$V_1 = 50 \times 0.5\,\lambda_2 = 31.25$ $\qquad$ $V_2 = 50 \times 0.5\,\lambda_2 = 31.25$

$M_1 = 18.75$ $\qquad$ $M_2 = 18.75$

$$r = \frac{M_1 + M_2}{C_1 + V_1 + C_2 + V_2} = \frac{18.75 + 18.75}{25 + 31.25 + 75 + 31.25} = 23.1\%$$

$r\,(C_1 + V_1) = 23.1\% \times 56.25 = 13$ $\qquad$ $r\,(C_2 + V_2) = 23.1\% \times 106.25 = 24.54$

$(C_1 + V_1) + r\,(C_1 + V_1) = 56.25 + 13 = 150\,p_1$

and $p_1 = 0.46$ hours

$(C_2 + V_2) + r\,(C_2 + V_2) = 106.25 + 24.5 = 100\,p_2$

and $p_2 = 1.31$ hours.

Evidently, the total value added is unchanged (since the total surplus value and the total variable capital remained the same) and is equal to the *total* direct labour time = 100 hours.

This example illustrates how the total surplus value is distributed over the two sectors. In the sector with a higher C/V (what Marx calls the “value composition of capital”, discussed in more detail in Chapter 5), profits will be higher than the surplus value produced, while in the sector with lower C/V, profits will be lower than the surplus value produced. In the general case where there are many sectors, according to Marx, profits will be equal to the total surplus value produced in the sector where C/V is average (Marx, 1981, p. 264).

2.2.2 Production prices proper

The above procedure involving transforming labour values into prices of production is the one that Marx adopts in Volume 3 of *Das Kapital*. He mentions that his procedure is not completely correct, since *only* outputs and not the inputs are expressed in prices of production:

> It was originally assumed that the cost price of a commodity equalled the *value* of the commodities consumed in its production. But for the buyer of a commodity, it is the price of production that constitutes its cost price and can

> thus enter into forming the price of another commodity. (. . .) It is necessary to bear in mind this modified significance of the cost price, and therefore to bear in mind too that if the cost price of a commodity is equated with the value of the means of production used up in producing it, it is always possible to go wrong. Our present investigation does not require us to go into further detail on this point.
>
> (Marx, 1981, pp. 264–265)

Yet, with some knowledge of linear algebra and preliminary manipulation of the production data, it is not particularly difficult to calculate the correct prices of production (production prices proper) in the above example.

We already know that in the above numerical example, the following technologically determined relationships hold:

$$
\begin{array}{lll}
50\,\text{kg iron} & & +\ 50\text{ hours labour} \rightarrow 150\,\text{kg iron} \\
\underline{50\,\text{kg iron}} & +\ \underline{40\,\text{kg wheat}} & +\ 50\text{ hours labour} \rightarrow 100\,\text{kg wheat} \\
100\,\text{kg iron} & \ \ 40\,\text{kg wheat} &
\end{array}
$$

The "net product" of this economy therefore consists of 50 kg of iron (150 kg output − 100 kg inputs) and 60 kg of wheat (100 kg output − 40 kg inputs). We also know that for each hour of labour time spent, half a kilogram of wheat is needed for the normal reproduction of the worker and his family, i.e. 25 kg of wheat in sector 1 and 25 kg of wheat in sector 2. Combining the above technological information on material input requirements with that on the required inputs of necessary consumption leads to the following relationships:

$$
\begin{array}{ll}
50\,\text{kg iron} & +\ 25\,\text{kg wheat} \rightarrow 150\,\text{kg iron} \\
\underline{50\,\text{kg iron}} & +\ \underline{65\,\text{kg wheat}} \rightarrow 100\,\text{kg wheat} \\
100\,\text{kg iron} & \ \ 90\,\text{kg wheat}
\end{array}
$$

What remains of the "net product" after satisfying the socially necessary consumption of the workers Marx calls the surplus product. In the above example, this surplus product consists of 50 kg of iron (150 – 100) and 10 kg of wheat (100 – 90).

The system of production prices proper can now be written as follows:

$$(50\,p_1 + 25\,p_2)\,(1 + r) = 150\,p_1$$

$$(50\,p_1 + 65\,p_2)\,(1 + r) = 100\,p_2$$

with p_1 and p_2 the production prices and r the uniform rate of profits. If we want to express these prices in hours of labour time, comparable with labour values, the following equality is added, which indicates the equality of the total direct labour spent and the total value of the net product:

$$100\text{ hours of labour} = 50\,p_1 + 60\,p_2$$

Without going into exactly how to solve this system of equations mathematically, the economically sensible solution is:

$r = 21.8\%$

$p_1 = 0.437$ hours (according to Marx's procedure: 0.46 hours)

$p_2 = 1.302$ hours (according to Marx's procedure: 1.31 hours)

Thus, the procedure followed by Marx leads to only an approximately correct solution, and we could ask how labour values and production prices can be linked and whether such a link is possible at all. This issue is known in the economic literature as the transformation problem, and since the end of the nineteenth century has constituted a Calvary for Marxist economists.[8]

2.2.3 The mathematical formulation of values and prices

Until now, we have used numerical examples to elucidate the most important propositions of Marx's theory of value. We will have to leave this approach temporarily in the interests of arriving at a better understanding of the discussions on the transformation problem.

It was explained above that, viewed from a mathematical perspective, the basics of Marx's theory of value can be expressed as systems of equations, where the solution consists of a set of labour values or prices of production. We will now return to our system of labour values:

$$\begin{aligned} 50\,\lambda_1 \qquad\qquad\;\; + 50 &= 150\,\lambda_1 \\ 50\,\lambda_1 + 40\,\lambda_2 + 50 &= 100\,\lambda_2 \end{aligned}$$

which, per unit of output, leads to the system (1):

$$\begin{aligned} \frac{50}{150}\lambda_1 + \frac{50}{150} &= \lambda_1 \\ \frac{50}{100}\lambda_1 + \frac{40}{100}\lambda_2 + \frac{50}{100} &= \lambda_2 \end{aligned} \tag{1}$$

Using matrix algebra, the value system (1) is written as:

$$\begin{bmatrix} \frac{50}{150} & 0 \\ \frac{50}{100} & \frac{40}{100} \end{bmatrix} \times \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} + \begin{pmatrix} \frac{50}{150} \\ \frac{50}{100} \end{pmatrix} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}$$

or, in general:

$$\mathbf{A}\boldsymbol{\lambda} + \mathbf{l} = \boldsymbol{\lambda}$$

with: **A**: a square matrix of technologically necessary inputs of means of production per unit of output, $\begin{bmatrix} \frac{50}{150} & 0 \\ \frac{50}{100} & \frac{40}{100} \end{bmatrix}$

$\boldsymbol{\lambda}$: a column vector of labour values λ_1 and λ_2, $\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}$

$\mathbf{l}$: a column vector of technologically necessary direct labour inputs per unit of output, $\begin{pmatrix} \frac{50}{150} \\ \frac{50}{100} \end{pmatrix}$.

This system of equations can be solved for the vector $\boldsymbol{\lambda}$ of labour values after the following transformations:

$$\mathbf{l} = (\boldsymbol{\lambda} - \mathbf{A}\,\boldsymbol{\lambda})$$

$$\text{or } \mathbf{l} = (\mathbf{I} - \mathbf{A})\,\boldsymbol{\lambda}$$

with **I** the unit matrix:

$$\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

in the above example with two spheres of production.

By bringing **(I – A)** to the other side, we find the solution to the vector $\boldsymbol{\lambda}$. In simple (so-called scalar) algebra, this is done by dividing both sides of the equations by **(I – A)**. The equivalent in matrix algebra is to pre-multiply both sides by the matrix $(\mathbf{I} - \mathbf{A})^{-1}$, such that $(\mathbf{I} - \mathbf{A})^{-1}(\mathbf{I} - \mathbf{A}) = \mathbf{I}$.[9] This leads to the solution:

$$(\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{l} = \boldsymbol{\lambda}$$

Since the elements of **A** are input coefficients, it can be shown that:

$$(\mathbf{I} - \mathbf{A})^{-1} = (\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \ldots), \text{ such that}$$

$$(\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \ldots)\,\mathbf{l} = \boldsymbol{\lambda},$$

$$\text{or else: } \mathbf{l} + \mathbf{A}\,\mathbf{l} + \mathbf{A}^2\mathbf{l} + \mathbf{A}^3\mathbf{l} + \ldots = \boldsymbol{\lambda}$$

This solution to $\boldsymbol{\lambda}$ expresses what we previously showed numerically: each labour value is the sum of the necessary direct labour time per unit of output ($\mathbf{l}$) and the necessary labour time that is contained in the directly required means of production, in the means of production to produce these directly required means of production, etc. $(\mathbf{A}\mathbf{l} + \mathbf{A}^2\mathbf{l} + \mathbf{A}^3\mathbf{l} + \ldots)$ (see Georgescu-Roegen, 1950, p. 217; Cameron, 1952, pp. 191–197).

Turning now to the system of production prices:[10]

$$(50\,p_1 + 25\,p_2)\,(1 + r) = 150\,p_1$$
$$(50\,p_1 + 65\,p_2)\,(1 + r) = 100\,p_2$$

or per unit of output:

$$\left(\frac{50}{150}\,p_1 + \frac{25}{150}\,p_2\right)(1 + r) = p_1 \qquad (2)$$
$$\left(\frac{50}{100}\,p_1 + \frac{65}{100}\,p_2\right)(1 + r) = p_2$$

In matrix form (2) is:

$$\begin{bmatrix} \frac{50}{150} & \frac{25}{150} \\ \frac{50}{100} & \frac{65}{100} \end{bmatrix} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} (1+r) = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$$

or in matrix notation:

$$\mathbf{B}\,\mathbf{p}\,(1 + r) = \mathbf{p}$$

with **p** the column vector of prices of production.

The reader will notice that Matrix **B** is nothing but Matrix **A**, to which is added in the column corresponding with the necessary wheat inputs per unit of output (0 and 40/100 kg, respectively) the necessary wheat consumption for the workers (25 kg of wheat for 50 hours of labour, or 25/150 kg per unit of output in both sector 1 and sector 2).

In addition, the reader will recall that the value of labour power corresponds with half a kilogram of wheat per hour of labour time performed, but that it contains no iron:

$$\begin{pmatrix} 50 \text{ hours} \\ 50 \text{ hours} \end{pmatrix} \times \begin{pmatrix} 0\,\text{kg wheat} & 0.5\text{kg wheat} \end{pmatrix}$$

or per unit of output:

$$\begin{pmatrix} \frac{50}{150} \\ \frac{50}{100} \end{pmatrix} \begin{pmatrix} 0 & 0.5 \end{pmatrix} = \begin{bmatrix} 0 & \frac{25}{150} \\ 0 & \frac{25}{100} \end{bmatrix}$$

This is in matrix notation: **l c** (**c** is the row vector of necessary consumption per hour of labour time). It thus holds that:

$$\mathbf{B} = \mathbf{A} + \mathbf{l}\,\mathbf{c}$$

$$\text{and} \quad (\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\mathbf{p}\,(1 + r) = \mathbf{p}$$

Mathematically, if r is calculated (a somewhat long-winded procedure which consists of finding the largest root $1/(1 + r)$ of a power function; only this root is associated with non-negative prices),[11] the production prices system can be solved. This system can also be decomposed into:

$$\mathbf{A}\,\mathbf{p} + \mathbf{l}\,\mathbf{c}\,\mathbf{p} + r\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\mathbf{p} = \mathbf{p}$$

with *per unit of output* and in prices **p**:

$\mathbf{A}\,\mathbf{p}$:	the constant capital of the respective sectors
$\mathbf{l}\,\mathbf{c}\,\mathbf{p}$:	the respective sectoral variable capital
$r\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\mathbf{p}$:	the profits on the invested capital, based on the rate of profits r.

The reader will now better see the difference with Marx's procedure in calculating **p**, i.e.

$$\mathbf{l}\,\boldsymbol{\lambda} + \mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} + r\,(\mathbf{l} + \mathbf{l}\,\mathbf{c})\,\boldsymbol{\lambda} = \mathbf{p}$$

In the next section we investigate the relationship between the production prices proper and the labour values or, in other words, the relationship between vector **p** and vector $\boldsymbol{\lambda}$ in the general case of n sectors of production ($n \geq 2$).[12]

2.3 Elements of a formal solution to the transformation problem: corrections of Marx's procedure

This section deals with developments in respect of finding the solution to the transformation problem in the second half of the last century. The reader will remember that the labour value of a good or service is made up of all labour time spent on its production, either directly or indirectly. The section shows that the same holds for prices of production, albeit that the labour time inputs are "weighted". Next, we elaborate on Piero Sraffa's contribution to solving the transformation problem. Finally, we show that Marx's procedure for calculating prices of production is, in fact, the first step in an iterative process of adjusting labour values, leading to the production prices proper.

2.3.1 Prices of production as the sum of dated labour time

In section 2 it was stated – however, without mathematical proof – that the labour value of each good or service can be considered to be the sum of the labour time spent on its production and that the vector of labour values $\boldsymbol{\lambda}$ could be written as:

$$(\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \ldots)\, \mathbf{l} = \boldsymbol{\lambda},$$

or else: $$\mathbf{l} + \mathbf{A}\,\mathbf{l} + \mathbf{A}^2\,\mathbf{l} + \mathbf{A}^3\,\mathbf{l} + \ldots = \boldsymbol{\lambda},$$

i.e. for each $\boldsymbol{\lambda}$ as the sum of all direct ($\mathbf{l}$) and indirect labour ($\mathbf{A}\,\mathbf{l} + \mathbf{A}^2\,\mathbf{l} + \mathbf{A}^3\,\mathbf{l} + \ldots$) necessary to produce the good or service at issue. The objective is now to construct an analogous expression for the vector of production prices **p**. This was Piero Sraffa's feat in his slim but ground-breaking book *Production of Commodities by Means of Commodities* (1960).

Sraffa's procedure uses the wage rate ω as "numéraire", i.e. a unit to express the prices of production (labour values are measured and expressed in units of labour time). Knowing that **c** represents the vector of workers' necessary consumption per hour of labour time performed, the wage rate ω is therefore the necessary consumption per hour of labour time, valued in prices. Thus:

$$\mathbf{c}\,\mathbf{p} = \omega = 1$$

The system of production prices:

$$(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\mathbf{p}\,(1 + r) = \mathbf{p}$$

or $$\mathbf{A}\,\mathbf{p}\,(1 + r) + \mathbf{l}\,\mathbf{c}\,\mathbf{p}\,(1 + r) = \mathbf{p}$$

becomes:

$$\mathbf{A}\,\mathbf{p}\,(1 + r) + \mathbf{l}\,(1 + r) = \mathbf{p} \tag{3}$$

(since $\mathbf{c}\,\mathbf{p} = 1$).

This leads to:

$$\mathbf{l}\,(1 + r) = \mathbf{p} - \mathbf{A}\,\mathbf{p}\,(1 + r) = [\mathbf{I} - \mathbf{A}(1 + r)]\,\mathbf{p}$$

and after post-multiplication of both sides with $[\mathbf{I} - \mathbf{A}(1 + r)]^{-1}$ to:

$$[\mathbf{I} - \mathbf{A}(1 + r)]^{-1}\,\mathbf{l}\,(1 + r) = \mathbf{p}$$

or, analogously, with the expression of labour values as a sum of direct and indirect labour time:

$$[\mathbf{I} + \mathbf{A}(1 + r) + \mathbf{A}^2(1 + r)^2 + \mathbf{A}^3(1 + r)^3 + \ldots]\,\mathbf{l}\,(1 + r) = \mathbf{p}$$

Prices of production can thus be reduced to a sum of direct and indirect labour time spendings, with the spendings in the past weighted by a factor that is higher the further back we go in time by considering earlier production stages:

$$\mathbf{l}(1 + r) + \mathbf{A}\,\mathbf{l}(1 + r)^2 + \mathbf{A}^2\,\mathbf{l}(1 + r)^3 + \mathbf{A}^3\,\mathbf{l}(1 + r)^4 + \ldots = \mathbf{p}$$

This is in contrast to labour values where no weighting is applied. In prices of production, the indirect labour incorporated into the used means of production is weighted more heavily than the indirect labour that passes through few stages. The logic is that in prices, the profits on the labour spendings of the successive stages are accumulated: five hours of labour time spent three periods or stages ago will have yielded a profit during these three periods. If the average rate of profits is $r = 20\%$, this spending of labour time will represent in the price of production: 5 hours $\times (1.20)^3 = 8.64$ hours. With every transfer of this amount of labour time to the next production stage, an additional profit charge will accrue. The successive charges are cumulative as if each spending of labour time is put out at a compound interest rate r. An actuary will recognise the production price as the "end value" of "deposit" of labour time in the successive production and supply stages, each of which is put out at an interest rate r. Evidently this is in contrast to labour values where the labour time of all stages is simply added up.

For the transformation of labour values into prices of production, the time structure of the indirect labour input supplies is clearly of the utmost importance. In contrast to what Marx assumed and can be deduced from his transformation procedure (see section 2.2.2), the sectoral inequalities of the capital intensity (the physical inputs of means of production per worker, or what Marx calls the organic composition of capital) only partially explain the deviations of prices of production from labour values. That the means of production are, in turn, produced in sectors with high or low capital intensity also plays a role, as does the capital intensity of the suppliers of the suppliers, etc. It can thus be concluded that it is not the "direct" capital intensity (i.e. of the sector being considered) but rather the "direct and indirect" capital intensity that can be held responsible for the deviations in prices of production from labour values. Sraffa's formula of $\mathbf{p}$ indicates that the transformation problem has a solution, although a very complex one.

2.3.2 Marx's "average sector" and Sraffa's "standard system"

Apart from transforming undated (unweighted) into dated (or weighted with a compound rate of profits) labour time spendings, did Sraffa's *Production of Commodities by Means of Commodities* make a further contribution to the formal solution of the transformation problem? Sraffa constructed a set (vector) of weights that, when applied to the output scales of the economy, lead to an imaginary "sub-system" – Sraffa's standard system – in which there is a strict linear relationship between the wage rate (if paid at the end of each production period) and the rate of profits (see Figure 2.1).

Sraffa's aim was to offer a solution to the theoretical problem with which David Ricardo had struggled, i.e. to find a unit of measurement of prices such that with a change in the wage rate in all sectors of the economy, its effect on each price can be unambiguously distinguished from the induced effect on the prices of the supplying sectors. It appears that Sraffa's standard system provided such an "invariable measure of value". Ronald Meek (1961, pp. 177ff.;

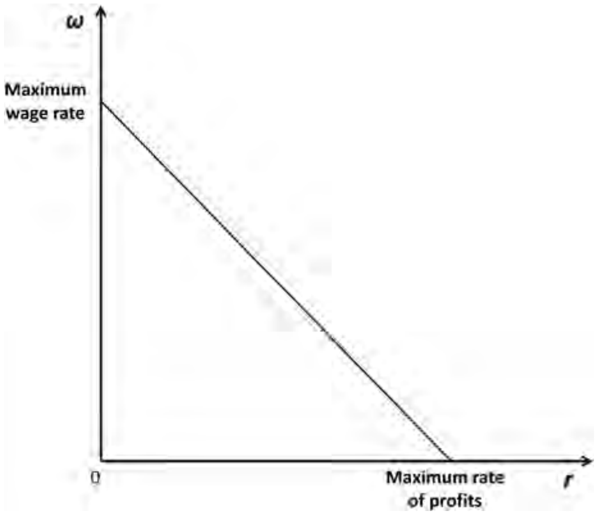

Figure 2.1 Linear relationship between the wage rate and the rate of profits in Sraffa's standard system

see also Dobb, 1961, p. 490)[13] pointed out that this imaginary "sub-system" corresponds, in fact, with Marx's proposed "sector with average organic composition of capital" in terms of which profits (as redistributed surplus value) and surplus value are equal, as well as the rate of profits and the ratio of surplus value to the value of the invested capital, *and* the value of output expressed in labour values and prices of production.[14]

In Volume 3 of *Das Kapital,* Marx gives the following example to illustrate the role of this average sector:

> How these capitals function after the average rate of profit is established, on the assumption of one turnover in the year, is shown by the following table, in which capital I represents the average composition, with an average rate of profit of 20 per cent.
>
> I. 80c + 20v + 20s. Rate of profit = 20 per cent. Price of the product = 120. Value = 120.
> II. 90c + 10v + 10s. Rate of profit = 20 per cent. Price of the product = 120. Value = 110.
> III. 70c + 30v + 30s. Rate of profit = 20 per cent. Price of the product = 120. Value = 130.
>
> Commodities produced by capital II thus have a value less than their price of production, and those produced by capital III have a price of production less than their value. Only for capitals such as I, in branches of production whose

> composition chanced to coincide with the social average, would the value and the price of production be the same.
>
> (Marx, 1981, p. 264)

It now appears that no such "social average" can be assumed to exist and that Sraffa's standard system, in fact, represents this "social average". It re-proportions, as a thought experiment, the production scale of the sectors of the economy, such that in *each* sector the same proportion (1 + R) will be found between the total quantity of output produced and what is supplied as input to the own and the other sectors. R is called the "Standard Ratio".

The reader will remember from Chapter 1 that in our imaginary economy, 150 kg of iron and 100 kg of wheat were produced. Taking the necessary consumption of the workers in the respective sectors into account, Sraffa's standard system was in a matrix notation:

$$\mathbf{q}\,\mathbf{B}\,(1+R) = \mathbf{q}$$

$$\text{or else:}\quad \mathbf{q}\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,(1+R) = \mathbf{q} \tag{4}$$

to which the scaling equality was added:

$$\mathbf{q}\,\mathbf{l} = 100$$

It now appears that Sraffa's standard system is the "dual" of the prices of production system:

$$(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\mathbf{p}\,(1+r) = \mathbf{p}$$

$$\mathbf{c}\,\mathbf{p} = 1$$

When the equations of the standard system in our imaginary economy were solved, it was found that:

R = 21.8%

q_1 = 123.8 kg of iron (instead of 150 kg)

q_2 = 118.5 kg of wheat (instead of 100 kg).

The importance of Sraffa's standard system for the transformation problem resides in the profits, as well as the value of output, being calculated on the basis of labour values and prices of production being equal. Put differently: in this standard system, the transformation of labour values into prices of production leaves the value of total output in both price systems unchanged, including the surplus value/profits. In addition, the rate of profits in the production prices system is equal to that calculated based on labour values.

Starting from the labour values system:

$$\mathbf{A}\,\boldsymbol{\lambda} + \mathbf{l} = \boldsymbol{\lambda},$$

which can be proven as follows:

Assuming a uniform rate of surplus value σ in the various sectors, the labour value system can be rewritten as:

$$\mathbf{A}\,\boldsymbol{\lambda} + \mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} + \sigma\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} = \boldsymbol{\lambda}$$

The total labour value of output in the standard system is equal to:[15]

$$\mathbf{q}\,\boldsymbol{\lambda} = \mathbf{q}\,\mathbf{A}\,\boldsymbol{\lambda} + \mathbf{q}\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} + \sigma\,\mathbf{q}\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} = \mathbf{q}\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,(1 + R)\,\boldsymbol{\lambda}$$

or

$$\mathbf{q}\,\boldsymbol{\lambda} = \mathbf{q}\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\boldsymbol{\lambda} + \sigma\,\mathbf{q}\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} = \mathbf{q}\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\boldsymbol{\lambda} + R\,\mathbf{q}\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\boldsymbol{\lambda},$$

such that also: $\sigma\,\mathbf{q}\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} = R\,\mathbf{q}\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\boldsymbol{\lambda} = \mathbf{S}\,\boldsymbol{\lambda},$

i.e. the total surplus value is equal to the total value of the surplus products, with $R\,\mathbf{q}\,(\mathbf{A} + \mathbf{l}\,\mathbf{c}) = \mathbf{S}$ being the row vector of the surplus products.

On the other hand, the total output value expressed in prices of production and total profits in the standard system are:[16]

$$\mathbf{q}\,\mathbf{p} = \mathbf{q}\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,(1 + R)\,\mathbf{p} = \mathbf{q}\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,(1 + r)\,\mathbf{p},$$

such that: $R\,\mathbf{q}\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\mathbf{p} = r\,\mathbf{q}\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\mathbf{p} = \mathbf{S}\,\mathbf{p}.$

It thus holds that:

$$r = R = \frac{\sigma\,\mathbf{q}\,\mathbf{c}\,\boldsymbol{\lambda}}{\mathbf{q}\left(\mathbf{A} + \mathbf{l}\,\mathbf{c}\right)\boldsymbol{\lambda}} = \frac{\text{surplus value standard system}}{\left(\mathrm{C} + \mathrm{V}\right)\text{standard system}}$$

Accepting the condition that the transformation of labour values into prices of production should not change the value of the surplus product in the standard system, it also holds that:

$$\sigma\,\mathbf{q}\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} = r\,\mathbf{q}\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\mathbf{p} = \mathbf{S}\,\mathbf{p},$$

and therefore that:

$$R\,\mathbf{q}\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\boldsymbol{\lambda} = R\,\mathbf{q}\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\mathbf{p},$$

as well as: $\mathbf{q}\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\boldsymbol{\lambda} = \mathbf{q}\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\mathbf{p},$

and: $\mathbf{q}\,\boldsymbol{\lambda} = \mathbf{q}\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\boldsymbol{\lambda} + \sigma\,\mathbf{q}\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} = \mathbf{q}\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\mathbf{p} + R\,\mathbf{q}\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\mathbf{p} = \mathbf{q}\,\mathbf{p}.$

These equalities show that Sraffa's standard system allows a perfect transformation of labour values into prices of production and that it is a generalised version of Marx's "sector with average organic composition of capital". It should be clear, however, that this "average sector" is not a specific sector but rather a weighted average of all sectors that are producing means of production and/or necessary consumer goods and services (see also Morishima, 1973, Ch. 7 and Ch. 12; Eatwell, 1975; Medio, 1972).

Of course, if we do not mind that total output and the surplus product, when measured in prices of production, differ from their counterparts expressed in labour values, there is no need to apply a Sraffa-like standard system to the transformation problem or to assume for analytical purposes – as we frequently do in subsequent chapters – that the capitalists' consumption can be neglected and that the economy is on its balanced growth path. This opinion is expressed clearly by Pierangelo Garegnani in his discussion on Marx's procedure for calculating prices of production:

> It is not difficult to see where lay the fault in the notion of a redistribution of surplus value (. . .). Unlike the 5 sacks of corn which do not change in size relative to the 10 people in the course of the redistribution, the size of the social surplus value does so change relative to capital. This surplus value is in fact the price of production of the surplus product, and cannot but change relative to that of social capital when, with the redistribution of surplus value, relative "prices" in general come to diverge from relative "values". As we saw (. . .) the profit rate is but the relative value of those two composite commodities and it will not be equal to the ratio between the quantities of labour embodied in them any more than the relative price of any two commodities.
>
> (Garegnani, 1984, p. 308)

2.3.3 Prices of production as the outcome of iterative adjustments, starting from labour values

We showed that in Volume 3 of *Das Kapital* Marx calculates prices of production, starting from the inputs in the production processes expressed in labour values to which he applies a uniform rate of profits M/(C + V). Although Marx's procedure was not correct (and he was aware of this), the same procedure was followed afterwards. Michio Morishima demonstrated, with some necessary assumptions (1974), that:

- A standard system à la Sraffa (or an outputs vector) is obtained as a step in a process of iterations.

  In our notation, this means that given:

$$\mathbf{q}_t = \frac{\mathbf{q}_{t-1}\lambda}{\mathbf{q}_{t-1}(\mathbf{A} + \mathbf{l}\,\mathbf{c})\lambda} \times \mathbf{q}_{t-1}(\mathbf{A} + \mathbf{l}\,\mathbf{c}),$$

we can go back after t – 1 steps to:

$$q = q_0 = \frac{\mathbf{q_0}\,\boldsymbol{\lambda}}{\mathbf{q_0(A + l\ c)}\boldsymbol{\lambda}} \times \mathbf{q_0(A + l\ c)}$$

It holds that $\mathbf{q_{t-1}}\lambda/(\mathbf{q_{t-1}(A + l\ c)}\lambda)$ is the Marxian rate of profits factor in period t –1 and that $\mathbf{q}\lambda/(\mathbf{q(A + l\ c)}\lambda) = 1 + r = 1 + R$. The Marxian rate of profits thus converges to the real rate of profits in an economic system which is expanding at the maximum rate of growth *R* (i.e. if there is no capitalist consumption of the surplus product).

- For a given rate of profits *r* in a standard system à la Sraffa (or an economic system along a balanced maximum growth path), prices of production are the result of an infinite adjustment process which starts with labour values.

  Stated in our matrix notation, this implies that given:

  $$\mathbf{p_t} = (1 + r)\ \mathbf{(A + l\ c)p_{t-1}}$$

  and starting from $\mathbf{p_0} = \lambda$, $\mathbf{p}$ will ultimately be obtained.

Morishima interprets vector **q** as being the outputs vector at the balanced maximum growth rate *R*. However, if **q** is interpreted as the vector of output proportions of the standard system of the economy, the rationale for considering the first process of adjustment as an economic process disappears and it becomes difficult to give it an economic interpretation. Hence, the second iteration process (which starts from λ) will only lead to **p** *if r (the real rate of profits of the production prices system) is known and given.* In other words, in each economic system that is not expanding at the maximum possible rate, the logical link to the rate of profits of the production prices system is lacking. The point is to define an adjustment process, which starts from λ and the Marxian rate of profits M/(C + V) with prices and the rate of profits changing *simultaneously* such that ultimately the real prices of production **p** and the real rate of profits will be obtained. Such adjustment processes were independently embellished by Anwar Shaikh and ourselves (Shaikh, 1977; Cuyvers, 1980).

Our algorithm proceeds as follows:

- Because of the unequal rate of profits between the sectors, the adjustment process starts from the constant and variable capital in labour values.
- The rate of profits that is realised *at the end of the last period* is used to calculate, with given constant and variable capital, the prices *at the end of the current period*.
- Whatever the outputs produced, in each sector and at the end of period t, the realised rate of profits is equal to the difference between total sales proceeds and the necessary replacement of constant and variable capital, valued at prices at the end of period t.

In our matrix notation, this algorithm can be written as follows:

$$\mathbf{p}_{t+1} = (1 + r_t)\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\mathbf{p}_t$$

$$r_t = \frac{\mathbf{q}_0\mathbf{p}_t - \mathbf{q}_0(\mathbf{A} + \mathbf{l}\,\mathbf{c})\mathbf{p}_t}{\mathbf{q}_0(\mathbf{A} + \mathbf{l}\,\mathbf{c})\mathbf{p}_t}$$

$$\mathbf{p}_0 = \boldsymbol{\lambda}$$

with $\mathbf{q}_0$ being the given outputs vector (not necessarily identical to $\mathbf{q}$). After an infinite number of adjustments, our algorithm leads to the real prices of production and the real rate of profits. These evidently differ from the outcome of Marx's algorithm, which is but the first step in the algorithm:

$$\mathbf{p}_1 = (1 + r_0)\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\boldsymbol{\lambda}$$

$$r_0 = \frac{\mathbf{q}_0\boldsymbol{\lambda} - \mathbf{q}_0(\mathbf{A} + \mathbf{l}\,\mathbf{c})\boldsymbol{\lambda}}{\mathbf{q}_0(\mathbf{A} + \mathbf{l}\,\mathbf{c})\boldsymbol{\lambda}}$$

As Sraffa teaches, equality between r_0 and r is, from the start of the adjustment process, only possible in a "weighted average sector" – a thought construct that uses the output proportions in vector $\mathbf{q^*}$, Sraffa's standard system. The fundamental issue remains, however: why start from labour values? This is dealt with in the next section.

2.4 What is the logic in transforming values into prices of production?

Why labour values should be transformed into prices of production is a question that is not adequately answered by most Marxists. Often, labour values are presented as a determining factor operating below the surface of capitalist reality, while prices of production are a phenomenon "at the surface", with reference to the other, more plausible thesis that surplus labour is at the root of profits (see, for example, Baumol, 1974). In asking the question why this is so, different answers are given. These questions generally prompt semantic and philosophical discussions, which few – if any – economists with a genuine interest in Marx have found convincing. On the contrary, many economists favourably disposed toward Marx consider these discussions to be "metaphysical" and "unscientific", and they consequently reject the theoretical concept of labour value (or avoid using it) (see, for example, Robinson, 1942, Preface). Nobel Prize laureate Paul Samuelson characterised the Marxian transformation problem as completely redundant, since it boils down to thinking away the *real prices* and then allowing them back into the picture after having introduced labour values, i.e. "prices" that have no existence in the real world and cannot be reconciled with the normal functioning of a capitalist production system. He called the labour values of Volume 1 of *Das Kapital* an “unnecessary detour”. When Marx analyses competitive pricing in Volume 3 of *Das Kapital*, labour values disappear in the juggler’s hat again – so Samuelson stated (Samuelson, 1970, pp. 423–425; Samuelson, 1971, pp. 399–431; see also Robinson, 1950, p. 362).[17]

In this section we endeavour to present some logical arguments in favour of the labour theory of value. We first look into Morishima’s “Fundamental Marxian Theorem”, after which we present our interpretation of labour values as being the other side of the coin that shows prices of production.

2.4.1 The rate of surplus value determines the rate of profits, not the other way round (Morishima’s “Fundamental Marxian Theorem”)

We have shown that in a standard system, the Marxian rate of profits M/(C + V) corresponds exactly with the real rate of profits of the system of production prices:

$$r = \frac{\sigma \, \mathbf{q} \, \mathbf{l} \, \mathbf{c} \, \boldsymbol{\lambda}}{\mathbf{q} \, (\mathbf{A} + \mathbf{l} \, \mathbf{c}) \, \boldsymbol{\lambda}}$$

where, σ is the rate of exploitation/rate of surplus value, $\mathbf{q}\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda}$ is the variable capital V and $\mathbf{q}\,\mathbf{A}\,\boldsymbol{\lambda}$ is the constant capital C in the standard system.

This equality implies that with a constant share of the variable capital $\mathbf{q}\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda}$ in the total capital $\mathbf{q}\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\boldsymbol{\lambda}$, changes in the rate of exploitation σ will result in changes in the rate of profits r. Since the share of variable capital in total capital is less than 100% (or one), the rate of exploitation will be higher than the rate of profits. However, there is more: without exploitation ($\sigma = 0$), profits will be zero ($r = 0$). Stated differently: *a positive rate of exploitation is a necessary condition for a positive rate of profits* (Morishima, 1973, pp. 63–68). The proof of this theorem depends completely on the special character of vector **q**, i.e. that the reasoning is within a dual “sub-system” (Sraffa’s standard system or a variant) of the economy.

Morishima reformulated the necessity condition of exploitation for positive profits in his discussion with Paul Samuelson (Morishima, 1974, pp. 71–74). “Translated” into our matrix language, Morishima’s “Fundamental Marxian Theorem” goes as follows.

Taking the wage rate as “numéraire” ($\omega = \mathbf{c}\,\mathbf{p} = 1$), we know (see section 2.3.1) that the prices of production can be written as a sum of dated quantities of labour:

$$\mathbf{p} = [\mathbf{I} + \mathbf{A}(1 + r) + \mathbf{A}^2(1 + r)^2 + \ldots]\, \mathbf{l}\, (1 + r)$$

$$\text{or } \mathbf{p} = [\mathbf{I} - \mathbf{A}(1 + r)]^{-1}\, \mathbf{l}\, (1 + r)$$

The “numéraire” is therefore:

$$\omega = \mathbf{c}\,\mathbf{p} = \mathbf{c}\,[\mathbf{I} - \mathbf{A}(1 + r)]^{-1}\, \mathbf{l}\, (1 + r) = 1$$

Using this equality, the rate of profits r can be calculated.[18] This rate of profits will *only* be positive if:

$$\mathbf{c}\,[\mathbf{I}-\mathbf{A}]^{-1}\,\mathbf{l} < 1$$

On the other hand, it was also pointed out (see section 2.3.2) that the system of labour values $\mathbf{A}\,\boldsymbol{\lambda} + \mathbf{l} = \boldsymbol{\lambda}$ can be written as:

$$\mathbf{A}\,\boldsymbol{\lambda} + \mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} + \sigma\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} = \boldsymbol{\lambda}$$

$$\text{or } \mathbf{A}\,\boldsymbol{\lambda} + (1+\sigma)\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} = \boldsymbol{\lambda}$$

using a uniform rate of exploitation σ. This system of equations is only in conformity with the system of labour values from which we started if $(1+\sigma)$ $\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} = \mathbf{l}$, or:

$$(1+\sigma)\,\mathbf{c}\,\boldsymbol{\lambda} = 1.$$

or, stated plainly, if the value of labour power $\mathbf{c}\,\boldsymbol{\lambda}$ in all sectors is equal to $1/(1+\sigma)$. It can then be concluded that prices are only labour values if:

1. In all sectors the same ratio σ is found between profits and wages.
2. Prices are expressed in units of labour time (i.e. paid and unpaid labour, taken together).[19]

Since $\boldsymbol{\lambda} = [\mathbf{I}-\mathbf{A}]^{-1}\,\mathbf{l}$ (see section 2.2.3), the above condition for a positive rate of profits equates to:

$$\mathbf{c}\,\boldsymbol{\lambda} < 1$$

and because for the labour value system it holds that $(1+\sigma)\,\mathbf{c}\,\boldsymbol{\lambda} = 1$, the condition $\mathbf{c}\,\boldsymbol{\lambda} < 1$ is always fulfilled if the rate of surplus value is positive ($\sigma > 0$). Morishima's "Fundamental Marxian Theorem" thus states: the rate of profits r (of the production prices system) is only positive if the rate of surplus value (of the labour values system) is positive. It unambiguously implies that a labour theory of value, which considers profits as the result of unpaid labour (ergo as exploitation of the workers), logically precedes the production prices theory, such as the one underlying $(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\mathbf{p}\,(1+r) = \mathbf{p}$.

2.4.2 The transformation of labour values into prices of production: logic or reality?

It has been argued that production prices yield a uniform rate of profits and therefore are inherent in the capitalist system of production. Labour values, in contrast, might be considered to be prices that would hold in a system of simple (a-capitalistic) commodity production. The transformation of labour values into production prices is then a "logical transformation" which intends to show how – if at all – capitalist production influences the conditions of surplus value production, what the conditions are for reproduction of the production system, etc. Labour values are thus a logical step toward the full understanding of the phenomenon of capitalist prices as *they logically precede capitalist prices of production*.[20] The nagging question that remains, however, is whether the labour theory of value provides an operational concept that promotes the analysis of capitalism. Stated differently: what are the insights and elements that the labour theory of value delivers which, in the absence of such a theory, would not be detected?[21]

Marx makes an interesting observation in this respect by highlighting a tendency of the rate of exploitation to equalise across sectors. In Volume 3 of *Das Kapital* he writes:

> If capitals that set in motion unequal quantities of living labour produce unequal amounts of surplus-value, this assumes that the level of exploitation of labour, or the rate of surplus value, is the same, at least to a certain extent, or that the distinctions that exist here are balanced out by real or imaginary (conventional) grounds of compensation. This assumes competition among the workers, and an equalization that takes place by their constant migration between one sphere of production and another. We assume a general rate of surplus-value of this kind, as a tendency, like all economic laws, and as a theoretical simplification (. . .). In reality, this is only an approximation; but the approximation is all the more exact, the more the capitalist mode of production is developed.
>
> (Marx, 1981, p. 275)

However, that labour mobility leads to a uniform rate of surplus value is far from obvious. Evidently, when all labour is the same, labour mobility will give rise to a uniform wage rate. In addition, labour intensity and the length of the working day will tend to equalise. But this process does not by itself lead to a uniform rate of exploitation across sectors; moreover, as our previous numerical examples showed, a uniform daily or hourly wage rate will not hamper the process of the sector rate of profits equalising. Marx's approach in the above extract does not offer a solution: for the rates of exploitation to equalise through labour mobility, workers need to have knowledge about the labour time required to produce their necessary consumption. They should react "with their feet" to deviations in *the value of labour power* across activities and sectors (which they do not know exactly) and not to deviations in the wage rate (which they do know). This is quite a dubious assumption.

What will happen, though, if workers are not only enforcing equal pay, but are *simultaneously* paying attention to the profits of their bosses compared to the wages paid?[22] Assume that a tendency for the ratio of profits to wages to be equalised across sectors was based on, for example, the workers' knowledge of the profits and wages of the last production period. At the end of the new period, the output of the respective sectors would then be sold at their labour values (or at prices that are strictly proportional to the labour values). Since unequal rates of profits would prevail, the capitalists would leave the less profitable sectors. After a series of adjustments, prices of production à la Marx would finally be reached. It will start all over again in the next production period, with the workers reacting to the unequal profits–wages ratio of the last period and the capitalists reacting to the unequal rates of profits that the workers' reactions will provoke . . . .

According to this reasoning, labour values do not only logically precede prices of production but they acquire the same status as prices of production. If the workers in the various industrial sectors pay sufficient attention to deviations from a normal profits–wages ratio, there will be a tendency for normal supply prices to appear which are equal or proportionate to the labour values. However, since capitalists will transfer capital to the sectors that promise the highest return on their investments, there will at the same time be a tendency toward prices of production. Labour values and prices of production should be considered to be the different vectors of exchange values, determined by a tendency toward equalisation of the rate of exploitation and the rate of profits, respectively. Thus, based on this reasoning, the argument is no longer that labour values just "logically precede" prices of production. Labour values and prices of production are the two sides of the same coin, i.e. two different sets of normal, long-term prices for the same real world but neither ever fully realised as they are the result of two conflicting tendencies. Both are normal, long-term prices which allow the physical reproduction of the production system (including the expanded reproduction following a path of balanced economic expansion) based on the normal physical input-output relationship between the sectors of the economy.

2.5 A short digression into technological coefficients, subsistence wages and the "law of value"

We have shown that prices of production should be reduced to "dated quantities of labour". The next issue to consider is how changes in these spendings of labour and in labour productivity in the various production processes and sectors lead to an expansion or contraction of output. This is what Marx calls the "law of value".

In both the previous and present chapter, we have used systems of equations, the solutions for which are a set of prices of production $\mathbf{p}$ or labour values λ, or else, of output quantities $\mathbf{q}$. In the present section, we will deal briefly with the meaning of Matrix $(\mathbf{A} + \mathbf{l}\,\mathbf{c})$ and some of its implications.

Matrix $(\mathbf{A} + \mathbf{l}\,\mathbf{c})$ is the result of the summation of Matrices $\mathbf{A}$ and $\mathbf{l}\,\mathbf{c}$. Matrix $\mathbf{A}$ consists of physical (input-) coefficients. In our numerical example, 50 kg of iron (in the form of tools and other means production) and 40 kg of wheat (in the form of seed for sowing) are needed to produce 100 kg of wheat efficiently. For each kilogram of wheat, this amounts to 0.5 kg of iron and 0.4 kg of wheat. These quantities of inputs per unit of output are technologically determined. They tell us what is necessary in order to produce wheat, given the present state of knowledge and production techniques. These input coefficients also determine the labour that is directly and indirectly required to produce one unit of the respective outputs via the labour values system:

$$\mathbf{A}\boldsymbol{\lambda} + \mathbf{l} = \boldsymbol{\lambda}$$

with the solution:

$$\boldsymbol{\lambda} = (\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \ldots)\,\mathbf{l}$$

This amounts to what Marx says is, on average, socially necessary to produce a given good or service – in our example, 1 kg of wheat: “The value of any commodity – and thus also of the commodities which capital consists of – is determined not by the necessary labour-time that it itself contains, but by the *socially* necessary labour-time required for its reproduction” (Marx, 1981, p. 238).

In Volume 1 of *Das Kapital* Marx also states:

> Lastly – and for this purpose our friend [the capitalist] has a penal code of his own – all wasteful consumption of raw material or instruments of labour is strictly forbidden, because what is wasted in this way represents a superfluous expenditure of quantities of objectified labour, labour that does not count in the product or enter into its value.
>
> (Marx, 1976, p. 303)

And in another passage in Volume 1 of *Das Kapital*, this is pointed out even more clearly:

> The value of a commodity is certainly determined by the quantity of labour contained in it, but this quantity is itself socially determined. If the amount of labour-time socially necessary for the production of any commodity alters – and a given weight of cotton represents more labour after a bad harvest than after a good one – this reacts back on all the old commodities of the same type, because they are only individuals of the same species, and their value at any given time is measured by the labour socially necessary to produce them, i.e. by the labour necessary under the social conditions existing at the time.
>
> (Marx, 1976, p. 318)

However, the technological coefficients of Matrix **A** are conceptually different from the elements of Matrix $\mathbf{l}\,\mathbf{c}$. The latter shows for each sector the labour time required per unit of output (which is also technologically determined), as compensated by the quantity of consumer goods that are socially necessary to allow the reproduction of the labour force. In our numerical example, 50 hours of direct labour are needed to produce 150 kg of iron and 100 kg of wheat, respectively, or 50/100 hours and 50/100 hours of labour to produce 1 kg of iron and 1 kg of wheat. We also assume that for the reproduction of the labour time used, half a kilogram of wheat is needed, or 25/150 kg of wheat to produce 1 kg of iron and 25/100 kg of wheat to produce 1 kg of wheat.

The elements of Matrix $\mathbf{l}\ \mathbf{c}$ are thus partially based on what is on average socially necessary (or what is considered to be necessary) to reproduce the labour force used. The workers receive a wage. During the first centuries of capitalist development, this wage could still be considered a *subsistence wage*, i.e. a wage that allowed the worker to maintain himself and his family physically. This has not been the case for quite some time.

Marx is clear on this. In Volume 3 of *Das Kapital* he writes, for instance:

> The actual value of his labour-power diverges from this physical minimum; it differs according to climate and the level of social development; it depends not only on physical needs but also on historically developed social needs, which become second nature. In each country, however, this governing average wage is a given quantity at a given time.
>
> (Marx, 1981, p. 999)

In discussing Marx's theory of subsistence wages, Roemer (1981, pp. 150ff.) argues that it is "tautological to speak of workers' subsistence being whatever they consume; at that level, Marx's argument loses its persuasion as an objective economic argument demonstrating the origin of profits" (Roemer, 1981, p. 150)

Quoting Marx's *Wages, Price and Profit*,[23] he argues further, "because of the peculiar nature of another law of capitalist development (labor-saving technical innovation), the class power of workers is not sufficient to win them more than the physical subsistence level" (Roemer, 1981, p. 152).

It is certainly true that if following Marx's model, labour-saving innovations are introduced by the capitalists for putting downward pressure on the wages, the outcome would be as Roemer predicts. However, as we will see in Chapter 5, it is Marx's belief that such innovations are also, if not most importantly, the individual capitalist's instrument in the competitive struggle with his capitalist rivals and a source of super-profits. Yet, even this factor can only be part of the story. The historical process determining the evolution of the "traditional standard of life" (Marx, 1969, p. 27) and of the subsistence wage rate is much more complex and multi-dimensional. During this process also important product innovations take place, with new products and services developed which enter the workers' consumption, and also new needs, both physical and cultural, will ultimately lead to changes in this consumption.

Today, the average wage rate, at least in the developed economies, bears no relationship to the consumption required for mere survival and reproduction of the labour force. The wage earners spend their wages on goods and services according to historically determined and evolving average consumption patterns, which are unrelated to any physical bundle of consumer goods. Is this an insurmountable theoretical problem? No. However, because the workers' consumption is largely unrelated to what is "required", the composition of the basket of consumption goods should be taken as similar to what in many industrial countries is used for calculating the index of consumption prices. The important point here is that the real wage (or the value of labour power, as Marx puts it) is fixed during the period under consideration and is governed by conditions other than those determining total output and the income shares in the net product, other than that of labour (Garegnani, 1984, pp. 295–296).

In fact, $\mathbf{l}\,\mathbf{c}$ plays no role whatsoever in the determination of labour values – only in the determination of the prices of production. Furthermore, even in the production prices system it could be replaced by the amount of money invested as variable capital, although expressed per unit of output. The total socially necessary consumption of the workers is, however, of great relevance, since its volume influences the conditions of simple or expanded reproduction.

How the "law of value" works is easily understood if in each sector one specific technology is applied. This technology will determine the elements of $\mathbf{A}$, and the efficient use of the various inputs will lead us to the average socially necessary labour and, indirectly, to the price system which in the long run will yield a uniform rate of profits in the various sectors.

What if more technologies are available? Each technology can be identified by its specific technological input coefficients. If, for instance, an alternative technology exists for the production of wheat (which is different from that using 50/100 kg of iron and 40/100 kg of wheat, and which requires more means of production and less seed, i.e. 60/100 kg of iron and 30/100 kg of wheat), then, in terms of the mathematics involved, two equations will have to be constructed to solve for λ_2, p_2 or q_2. The system of equations will therefore contain more equations than unknowns[24] and can only be solved as a programming problem, after introducing specific constraints. Following this route, Michio Morishima showed that for such systems, labour values require a *minimum expenditure of direct labour time* (and not the summation of direct and indirect labour time spent) (Morishima and Catephores, 1978). Morishima, attempting to link this to the economic theory of Marx, refers to a passage in *Misère de la Philosophie*. If he had been looking for support in *Das Kapital* he would have found that, according to Marx, the capitalists are not introducing new production techniques voluntarily but rather because they are permitting a reduction in their supply price (Marx, 1981, p. 373). With the introduction of new technology, the expenditure of labour time is minimised. Marx also writes: "The price of the commodity is therefore reduced to a minimum through reducing to a minimum each part of the labour required to produce it" (Marx, 1981, p. 180).

We (Cuyvers, 1986) have extended Morishima's programming problem by introducing the constraint that the profits–wages ratio, to which the minimising of direct labour time expenditure would lead, should not be larger than the "normal" ratio in the capitalist economy under consideration. With this additional condition, we have shown that the growth-maximising and labour time-minimising choice of production techniques leads to the same mathematical solution to the programming problem. Thus, Marx's "law of value" operates with the capitalist choice of technology.

2.6 What about monopoly prices?

Marx writes in Volume 3 of *Das Kapital*:

> A monopoly price for certain commodities simply transfers a portion of the profit made by the other commodity producers to the commodities with the monopoly price. Indirectly, there is a local disturbance in the distribution of surplus-value among the various spheres of production, but this leaves unaffected the limit of the surplus-value itself. If the commodity with the monopoly price is part of the workers' necessary consumption, it increases wages and thereby reduces surplus-value, as long as the workers continue to receive the value of their labour-power. It could press wages down below the value of labour-power, but only if they previously stood above the physical minimum. In this case, the monopoly price is paid by deduction from real wages (i.e. from the amount of use-values that the worker receives for the same amount of labour) and from the profit of other capitalists. The limits within which monopoly price affects the normal regulation of commodity prices are firmly determined and can be precisely calculated.
>
> (Marx, 1981, p. 1001)

This passage relates to the real monopoly prices, but is also of special relevance for research on how the widely practised oligopolistic price setting functions. This price setting is based on decisions by the capitalists, the products of whom do not, for various reasons, allow "free competition". Such products have some uniqueness (e.g. a CD of Bruce Springsteen, which contractually may only be released by Columbia Records) or acquire their uniqueness in the eyes of the consumer through publicity (e.g. Coca Cola, Burton caps, Nike shoes, etc.). Alternatively, they respond to the needs of the buyer to differentiate (e.g. a brand-new Lamborghini Aventador LP700–4). Uniqueness can also relate to means of production (e.g. Apple computers, trucks, forklifts, etc.).

Paul Sweezy, who although being a Marxist economist also contributed in a fundamental way to the "mainstream economics" of oligopolistic and monopolistic competition (Sweezy, 1939), states in his pioneering *Theory of Capitalist Development* (1942) that from the moment monopoly formation became fully developed in the course of capitalist development, labour unions also emerged to largely (though not necessarily totally) prevent the reduction in wages, due to the monopolistic pricing of consumer goods (Sweezy, 1942, p. 273). On the other hand, the further spread of monopolies and oligopolies reinstates the tendency toward equalisation of the rates of profits. As quite a few sectors do not monopolise, a hierarchy of rates of profits emerges. Sweezy writes:

> The spreading process [of monopolies] works very unevenly, for there are always industries in which it is difficult or even impossible to effect a stable combination. These are the industries in which only a small capital investment is required, numerous firms are necessary to fill the demand and entry

> into the field is easy for any one with the required minimum of capital. Here competitive conditions persist despite the advantages to be had from combination. It follows that we can expect a general equalization of profit rates neither from the mobility of capital nor from the spreading of monopoly. We get instead a hierarchy of profit rates ranging from highest in the industries of large-scale production where close well-protected combinations are relatively easy to establish, to lowest in the industries of very small-scale production where numerous firms co-exist and the ease of entry precludes stable combinations.
>
> (Sweezy, 1942, pp. 273–274)

This is at the same time both correct and incorrect. In the sectors that are not monopolised, a tendency toward equalisation will probably continue to work, but in the monopolised sectors profits and the profit margin become relevant, rather than the rate of profits.[25] We will return to this issue in Chapter 7. Here we investigate the impact of monopolistic pricing after translating it into the mathematics of the linear production model. First of all, we assume that the coefficients of the **A** matrix and the **l** vector remain valid and constant. As we will see later, this is a well-founded assumption, based on the observation that under oligopolistic production conditions, unused production capacity exists such that the unit costs of production will not change with larger output – at least as long as full capacity utilisation is not reached.

We will now illustrate the effect of monopolistic prices on our imaginary economy. We assume that sector 1 (the iron-producing sector) is monopolised. The producers in that sector apply a mark-up of 50% on their unit costs.[26] The equations of the price system are now:

$$\left(\frac{50}{150}\,p_1 + \frac{25}{150}\,p_2\right)(1 + 0.50) = p_1 \tag{2b}$$

$$\left(\frac{50}{100}\,p_1 + \frac{65}{100}\,p_2\right)(1 + r) = p_2$$

The first equation becomes:

$$0.50\,p_1 + 0.25\,p_2 = p_1$$

$$\text{or:}\ 0.50\,p_1 = 0.25\,p_2$$

The second equation is:

$$0.50\,p_1 + 0.65\,p_2 = \frac{1}{1+r}\,p_2$$

in which the first can be substituted:

$$0.25\,p_2 + 0.65\,p_2 = \frac{1}{1+r}\,p_2$$

or:

$$0 = \frac{1}{1+r}\, p_2 - 0.90\, p_2$$

For a positive price of wheat, it should hold that:

$$\frac{1}{1+r} = 0.90$$

such that $r = 11.1\%$.

Taking the solution further, we add the condition that the value of the net product of our imaginary economy is equal to the total labour time spent in the period under consideration:

$$100 \text{ hours of labour} = 50\, p_1 + 60\, p_2$$

and because we know already that:

$$0.50\, p_1 = 0.25\, p_2$$

p_1 and p_2 can be solved from this system of two equations with two unknowns. The solution is:

$$p_1 = 0.588$$

$$p_2 = 1.176$$

It will be noticed that due to the monopolistic "mark-up" of 50% in sector 1, the rate of profits in sector 2 lies significantly below 21.8%, which would be the rate of profits if the prices of production were to hold. Therefore, the monopoly price of sector 1 is higher than the price of production and the resulting unit price of the not-monopolised sector 2 is lower than its price of production. The value of labour power (measured in actual prices) is per hour of labour time: p_2 c = 1.176 × 0.5 kg of wheat = 0.588 hours of labour, and in both sectors the 50 hours of value added is divided into 29.4 hours of necessary labour and 21.6 hours of surplus labour. The rate of surplus value is thus $\sigma = 21.6/29.4 = 0.735$.

Monopoly prices clearly give rise to a hierarchy of rates of profits and a redistribution of surplus value across the sectors, as well as a redistribution of the value added (measured in hours of labour time) between labour and capital. In essence, nothing changes in Marx's reasoning. The monopolisation of capitalism, however, does have profound implications for capital accumulation and economic growth, which we will discuss later.

2.7 Post-Keynesian views about the labour theory of value

Among Marxist economists it is customary to distinguish the "quantitative value problem" from the "qualitative value problem".[27] In this chapter, we have focused mainly on the quantitative value problem, which does not do full justice to Marx's theory. In fact, such neglect constricts and therefore mutilates the theory. In Marx's view, the exchange value as a quantitative relationship between products and services is not the core element but rather the outward manifestation of the social relations between the producers. It is for this reason that Marx views exchange value as an expression of social relations, as the product of human labour and human labour alone. However, mainstream economists and even many post-Keynesian economists are hostile toward this "qualitative" approach. In fact, they are even hostile toward the "quantitative value problem".

In the previous chapter we drew attention to the early – mostly Cambridge, UK – post-Keynesians' indebtedness to Marx's theory of reproduction and accumulation. At this point it is interesting to digress to their views about the labour theory of value.

In her 1942 *Essay on Marxian Economics*, Joan Robinson rejects the theory. She states that nothing of substance that Marx expressed in value is not much better expressed without recourse to it and that nothing of substance in Marx's economics depends on it (Robinson, 1942, pp. 20, 22). In *Economic Philosophy* (1962), she concludes the chapter devoted to the classical and Marxian theory of value by indicating that value has "no operational content" and is "just a word" (Robinson, 1962, p. 47). Still in 1965, five years after Sraffa published his *Production of Commodities by Means of Commodities*, she writes in the Preface to the second edition of her *Essay on Marxian Economics* (reprinted in 1966): "The concept of value seems to me to be a remarkable example of how a metaphysical notion can inspire original thought, though in itself it is quite devoid of operational meaning" (Robinson, 1942, p. xi).[28]

Other post-Keynesian neo-Marxist scholars have adopted a similar attitude with respect to labour values and have avoided building their theoretical arguments on it. As we pointed out in Chapter 1, Kalecki's use of vertically integrated sectors in his reproduction schemes allowed him to show how profits are "realised" by the capitalists' spending, without having any recourse to value or how surplus value is "produced". A privileged witness remembered Kalecki as being "allergic" to any discussion on the "law of value".[29] The use of vertically integrated sectors has also been of crucial importance in later applications by post-Keynesians such as Joan Robinson and Luigi Pasinetti.

Josef Steindl, who considered Kalecki to be his "inspiration and Guru" (Steindl, 1984, p. 7), devotes a chapter in *Maturity and Stagnation in American Capitalism* (1952) to Marx and the accumulation of capital. The subject of his book, so he admitted much later, was the result of a suggestion by Kalecki: "It was a very Marxian problem, but my methods of dealing with it were Kaleckian" (Steindl, 1984, p. 8).

Steindl's chapter on Marx elaborates on Marx's theory of wages and how it relates to accumulation and growth. It also links Marx's underconsumption thesis, rather than that of a tendentially declining rate of profits, to the long-run stagnation of capitalism. The latter thesis was revived by Paul Sweezy in his *Theory of Capitalist Development* (1942). Steindl's views on Marx have had a profound influence on the theoretical work of later neo-Marxist authors, among whom Paul A. Baran and Paul M. Sweezy figure prominently (Foster, 2013). Yet Steindl's chapter on Marx, while also dealing with exploitation, ignores the labour theory of value. He only mentions that according to Marx the value of labour power is found by applying Marx's rule that "commodities" have a "value" (Steindl's quotation marks) in terms of the "necessaries of life" (Steindl, 1952, p. 229). It can be assumed that Steindl probably shared Kalecki's "allergy" toward labour values. This becomes clear when Steindl writes in 1984 about the revival of Marxist economics in the late 1960s and 1970s:

> I find it regrettable that most of the renaissance of Marx – as far as economics is concerned – concentrated on the theory of value and the conundrums connected with it. This complex of questions is ultimately derived from Ricardo in so far as it relates to a competitive economy with equalization of profit rates and essentially without money. But there is a different side of Marx, a dynamic approach to economics (such as in the chapter on accumulation in *Capital*, Vol. I) which results from his aim to explain the development of capitalism, an approach to history with the tools of economic analysis.
>
> (Steindl, 1984, p. 12)

In addition, the attitudes of authors such as Paul A. Baran and Paul Sweezy toward the labour theory of value are not clear. In 1942, Sweezy explained at length and defended Marx's value theory in his *Theory of Capitalist Development* (Sweezy, 1942, pp. 33–34, Chapter III). Yet in 1957, Paul Baran argued in *The Political Economy of Growth* that the labour spent on the production of goods and services that meet no rational need, but are the result of publicity, is *not* value-creating labour but rather unproductive labour (Baran, 1957, p. 144). This means that such goods and services have no labour value. Such reasoning is also followed in Baran and Sweezy's *Monopoly Capital*. In both cases, no further use whatsoever is made of Marx's value concept, which leaves the impression that they abandoned the labour theory of value (Mandel, 1967).[30] The authors' analysis starts from the "economic surplus", not from exploitation on the factory floor.

Among the many present-day, post-Keynesian economists there are few who take the labour theory of value seriously. Notable exceptions are Geoffrey Harcourt and Luigi Pasinetti. No doubt, Sraffa's influence on them has been profound.[31] They rely on Sraffa's proof that the prices of production are "a sum of dated quantities of labour" and that, for a given Matrix $\mathbf{A}$ and Vector $\mathbf{l}$, there is a Sraffian solution to the transformation problem, although the transformation is a very complicated one. This made Joan Robinson finally come to terms, at least to some extent, with Marx's theory of value. In 1979, she wrote:

> With the light that Sraffa has thrown on the theory of value and Kalecki on the process of realisation of the surplus, we can develop a complete system, not of *neo* Marxism but of *intelligible* Marxism, and, what is more important,

> adapt it to the analysis of contemporary problems of capitalism, socialism and so-called "development"
>
> (Robinson, 1979, p. 253; Joan Robinson's italics)

a statement that prompted Harcourt and Kerr to comment that Sraffa's work had somehow enabled her to accept Marx's arguments (Harcourt and Kerr, 2009, p. 55).

Our point is that the "duality" of the system of outputs, enabling reproduction of the economy, and the system of associated prices (see, for example, Bródy, 1970; Morishima, 1973) – the vector of labour values with the vector of net products, and the vector of prices of production with the vector of surplus products – is crucial for understanding where Sraffa fits in. Having shown in Chapter 1 how Marx's schemes of reproduction relate to a Sraffa-like standard system, the same system is highly relevant for explaining the prices that allow reproduction of the economy. It is true that linking Sraffa's argument to Marx is only feasible if a vector of necessary consumption is introduced (which Sraffa did not do). The "standard system" that is then derived is evidently not identical to Sraffa's standard system (where, apart from wages being assumed as paid *post factum*, the money wage rate as such is postulated), but nevertheless is the mathematical "translation" of an economy (a closed economy, for sure) into maximum expanded reproduction or, as stated in the terminology of Michio Morishima, onto its maximum balanced growth path.[32] This standard system à la Sraffa will also be revisited in the following chapters.

Joan Robinson's basic model that she constructed in *The Accumulation of Capital* is similar to the Marx-Leontief linear model of which Sraffa also uses a variant (Cuyvers, 1979, pp. 328–330). She treats labour as the only unproduced factor of production, at least in the short run (Robinson, 1956, p. 67).[33] She writes: "from a long-run point of view, labour and natural resources are the factors of production in the economy as a whole, while capital goods and the time pattern of production are the means by which the factors are deployed" (Robinson, 1956, pp. 310–311).[34]

Next, Robinson assumes that all labour is alike (Robinson, 1956, pp. 64, 68, 115, 352) and that in each sector the given production technologies are each associated with rigid proportions between inputs and outputs (Robinson, 1956, pp. 65, 82–83n). In hindsight, it comes as no surprise that her "normal prices" are what Sraffa, hardly four years later, derived as "dated quantities of labour". This appears most strikingly in the following passage from *The Accumulation of Capital*:

> At any moment when work is being done today's labour is being added to the product of past labour, which in its own day was added to the product of still earlier labour. Under the capitalist rules of the game this shows itself in the element of interest in the cost of today's capital goods.
>
> (Robinson, 1956, p. 121)[35]

It should be quite clear that the Marx-Leontief model, of which Sraffa's system of equations is a variant and to which Robinson is referring, supports both the mathematical interpretation of Marx's theory of value and growth and the post-Keynesian (neo-Marxist) theory of growth.

However, the reactions of a number of "orthodox Marxists" to Sraffa's model have been hostile. They have questioned the assumption relating to the uniform rate of profits in the model (in spite of Marx himself using such a uniform rate) (see, for example, Langston, 1984, pp. 6ff.; Farjoun, 1984, pp. 13–16) or the use of the prices of production to express both inputs and outputs (see Kliman and McGlone, 1999; Freeman and Carchedi, 2000) or Sraffa's distinction between "basics" and "non-basics" (Farjoun, 1984, pp. 31ff.), etc. This is not the place to go into this controversy as it would lead us much too far, but it should be clear that the Sraffian interpretation of the Ricardian and Marxian theory of value and the "orthodox Marxists'" views on value are incompatible bedfellows.

2.8 To conclude

Despite the debate that has been raging for more than 100 years over the so-called transformation problem, it is evident from our discussion that Marx's labour theory of value, given a proper assumptions framework and provided it is properly formulated mathematically, is basically correct. Although the algorithm that Marx developed in *Das Kapital* to transform values into production prices is only the first step in the correct algorithm, his views on the operation of the "law of value" remain completely valid: not only is every production price the sum of all labour spendings (while taking into account the phasing-in time) so that the average socially necessary labour required for the production of the goods and services determines the production price, but the introduction of a technology choice in Marx's model leads to the minimisation of labour time spent in the production process.

We have also seen that a positive rate of exploitation is a precondition for a positive rate of profits, and that labour values "logically precede" production prices. The labour theory of value thus allows us to understand underlying processes and mechanisms. Furthermore, labour values and production prices are two sides of the same coin. We have indicated that labour values are actually "prices" with a similar status to production prices, i.e. exchange ratios that apply if there is a tendency toward equalisation within and across sectors in the rate of exploitation, such as would appear in concrete form to the workers as the ratio between profits and wages. Similarly, production prices are the exchange ratios brought about by capital mobility, which would finally induce a uniform rate of profits within and across sectors.

In the economic theory and criticism of Marx, production prices have apparently acquired the status of long-term prices which allow growth-maximising, expanded reproduction of the economic system. To us, this does not seem to be correct. Perhaps the labour theory of value has to be integrated again into contemporary linear production models because they also show a neglected but nonetheless fundamental tendency in the capitalist mode of production. From the standpoint of economic theory, this is more important for the scientific position of Marxism than for the ideological function performed by the labour theory of value – the emphasis being on the impact of social contradictions between the haves and the have-nots of the means of production, rather than on social harmony.

The early post-Keynesian neo-Marxist authors – Kalecki, Steindl and Joan Robinson – were highly critical of Marx's labour theory of value. Piero Sraffa, however, showed that the prices of production among the classical economists, as well as Marx, are a sum of dated labour time, with the rate of profits cumulatively applying to the labour time spent in the stages of production in the past. He also showed that the transformation of labour values – "undated quantities of labour time" – into prices of production is possible, although complicated. Because Joan Robinson, in her work on the theory of economic growth, basically used a similar model to that of Sraffa, she finally came around to admitting that the classical theory of value could be combined with Kalecki's and her views on how profits arise. Although most present-day post-Keynesian scholars neglect Marx's labour theory of value, notable exceptions such as Geoffrey Harcourt and Luigi Pasinetti are treating it seriously.

Notes

1 The reader will now better appreciate the importance of our assumption, made for the sake of simplicity, that apart from labour, only iron is needed to produce iron. This solves the equation $50\,\lambda_1 + 50$ hours of labour time $= 150\,\lambda_1$ for the unknown λ_1 and uses this solution in solving the equation of λ_2. This assumption can evidently be dropped easily, in which case we will have to solve a system of two linear equations with two unknowns: λ_1 and λ_2.

2 It is customary to assume linear depreciation of the capital stock in the various spheres of production. See e.g. Harris (1978). As Harcourt (1965) has shown, the assumption of linear depreciation is an arbitrary accountant's assumption, since real depreciation depends on the pattern of the output flows over the years. The accountant's rate of profits and the expected rate of profits in a situation of balanced growth (Robinson's Golden Age) deviate. However, we are making this assumption for the sake of the argument to simplify the input-output relations in the system of equations that determines the labour values.

3 David Ricardo is of the same opinion. He writes: "In speaking (. . .), however, of labour, as being the foundation of all value, and the relative quantity of labour as almost exclusively determining the relative value of commodities, I must not be supposed to be inattentive to the different qualities of labour, and the difficulty of comparing an hour's or a day's labour, in one employment, with the same duration of labour in another. *The estimation in which different qualities of labour are held, comes soon to be adjusted in the market with sufficient precision for all practical purposes, and depends much on the comparative skill of the labourer, and intensity of the labour performed.* The scale, when once formed, is liable to little variation. If a day's labour of a working jeweller be more valuable than a day's labour of a common labourer, it has long ago been adjusted, and placed in its proper position in the scale of value" (Ricardo, 1821, pp. 20–21; italics added).

4 To avoid circular reasoning, the transformation of concrete labour into abstract labour has to be independent of the relative wages of the various occupations. Following a suggestion by Hilferding, Rowthorn (1980) has devised a method for such a transformation. His transformation of skilled labour into simple (unskilled) labour involves

tracing back and adding together the amounts of socially necessary labour needed to produce skilled labour.

5 For a long time it has been argued that in the case of "joint production" (i.e. in the wheat sector, for instance, wheat will be produced simultaneously with straw, both of which can be sold), negative labour values are possible. See Steedman (1975, 1977). This is a reason for Roemer (1981, p. 52) "to reject the labor theory of value in its role as exchange theory, and reconstruct the theory of exploitation on a different basis". However, Kurz (1979, pp. 64ff.) has shown that Steedman's negative labour values are due to the assumption that all abstract labour has equal productivity, that, in fact, they are employment multipliers à la Keynes and Kahn, and that when an appropriate productivity index is attached to the labour inputs in the alternative techniques to produce the joint products, positive labour values are found. See also Harcourt (1979). We have dealt with the joint production riddle in Cuyvers (1983, pp. 129ff.). We have attempted to resolve the contradiction by (following Michio Morishima) setting out to find a positive solution to labour values and also considering alternative techniques of production. See Cuyvers (1986). For a later solution to the paradox with many supporting references to Marx, see Duménil and Lévy (1987) in which it is emphasised how important it is, for the condition for positive labour values, that the technological production system is "non-reductive" and "heterogeneous". Owing to the highly mathematical and technical nature of this discussion, we have chosen not to go into this further.

6 This does not imply that in all these societal systems, labour values would exist as long-term prices, but rather that the value added in the production processes can be split up between necessary labour time and surplus labour time.

7 For a brief review of how Marx's views evolved between *The Poverty of Philosophy* of 1847 and the late 1860s, see Meek (1977, pp. 99–104).

8 For a clear and succinct but thorough overview of the various "invariance" postulates used to transform labour values into prices of production, we refer to Laibman (1973), pp. 410–414. It is clearly shown that it is only under the very arbitrary assumption of equal "organic compositions of capital" that the total output and the total surplus in labour values and in prices of production will be equal simultaneously.

9 In scalar algebra, this is equivalent to $1 - a/(1 - a) = 1$.

10 As will become evident later, the prices we are considering here are expressed in labour hours, not in monetary units. The reason is that, at this stage, we are omitting from consideration the amount of money in circulation in the economy. Since the pioneering work by scholars such as Duménil and Foley, much has been written about the "monetary expression of labour time (MELT)". Duménil and Foley linked labour values in labour time with production prices in monetary units by choosing as MELT the total value added in money terms – the net national product – as compared to the total spending of labour time in the year in question. This insight can also be translated empirically. To the extent that this "New Interpretation" sets out to reason away the transformation problem, to us it looks like an attempt to conceal things. See Duménil (1980; 1983), Foley (1982; 1986). Other scholars have linked labour values to monetary prices of production by introducing the monetary stock. They assume a given velocity of circulation of money, a concept that we will criticise in Chapter 8 for both theoretical and operational reasons, based on the monetary theory of *Das Kapital*. We will return to the MELT in Chapter 8.

11 This follows from the so-called theorem of Perron and Frobenius in the matrix algebra of non-negative matrices (see Meyer, 2000, p. 667).

12 After von Böhm-Bawerk alleged that the transformation of values in prices of production was logically impossible, von Bortkiewicz showed how to do this for a three-sector economy (see Sweezy, 1949). Von Bortkiewicz's paper is published in the appendix of this book.

13 Independently of Sraffa a similar system was constructed in Morishima and Seton (1961).

14 For the total value of all output in labour values and prices of production to also be equal, it evidently suffices in an economy with only circulating capital that total surplus value equals total profits and that the rate of profits equals M/(C + V).

15 The system of equations $\mathbf{A}\,\boldsymbol{\lambda} + \mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} + \sigma\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} = \boldsymbol{\lambda}$ is pre-multiplied by the row vector **q** and system (4), $\mathbf{q}\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,(1 + R) = \mathbf{q}$ is post-multiplied by the column vector $\boldsymbol{\lambda}$.

16 $\mathbf{q}\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,(1 + R) = \mathbf{q}$ is post-multiplied by the column vector **p** and $(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\mathbf{p}\,(1 + r) = \mathbf{p}$ is pre-multiplied by the row vector **q**.

17 For a thorough overview and discussion of the development of Samuelson's views on Marx, see Harcourt (2006b). Laibman (2002, pp. 159–178), considers the transformation problem to be illusory and disputes its very existence: in a capitalist economy there are no labour values and labour values in their capitalist form are prices of production. This reasoning is evidently at odds with the reasoning we are following here in terms of the labour values to which prices will tend to veer if the workers in each sector are aspiring toward a uniform ratio of wages to profits.

18 The reader is reminded that only the smallest r for which this equality is satisfied will result in economically sensible prices $\mathbf{p} > 0$.

19 In other words, if $(1 + \sigma)\,\mathbf{c}\,\boldsymbol{\lambda} = 1$ is chosen as "numéraire" for the prices, and *not* $\mathbf{c}\,\boldsymbol{\lambda} = 1$ (only paid labour time, analogously with $\mathbf{c}\,\mathbf{p} = 1$, in order to show that prices of production are a sum of dated quantities of labour).

20 This thesis has been argued authoritatively in Meek (1973, pp. XXIVff., pp. XXXIVff.).

21 Morishima's "Fundamental Marxian Theorem" is doing exactly this: a positive rate of surplus value is a necessary condition for a positive rate of profits.

22 We advanced this assumption in Cuyvers (1986). It was only later that we realised that a similar assumption was made in Laibman (1973). Laibman's assumption was not made, however, to show an alternative price system of labour values. Ronald Meek has responded to Laibman's assumption that he cannot find "an atom of evidence in Marx's own writings that this was what he himself had in mind" (see Meek, 1977, p. 119). This is certainly true if we want the uniform rate of exploitation to play a role as "invariance postulate" in the transformation of values in prices of production. However, as the passage in Marx's Volume 3 of *Das Kapital* (which we quoted above) shows, Marx had in mind a mechanism that tends to equalise this rate across sectors. This tendency, if fully worked out, leads to prices that are equal (or proportional) to labour values.

23 As mentioned earlier, we are solely relying on *Das Kapital*, Volume I of which was published in 1867. *Value, Price and Profit* (or *Wages, Price and Profit*) is the text of an address delivered by Karl Marx at two sessions of the General Council of the First International on 20 and 27 June 1865. It summarises some of the major theses of his labour theory of value and his theory of capitalist exploitation, which was going to be developed at length in *Das Kapital*. The text was written sometime between May and late June 1865. For reasons that are unknown, the document was never printed during Marx's life and was found in 1897 by Eleanor Marx after Engels' death. On *Value, Price and Profit*, Lapides (2008, pp. 167–168) notes: "the questions it raises have lingered unanswered and, perhaps, unanswerable. Addressed to a trade union audience on the subject of trade unions, it throws into relief the puzzling absence of any similar discussion in *Capital*".

24 Matrix **A** and therefore also $(\mathbf{A} + \mathbf{l}\,\mathbf{c})$ are becoming rectangular instead of square matrices. Moreover, the same holds when some sectors are producing "joint products", or when the economic lifetime of machines and other capital goods is introduced by considering as an input a machine of t years old, as well as an output of t + 1 years old.

25 As Sweezy rightly stated, the rate of profits and the monopoly price are indeterminate. Following, among others, Michał Kalecki, it can be assumed that the oligopolist is applying a "mark-up" in his price-setting decisions, on top of his unit costs. In this situation, it can also be assumed that the unit costs already include a "normal rate of profits", which in mainstream microeconomic theory is referred to as the cost of capital. The cost of capital, in turn, is determined by the prevailing interest rate.

When we review Marx's monetary theory in Chapter 8, it will appear that in his view, like in orthodox Keynesian theory, the interest rate is a monetary phenomenon, which evidently influences the investment behaviour of the entrepreneurs.

26 It will be noticed in this example that the monopolistic rate of profits and the applied "mark-up" are one and the same, because there is no fixed capital that is used in several production periods.

27 In the English language literature on Marx's theory of value, this was most prominently done by Sweezy, who separated one from the other so that Marx's theory could be better understood by "those brought up in the main tradition of economic thought" (Sweezy, 1942, p. 25).

28 Robinson's views on Marx's value theory after 1942 are authoritatively reviewed by Harcourt and Kerr (2009, pp. 51–56).

29 In their intellectual biography of Kalecki, Julio López G. and Michaël Assous write: "Kalecki did never (to our knowledge) refer to the 'Law of value', and he did not use it to explain the existence of a surplus accruing to capitalists. We may assume – though he never said it explicitly – that he agreed with Marx and classical economics that profits are indeed a surplus. Moreover, he did not start from values to arrive at production prices, but conceived prices as determined by firms which mark-up unit prime costs". In a later footnote the same authors mention that according to Kazimierz Laski, a close collaborator of Kalecki in the 1950s and 1960s and author of a chapter on the labour theory of value in the *Festschrift* for Josef Steindl, he was "allergic" to any discussion on the so-called "law of value" (López G. and Assous, 2010, p. 196 and p. 242 n.15, respectively).

30 See also Howard and King (1992, p. 120). For Sweezy's defence, see Sweezy (1974, pp. 31ff.).

31 See, for example, Harcourt and Massaro (1964), Harcourt (1972, pp. 193–195) and Harcourt (2006a, pp. 98 ff.). As for Pasinetti, we refer to, for example, Pasinetti (1988) and (1993). The procedure followed by Pasinetti is to calculate the labour content of a given vector of consumer goods for which he constructs, per product of this vector, the vertically integrated sector to produce it. This labour content is then the amount of labour that is directly, indirectly and "hyper-indirectly" required in the hyper-integrated sector in which the consumer goods are produced. However, this labour content is not equal to the Marxian labour value of the vector of consumer goods, because in Pasinetti's system of equations – like in Sraffa's solution to the vector of prices of production – the rate of profits is applied to the direct and indirect inputs.

32 It remains to be seen whether today, after many years of scholarly work, Sraffa's legacy can encourage further innovative research if we stick to the "letter" of his model. It has been convincingly argued what Sraffa's research questions were exactly and how a number of scholars used Sraffa's "spirit" (rather than the "letter" of his book) to trace out new developments. See Roncaglia (2009, Ch. 8). Although Sraffa was deeply influenced by Marx, his 1960 book was largely about a number of important Ricardian riddles. That his standard system offers a solution to Marx's transformation problem is merely a by-product. As already stated, the questions we want to tackle are different and relate to the reproduction of a capitalist system and the theory of value which explains prices allowing this reproduction. Reacting to Roncaglia (2009), who identified a Smithian, a Ricardian and a Marxian interpretation of Sraffa's work, Harcourt (2015) has argued that the Marxian stream is the most appropriate one for interpreting Sraffa and for further developments.

33 However, for long-term analysis, she treats natural resources in a similar way. See Robinson (1956, pp. 310–311).

34 She refers to a passage in Keynes's *General Theory* in support.

35 As mentioned by Geoffrey Harcourt, Robinson's indebtedness is to "the hints of what was to come" in Sraffa's introduction to the *Works and Correspondence of David Ricardo*. See Harcourt (1972, p. 14).

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3 Towards a better understanding of what will follow – long-term economic growth and dynamics

In Chapter 1, we discussed the reproduction of the economic system and how the reproduction schemes allowed Marx to analyse the process of reproduction. Yet we are faced with the reality that no economic system in the past was able to achieve long-term economic growth performance comparable to that of the last 200 years under capitalism. There is obviously a need for a theory on the factors that explain economic growth and its long-term dynamics. Starting with the insights emerging from Chapter 1, we will analyse this in the light of post-Keynesian and neo-Marxist theories of growth, and we will use the linear production model of the previous chapters to "translate" the concepts.

We should issue a warning to readers at this point. What follows in this chapter is not associated with Marx or the orthodox Marxists. Nevertheless, the insights will appear relevant to our more detailed investigation into the relationship between Marx's economics and the neo-Marxist and post-Keynesian views on the development of capitalism over an extended period of time. In Marx's economics, as in the neo-Marxist and post-Keynesian theory of economic growth, balanced reproduction and growth in a capitalist economic system is the exception rather than the rule. Moreover, the output levels in a standard system à la Sraffa will be of use in Chapter 4, where we analyse Marx's views on unproductive activities and the "*faux frais*" of production.

3.1 The "standard system" again

In Chapter 2, we emphasised the role of the standard system in solving the Marxian transformation problem, while in Chapter 1, we examined the relationship between Sraffa's standard system and the sectoral output proportions allowing maximum expansion of the economy. Although Sraffa developed his standard system for a fully stationary economy, it appears to be of relevance to the theory of economic growth that is in keeping with classical, Marxian and post-Keynesian economics. This is because the maximum rate of profits in this standard system is identical to the maximum rate of growth, which is attainable for given structural relations between inputs and outputs of the production system.

We will illustrate this with a numerical example of an economy that is producing iron and wheat but also a "commodity" (to use Sraffa's terminology) that is, in fact, not needed in the production system. As an example of the latter we will use Pekinese dogs which, during the *Belle Époque*, were the pets of many ladies of the European bourgeoisie. Pekinese dogs are not used as means of production; nor do they constitute necessary consumption. They are purely a luxury in the families of the capitalists. One female and one male specimen are evidently needed to breed Pekinese dogs. We will assume that wheat is the dog food that is required for Pekinese dog breeding.

The imaginary production system is as follows:

500 kg iron +			500 hours labour → 1500 kg iron
500 kg iron +	400 kg wheat +		500 hours labour → 1000 kg wheat
	100 kg wheat	300 Pekinese dogs +	200 hours labour → 600 Pekinese dogs
1000 kg iron	500 kg wheat	300 Pekinese dogs	1200 hours labour

A net product is produced that consists of 500 kg of iron (1500 – 1000), 500 kg of wheat (1000 – 500) and 300 Pekinese dogs (600 – 300).

Per unit of output the system is thus:

0.333 kg iron + 0.333 hours labour → 1 kg iron

0.500 kg iron + 0.400 kg wheat + 0.500 hours labour → 1 kg wheat

0.167 kg wheat + 0.500 Pekinese dogs + 0.333 hours labour → 1 Pekinese dog

The system of equations that determines the labour values is therefore:

$$\lambda_1 = 0.333\,\lambda_1 + 0\,\lambda_2 + 0\,\lambda_3 + 0.333$$

$$\lambda_2 = 0.500\,\lambda_1 + 0.400\,\lambda_2 + 0\,\lambda_3 + 0.500$$

$$\lambda_3 = 0\,\lambda_1 + 0.167\,\lambda_2 + 0.500\,\lambda_3 + 0.333$$

The first equation leads to $\lambda_1 = 0.333\,\lambda_1 + 0.333$, or: $0.667\,\lambda_1 = 0.333$, from which it follows that $\lambda_1 = 0.5$ hours of labour time. Substituting this result in the second equation gives: $\lambda_2 = (0.500 \times 0.5) + 0.400\,\lambda_2 + 0.500$, or: $\lambda_2 = 0.400\,\lambda_2 + 0.750$, or else: $0.600\,\lambda_2 = 0.750$, such that $\lambda_2 = 1.25$ hours of labour time. The solution for λ_2 is now substituted in the third equation: $\lambda_3 = (0.167 \times 1.25) + 0.500\,\lambda_3 + 0.333$, or: $\lambda_3 = 0.209 + 0.500\,\lambda_3 + 0.333$, hence: $\lambda_3 = 0.500\,\lambda_3 + 0.542$, which leads to: $0.500\,\lambda_3 = 0.542$ and to $\lambda_3 = 1.083$ hours of labour time.

It is also assumed that an average worker needs 0.25 kg of wheat per working hour as necessary consumption.[1] Thus, the compensation of labour in the iron and wheat sectors, respectively, amounts to 125 kg of wheat (500 hours × 0.25 kg), and in the breeding of Pekinese dogs to 50 kg of wheat (200 hours × 0.25 kg). The production system therefore becomes:

$$
\begin{array}{llll}
500\,\text{kg iron} + & 125\,\text{kg wheat} & & \rightarrow 1500\,\text{kg iron} \\
500\,\text{kg iron} + & 525\,\text{kg wheat} & & \rightarrow 1000\,\text{kg wheat} \\
 & 150\,\text{kg wheat} + & 300\,\text{Pekinese dogs} & \rightarrow 600\,\text{Pekinese dogs} \\
\hline
1000\,\text{kg iron} & 800\,\text{kg wheat} & 300\,\text{Pekinese dogs} &
\end{array}
$$

This production system will lead to a surplus product of 500 kg of iron (1500 – 500), 200 kg of wheat (1000 – 800) and 300 Pekinese dogs (600 – 300).

Per unit of output in each sector, this can be transformed into:

$$
\begin{array}{lll}
0.333\,\text{kg iron} + 0.083\,\text{kg wheat} & & \rightarrow 1\,\text{kg iron} \\
0.500\,\text{kg iron} + 0.525\,\text{kg wheat} & & \rightarrow 1\,\text{kg wheat} \\
 & 0.250\,\text{kg wheat} + 0.500\,\text{Pekinese dogs} & \rightarrow 1\,\text{Pekinese dog}
\end{array}
$$

The calculation of the prices of production will then follow on from the system of equations:

$$p_1 = (1 + r)\,[0.333\,p_1 + 0.083\,p_2 + 0\,p_3]$$
$$p_2 = (1 + r)\,[0.500\,p_1 + 0.525\,p_2 + 0\,p_3]$$
$$p_3 = (1 + r)\,[0\,p_1 + 0.250\,p_2 + 0.500\,p_3]$$

or, in our matrix notation:

$$\mathbf{p} = (1+r)\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\mathbf{p}$$

with p_1, p_2 and p_3 being the prices of production and r being the uniform rate of profits. For the prices of production to be expressed not just as exchange proportions but as amounts of labour time, an equation can be added which states, for example, that the total value added in labour time is equal to the total value of the net product:

$$1200\text{ hours} = 50\,p_1 + 50\,p_2 + 30\,p_3$$

The solution of this system of equations is:

$$r = 52.8\%$$
$$p_1 \approx 0.28\text{ hours}$$
$$p_2 \approx 1.07\text{ hours}$$
$$p_3 \approx 1.74\text{ hours}$$

It is known from the previous chapters that the standard system is a "sub-system" of the production system, in which outputs in *each* sector are in the same proportion $(1 + R)$ to the sector's input supplies.[2]

The standard system has to be proportioned such that:

$$q^*_1 = (1 + R)\,[0.333\, q^*_1 + 0.500\, q^*_2 + 0\, q^*_3]$$

$$q^*_2 = (1 + R)\,[0.083\, q^*_1 + 0.525\, q^*_2 + 0.250\, q^*_3]$$

$$q^*_3 = (1 + R)\,[0\, q^*_1 + 0\, q^*_2 + 0.500\, q^*_3]$$

or, in matrix notation:

$$\mathbf{q^*} = (1 + R)\, \mathbf{q^*}\, (\mathbf{A} + \mathbf{l}\, \mathbf{c}),$$

with $\mathbf{q^*}$ consisting of the elements q^*_1, q^*_2, and q^*_3, the output proportions in the standard system, and R being the so-called "Standard ratio". As before, an equation is added to solve this system unambiguously, e.g. stating that in the "sub-system" the same quantity of labour is spent as in the complete production system, i.e. 1200 hours. We know that 500 hours of labour are required to produce 1500 kg of wheat, which implies that to produce q^*_1, $500 \times q^*_1/1500$ is needed, or $0.333\, q^*_1$. Similarly, the amount of labour required to produce q^*_2 and q^*_3 can be expressed as $500 \times q^*_2/1000$ and $200 \times q^*_3/600$, respectively, or $0.500\, q^*_2$ and $0.333\, q^*_3$. Hence, the fourth equation of the standard system is:

$$1200 \text{ hours} = 0.333\, q^*_1 + 0.500\, q^*_2 + 0.333\, q^*_3$$

or, in matrix notation:

$$1200 \text{ hours} = \mathbf{q^*}\, \mathbf{l}$$

The solution of the standard system is then:

$$R = 52.8\%$$

$$q^*_1 \approx 1834 \text{ kg of iron}$$

$$q^*_2 \approx 1177 \text{ kg of wheat}$$

$$q^*_3 = 0 \text{ Pekinese dogs}$$

First of all, it can be remarked upon that in the standard system, no Pekinese dogs are raised. The reason is that in the Pekinese dogs sector both iron and wheat are required directly and/or indirectly, but no Pekinese dogs enter the iron or wheat sector as inputs. Pekinese dogs, therefore, should be considered "luxuries", in keeping with Sraffa's definition. We prefer, however, to call these "unnecessary products". They are "non-basics", further using Sraffa's terminology, i.e. they are price/value determined, not price/value determining.

$1 + R$ indicates that in each sector of the standard system, the output equals $1 + R$ times the supplies as input to the other sectors and itself. In our standard system, 1834 kg of iron are produced. An iron surplus of 52.8% of the iron supplied to the iron and wheat sector requires that 611 kg are used in the iron production sector and 589 kg in the wheat sector. In total, these supplies amount to 1200 kg of iron, leading to a surplus product of 634 kg of iron, i.e. 52.8% of the total supplies allowing simple reproduction.

In our standard system, 1177 kg of wheat are also produced. From this output, 152 kg are supplied to the iron sector and 618 kg to the wheat sector (as seed for sowing and for labour compensation), such that the total supplies of wheat in the standard system amount to 770 kg. The remaining quantity is the surplus product of 407 kg of wheat, which (in similar proportion to that of iron) represent 52.8% of the total supplies of wheat.

In the standard system, a surplus product is produced which consists of 634 kg of iron and 407 kg of wheat. If, in the next production period, the supplies to its own and the other sectors increase by 52.8%, the standard system's total output of iron and wheat at the end of that period will have increased proportionately. Stated differently: if the total surplus product of the standard system is used for expanded reproduction, an economic growth rate of 52.8% will be attained.

It has been established that $R = r$. This means that R can be interpreted as the *maximum rate of growth* g^* of the economic system. It is also identical to the uniform rate of profits of the economy, if and only if all labour and inputs – which are used up in the sector of the economy producing Pekinese dogs – are devoted to the production of necessary goods. The maximum rate of growth is achieved if none of the surplus product of the standard system is going to the "unproductive consumption" of the capitalists.

3.2 The maximum rate of growth and the attainable economic growth in case of non-necessary consumption

It was seen that the outputs in the standard system are proportioned such that maximum growth at g^* is possible in this "sub-system", provided none of its surplus product is devoted to unnecessary consumption in the form of, for instance, "unproductive consumption" by the capitalists.

In another economic system with other technologies and/or other necessary consumption patterns, and therefore with the elements of $(\mathbf{A} + \mathbf{l}\,\mathbf{c})$ differing from the above example, the maximum rate of growth will be higher or lower, while other proportions between the outputs will hold, making this maximum rate of growth possible. In the absence of unnecessary consumption, the maximum rate of growth is equal to the uniform rate of profits of the same economy. Figure 3.1 depicts, for different economies, the one-to-one relationship between their maximum rate of growth and the corresponding uniform rate of profits.

Among the many forms of unnecessary consumption, the capitalists' consumption features prominently. Since $r = g^*$ in the absence of unnecessary consumption, this equality is shown in Figure 3.1 by a 45 degree line. Each economy that is expanding at its maximum rate g^* will experience a rate of growth equal to the general rate of profits. If, in the absence of capitalists' and other unnecessary consumption, the rate of profits in a given economy falls, e.g. due to the introduction

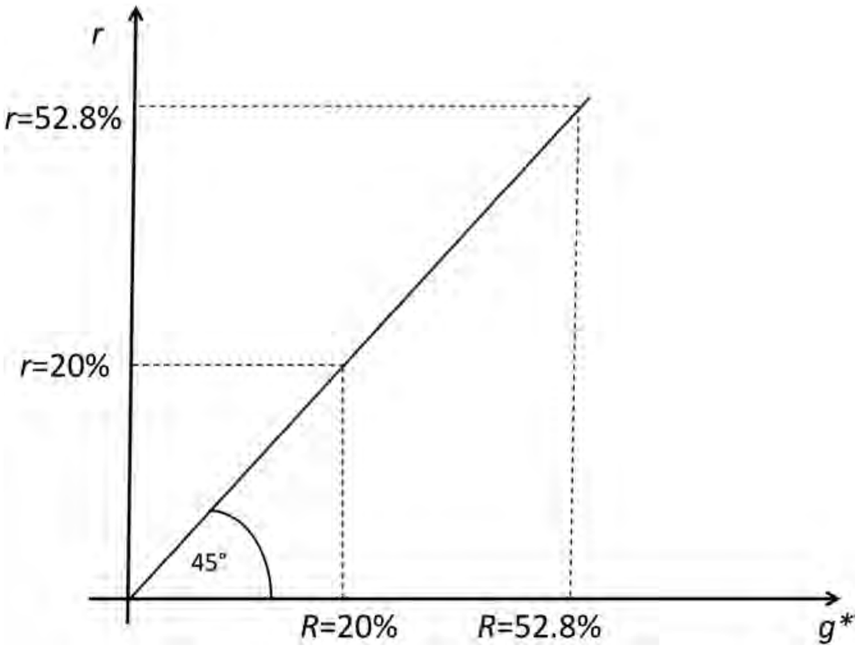

Figure 3.1 Rate of profits and maximum rate of growth in the absence of unnecessary consumption

of new labour-saving technologies, the rate of growth will also have to fall *pari passu*. This is shown by a movement along the 45 degree line of both the rate of profits and the maximum rate of growth, e.g. from 52.8% to 20%.

With positive capitalist consumption, the rate of growth will be lower than the rate of profits. The maximum rate of growth will not be attained. The surplus product does not consist solely of additional means of production and necessary consumption to be accumulated at the end of the production period; it consists partly of capitalist (unnecessary) consumption. Potential sources of accumulation are thus wasted. This consumption is, in fact, financed from profits and, therefore, lower profits remain for accumulation. Fewer investment opportunities are taken up and the rate of growth is lower than the maximum attainable rate of growth. To elucidate and illustrate this point, we should consider the standard system which led to the maximum rate of growth $g^* = 52.8\%$, but with capitalist consumption. Assume, therefore, that the capitalists buy 10% of the wheat output and 40% of the Pekinese dogs. We can then write the following system of equations:

$$q_1 = (1 + g)\,[0.333\,q_1 + 0.500\,q_2 + 0\,q_3]$$
$$q_2 = (1 + g)\,[0.083\,q_1 + 0.525\,q_2 + 0.250\,q_3] + 0.100\,q_2$$

$$q_3 = (1 + g)\ [0\ q_1 + 0\ q_2 + 0.500\ q_3] + 0.400\ q_3$$

$$1200 \text{ hours} = 0.333\ q_1 + 0.500\ q_2 + 0.333\ q_3$$

In matrix notation, this system is:

$$\mathbf{q} = (1+g)\ \mathbf{q}\ (\mathbf{A} + \mathbf{l}\ \mathbf{c}) + \mathbf{q}\ \boldsymbol{\varepsilon}$$

$$1200 \text{ hours} = \mathbf{q}\ \mathbf{l}$$

with $(\mathbf{A} + \mathbf{l}\ \mathbf{c})$ the customary matrix of necessary inputs per unit of output and $\boldsymbol{\varepsilon}$ a matrix with the proportionate shares of the capitalists' consumption of the respective goods on the diagonal and zeros elsewhere:

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.100 & 0 \\ 0 & 0 & 0.400 \end{pmatrix}$$

In this new system of equations, we add to the wheat sector the purchase by the capitalists of 0.100 q_2 and to the Pekinese dogs sector the purchase of 0.400 q_3, or 10% of q_2 and 40% of q_3. Also, the maximum rate of growth R is replaced by g, which is the rate of growth that is attainable, given the capitalists' consumption pattern. Of each kilogram of wheat that is now produced, 10% goes to the capitalists; of the remaining 90%, a portion is supplied to sectors 1, 2 and 3 (as well as to the workers in these sectors) in proportion to the quantities of sectoral output. What remains is just sufficient to supply the three sectors with the additional quantity of wheat that will allow each sector to increase output by $g \times 100\%$. The same holds for the sector where pet Pekinese dogs are raised: 40% of the annual litter is sold to the capitalists while the other 60% is just sufficient to grow the litter in the next period by $g \times 100\%$.

Solving the system of equations, we get:

$g = 20\%$

$q_1 = 1174$ kg iron

$q_2 = 1174$ wheat

$q_3 = 665$ Pekinese dogs

The next physical reproduction scheme is as follows:

391.3 kg iron +	97.4 kg wheat		→ 1174 kg iron
587.0 kg iron +	616.4 kg wheat		→ 1174 kg wheat
	166.2 kg wheat +	332 Pekinese dogs	→ 665 Pekinese dogs
978.3 kg iron	880.0 kg wheat	332 Pekinese dogs	

In the next period, the criteria for simple reproduction first have to be met, which requires putting aside 978.3 kg iron + 880.0 kg wheat + 332 Pekinese dogs. The surplus product consists of 195.7 kg of iron, 294 kg of wheat and 332 Pekinese dogs.[3]

So as not to complicate the reasoning unnecessarily, we proceed by using labour values. The surplus product, aggregated in labour values, equals total surplus value, i.e.

$$195.7\,\lambda_1 + 294\,\lambda_2 + 332\,\lambda_3 = \mathrm{M}$$

or: $$(195.7 \times 0.5 \text{ hours}) + (294 \times 1.25) + (332 \times 1.083) = \mathrm{M}$$

$$97.5 \text{ hours} + 367.5 \text{ hours} + 360.0 \text{ hours} = 825 \text{ hours} = \mathrm{M}$$

In matrix form, this system is:

$$\mathbf{q}\ [\mathbf{I}\text{–}(\mathbf{A} + \mathbf{l}\ \mathbf{c})]\ \boldsymbol{\lambda} = \mathrm{M}$$

On the other hand, the value of the total capital used is:

$$978.3\,\lambda_1 + 880.0\,\lambda_2 + 332\,\lambda_3 = \mathrm{C} + \mathrm{V}$$

or: $$(978.3 \times 0.5) + (880 \times 1.25) + (332 \times 1.083) \approx 1949 \text{ hours} = \mathrm{C} + \mathrm{V}$$

The general rate of profits in the economy is thus:

$$\frac{\mathrm{M}}{\mathrm{C}+\mathrm{V}} = \frac{825}{1949} = 42.3\% = r$$

The capitalists have already consumed 0.10 q_2 and 0.40 q_3, or 117.4 kg of wheat and 266 pet Pekinese dogs. Expressed in terms of labour values, this consumption corresponds with 117.4 λ_2 + 266 λ_3, or 435 hours, i.e. 52.7% of the surplus value M. In other words, the capitalists can accumulate 47.3% of M, or a value of 390 hours. The same value is evidently found if we calculate what remains of the value of the surplus product after the capitalists' consumption, which is accumulated and thus gives rise to Δ(C + V):

$$195.7\,\lambda_1 + (294 - 117.4)\,\lambda_2 + (332 - 266)\,\lambda_3 = 390 \text{ hours} = \Delta(\mathrm{C} + \mathrm{V})$$

The rate of increase of the invested capital (the rate of accumulation) is therefore in our imaginary economy:

$$\frac{\Delta\mathrm{C}+\mathrm{V}}{\mathrm{C}+\mathrm{V}} = \frac{390}{1949} = 20\% = g$$

The share of the surplus value that the capitalists reserve for accumulation – in our example 47.3% of the surplus value – we call the capitalists' savings rate s_c, which is fully invested. The value of the accumulated surplus value is:

$$s_c \times M = \Delta(C + V)$$

$$\text{such that:} \quad s_c \times \frac{M}{C+V} = \frac{\Delta(C+V)}{(C+V)} = g$$

In terms of the above example:

$$47.3\% \times 42.32\% = 20\% = g$$

After rearrangement, we find:

$$r = g/s_c$$

We will return to this formula when we consider the role of the realisation of surplus value in the process of accumulation.

It appears that all economies with the same capitalists' savings rate will show the same relationship between the rate of profits and the possible rate of accumulation. As in Figure 3.1, all the combinations of the rate of profits r and the rate of accumulation g lie on the same straight line, although no longer on the 45 degree line.

The capitalists' savings rate can also be determined differently. We know that the capitalists consume $\mathbf{q\varepsilon}$, of which the value is equal to $\mathbf{q}\,\boldsymbol{\varepsilon}\,\boldsymbol{\lambda}$ and is deducted from the total surplus value $M = \mathbf{q}\,[\mathbf{I} - (\mathbf{A} + \mathbf{l}\,\mathbf{c})]\,\boldsymbol{\lambda}$. The difference between M and $\mathbf{q}\,\boldsymbol{\varepsilon}\,\boldsymbol{\lambda}$ is the amount of S_c which the capitalists save:

$$S_c = \mathbf{q}\,[\mathbf{I} - (\mathbf{A} + \mathbf{l}\,\mathbf{c})]\,\boldsymbol{\lambda} - \mathbf{q}\,\boldsymbol{\varepsilon}\,\boldsymbol{\lambda}$$

$$\text{and} \quad s_c = \frac{\mathbf{q}\left[\mathbf{I} - \left(\mathbf{A} + \mathbf{l}\,\mathbf{c}\right)\right]\boldsymbol{\lambda} - \mathbf{q}\,\boldsymbol{\varepsilon}\,\boldsymbol{\lambda}}{\mathbf{q}\left[\mathbf{I} - \left(\mathbf{A} + \mathbf{l}\,\mathbf{c}\right)\right]\boldsymbol{\lambda}}$$

From $r = g/s_c$ it follows that if the total surplus value is used for accumulation, $s_c = 1$ and $r = g$, as Figure 3.1 shows, leading to a slope of 45 degrees:

$$\frac{\Delta r}{\Delta g} = \frac{1}{s_c} = 1 \text{ (if } s_c = 1)$$

$$\text{and} \quad \frac{\Delta r}{\Delta g} = \frac{1}{s_c} > 1 \text{ (if } 0 < s_c < 1)$$

Hence, if $0 < s_c < 1$, the curve that shows the relationship between r and g lies *above* the 45 degree line, as Figure 3.2 shows. This curve indicates that the same rate of profits as before is now associated with a lower rate of accumulation,

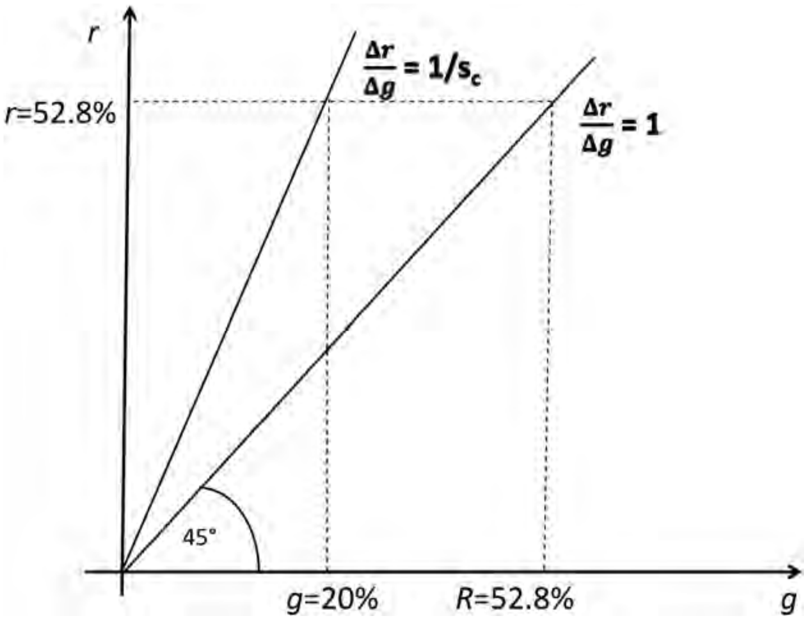

Figure 3.2 Rate of profits–rate of growth combinations for different capitalists' savings rates

and this even more so with a proportionate increase in capitalists' consumption out of profits. Since it is assumed that there is no fixed capital (or alternatively, that all capital circulates during one period of production), the rate of accumulation is also equal to the rate of expansion of output, and thus equal to the rate of economic growth in the economy.

We can see that g, the rate of economic growth, is such that inputs, employment and production capacity in each sector are expanding at the same rate of $g \times 100\%$. This g is solved from the system of equations:

$$\mathbf{q} = (1 + g)\,\mathbf{q}\,(\mathbf{A} + \mathbf{l}\,\mathbf{c}) + \mathbf{q}\,\boldsymbol{\varepsilon}$$

$$1200 \text{ hours} = \mathbf{q}\,\mathbf{l}$$

and is therefore not just the rate of growth based on value aggregates. It is the *physical percentage rate of expansion* of each sector, and thus for the economy as a whole. Each element in our production and reproduction system will increase at a rate of 20% in the next production period. Again, in our imaginary economy, we have a process of proportional expansion, with rates of expansion in the three sectors equal to g:

$$g_1 = g_2 = g_3 = g = 20\%$$

In the next production period, the situation is as follows:

$$
\begin{array}{llll}
496.6\,\text{kg iron} + 116.9\,\text{kg wheat} & & & \rightarrow 1408.8\,\text{kg iron} \\
704.4\,\text{kg iron} + 739.7\,\text{kg wheat} & & & \rightarrow 1408.8\,\text{kg wheat} \\
\underline{\hphantom{1174.0\,\text{kg iron}}} & \underline{199.4\,\text{kg wheat}} + & \underline{399.0\ \text{Pekinese dogs}} & \rightarrow 798\ \text{Pekinese dogs} \\
1174.0\,\text{kg iron} & 1056.0\,\text{kg wheat} & 399.0\ \text{Pekinese dogs} &
\end{array}
$$

Similarly, the total number of hours worked increases at a rate of 20% to 1440 hours, which is made possible by the additional employment of labour at the same real wage rate (0.25 kg of wheat per man hour worked). The extension of the working day without an increase in the rate of exploitation σ is also a possibility. This increases the number of hours that need to be worked, as well as the necessary consumption of the workers. In contrast, however, an extension of the working day without an accompanying increase in the necessary consumption will prevent an economic growth rate of 20%. Although less wheat would be used for the reproduction of labour, the quantity of wheat that would become available could neither be used in the short run in the production of iron, nor in the production of pet Pekinese dogs. In the long run, the induced decline in the real wage rate (the increase in the rate of exploitation due to the extension of the working day) *could* lead to a rise in the rate of profits – if certain conditions are fulfilled. This will be discussed in section 3.3.

3.3 Full versus incomplete realisation of surplus value

From Chapter 1 it will be remembered that the condition for a normal reproduction of the capitalist system of production is that the total output is entirely sold or, stated differently, that total output corresponds with the effective demand. Only then will the surplus value produced in the various economic sectors be reaped by the capitalists. Marx calls this a situation of full (or complete) surplus value realisation. If this condition is not fulfilled, we are confronted by incomplete surplus value realisation.

It can easily be demonstrated that the degree of realisation of the surplus value depends on the capitalists' decisions to accumulate; thus, further analysis is possible following the neo-Marxist models of economic growth provided by Michał Kalecki and Joan Robinson.

For the equality in our previous formula:

$$s_c \times \frac{M}{C+V} = \frac{\Delta(C+V)}{(C+V)} = g$$

to hold, we have to assume that:

$$s_c \times M = \Delta(C+V)$$

or, in other words, that *the not-consumed part of the surplus value is integrally accumulated* and that, as a result, the capitalists' savings rate and the rate of accumulation are identical:

$$s_c = \frac{\Delta(C+V)}{M}$$

It then follows that if less is accumulated than the capitalists are saving,[4] the capitalists' savings rate will exceed the rate of accumulation:

$$s_c > \frac{\Delta(C+V)}{M}$$

Part of the capitalists' savings will not end up in the reproduction process, and the surplus product is not entirely sold. Let us consider our numerical example again, but with the capitalists not entirely accumulating what they save. The surplus product consists of (rounded off) 196 kg of iron + 294 kg of wheat + 332 pet Pekinese dogs, corresponding with a total surplus value of 825 hours of surplus labour. The capitalists' consumption is composed of:

117.4 kg of wheat + 266 pet Pekinese dogs

which corresponds with:

$$117.4\,\lambda_1 + 266\,\lambda_2 = 435 \text{ hours}$$

What is left of the surplus product after this consumption is:

196 kg of iron + 176.6 kg of wheat + 66 pet Pekinese dogs

corresponding with 390 hours of surplus labour. We now assume that what is actually accumulated by the capitalists consists of:

147 kg of iron + 132.5 kg of wheat + 50 Pekinese dogs

which, expressed in value terms, equals:

$$147\,\lambda_1 + 132.5\,\lambda_2 + 50\,\lambda_3 = 293 \text{ hours of accumulated surplus value}$$

The produced surplus value is not completely realised. Of the 825 hours of surplus labour, only 728 hours are realised by the capitalists' consumption, equivalent to 435 hours, and accumulation, corresponding with 293 hours. *The thus realised rate of profits will be lower than the expected rate of profits*, i.e.

$$\frac{\text{Realised surplus value}}{C+V} = \frac{728\,\text{hours}}{1949\,\text{hours}} = 37.4\%$$

Most likely, the capitalists' accumulation will decrease further in the next period and with that, also the realised surplus value and the actual (realised) rate of profits. This scenario could be prevented by an increase in accumulation, but it would be conceivable only if the capitalists had regard for the social and macroeconomic realities rather than their individual interests which prescribe that they invest less if the rate of profits falls. That (at the macroeconomic level) the rate of profits will decline further if they follow their individual interests will not be recognised by the capitalists.

Alternatively, the not-accumulated part of the surplus product might be consumed. In this way, the realised and produced surplus value could be equal at 825 hours, such that the rate of profits of 42.3% could once more be obtained. The increased consumption by the capitalists would reduce their savings rate from s_c to s_c', which, in turn, would increase the slope of the line depicting the combinations of the rates of profits and the rates of accumulation to:

$$1/s_c' > 1/s_c$$

This is shown in Figure 3.3.

This reasoning can be reversed. To the extent that the capitalists consume proportionately less and accumulate the surplus product, the rate of accumulation will increase, given the realised rate of profits. This implies that in Figure 3.3

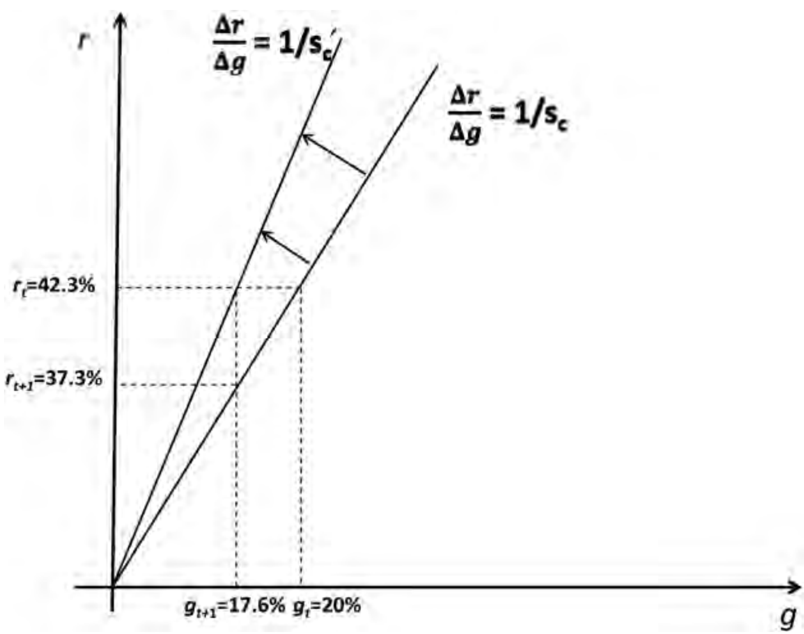

Figure 3.3 Effect of increased consumption by capitalists in a situation of incomplete realisation of the surplus value

a new line of *r-g* combinations has to be drawn, with a smaller slope than $1/s_c$.[5] Moreover, in a situation of unemployment and unused production capacity, boosting investment will increase employment, as well as sales and profits. The rise in profits, in turn, will lead to a further increase in the capitalists' accumulation. This investment-triggered mechanism of economic expansion is based on the macroeconomic policies advocated by John Maynard Keynes, for which he laid the theoretical foundation in his *The General Theory of Employment, Interest and Money* (Keynes, 1936). These policies will evidently also bear fruit if, for whatever reason, the economy shows a persistent tendency to plunge into stagnation (Steindl, 1952, p. 237). Such a tendency, if active, will in the long run reduce both the produced and the realised surplus value to a level below the *potential surplus value* (at full employment and full capacity utilisation).[6] We will return to this in a later chapter.

3.4 What about socially necessary consumption?

In the above analysis we have consistently taken into account the socially necessary consumption of the workers engaged in production. Earlier it was assumed that this socially necessary consumption consists of a basket of goods and services that are needed for the reproduction of the labour power, i.e. for the reproduction of the workers, allowing them to perform their labour in the next production period, but also for the reproduction of their family such that the next generation of workers is "produced".

This approach is appropriate when the wages earned by the workers are exactly sufficient for this reproduction of labour power, in which case these wages are called *subsistence wages*. When Marx was writing *Das Kapital*, wages were on average at the level that was required for the subsistence of the workers and their families. Periodically, significant unemployment created an "industrial reserve army", which prevented the progressive rise in these subsistence wages.

In his much-heralded standard work, *Traité d'économie marxiste* (1962), Ernest Mandel presented the orthodox Marxist position as follows:

> In the capitalist mode of production, labour-power has become a commodity. Like that of any other commodity, the value of this labour-power is determined by the amount of labour socially necessary to produce it. The value of labour-power is thus the cost of reconstituting this labour-power *in a given social setting* (food, clothing, housing, etc.). Because the worker has only his labour-power to sell in order to buy what he and his family need to live, and because of the presence of the industrial reserve army, wages vary around a *subsistence minimum*.
>
> (Mandel, 1968, Volume 1, p. 143; Mandel's italics)

Today there can be no doubt that wages are higher than the subsistence minimum. It is even questionable whether wages can still be regarded as subsistence wages. Of course, subsistence wages are to a certain extent determined by habits and customs that induce the workers and their families to follow an average consumption pattern. However, it seems to stretch reality too much to equate this consumption pattern with a well-specified bundle of goods and services per unit of labour time spent, or with the vector **c** in our matrix notation. The consumption pattern of the workers is to a lessening extent determined by what is really needed, and to a growing extent by what is suggested through marketing and publicity. An average blue or white collar worker's family has a TV set and books, buys comics (or collects rare, old ones), goes to concerts and takes a couple of weeks off each year to go abroad, etc. The socially determined component of workers' consumption has expanded, while the importance of what is required for subsistence has diminished proportionately. In addition, white and blue collar workers are saving part of their income, which allows them deferred consumption but, more importantly, will yield interest when invested, i.e. part of the surplus value. A proportion of the workers' savings is channelled via the banking sector into capitalist companies for investment, thus also contributing to the realisation of surplus value.

Marglin (1984) has devoted a lot of attention to this issue and writes, when taking into account the workers' possibility of substituting one good of **c** for another, that the subsistence consumption should be considered

> as a nominal basket of goods that workers struggle about but do not necessarily consume. That is, workers and capitalists may be assumed to bargain over a real wage that reflects a conventional consumption bundle in a given historical situation. The real wage emerges as a consequence of this struggle. Commodity prices must allow workers to purchase the "conventional" consumption bundle, the product of history and class struggle, with their nominal wages whether or not any particular worker wants to consume the conventional bundle.
>
> (Marglin, 1984, p. 268)

What if we follow Piero Sraffa and consider part of the wages earned today as *surplus wages* ? In Sraffa's words:

> We have up to this point regarded wages as consisting of the necessary subsistence of the workers and thus entering the system on the same footing as the fuel for the engines or the feed for the cattle. We must now take into account the other aspect of wages since, besides the ever-present element of subsistence, they may include a share of the surplus product.
>
> (Sraffa, 1960, p. 9)

The consequence of considering part of the wages as surplus wages is that the corresponding consumption of goods and services is consumption of the surplus product. This, like the not-necessary consumption of the capitalists, reduces the rate of growth of the economy below the maximum rate. Moreover, if this consumption out of surplus wages is consumption of goods and services – which in no possible way are inputs, i.e. neither means of production, nor necessary workers' consumption – the goods and services involved are not present in Matrix $(\mathbf{A} + \mathbf{l}\,\mathbf{c})$ and a change in the way these goods and services are produced will have no influence on the prices and quantities of the standard system, or on the rate of profits.

Denoting the surplus wages per hour of labour time by ω_s and the total number of hours worked (1200 hours in our example above) by L, the total wage bill will consist of two parts: the socially necessary consumption $\mathbf{c}\,\mathbf{p}\,\mathrm{L}$ and the surplus wages $\omega_s\,\mathrm{L}$ or, looked at per unit of output in each sector: $\mathbf{l}\,\mathbf{c}\,\mathbf{p} + \mathbf{l}\,\omega_s$. In our matrix notation, the price system is then:

$$\mathbf{p} = [(\mathbf{A} + \mathbf{l}\,\mathbf{c})\mathbf{p} + \mathbf{l}\,\omega_s](1+r)$$

or, if for simplicity's sake it is assumed that the surplus wages are paid at the end of the period and therefore are not advanced (and that no rate of profits applies to that part of the wage bill):

$$\mathbf{p} = (\mathbf{A} + \mathbf{l}\,\mathbf{c})\mathbf{p}(1 + r) + \mathbf{l}\,\omega_s$$

Choosing ω_s as "numéraire" of prices, we can write this as:

$$\frac{\mathbf{p}}{\omega_s} = (\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\frac{\mathbf{p}}{\omega_s}\,(1 + r) + \mathbf{l}$$

from which it follows:

$$[\mathbf{I} - (\mathbf{A} + \mathbf{l}\,\mathbf{c})\,(1 + r)]\,\frac{\mathbf{p}}{\omega_s} = \mathbf{l}$$

and

$$\frac{\mathbf{p}}{\omega_s} = [\mathbf{I} - (\mathbf{A} + \mathbf{l}\,\mathbf{c})\,(1 + r)]^{-1}\,\mathbf{l}$$[7]

Our Sraffa-like standard system remains:

$$\mathbf{q}^* = \mathbf{q}^*\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,(1 + R)$$

The surplus for the economy at its maximum balanced growth path is then:

$$R\,\mathbf{q}^*\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\mathbf{p} = r\,\mathbf{q}^*\,(\mathbf{A} + \mathbf{l}\,\mathbf{c})\,\mathbf{p} + \mathbf{q}^*\,\mathbf{l}\,\omega_s,$$

i.e. the sum of total profits and the surplus wage bill.

But does this add anything useful? Looking at it from the perspective of the reproduction of the economic system and the physical input-output relations across the sectors, considering part of the wages as surplus does not seem to offer any better insight. Even if we were not using Marx's concept of socially necessary consumption but were starting from a money wage rate, in each geographical and historical context in which the workers consume their wages and for a sufficiently long period, an average consumption pattern **c** per working hour would hold.[8] This, in turn, allows us to work with the Matrix **l c**, the elements of which are for each sector the various workers' necessary consumption inputs per unit of output. Adding **l c** to the physical inputs Matrix **A**, the matrix **(A + l c)** will lead to the vector **q***, the outputs of the standard system (or, viewed differently, the outputs along a maximum balanced growth path), and to R, the maximum rate of growth.

Taking into account capitalist consumption, the system defining outputs expanding at $g \times 100\%$ along the balanced growth path is:

$$\mathbf{q} = (1 + g)\, \mathbf{q}\, (\mathbf{A} + \mathbf{l}\, \mathbf{c}) + \mathbf{q}\, \boldsymbol{\varepsilon}$$

with **q** the vector of outputs and g the rate of growth of the economy. The concept of surplus wages might be relevant for the system of price equations, but it actually blurs the image of the dual system of output determination (at least if we consider the system of equations during a sufficiently long time period, such that the coefficients correspond with the underlying average input-output relations between the sectors, allowing normal expanded reproduction). We are convinced that Marx, as an economist in the classical tradition, would be much more at ease with this approach than with that of Sraffa's surplus wages.[9]

3.5 The contribution of the post-Keynesian neo-Marxists

In Volume 1 of *Das Kapital*, Marx concurs with the classical economists in the way that they regarded the capitalists' historical mission:

> Accumulate, accumulate! That is Moses and the prophets! (. . .) Therefore save, save, i.e. reconvert the greatest possible portion of surplus-value or surplus product into capital! Accumulation for the sake of accumulation, production for the sake of production: this was the formula in which classical economics expressed the historical mission of the bourgeoisie in the period of its domination. Not for one instant did it deceive itself over the nature of wealth's birth-pangs. (. . .) But what use is it to lament a historical necessity? If, in the eyes of classical economics, the proletarian is merely a machine for the production of surplus-value, the capitalist too is merely a machine for the transformation of this surplus-value into surplus capital. Classical economics takes the historical function of the capitalist in grim earnest.
>
> (Marx, 1976, p. 742)

He emphasised the importance of the capitalist urge to accumulate further as follows:

> Only as a personification of capital is the capitalist respectable. As such, he shares with the miser an absolute drive towards self-enrichment. But what

> appears in the miser as the mania of an individual is in the capitalist the effect of a social mechanism in which he is merely a cog. Moreover, the development of capitalist production makes it necessary constantly to increase the amount of capital laid out in a given industrial undertaking, and competition subordinates every individual capitalist to the immanent laws of capitalist production, as external and coercive Iaws. It compels him to keep extending his capital, so as to preserve it, and he can only extend it by means of progressive accumulation.
>
> (Marx, 1976, p. 739)

The existence of the individual capitalist as a member of his social class surrounded by rivals depends on his capital accumulation. In this way, capitalism carries huge potential for economic expansion. In order to accumulate as much as possible, each individual capitalist producer will aim to reap the highest possible profits:

- by making the employed labour force work maximally, which leads to maximum exploitation;
- by introducing technological improvements, which will force the price of his products below that of his competitors, thereby yielding surplus profits.

This has important consequences for the dynamics of capitalism, both in the short and long run. In the short run it leads to a business cycle (which we will discuss later in Chapter 8) and in the long run to a high pace of economic expansion (which is to some extent dealt with in this chapter).

Moreover, so Marx stresses, the wage rate is, in the long run, a function of the rate of accumulation, not the other way round. When wages rise faster than the capitalists' accumulation needs allow, the profit incentive is dulled and accumulation drops until the downward pressure on wages has generated sufficient profits, i.e. accumulation resources (Marx, 1976, pp. 770–771).

In the preceding section we saw how the linear production model à la Marx, Leontief, von Neumann and Sraffa – using the technologically determined input coefficients and the vector of necessary consumer goods – not only allows the calculation of labour values and prices of production, but also the general rate of profits and the maximally attainable rate of growth of the economic system. Both the rate of profit and the attainable rate of growth are thus structurally linked to the system of inter-sectoral input flows. We also found the relationship between the rate of accumulation and the rate of profits to be the translation in terms of the linear model of what is known in Marxist economics as the surplus value realisation mechanism: when the capitalist class accumulates the entire surplus product, this also realises the surplus value via the purchases of additional C and V. When part of the surplus product consists of consumer goods that are consumed by the capitalists, the rate of economic growth will fall below the maximally attainable rate of growth; nevertheless, the surplus value is realised by this consumption.

Similar mechanisms were investigated by the post-Keynesian neo-Marxists. Among these was Joan Robinson who, apart from the profit realisation mechanism, conducted a more macroeconomic study, devoting attention to the formal analysis of the capitalists' inherent urge to accumulate.[10] Robinson states that the strength of the urge to accumulate (which she calls the "animal spirits", following Keynes) determines the preparedness of the capitalists to accumulate much or little, given the expectations about the rate of profits. The higher the rate of profits that is expected in the long run, the more the capitalists will want to accumulate. Further increases in the expected rate of profits are, however, accompanied by more uncertainty. As a result, the rate of accumulation (or, for a given capital–output ratio, the rate of economic growth) increases further, but less than proportionately. The mathematical relationship between the expected rate of profits and the desired rate of accumulation is expressed in the *Robinsonian investment curve*. This should not be confused with the relationship between the rate of profits and the rate of accumulation/rate of growth for given capitalist savings rates. We call this the *realisation curve*, the functional form of which is nothing but $r = g/s_c$ (Robinson, 1956, p. 272; 1962, p. 61).

In Figure 3.4 below, the straight line depicts the realisation curve, while the convex shaped curve is the Robinsonian investment curve, given the capitalist urge to accumulate.

At a given moment in time, the rate of growth of the capital stock (the rate of accumulation) is g'. The capitalists' savings rate s_c, via this rate of accumulation, generates the realised rate of profits r'. Figure 3.4 indicates, however, that when the capitalists expect r', they are eager to accumulate more, i.e. g^+. Once this higher rate of accumulation is reached, a higher rate of profits is also realised. Some kind of equilibrium is reached at the rate of accumulation g^* and the rate of profits r^*, when the desired rate of accumulation becomes equal to the actual rate. Moreover, given the capitalist urge to accumulate, g^* is the maximum rate of accumulation that is reachable in the long run.

Often the maximal rate of accumulation g^* cannot be reached due to labour scarcity. The reserves of labour that are available in the long run determine whether the additional capital goods and other means of production can be manned or not. An intense capitalist urge to accumulate – the normal situation in the competitive phase of capitalism – will cause the capitalists to do their utmost to reach the desired rate of accumulation. In a concrete sense, the length of the working day or labour intensity will be increased, or else: new labour-saving techniques of production will be introduced. These methods pertaining to labour scarcity[11] involve what Marx calls the creation of absolute and/or relative surplus value, and will be discussed further in Chapters 5 and 7.

How labour scarcity prevents the desired rate of accumulation being reached is illustrated in Figure 3.5. The maximum attainable rate of accumulation is g'. This rate will allow the realisation of the rate of profits r'. However, if this r' is also expected in the future, the capitalists will aim at a proportionate increase in their invested capital at g^+. Therefore, labour intensity will be increased and/or new technology will be introduced until g^* can be reached.[12]

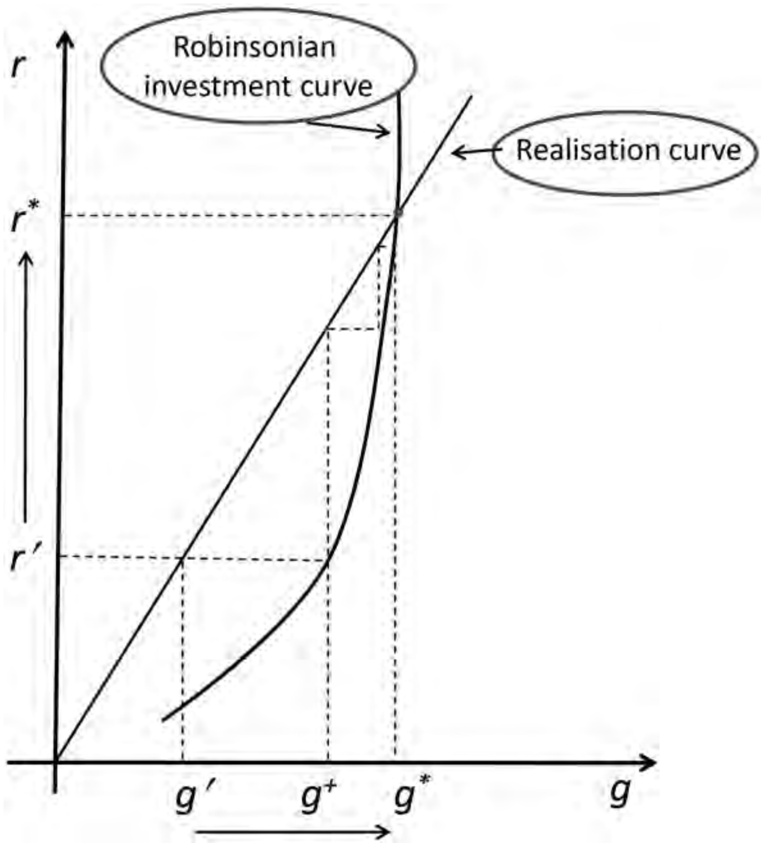

Figure 3.4 The interaction between profits realisation, profits expectations and the capitalist urge to accumulate

Apart from labour scarcity, the organised labour movement can also constrain the maximum attainable rate of growth. Robinson calls the wage rate that plays this role the inflation barrier. It can be considered the "minimum acceptable real wage" (Robinson, 1962, p. 58), "a limit to the level to which real-wage rates can fall without setting up a pressure to raise money-wage rates" (Robinson, 1956, p. 48), or else: "the level of real wages, that the workers are willing to accept and able to enforce" (Robinson, 1956, pp. 83–84). Once the inflation barrier is reached, the actual rate of accumulation cannot move any closer to the desired rate of accumulation. Each attempt to increase the actual rate will merely lead to inflation.

Marx's assumption about the capitalist urge to accumulate – which was subsequently adopted by Joan Robinson – has far-reaching consequences for the analysis of the capitalist economic growth and development process. After all,

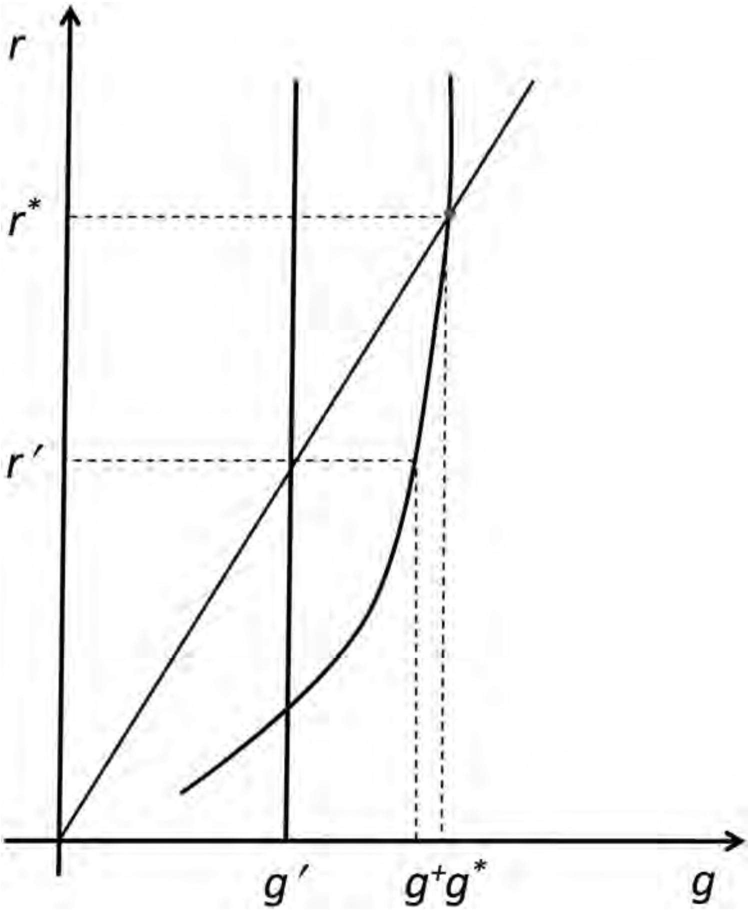

Figure 3.5 Maximum attainable rate of accumulation g' which is lower than the desired rate of accumulation g*

it is this urge to accumulate that, in the long run, leads to a positive rate of accumulation and to economic growth. If the existence of this urge to accumulate is rejected on theoretical grounds, other factors have to be thought of that can explain expanded reproduction under capitalist conditions. Rosa Luxemburg (1951, p. 344) denied the existence of this urge and emphasised the role played by the "external markets" in the continuation of the capitalist accumulation process. According to Luxemburg, "external markets" are the outlets offered in the non-capitalist part of the globe, which is also important as the "periphery to be developed" using existing reserves of raw materials and cheap labour. In addition, theoretical recourse to this Luxemburgist thesis provides interesting insights into the nature of imperialism, such as, for example, the struggle between the imperialist powers for these “external markets”. Another theoretical “by-product” is that in the absence of “external markets”, the development process logically unravels into general economic stagnation and finally into the “breakdown” of the capitalist system.

However, factors other than Luxemburg’s “external markets” can stimulate capitalist development. For instance, an important new invention might lead to large investments for the replacement and renewal of the existing capital stock, thus generating a process that Joseph Schumpeter termed “creative destruction”, which echoed Marx’s analysis of the process (Schumpeter, 1994, pp. 81ff.). Such investments foster the realisation of the surplus value and enhance economic growth.

The theoretical consequence of “external markets” or inventions and innovations is that the growth potential of capitalism depends on external factors, or at least on factors that are hardly – if at all – influenced by the dynamics of capitalist development.[13] In contrast, the assumption that an inherent capitalist urge to accumulate exists serves to underpin the view that capitalism expands *by itself* and introduces permanent new technologies in the spheres of production. We will return to some of these discussions in Chapter 9, which explores the role of effective demand in the long run dynamics of capitalism.

***

The above analysis of how realised profits are determined by the rate of accumulation rests on the assumption that workers do not save. The upshot of the analysis can be summed up in the following witticism attributed to Michał Kalecki: “Workers spend what they earn. Capitalists earn what they spend”. The post-Keynesian reasoning goes as follows:

When only capitalists save a proportion s_c of their profits M, and provided these savings are subsequently accumulated as capital, it holds that:

$$s_c \, M = \Delta\,(C + V)$$

or

$$s_c \frac{M}{C+V} = \frac{\Delta C + V}{C+V}$$

or else:

$$s_c \, r = g$$

Since the surplus value realisation mechanism implies that it is the accumulation of capital that leads to the realisation of the surplus value and not the other way round, this equality can be transformed into the equation:

$$r = g/s_c$$

When both the capitalists and the workers save, the reasoning becomes somewhat more complicated since we have to start from equality:

$$s_c M + s_w V = \Delta (C + V)$$

with s_w the savings rate of the workers and $s_w V$ the total savings of the working class. Bearing in mind that by assumption the total value added Y in the economy consists of wages and profits, and therefore that Y = M + V, this equality can be written as:

$$s_c M + s_w (Y - M) = s_w Y + (s_c - s_w)M = \Delta (C + V)$$[14]

or:

$$s_c \frac{Y}{(C+V)} + (s_c - s_w)\frac{M}{(C+V)} = \frac{\Delta(C+V)}{(C+V)}$$

or else:

$$\frac{s_w}{k} + (s_c - s_w)r = g$$

with $k = (C+V)/Y$, the capital–output ratio.

After rearranging the terms of this equality, it follows that:

$$r = \frac{g}{s_c - s_w} - \frac{s_w}{(s_c - s_w)k}$$

It will be noticed that when $s_w = 0$, the first equality $r = g/s_c$ holds. It must be assumed that $s_c > s_w \geq 0$; if not, the slope of the realisation curve is negative. This seems to be a justified assumption.[15] The following numerical example clarifies the impact of $s_w > 0$.

Assume that originally the savings ratio of the capitalists is $s_c = 0.9$ and that of the workers is $s_w = 0$. With a rate of accumulation $g = 16\% = 0.16$, this will lead to a rate of profits of $r = 17.8\%$. We assume next that g and s_c remain the same, but $s_w = 0.1$ and also $k = 0.8$.[16] If these values are put into the above formula, we will find:

$$r = \frac{0.16}{0.9 - 0.1} - \frac{0.1}{(0.9 - 0.1)0.8} = 0.20 - 0.156 = 4.4\%$$

It appears that, together with the capital–output ratio (which reflects the average capital intensity of production), the introduction of positive workers' savings has a huge impact on the rate of profits. *Ceteris paribus*, $s_c > s_w > 0$ not only increases the slope of the realisation curve, as compared to the situation when $s_w = 0$, but also moves the realisation curve of Figures 3.4 and 3.5 in a south-easterly direction.

Since the purpose of this chapter was simply to explore the relationship between the Marxian theory of value and growth and the early post-Keynesian theory of growth, as exemplified by Joan Robinson's version (which in many ways can also be regarded as neo-Marxist), we are not going to delve any deeper into this issue. Although important for the integration of both, it would quickly lead us into rather technical and theoretical discussions which are beyond the scope of this book. In any case, we hope it is clear by now that Marx's insights into the accumulation of capital and the expanded reproduction process can be revived and updated by the post-Keynesian views, thereby allowing such insights to find followers once more in some of today's discussions on political economy.

Notes

1 In Chapter 1, the assumption was that 0.5 kg of wheat is required. If we stick to this assumption in the present numerical example, there will be a shortage of wheat: 1000 kg of wheat is produced, of which 500 kg is needed as seed for sowing in the wheat sector and as dog food in the raising of Pekinese dogs. Since the labour time spent is 1200 hours, a necessary consumption of 0.5 kg of wheat per hour would require 600 kg of wheat in total. The total wheat output would be insufficient to allow simple reproduction.

2 It should be stressed again that the standard system that is used here is different from Sraffa's where wages are assumed to be paid at the end of the production period and not advanced by the capitalists at the start of that period, and where real wages are not expressed as a vector of subsistence consumption.

3 A total of 332.5 Pekinese dogs are required to breed 665 pet dogs. As 332.5 dogs are evidently impossible, this is rounded off to 332.

4 Capitalists' savings = surplus value – capitalists' consumption.

5 Because of the capitalists' lower consumption levels, the savings rate s_c' is now higher than s_c and is therefore $1/s_c' < 1/s_c$.

6 Not surprisingly, what Paul A. Baran denotes as "potential economic surplus" is playing a crucial role in the post-Keynesian neo-Marxist theory of economic growth in monopoly capitalism, which, for a number of reasons, is highly prone to stagnation (Baran, 1957, pp. 133–134).

7 If surplus wages are advanced, the solution is $p/\omega_s = [\mathbf{I} - (\mathbf{A} + \mathbf{l}\,\mathbf{c})\,(1 + r)]^{-1}\,\mathbf{l}\,(1 + r)$, with $\omega_s = 1$ ("numéraire"). This solution closely resembles $\mathbf{p} = [\mathbf{I} - \mathbf{A}(1 + r)]^{-1}\,\mathbf{l}\,(1 + r)$ in the absence of surplus wages, and with $\omega = \mathbf{c}\,\mathbf{p} = 1$ as "numéraire" (see Chapter 2, § 2.3.1), be it that in the former case the relevant matrix is $(\mathbf{A} + \mathbf{l}\,\mathbf{c})$ and in the latter only $\mathbf{A}$. Sraffa, in fact, suggests making the rate of profits the exogenous factor, determined by the monetary rate of interest (Sraffa, 1960, p. 33). To regard the rate of interest not as an endogenous variable but as exogenously given is also characteristic of some of the strands of post-Keynesian macroeconomic theory. See Chapter 8 § 8.5. We do not follow this procedure here.

8 At least when assuming that all labour is of the same quality: homogeneous abstract labour. The reasoning becomes more complex without this assumption.

9 Another important point here is that, as remarked by Goodwin (Goodwin, 1986, p. 127), unless the way that surplus wages can be fixed is specified, the distribution of the surplus between wages and profits is undetermined and so is the rate of exploitation. A comparison between the classical and Marx's views on subsistence wages and Sraffa's surplus wages, and their place in the surplus approach is made, for example, in Garegnani (1984, pp. 321–323). Sraffa's treatment of subsistence and surplus wages between the 1920s and the publication of *Production of Commodities* in 1960 is, judging from available archive material, wonderfully analysed in Picchio (2011).

10 As we will discuss in Chapter 8, there is no unanimity among the post-Keynesian neo-Marxists concerning the inherent urge to accumulate. Kalecki and Joan Robinson have a different opinion.

11 And thus the possibility of surplus value production since, according to the labour theory of value, given the rate of exploitation each additional worker will produce an additional surplus value. The impact of labour scarcity on the business cycle is discussed in Chapter 8.
12 The capitalists will probably introduce more labour-saving technologies than are required to reach g^*, thus creating a labour surplus – the "industrial reserve army" – which puts the working class in an unfavourable negotiating position.
13 This is not so when, following Marx, technological innovations and/or their introduction in production processes are considered to be an endogenous phenomenon, i.e. depending on factors like the rhythm with which the accumulation of capital takes place.
14 This is the "Keynesian" identity of savings and investments which Nicholas Kaldor (1956) introduced. Luigi Pasinetti (1962) refined this further by more appropriately taking into account the savings of the working class.
15 Since the rate of profits r is determined by the rate of accumulation g, it is justified to ask whether a positive rate of profits can exist in a "stationary state", i.e. when $g = 0$. Harris (1978, p. 185) has argued that to explain a positive r, going together with $g = 0$, the rentier class has to be introduced in the model.
16 This means that C + V is purely "circulating capital", which in each production period is entirely transferred to the national income Y and increased by M. With fixed capital, k can evidently be larger than 1.

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4 Productive and unproductive labour

A source of much unabated discussion among Marxists and scholars interested in the history of economic thought is how Marx dealt with commerce, banking and many other services relating to the so-called sphere of circulation. As a significant corollary of his labour theory of value, Marx considers such economic activities to be unproductive. We will probe this topic by looking at the relationship between the spending of the surplus value and economic growth, as outlined in the previous chapter, since it is important to assess the extent to which resources of accumulation are wasted (or otherwise) in a capitalist economy.

The present chapter deals with Marx's views on what should be considered productive and unproductive, respectively.[1] The distinction between productive and unproductive activities is far from trivial or unimportant, as in Marx's model economic activities, when unproductive, reduce the available surplus value that is redistributed as profits and therefore affect the rate of profits. That butlers and household servants of the capitalists are paid out of surplus value is easy to grasp. The same probably holds true at the macroeconomic level for the military, clergy, etc. Yet the manner in which commercial, accounting, advertising, banking or consulting services should be dealt with is different. Such activities are – like physical inputs in production processes – cost-increasing, but in contrast to the inputs of, for instance, raw materials, energy, machine equipment, etc., Marxist scholars generally argue that commerce, banking, accounting, advertising, etc. – however socially necessary they might be – do not transfer any value to the value of the use value (product or service) that is produced. Whether such cost-increasing inputs are unproductive therefore seems to depend on whether or not they transfer value or create a use value. As will be seen shortly, this is equivalent to saying that they are not generating surplus value.

First, we will show, by using a reproduction scheme, that the capitalists' consumption is unproductive, and that the same argument can be advanced in respect of other activities that are paid for out of the surplus value. Next, we will turn to the various cost-increasing inputs that Marx also considers to be unproductive. How these cost-increasing inputs are viewed is important for the discussion about the secular behaviour of the rate of profits and about the rise to prominence of the service sector in "monopoly capitalism".[2] It is evident that a number of services belong to what constitutes the necessary consumption of the workers, although some services are based on the spending of surplus wages. Clearly, it is not correct to say that Marx considers all services to be “unproductive”. We will see that storage and transportation of goods, two important service activities that are directly related to sales and therefore to the realisation of the value of the output produced in distribution, are not unproductive.

The past 30 years have witnessed a renewed interest in these cost-increasing inputs. An almost infinite number of books and articles have been published – some theoretical, others empirical, still others contemplative, some providing commentary, some simply restating. It is not our intention to attempt an overview of this vast literature.[3] Rather, we will be looking at cost-increasing inputs from a dual analytical perspective, i.e. as creating value (or not) and as giving rise (or not) to a surplus that can be accumulated in the next cycle of expanded reproduction.

In order to investigate this dual character, we will revisit the mathematical interpretation of unproductive labour, based on what David Laibman has called the “analytical definition” (Laibman, 1992, pp. 76–78), using the linear production model which was explored and developed by as diverse a collection of scholars as John von Neumann, Piero Sraffa, Michio Morishima, András Bródy and others. A first attempt to “translate” Marx’s “analytical” approach to unproductive labour associated with cost-increasing inputs into the vocabulary of this model was made by Cuyvers (1978). The aim of this chapter is to explore and further develop this mathematical approach.

4.1 Spending out of surplus value

For the sake of the argument, the two-sector reproduction scheme that we discussed earlier is augmented by a so-called “parasite” sector, the “output” of which is fully paid out of surplus value produced in sectors 1 and 2, either directly (e.g. notaries) or indirectly via the state (judicial system, military, arms spending, etc.).

$C_1 + V_1 + M_1 = W_1$ (sector 1: means of production)

$C_2 + V_2 + M_2 = W_2$ (sector 2: consumer goods)

....

$C_3 + V_3 + M_3 = W_3$ (sector 3: arms production, “law and order” infrastructure,[4] etc.)

$V'_3 = W'_3$ (sector 3': military personnel, judges, notaries, etc.)

The surplus value that is produced in each sector is spent on additional constant and variable capital as well as on unproductive consumption by the capitalists. Also, part of this surplus value is transferred directly or indirectly (via the state) to sectors 3 and 3'. This is most clearly seen when we assume that all surplus value is spent on the “output” of sectors 3 and 3', i.e. there is no capital accumulation or capitalist consumption. In that case:

$$W_3 + W'_3 = M_1 + M_2 + M_3$$

$$\text{or:} \quad C_3 + V_3 + V'_3 = M_1 + M_2$$

This transfer of surplus value effectively illustrates that the sectors 3 and 3' – even when material output is produced, such as arms – are not adding any value. The value of the "output" of sectors 3 and 3' is already counted as surplus value of sectors 1 and 2. Assessed from an accumulation perspective, the spendings involved are wasted (Cuyvers, 1972).

The implied definition of unproductive activities as activities that are based on the spending of the surplus value which does not create any additional value (including the consumer goods of the capitalists) is sometimes at odds with another definition used by Marx, i.e. that unproductive activities do not create any use value. This is discussed in the next section.

4.2 Cost-increasing inputs, value creation and the technological inputs structure

Marx's theoretical approach to the unproductive activities in capitalist commodity production and circulation – the "*faux frais*" – is straightforward: labour and means of production are unproductively deployed if they are not creating use value, or are not contributing to the increase or maintenance of use value. In contrast, labour that creates or increases use value (material production, transportation, etc.) or prevents a decrease or a deterioration in use value (e.g. storage) is productive. Commerce, i.e. the mere selling and buying of commodities, does not increase the use value of the commodities that are traded; nor does it create any use value as such. It is therefore unproductive.

Marx devoted pages to making this point. Regarding the mere commercial circulation activities, he stresses:

> The purely commercial costs of circulation (i.e. excluding the costs of dispatch, transport, storage, etc.) are the costs that are necessary to realize the value of the commodity, whether transforming it from commodity into money or from money into commodity (. . .). *All these costs are incurred not in the production of the commodities' use-value, but rather in the realization of their value; they are pure costs of circulation. They do not come into the immediate production process, but they do come into the circulation process and hence into the overall process of reproduction.*
>
> (Marx, 1981, pp. 402–403; our italics)

> Those circulation costs that proceed from the mere change in form of value, from circulation in its ideal sense, *do not enter into the value of commodities*.
>
> (Marx, 1978, p. 214; our italics)

The same argument is advanced when Marx considers bookkeeping activities, which he contrasts with warehousing and transportation. In the latter case:

> [T]he value of the commodities is conserved, or increased, only because the use-value, the product itself, is transferred under certain objective conditions that cost an outlay of capital, and subjected to operations in which additional labour works on the use-values. *The calculation of the commodity values (the book-keeping for this process) and the buying and selling, on the contrary, do not operate on the use-value in which the commodity value exists.* They are only concerned with its form. Thus although in the case assumed here these expenses of stock formation (which is here involuntary) arise purely from a delay in the change of form and from the necessity for this change; their actual object is not the formal transformation of value, but the conservation of the value which exists in the commodity as a product, a use-value, and hence can be conserved only by conserving the product, the use-value itself. *The use-value is not increased or raised; on the contrary, it declines. But its decline is restricted, and it itself is conserved.*
>
> (Marx, 1978, pp. 216–217; our italics)

It will become clear that, when stressing the difference between productive and unproductive labour (in relation to labour creating value and producing use values), Marx's views, when translated into matrix algebra, underpin the dual character of the system of equations that determines labour values $\boldsymbol{\lambda}$ and its dual determining output proportions $\mathbf{q}$. As before, we will use a simplified model of a capitalist economy to illustrate the impact of unproductive labour and cost-increasing inputs. Thereafter, we will generalise.

We start with our imaginary economy in which iron and wheat are produced but bookkeeping is also carried out, the "output" of which are entries in the accounts. In a feudal society, bookkeeping was an activity that allowed the feudal lord to keep track of what was produced and what was due to him. In capitalism, bookkeeping allows the capitalist to ascertain the costs of production, the wages paid, the sales, the profits made, etc. It is therefore a management tool on which management decision making is based.

The production system of the economy is set out below, with an output of iron and wheat equal to that of the standard system (rounded off).[5]

$$
\begin{array}{llll}
717\,\text{kg iron} & +\,8608\text{ entries} & +\,717\text{ hours labour} & \rightarrow 2152\,\text{kg iron} \\
283\,\text{kg iron} + 14.2\,\text{kg wheat} & +\,1704\text{ entries} & +\,283\text{ hours labour} & \rightarrow 568\,\text{kg wheat} \\
1.15\,\text{kg iron} & +\,1146\text{ entries} & +\,286\text{ hours labour} & \rightarrow 11\,458\text{ entries}
\end{array}
$$

It will be noticed that the total spending of labour equals 1286 hours of labour.

The production processes for iron and wheat are technologically determined. Strictly speaking, bookkeeping entries are socially necessary but not technologically required for the production of the use values. It thus makes sense to distinguish the *total inputs structure* from the *technological inputs structure*, the latter being a sub-system of the total inputs structure.

With reference to Marx's axiom about unproductive labour, the labour spent on bookkeeping does not create or preserve any use value and is unproductive. When reduced to the technological "core", the technological inputs structure of our imaginary economy is:[6]

$$717\,\text{kg iron} \qquad\qquad + 717 \text{ hours labour} \rightarrow 2152\,\text{kg iron}$$

$$283\,\text{kg iron} + 14.2\,\text{kg wheat} + 283 \text{ hours labour} \rightarrow 568\,\text{kg wheat}$$

The total labour time spent in the sectors of the technological "core" of our economy is 1000 hours (717 + 283).

Using the technological inputs structure, the labour values of iron and wheat can be calculated. We do this by considering the technological inputs per unit of output of iron and wheat:

$$0.333\,\text{kg iron} \qquad\qquad + 0.333 \text{ hours labour} \rightarrow 1\,\text{kg iron}$$

$$0.500\,\text{kg iron} + 0.025\,\text{kg wheat} + 0.500 \text{ hours labour} \rightarrow 1\,\text{kg wheat}$$

from which the system of labour values can be derived:

$$0.333\,\lambda_1 \qquad\qquad + 0.333 \text{ hours} = \lambda_1$$

$$0.500\,\lambda_1 + 0.025\,\lambda_2 + 0.500 \text{ hours} = \lambda_2$$

This is the same system of equations that we considered in Chapter 2.

In our matrix notation:

$$\mathbf{A}\boldsymbol{\lambda} + \mathbf{l} = \boldsymbol{\lambda}$$

which leads to the solution:

$$(\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{l} = \boldsymbol{\lambda}$$

In our example, the first equation can be solved directly:

$$0.333 = (1 - 0.333)\,\lambda_1$$

such that:

$$\lambda_1 = \frac{0.333}{0.667} = 0.500 \text{ hours}$$

This value of λ_1 is then substituted in the equation of the wheat sector:

$$0.500 \times (0.500) + 0.025\,\lambda_2 + 0.500 \text{ hours} = \lambda_2$$

or: $0.250 + 0.500 = (1 - 0.025)\,\lambda_2$

such that: $\dfrac{0.750}{0.975} = \lambda_2 = 0.769$ hours

We have chosen our imaginary economy on the basis that it coincides with the standard system, or, in matrix notation:

$$\mathbf{q^*} = (1 + R)\mathbf{q^*}(\mathbf{A} + \mathbf{l\ c}) = (1 + R)\mathbf{q^*B}$$ [7]

where it is assumed that the necessary workers' consumption consists of 0.25 kg of wheat per working hour. For our economy, the output system can be written out in full as:[8]

$$q^*_1 = (1 + R)\,(0.333\,q^*_1 + 0.500\,q^*_2)$$
$$q^*_2 = (1 + R)\,(0.083\,q^*_1 + 0.150\,q^*_2)$$

In addition, we define the equality:

$$0.333\,q^*_1 + 0.500\,q^*_2 = 1000 \text{ hours of labour}$$

i.e. total output is such that it is produced by the total productive labour spent.

The solution leads to $R = 115.1\%$, $q^*_1 = 2152$ kg iron and $q^*_2 = 568$ kg wheat (rounded off).

The above reproduction scheme of the standard system shows that 1000 kg of iron, 14.2 kg of wheat and 1000 hours of labour are required as inputs to produce a gross output of 2152 kg of iron and 568 kg of wheat. This implies that a net product of 1152 kg iron and 553.8 kg of wheat is produced, the value of which is 1000 hours (rounded off). Knowing the necessary consumption of 0.25 kg of wheat per working hour, we can calculate the value of the necessary consumption, the value of variable capital and constant capital, and the total surplus value in this standard system. The value of the constant capital amounts to: $1000\,\lambda_1 + 14.2\,\lambda_2 \approx 510.9$ hours. We also know that the total value added is equivalent to 1000 hours of labour time, i.e. V + M = 1000. Based on the necessary consumption of 0.25 kg of wheat per working hour, the total necessary consumption amounts to 1000 hours × 0.25 kg of wheat = 250 kg of wheat, while the labour value of the total variable capital in the standard system: $V = 250\,\lambda_2 = 192.3$ hours. Hence, considering that the total value added is V + M = 1000, M = 807.7 hours of labour time. The rate of surplus value/rate of exploitation is:

$$\sigma = \frac{\mathrm{M}}{\mathrm{V}} = \frac{807.7}{192.3} = 420\%$$

In the iron sector 717 hours are worked, for which 179.3 kg of wheat are paid as necessary consumption, equivalent to 137.9 hours. At a rate of exploitation of 420%, the surplus value in the sector is: 420% × 137.9 hours = 579.1 hours, such that V + M equals 717 hours. In the wheat sector 283 hours are worked and the necessary consumption is 70.8 kg of wheat, with a labour value of 54.4 hours. At the given rate of exploitation, the surplus value in the wheat sector is 228.6 hours. The total surplus value of both sectors is therefore equal to 807.7 hours.

According to Marx, the creation of exchange value and use value are linked, from which it logically follows that the genesis of the surplus value is in the creation of use values. This implies that, when investigated using the linear production model, the surplus value has to be calculated using the value added and the necessary consumption in the technological inputs structure **A**.

The total surplus value of the standard system can also be calculated based on a given and known rate of exploitation σ, starting from post-multiplication of $\mathbf{q^*} = (1 + R)\mathbf{q^*(A + l\ c)}$ with $\boldsymbol{\lambda}$:

$$\mathbf{q^*}\,\boldsymbol{\lambda} = (1 + R)\,\mathbf{q^*(A + l\ c)}\,\boldsymbol{\lambda}$$

After rearrangement this becomes:

$$\mathbf{q^*}\,\boldsymbol{\lambda} - \mathbf{q^*}\,\mathbf{(A + l\ c)}\,\boldsymbol{\lambda} = R\,\mathbf{q^*}\,\mathbf{(A + l\ c)}\,\boldsymbol{\lambda} = \mathrm{M} \tag{5}$$

On the other hand, pre-multiplication of $\boldsymbol{\lambda} = \mathbf{A}\,\boldsymbol{\lambda} + \mathbf{l}$ with $\mathbf{q^*}$ gives:

$$\mathbf{q^*}\,\boldsymbol{\lambda} = \mathbf{q^*}\,\mathbf{A}\,\boldsymbol{\lambda} + \mathbf{q^*}\,\mathbf{l} \tag{6}$$

From Chapter 2 it will be remembered that:

$$\mathbf{l} = (1 + \sigma)\,\mathbf{l\ c}\,\boldsymbol{\lambda}$$

such that (6) can be rewritten as:

$$\mathbf{q^*}\,\boldsymbol{\lambda} = \mathbf{q^*}\,\mathbf{A}\,\boldsymbol{\lambda} + \mathbf{q^*}\,\mathbf{l\ c}\,\boldsymbol{\lambda} + \sigma\,\mathbf{q^*}\,\mathbf{l\ c}\,\boldsymbol{\lambda} \tag{6b}$$

and:

$$\mathbf{q^*}\,\boldsymbol{\lambda} - \mathbf{q^*}\,\mathbf{(A + l\ c)}\,\boldsymbol{\lambda} = \sigma\,\mathbf{q^*}\,\mathbf{l\ c}\,\boldsymbol{\lambda} \tag{5b}$$

A comparison with the previous expression of $\mathbf{q^*}\,\boldsymbol{\lambda}$ leads to:

$$\sigma\,\mathbf{q^*}\,\mathbf{l\ c}\,\boldsymbol{\lambda} = R\,\mathbf{q^*}\,\mathbf{(A + l\ c)}\,\boldsymbol{\lambda} = \mathbf{S}\,\boldsymbol{\lambda} = \mathrm{M}$$

i.e. the total value of the surplus products or the total surplus value of the standard system. This equality will be used later in this chapter.

In our numerical example based on the technological matrix, this is:[9]

$$4.2\begin{pmatrix}2152 & 568\end{pmatrix}\begin{pmatrix}0.333\\0.500\end{pmatrix}\begin{pmatrix}0 & 0.25\end{pmatrix}\begin{pmatrix}0.500\\0.769\end{pmatrix} = 1.151\ \begin{pmatrix}2152 & 568\end{pmatrix}$$
$$\begin{pmatrix}0.333 & 0.083\\0.500 & 0.150\end{pmatrix}\begin{pmatrix}0.500\\0.769\end{pmatrix} = \mathrm{M}$$

Since $\mathbf{l}\ \mathbf{c} = \begin{pmatrix}0.333\\0.500\end{pmatrix}(0\ \ 0.25) = \begin{pmatrix}0 & 0.083\\0 & 0.125\end{pmatrix}$ is $\mathbf{l}\ \mathbf{c}\ \boldsymbol{\lambda} = \begin{pmatrix}0.064\\0.096\end{pmatrix}$ and total surplus value equals:

$$\mathrm{M} = \sigma\ \mathbf{q^*}\ \mathbf{l}\ \mathbf{c}\ \boldsymbol{\lambda} = (4.2 \times 2152 \times 0.064) + (4.2 \times 568 \times 0.096) \approx 807.7 \approx R\ \mathbf{q^*}\ \mathbf{(A + l\ c)}\ \boldsymbol{\lambda} = 1.151 \times [(2152 \times 0.333 \times 0.500) + (2152 \times 0.083 \times 0.769] + 1.151 \times [(568 \times 0.500 \times 0.500) + (568 \times 0.150 \times 0.769)]$$

4.3 The translation of unproductive inputs and outputs in the linear model of production

We will now “translate” Marx’s views on unproductive activities and unproductive labour in terms of our linear model of production.[10] We consider **A**, the technologically determined matrix of inter-sectoral input coefficients of inputs from Marx’s productive sectors into the productive sectors, i.e. the sectors in which the use values are produced. Together with **l**, the vector of direct labour input coefficients of the productive sectors, the usual system of labour values can be written as:

$$\mathbf{A}\,\boldsymbol{\lambda} + \mathbf{l} = \boldsymbol{\lambda}$$

as well as its dual system of use values:[11]

$$\mathbf{q}\,\mathbf{A} + \mathbf{N} = \mathbf{q}$$

with **N** a row vector of net products of the productive sectors, and $\boldsymbol{\lambda}$ the column vector of labour values associated with **q**, the row vector of use values.

Pre-multiplication of the labour values system with **q** and post-multiplication of the output system with $\boldsymbol{\lambda}$ gives:

$$\mathbf{q}\,\mathbf{A}\,\boldsymbol{\lambda} + \mathbf{q}\,\mathbf{l} = \mathbf{q}\,\boldsymbol{\lambda}$$

and: $\mathbf{q}\,\mathbf{A}\,\boldsymbol{\lambda} + \mathbf{N}\,\boldsymbol{\lambda} = \mathbf{q}\,\boldsymbol{\lambda}$

respectively.

Therefore, the value of the net product is equal to the labour time spent:

$$\mathbf{N}\,\boldsymbol{\lambda} = \mathbf{q}\,\mathbf{l}$$

A similar approach can be adopted for the unproductive activities. Since the productive sectors supply inputs to the unproductive sectors, we can write:

$$\mathbf{A}_{12}\boldsymbol{\lambda} + \mathbf{l'} = \boldsymbol{\lambda'}$$

with $\mathbf{A}_{12}$ and $\mathbf{l'}$, respectively, being the matrix of input coefficients relating to the supplies of the productive sectors to the unproductive sectors, and the labour input coefficients in the respective unproductive sectors. The column vector $\boldsymbol{\lambda'}$ contains the labour values of the outputs of the unproductive sectors.

Evidently, the unproductive sectors also supply cost-increasing inputs to the productive sectors, such that:

$$\mathbf{qA}_{21} + \mathbf{N'} = \mathbf{q'}$$

with the elements of $\mathbf{A}_{21}$ showing – per unit of output in the productive sectors – the supplies of the unproductive sectors to these, and with $\mathbf{q'}$ and $\mathbf{N'}$ the row vectors of unproductive outputs and the net products of the unproductive sectors, respectively.

The total inputs structure is therefore as follows:

$$\begin{pmatrix} \mathbf{A} & \mathbf{A}_{21} \\ \mathbf{A}_{12} & \mathbf{A}_{22} \end{pmatrix}\begin{pmatrix} \boldsymbol{\lambda} \\ \boldsymbol{\lambda}' \end{pmatrix} + \begin{pmatrix} \mathbf{l} \\ \mathbf{l}' \end{pmatrix} = \begin{pmatrix} \boldsymbol{\lambda} \\ \boldsymbol{\lambda}' \end{pmatrix} \qquad (7)$$

with which a dual system corresponds:

$$(\mathbf{q} \;\; \mathbf{q}')\begin{pmatrix} \mathbf{A} & \mathbf{A}_{21} \\ \mathbf{A}_{12} & \mathbf{A}_{22} \end{pmatrix} + (\mathbf{N}\;\mathbf{N'}) = (\mathbf{q} \;\; \mathbf{q'}) \qquad (8)$$

When there are n productive sectors and m unproductive sectors, it will be seen that $\mathbf{A}$, $\mathbf{A}_{21}$, $\mathbf{A}_{12}$ and $\mathbf{A}_{22}$ are partitions of the $(n + m) \times (n + m)$ matrix showing the total inputs structure, and that $\mathbf{l}$ and $\mathbf{l'}$ are $(n \times 1)$ and $(m \times 1)$ partitions, respectively, of the column vector of the labour input coefficients of the productive and the unproductive sectors. $\mathbf{N}$ and $\mathbf{N'}$, and $\mathbf{q}$ and $\mathbf{q'}$, are $(1 \times n)$ and $(1 \times m)$ partitions of the vectors of net output and total output, respectively, of the productive and the unproductive sectors.

This presentation reveals some striking contradictions, which can only be removed by adopting Marx's solution to the riddle. Writing out the expression of the upper partitions of the $\begin{pmatrix} \boldsymbol{\lambda} \\ \boldsymbol{\lambda}' \end{pmatrix}$ system gives:

$$\mathbf{A}\boldsymbol{\lambda} + \mathbf{A}_{21}\boldsymbol{\lambda'} + \mathbf{l} = \boldsymbol{\lambda}$$

which clearly contradicts $\mathbf{A}\boldsymbol{\lambda} + \mathbf{l} = \boldsymbol{\lambda}$.

One of these expressions is plainly wrong, or $\mathbf{A}_{21}\boldsymbol{\lambda'} = \mathbf{0}$ (either because $\mathbf{A}_{21} = \mathbf{0}$ or $\boldsymbol{\lambda'} = \mathbf{0}$).

Following Marx's argument, the system of equations determining the labour values based on the total inputs structure should be:

$$\begin{pmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}\begin{pmatrix} \lambda \\ \lambda' \end{pmatrix} + \begin{pmatrix} \mathbf{l} \\ \mathbf{0} \end{pmatrix} = \begin{pmatrix} \lambda \\ \lambda' \end{pmatrix}$$

A similar argument can be advanced if the upper partitions of the $(\mathbf{q} \;\; \mathbf{q}')$ system are considered:

$$\mathbf{q}\,\mathbf{A} + \mathbf{q}'\,\mathbf{A}_{12} + \mathbf{N} = \mathbf{q}$$

whereas before, it was found that $\mathbf{q}\,\mathbf{A} + \mathbf{N} = \mathbf{q}$. Again, this contradiction is only solved if $\mathbf{q}'\,\mathbf{A}_{12} = \mathbf{0}$.

That $\mathbf{A}_{21}\,\lambda' = \mathbf{0}$ and $\mathbf{q}'\,\mathbf{A}_{12} = \mathbf{0}$ is the solution to the contradictions of a simple translation of cost-increasing inputs in the language of the linear production model appears to be Marx's opinion as well. The above quotations from *Das Kapital* reveal that, according to Marx's axiom, unproductive activities are not value creating and therefore have no labour value ($\lambda' = \mathbf{0}$) and create no use value ($\mathbf{q}' = \mathbf{0}$). Moreover, the inputs $\mathbf{A}_{12}$ and $\mathbf{l}'$ do not contribute to the production of use value and should be clearly distinguished from $\mathbf{A}$ and $\mathbf{l}$, which do so. In addition, Marx suggests in *Das Kapital* that such cost-increasing inputs in the productive spheres (such as those of commerce, bookkeeping, etc.) are not technologically determined, in contrast to the productive inputs:

> It lies in the nature of the thing that a labour that consists simply in intermediary operations, involving partly the calculation of values, partly their realization, and partly again the transformation of the money realised back into means of production, a labour whose scope thus depends on the magnitude of values produced and to be realized – that a labour of this kind functions not as the cause of the respective magnitudes and amounts of these values, as does directly productive labour, but is rather a consequence of them. It is similar with the other costs of circulation. If there is much to be weighed, measured, packed and transported, there must be plenty there in the first place. The amount of packing and transport work, etc. depends on the mass of the commodities that are objects of this activity and not the other way round.
>
> (Marx, 1981, p. 414)

This would imply that $\mathbf{A}_{21} = \mathbf{0}$. Moreover, cost-increasing inputs are neither individually consumed, nor can they be stored for use in the next production period, such that $\mathbf{N}' = \mathbf{0}$ and therefore also the vector of surplus products of the unproductive sectors $\mathbf{S}' = \mathbf{0}$.

4.4 Prices and cost-increasing inputs in the total inputs structure

In Marx's economic theory and model of value, the total surplus value produced in each sector shows the same ratio to the value of labour power, i.e. the degree of exploitation σ is assumed to be equal within and between the sectors. However, in the capitalist system, prices are established that allow the same rate of profits r as a ratio of the value of the capital invested. These are Marx's prices of production. If this is not the case, then capital in the less profitable sectors will "migrate" to the more profitable sectors. This, in turn, will affect the respective outputs, such that the former sectors will produce less than is required for reproduction, and the latter more. As a result, output prices in the former sectors will rise and those in the latter sectors will fall, until in each sector the same rate of profits r is obtained and the total surplus value is distributed over the sectors in such a way that in each sector the same rate of profits is earned.[12]

Capitalists have also invested capital in the unproductive sectors, which should yield the rate of profits r. However, as shown in Marx's theory of value and prices, the expenditure of unproductive labour time and of the cost-increasing inputs have to be deducted from total surplus value (see also Cuyvers, 1978), and so the amount of surplus value that is redistributed as profits, *ceteris paribus*, will be lower. This is, for example, clearly explained by Marx in his analysis of commercial capital:

> In connection with commercial capital (. . .), we are dealing with a capital that takes a share in profit without participating in production. (. . .) Commercial capital thus contributes to the formation of the general rate of profit according to the proportion it forms in the total capital. (. . .) We thus obtain a stricter and more accurate definition of the production price. By price of production we still understand, as before, the price of the commodity as equal to its cost (i.e. the value of the constant and variable capital it contains) plus the average profit on this. But this average profit is now determined differently. It is determined by the total profit that the total productive capital produces; but it is not calculated just on this total productive capital alone (. . .); it is calculated, rather, on the total productive and commercial capital (. . .). [T]he price at which the industrial capitalist class sells (. . .) is less than their value.
>
> (Marx, 1981, pp. 397–399)

Owing to the introduction of unproductive cost-increasing inputs in the analysis, prices (prices of production) will deviate *fundamentally* from the equilibrium prices that we considered previously. Although, in Marx's view, these "*faux frais*" have no impact on the value-creation process, they affect the price-creation process. Stated differently, whereas labour values are established given the technological inputs structure, prices of production are now determined through the total inputs structure.

Starting again with the standard system:[13]

717 kg iron +		8608 entries	+ 717 hours labour → 2152 kg iron
283 kg iron +	14.2 kg wheat	+ 1704 entries	+ 283 hours labour → 568 kg wheat
1.2 kg iron		+ 1146 entries	+ 286 hours labour → 11 458 entries
1001.2 kg iron	14.2 kg wheat	11 458 entries	1286 hours labour

The net product consists of 1150.8 kg of iron and 553.8 kg of wheat.[14] The value of this net product in production prices will have to be equal to the value added in the productive sectors, i.e. 1000 hours of labour time.

When due account is taken of the workers' necessary consumption, the "surplus product" will consist of 1150 kg of iron and 303.8 kg of wheat. The corresponding surplus value will be equal to 808.5 hours of labour time (rounded off), which is the surplus value that, according to Marx, has to be distributed among all sectors of the economy.

We now consider the following price system:

$$(1+r)\begin{pmatrix} \mathbf{B} & \mathbf{B}_{21} \\ \mathbf{B}_{12} & \mathbf{B}_{22} \end{pmatrix}\begin{pmatrix} \mathbf{p} \\ \mathbf{p'} \end{pmatrix} = \begin{pmatrix} \mathbf{p} \\ \mathbf{p'} \end{pmatrix} \tag{9}$$

in which **p** is the partition of $\begin{pmatrix} \mathbf{p} \\ \mathbf{p'} \end{pmatrix}$ with unit prices of the outputs of the productive sectors and **p'** the partition with unit prices of the cost-increasing inputs. At the same time:

$$\begin{pmatrix} \mathbf{B} & \mathbf{B}_{21} \\ \mathbf{B}_{12} & \mathbf{B}_{22} \end{pmatrix} = \begin{pmatrix} \mathbf{A} & \mathbf{A}_{21} \\ \mathbf{A}_{12} & \mathbf{A}_{22} \end{pmatrix} + \begin{pmatrix} \mathbf{l\,c} & \mathbf{l\,c'} \\ \mathbf{l'c} & \mathbf{l'c'} \end{pmatrix}$$

with **l** the productive labour inputs per unit of output in the productive sectors and **l'** the unproductive labour inputs in the unproductive sectors. We assume that the necessary consumption per working hour of productive and unproductive labour is the same (0.25 kg of wheat) and that no necessary consumer goods (or services) are produced by the unproductive sectors ($\mathbf{c'} = \mathbf{0}$), such that:

$$\begin{pmatrix} \mathbf{B} & \mathbf{B}_{21} \\ \mathbf{B}_{12} & \mathbf{B}_{22} \end{pmatrix} = \begin{pmatrix} \mathbf{A} & \mathbf{A}_{21} \\ \mathbf{A}_{12} & \mathbf{A}_{22} \end{pmatrix} + \begin{pmatrix} \mathbf{l\,c} & \mathbf{0} \\ \mathbf{l'c} & \mathbf{0} \end{pmatrix}$$

Written out per partition, the system (9) is:

$$(1 + r)\,(\mathbf{B\,p} + \mathbf{B}_{21}\,\mathbf{p'}) = \mathbf{p} \tag{9b}$$

$$(1 + r)\,(\mathbf{B}_{12}\,\mathbf{p} + \mathbf{B}_{22}\,\mathbf{p'}) = \mathbf{p'} \tag{9c}$$

The dual of the system of production prices is the outputs system:

$$(1+r)(\mathbf{Q}\ \ \mathbf{Q'})\begin{pmatrix} \mathbf{B} & \mathbf{B}_{21} \\ \mathbf{B}_{12} & \mathbf{B}_{22} \end{pmatrix} = (\mathbf{Q}\ \ \mathbf{Q'}) \tag{10}$$

The (**Q, Q'**) system (10) can be interpreted as leading to outputs in each of the sectors (productive and unproductive alike), which are such that they amount to $(1 + r)$ times the inputs that each sector supplies to all sectors. In other words, (**Q, Q'**) allows all sectors to expand by $r \times 100\%$. As a consequence, the economy moves along its "equilibrium growth path".[15]

Post-multiplication of (**Q, Q'**) by $\begin{pmatrix} \lambda \\ \lambda' \end{pmatrix}$ leads to:

$$(1+r)(\mathbf{Q}\ \mathbf{Q'})\begin{pmatrix} \mathbf{B} & \mathbf{B}_{21} \\ \mathbf{B}_{12} & \mathbf{B}_{22} \end{pmatrix}\begin{pmatrix} \lambda \\ \lambda' \end{pmatrix} = (\mathbf{Q}\ \mathbf{Q'})\begin{pmatrix} \lambda \\ \lambda' \end{pmatrix}$$

in which, however, according to Marx's axiom, $\boldsymbol{\lambda'} = \mathbf{0}$.

Therefore:

$$(1 + r)\,[\mathbf{Q}\,\mathbf{B} + \mathbf{Q'}\,\mathbf{B}_{12}]\,\boldsymbol{\lambda} = \mathbf{Q}\,\boldsymbol{\lambda} \tag{10b}$$

The $\boldsymbol{\lambda}$ system with a uniform rate of exploitation σ can now be simplified to:

$$\mathbf{B}\,\boldsymbol{\lambda} + \sigma\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} = \boldsymbol{\lambda}$$

from which it follows that:

$$\mathbf{Q}\,\mathbf{B}\,\boldsymbol{\lambda} + \sigma\,\mathbf{Q}\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} = \mathbf{Q}\,\boldsymbol{\lambda}$$

and therefore:[16]

$$\mathbf{Q}\,\boldsymbol{\lambda} - \mathbf{Q}\,\mathbf{B}\,\boldsymbol{\lambda} = \sigma\,\mathbf{Q}\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} = r\,\mathbf{Q}\,\mathbf{B}\,\boldsymbol{\lambda} + (1 + r)\,\mathbf{Q'}\,\mathbf{B}_{12}\,\boldsymbol{\lambda} = \mathrm{M}$$

i.e. the surplus value in the (**Q, Q'**) system.

Writing out the (**Q, Q'**) system of equations (10), we find:

$$(1 + r)\,\mathbf{Q}\,\mathbf{B} + (1 + r)\,\mathbf{Q'}\,\mathbf{B}_{12} = \mathbf{Q} \tag{10c}$$

$$(1 + r)\,\mathbf{Q}\,\mathbf{B}_{21} + (1 + r)\,\mathbf{Q'}\,\mathbf{B}_{22} = \mathbf{Q'} \tag{10d}$$

From the previous section, we know that $(1 + R)\,\mathbf{q^*}\,\mathbf{B} = \mathbf{q^*}$, such that if $r = R$, in which case it holds for the first sub-system (10c), $\mathbf{Q'} = \mathbf{0}$ and $\mathbf{Q} = \mathbf{q^*}$. In other words, if $r = R$, the productive outputs vector will be equal (or proportional) to the outputs vector of the productive sectors along its balanced equilibrium growth path. This is logical: in each sector, $1 + R$ is the ratio of produced output and its total inter-sectoral supplies. If it is assumed that in the cost-increasing, inputs-supplying unproductive sectors, no surplus product is produced, i.e. its corresponding vector of surplus products $\mathbf{S'} = \mathbf{0}$ (and a fortiori also its vector of corresponding net products $\mathbf{N'} = \mathbf{0}$), the equality condition of sub-system (10d) is satisfied if, and only if, $\mathbf{Q'} = \mathbf{0}$.

Positive outputs $\mathbf{Q} = (q_1\ q_2)$ and $\mathbf{Q'} = q_3$ are also possible for $r < R$. We illustrate this situation using our numerical example:

$$(q_1\ q_2\ q_3) = (1 + r)\ (q_1\ q_2\ q_3) \begin{pmatrix} 0.333 & 0.083 & 4 \\ 0.500 & 0.150 & 3 \\ 0.0001 & 0.0125 & 0.10 \end{pmatrix}$$

such that $\mathbf{B} = \begin{pmatrix} 0.333 & 0.083 \\ 0.500 & 0.150 \end{pmatrix}$, $\mathbf{B}_{12} = (0.0001\ 0.0125)$, $\mathbf{B}_{21} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\mathbf{B}_{22} = 0.10$.

Adding the equality:

$$0.333\ q_1 + 0.500\ q_2 + 0.025\ q_3 = 1286 \text{ hours of labour time}$$

the solution of the system of equations is $r = 86\%$, $q_1 = 1540.9$ kg iron, $q_2 = 626.6$ kg wheat and $q_3 = 18\,384.3$ bookkeeping entries.[17] This leads to the following scheme of reproduction:

513.3 kg iron		+ 6163.6 entries +	513.3 hours	→ 1540.9 kg iron
313.3 kg iron +	15.7 kg wheat	+ 1879.8 entries +	313.3 hours	→ 626.6 kg wheat
1.8 kg iron		+ 1838.4 entries +	459.5 hours	→ 18384.3 entries
828.4 kg iron	15.7 kg wheat	9854.8 entries	1286 hours	

In this economic system, a net product is produced of 712.5 kg of iron, 610.9 kg of wheat and 8529.5 entries. It should be noted that we will now deviate from the assumption that the output of the unproductive sectors that are used as cost-increasing inputs in the productive sectors is immediately produced and used up, and therefore cannot be transferred to the next production cycle of expanded reproduction and is not part of it. In the production period under consideration, 18 384.3 bookkeeping entries are made, of which only 9854.8 are, strictly speaking, required. The remaining 8529.5 entries relate to economic transactions in the next production period. This result evidently stems from the system of equations that allows the unproductive activities to increase at a rate of 86%, while it is implicitly assumed that bookkeeping services can be accumulated, which is completely nonsensical. Obviously, bookkeeping entries cannot be made before the underlying economic transactions take place. Nevertheless, bookkeeping, like many services, is *immediately performed* and it suffices to set aside the required additional labour power and variable capital (in this case, wheat), as well as the required constant capital, such that bookkeeping activities could increase in the next production period by 86%, together with the productive activities. We will return to this in section 4.7.

On the other hand, a number of cost-increasing inputs (thus considered by Marx to be unproductive) can indeed be stored and accumulated as capital. They are therefore part of the net product and the surplus product of the economy. In 2014, Coca-Cola spent US$ 3.5 billion on advertising. This gigantic spending included the newly designed advertising campaigns for the next production period (e.g. the following year), which in other words is part of the expanded reproduction. A considerable proportion of this advertising expenditure was on physical output, such as bottles and cans of the newest design and printed with the newest publicity “eye-catcher”. This blue-printed advertising campaign for the next year can be considered, as a surplus product, for use in the next cycle of expanded reproduction. It can even be argued, without stretching the facts, that by providing the new packaging materials bearing the latest, newly designed slogans and colourful images, a new use value is created.

The balanced equilibrium growth rate r is also the rate of profits in the system of production prices based on the total inputs structure:

$$\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = (1+r) \begin{pmatrix} 0.333 & 0.083 & 4 \\ 0.500 & 0.150 & 3 \\ 0.0001 & 0.0125 & 0.10 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}$$

or, in our previous notation:

$$\begin{pmatrix} \mathbf{p} \\ \mathbf{p'} \end{pmatrix} = (1+r) \begin{pmatrix} \mathbf{B} & \mathbf{B}_{21} \\ \mathbf{B}_{12} & \mathbf{B}_{22} \end{pmatrix} \begin{pmatrix} \mathbf{p} \\ \mathbf{p'} \end{pmatrix} \tag{9}$$

with $\mathbf{B} = \begin{pmatrix} 0.333 & 0.083 \\ 0.500 & 0.150 \end{pmatrix}$, $\mathbf{B}_{12} = (0.0001\ 0.00625)$, $\mathbf{B}_{21} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\mathbf{B}_{22} = 0.10$.

We are normalising prices by adding the condition that the total profits have to be equal to the total surplus value.

Based on the equality:

$$\mathbf{Q}\,\mathbf{l} - \mathbf{Q}\,\mathbf{l}\,\mathbf{c}\,\lambda = \mathrm{M} = \sigma\,\mathbf{Q}\,\mathbf{l}\,\mathbf{c}\,\lambda$$

M = 668 hours of surplus labour and the rate of surplus value $\sigma \approx 420\%$.

However, in the next section, M is calculated to be 579.2 hours of surplus labour, such that:

$$r\,(\mathbf{QB} + \mathbf{Q'B}_{12})\,\mathbf{p} + r\,(\mathbf{QB}_{21} + \mathbf{Q'B}_{22})\,\mathbf{p'} = 579.2$$

or: $712.7\,p_1 + 289.8\,p_2 + 8502.6\,p_3 = 579.2$ hours[18]

Solving system (9) gives $r = 86\%$, $p_1 \approx 0.44$, $p_2 \approx 0.64$ and $p_3 \approx 0.009$. In comparison, it will be remembered that $\lambda_1 = 0.500$, $\lambda_2 = 0.769$ and $\lambda_3 = 0$. Hence, $\lambda > p$, which was already signalled by Marx (1981, p. 398). It is not clear whether in his analysis of, for example, commercial capital, Marx was sufficiently aware of its implications for the transformation of labour values into prices of production. In fact, we are again confronted by the transformation problem, which we had hoped to solve by using the outputs of the standard system: although total profits are equal to the surplus value that is distributed as profits, and the value (in labour value terms) of the net product of our imaginary economy is equal to the labour spent in productive sectors:

$$\mathbf{N}\,\boldsymbol{\lambda} = \mathbf{Q}\,\mathbf{l} = 826 \text{ hours}$$

the value of the net product, expressed in prices of production, deviates from this:

$$\mathbf{N}\,\mathbf{p} + \mathbf{N'}\,\mathbf{p'} = 785.3 \text{ hours}$$

Total output expressed in labour values and in prices of production are, however, equal:

$$\mathbf{Q}\,\boldsymbol{\lambda} = \mathbf{Q}\,\mathbf{p} + \mathbf{Q'}\,\mathbf{p'} = 1252.3 \text{ hours}$$

If we want to avoid the transformation problem, the "labour values"[19] $\boldsymbol{\lambda}'$ of the outputs of the unproductive sectors also have to be taken into account. This implies solving the labour values system:

$$\begin{pmatrix} \mathbf{A} & \mathbf{A}_{21} \\ \mathbf{A}_{12} & \mathbf{A}_{22} \end{pmatrix}\begin{pmatrix} \lambda \\ \lambda' \end{pmatrix} + \begin{pmatrix} \mathbf{l} \\ \mathbf{l}' \end{pmatrix} = \begin{pmatrix} \lambda \\ \lambda' \end{pmatrix}$$

or:

$$\mathbf{A}\,\boldsymbol{\lambda} + \mathbf{A}_{21}\,\boldsymbol{\lambda}' + \mathbf{l} = \boldsymbol{\lambda}$$

$$\mathbf{A}_{12}\,\boldsymbol{\lambda} + \mathbf{A}_{22}\,\boldsymbol{\lambda}' + \mathbf{l}' = \boldsymbol{\lambda}'$$

In this system $\boldsymbol{\lambda} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}$ and $\boldsymbol{\lambda}'$ is the vector of "labour values" of the unproductive cost-increasing inputs, while **A** is the technological matrix of physical inputs (in other words, the inputs of the productive sectors in themselves and in the other productive sectors), $\mathbf{A}_{12}$ the matrix (or vector) of the physical inputs of the productive sectors in the unproductive sectors, $\mathbf{A}_{21}$ the matrix of the cost-increasing inputs from the unproductive sectors in the productive sectors, and $\mathbf{A}_{22}$ the supplies of the unproductive sectors to each other. In our imaginary economy, the above system is the following system of equations:

$$0.333\,\lambda_1 + 4\,\lambda_3 + 0.333 = \lambda_1$$

$$0.500\,\lambda_1 + 0.025\,\lambda_2 + 3\,\lambda_3 + 0.500 = \lambda_2$$

$$0.0001\,\lambda_1 + 0.1\,\lambda_3 + 0.025 = \lambda_3$$

Solving the system leads to: $\lambda_1 = 0.667$, $\lambda_2 = 0.941$ and $\boldsymbol{\lambda'} = \lambda_3 = 0.028$.

Based on what we discussed in Chapter 2, it can be easily shown using the solutions **Q** and **Q'**, and **p** and **p'**, that if the total surplus value of the productive and unproductive sector is equal to total profits in the standard system with $\mathbf{Q'} > \mathbf{0}$:

$$\mathbf{Q}\,(\mathbf{I} - \mathbf{B})\,\boldsymbol{\lambda} - \mathbf{Q'}\,\mathbf{B}_{12}\,\boldsymbol{\lambda} - \mathbf{Q}\,\mathbf{B}_{21}\,\boldsymbol{\lambda'} + \mathbf{Q'}\,(\mathbf{I} - \mathbf{B}_{22})\,\boldsymbol{\lambda'}$$
$$= \sigma\,(\mathbf{Q}\,\mathbf{a} + \mathbf{Q'}\,\mathbf{a'})\,\mathbf{c}\,\boldsymbol{\lambda} = \mathrm{M} \tag{11}$$

$$= r\,(\mathbf{Q}\,\mathbf{B} + \mathbf{Q'}\,\mathbf{B}_{12})\,\boldsymbol{\lambda} + r\,(\mathbf{Q}\,\mathbf{B}_{21} + \mathbf{Q'}\,\mathbf{B}_{22})\,\boldsymbol{\lambda'} \tag{12}$$

$$= r\,(\mathbf{Q}\,\mathbf{B} + \mathbf{Q'}\,\mathbf{B}_{12})\,\mathbf{p} + r\,(\mathbf{Q}\,\mathbf{B}_{21} + \mathbf{Q'}\,\mathbf{B}_{22})\,\mathbf{p'} \tag{13}$$

It then follows that: $(\mathbf{Q}\,\mathbf{B} + \mathbf{Q'}\,\mathbf{B}_{12})\,\boldsymbol{\lambda} + (\mathbf{Q}\,\mathbf{B}_{21} + \mathbf{Q'}\,\mathbf{B}_{22})\,\boldsymbol{\lambda'} = (\mathbf{Q}\,\mathbf{B} + \mathbf{Q'}\,\mathbf{B}_{12})\,\mathbf{p} + (\mathbf{Q}\,\mathbf{B}_{21} + \mathbf{Q'}\,\mathbf{B}_{22})\,\mathbf{p'}$ and also that: $\mathbf{Q}\,\boldsymbol{\lambda} + \mathbf{Q'}\,\boldsymbol{\lambda'} = \mathbf{Q}\,\mathbf{p} + \mathbf{Q'}\,\mathbf{p'}$. Thus, allowing that $\boldsymbol{\lambda'} > \mathbf{0}$, the standard system enables the transformation of surplus value into profits without the value of output in labour values deviating from the value in prices of production. That these "labour values" impact on the surplus value or on the total value of output solely depends on whether $\mathbf{S'} > \mathbf{0}$, in which case $\mathbf{Q'} > \mathbf{0}$.[20] Cost-increasing inputs that do not lead to a surplus product ($\mathbf{S'} = \mathbf{0}$) give rise to a corresponding vector $\mathbf{Q'} = \mathbf{0}$, by which the simultaneous equality of profits and surplus value, and the value of output in labour values and prices of production in our standard system become:

$$\mathbf{Q}\,(\mathbf{I} - \mathbf{B})\,\boldsymbol{\lambda} - \mathbf{Q}\,\mathbf{B}_{21}\,\boldsymbol{\lambda'} = \sigma\,\mathbf{Q}\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} = r\,\mathbf{Q}\,\mathbf{B}\,\boldsymbol{\lambda} + r\,\mathbf{Q}\,\mathbf{B}_{21}\,\boldsymbol{\lambda'}$$

$$= r\,\mathbf{Q}\,\mathbf{B}\,\mathbf{p} + r\,\mathbf{Q}\,\mathbf{B}_{21}\,\mathbf{p'}$$

such that: $\mathbf{Q}\,\mathbf{B}\,\boldsymbol{\lambda} + \mathbf{Q}\,\mathbf{B}_{21}\,\boldsymbol{\lambda'} = \mathbf{Q}\,\mathbf{B}\,\mathbf{p} + \mathbf{Q}\,\mathbf{B}_{21}\,\mathbf{p'}$

and: $(1 + r)\,\mathbf{Q}\,\mathbf{B}\,\boldsymbol{\lambda} + (1 + r)\,\mathbf{Q}\,\mathbf{B}_{21}\,\boldsymbol{\lambda'} = (1 + r)\,\mathbf{Q}\,\mathbf{B}\,\mathbf{p} + (1 + r)\,\mathbf{Q}\,\mathbf{B}_{21}\,\mathbf{p'}$

and therefore: $\mathbf{Q}\,\boldsymbol{\lambda} = \mathbf{Q}\,\mathbf{p}$.

4.5 Unproductive cost-increasing inputs and the rate of profits

We will now return to Marx's axiom that $\boldsymbol{\lambda'} = \mathbf{0}$, mindful that the unproductive sectors leave a surplus product such that $\mathbf{Q'} > \mathbf{0}$, and will first elaborate on the relationship between unproductive cost-increasing inputs and the rate of profits. We know that the rate of profits comes as a solution to the prices of production system and also, in the absence of capitalist consumption, that it equals the equilibrium rate of growth in the dual outputs system:

$$(1+r)(\mathbf{Q}\ \ \mathbf{Q}')\begin{pmatrix}\mathbf{B} & \mathbf{B}_{21}\\ \mathbf{B}_{12} & \mathbf{B}_{22}\end{pmatrix} = (\mathbf{Q}\ \ \mathbf{Q}') \tag{10}$$

or:

$$\mathbf{Q} = (1+r)\,\mathbf{Q}\,\mathbf{B} + (1+r)\,\mathbf{Q}'\,\mathbf{B}_{12}$$

$$\mathbf{Q}' = (1+r)\,\mathbf{Q}\,\mathbf{B}_{21} + (1+r)\,\mathbf{Q}'\,\mathbf{B}_{22}$$

Post-multiplying these two sub-systems of equations by $\boldsymbol{\lambda}$ and $\boldsymbol{\lambda}'$, and bearing in mind that $\boldsymbol{\lambda}' = \mathbf{0}$, will give:

$$\mathbf{Q}\,\boldsymbol{\lambda} = (1+r)\,\mathbf{Q}\,\mathbf{B}\,\boldsymbol{\lambda} + (1+r)\,\mathbf{Q}'\,\mathbf{B}_{12}\,\boldsymbol{\lambda}$$ [21]

or, after rearranging the terms:

$$\mathbf{Q}\,\boldsymbol{\lambda} - \mathbf{Q}\,\mathbf{B}\,\boldsymbol{\lambda} = r\,\mathbf{Q}\,\mathbf{B}\,\boldsymbol{\lambda} + (1+r)\,\mathbf{Q}'\,\mathbf{B}_{12}\,\boldsymbol{\lambda}$$

Since $\boldsymbol{\lambda} = \mathbf{A}\,\boldsymbol{\lambda} + \mathbf{l}$, $\mathbf{Q}\,\boldsymbol{\lambda}$ can also be written as $\mathbf{Q}\,\mathbf{A}\,\boldsymbol{\lambda} + \mathbf{Q}\,\mathbf{l}$. As $\mathbf{B} = (\mathbf{A} + \mathbf{l}\,\mathbf{c})$, this equality becomes:

$$\begin{aligned}\mathbf{Q}\,\boldsymbol{\lambda} - \mathbf{Q}\,\mathbf{B}\,\boldsymbol{\lambda} &= \mathbf{Q}\,\mathbf{A}\,\boldsymbol{\lambda} + \mathbf{Q}\,\mathbf{l} - \mathbf{Q}\,\mathbf{A}\,\boldsymbol{\lambda} - \mathbf{Q}\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} = \mathbf{Q}\,\mathbf{l} - \mathbf{Q}\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda}\\ &= r\,\mathbf{Q}\,\mathbf{B}\,\boldsymbol{\lambda} + (1+r)\,\mathbf{Q}'\,\mathbf{B}_{12}\,\boldsymbol{\lambda}\end{aligned} \tag{14}$$

As we know that for each sector it holds that:

$$\mathbf{l} = (1+\sigma)\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} = \mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} + \sigma\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda}$$

i.e. that total labour time in each sector consists of paid ($\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda}$) and unpaid ($\sigma\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda}$) labour time, it also holds that:

$$\mathbf{Q}\,\mathbf{l} - \mathbf{Q}\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} = \mathrm{M} = \sigma\,\mathbf{Q}\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda}$$

with σ the uniform rate of surplus value and M the total surplus value.

Based on (14), it is found that:

$$\mathrm{M} = \sigma\,\mathbf{Q}\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} = r\,\mathbf{Q}\,\mathbf{B}\,\boldsymbol{\lambda} + (1+r)\,\mathbf{Q}'\,\mathbf{B}_{12}\,\boldsymbol{\lambda}$$

or: $\mathrm{M} = \sigma\,\mathbf{Q}\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} = r\,\mathbf{Q}\,\mathbf{B}\,\boldsymbol{\lambda} + \mathbf{Q}'\,\mathbf{B}_{12}\,\boldsymbol{\lambda} + r\,\mathbf{Q}'\,\mathbf{B}_{12}\,\boldsymbol{\lambda}$

Therefore: $\sigma\,\mathbf{Q}\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda} - \mathbf{Q}'\,\mathbf{B}_{12}\,\boldsymbol{\lambda} = r\,\mathbf{Q}\,\mathbf{B}\,\boldsymbol{\lambda} + r\,\mathbf{Q}'\,\mathbf{B}_{12}\,\boldsymbol{\lambda}$

and:
$$r = \frac{\sigma\,\mathbf{Q\,l\,c}\,\boldsymbol{\lambda} - \mathbf{Q'\,B}_{12}\,\boldsymbol{\lambda}}{\mathbf{Q\,B}\,\boldsymbol{\lambda} + \mathbf{Q'\,B}_{12}\,\boldsymbol{\lambda}} \tag{15}$$

Hence, with $\boldsymbol{\lambda}' = \mathbf{0}$ and $\mathbf{Q'} > \mathbf{0}$, the rate of profits is the ratio of the surplus value, which is distributed as profits (i.e. the total surplus value net of the value of the supplies of the productive to the unproductive sectors), to the value of the productive capital of the productive and the unproductive sectors. This formula (15) is similar to that used by most Marxist scholars since the pioneering empirical work of Joseph Gillman (1956) in many calculations of the systemic rate of profits. We denote (15) as the Gillman rate of profits. At the same time, this rate of profits is easily derived from the system of production prices as the ratio of the total profits in all sectors and the total capital of both the productive and unproductive sectors.[22]

The total surplus value, distributed as profits, is:

$$\sigma\,\mathbf{Q\,l\,c}\,\boldsymbol{\lambda} - \mathbf{Q'\,B}_{12}\,\boldsymbol{\lambda} = r\,(\mathbf{Q\,B}\,\boldsymbol{\lambda} + \mathbf{Q'\,B}_{12}\,\boldsymbol{\lambda}) = r\,(\mathbf{Q\,B\,p} + \mathbf{Q'\,B}_{12}\,\mathbf{p} + \mathbf{Q\,B}_{21}\,\mathbf{p'} + \mathbf{Q'\,B}_{22}\,\mathbf{p'})$$

In our numerical example with outputs $\mathbf{Q}$ and $\mathbf{Q'}$, this is:

$$\mathrm{M} = \sigma\,\mathbf{Q\,l\,c}\,\boldsymbol{\lambda} = \mathbf{Q}\,\boldsymbol{\lambda} - \mathbf{Q\,B}\,\boldsymbol{\lambda} = 669 \text{ hours} = r\,\mathbf{Q\,B}\,\boldsymbol{\lambda} + (1 + r)\,\mathbf{Q'\,B}_{12}\,\boldsymbol{\lambda}$$

and: $\mathbf{Q'\,B}_{12}\,\boldsymbol{\lambda} = 89.3$ hours

The surplus value that needs to be distributed as profits then equals:

$$\sigma\,\mathbf{Q\,l\,c}\,\boldsymbol{\lambda} - \mathbf{Q'\,B}_{12}\,\boldsymbol{\lambda} = 579.2 \text{ hours}$$

The value of total capital is:

$$\mathbf{Q\,B}\,\boldsymbol{\lambda} + \mathbf{Q'\,B}_{12}\,\boldsymbol{\lambda} = 673.1$$

and: $r = 86\%$

However, from an orthodox Marxist point of view, this all leads to the contradictory situation that although $\boldsymbol{\lambda}' = \mathbf{0}$ and $\mathbf{Q'} > \mathbf{0}$, the rate of profits can, as a matter of fact, simply be derived from the system of prices of production. The rate of profits, which is based on the labour values involved, is (to paraphrase Joan Robinson's famous dictum about labour values) "devoid of operational meaning" (Robinson, 1942, p. xi) and has no empirical relevance. If we wish to link the rate of profits to the redistribution of surplus value over the sectors, then we are forced to introduce the concepts of productive and unproductive labour being redundant as far as the rate of profits is concerned.

As stressed, the above formula for the rate of profits depends on the assumption that the unproductive activities generate a surplus product. Only then is $\mathbf{Q'} > \mathbf{0}$ and $r < R$. However, if it is assumed that the outputs of the unproductive activities are not storable and cannot be accumulated as capital, then $\mathbf{Q'} = \mathbf{0}$ and $r = R$. To the extent that these unproductive activities are immediately used, the necessary constant and variable capital has to be accumulated sufficiently to allow their increase in the next production period. Anyway, if we adopt Marx's axiom that $\boldsymbol{\lambda}' = \mathbf{0}$ and if the outputs of the unproductive sectors are not storable and cannot be accumulated, then they have no influence whatsoever on the surplus value and the total profits. The previous equality:

$$\sigma\, \mathbf{Q\, l\, c}\, \boldsymbol{\lambda} - \mathbf{Q'\, B}_{12}\, \boldsymbol{\lambda} = r\,(\mathbf{Q\, B}\, \boldsymbol{\lambda} + \mathbf{Q'\, B}_{12}\, \boldsymbol{\lambda}) = r\,(\mathbf{Q\, B\, p} + \mathbf{Q'\, B}_{12}\, \mathbf{p} + \mathbf{Q\, B}_{21}\, \mathbf{p'} + \mathbf{Q'\, B}_{22}\, \mathbf{p'})$$

then transforms into (since $\mathbf{Q'} = \mathbf{0}$):

$$\sigma\, \mathbf{Q\, l\, c}\, \boldsymbol{\lambda} = r\, \mathbf{Q\, B}\, \boldsymbol{\lambda} = r\,(\mathbf{Q\, B\, p} + \mathbf{Q\, B}_{21}\, \mathbf{p'})$$

and the rate of profits:

$$r = \frac{\sigma\, \mathbf{Q\, l\, c}\, \boldsymbol{\lambda} - \mathbf{Q'\, B}_{12}\, \boldsymbol{\lambda}}{\mathbf{Q\, B}\, \boldsymbol{\lambda} + \mathbf{Q'\, B}_{12}\, \boldsymbol{\lambda}} \tag{15}$$

reduces to:

$$r = \frac{\sigma\, \mathbf{Q\, l\, c}\, \boldsymbol{\lambda}}{\mathbf{Q\, B}\, \boldsymbol{\lambda}} = R \tag{16}$$

The rate of profits is then equal to the ratio of the surplus value produced in the productive sectors to the value of the constant and variable capital in these sectors only. This rate of profits also equals R of the standard system. It is solely determined by the conditions of production of the productive sectors, i.e. by the technological inputs structure. The status of these cost-increasing inputs is analogous to Sraffa's "luxuries" or "non-basics" (1960, pp. 6–7), the "Pekinese pet dogs" of Chapter 3.

For our final evaluation of Marx's axiom ($\boldsymbol{\lambda}' = \mathbf{0}$), we should bear in mind at this stage that:

1. To the extent that the outputs of unproductive sectors are by their nature not storable and cannot be accumulated as capital ($\mathbf{Q'} = \mathbf{0}$), they have no influence on the general rate of profits. The surplus value produced in the productive sectors is then equal to the profits on the capital of these sectors, including the cost-increasing inputs there. Marx's statement that only productive labour leads to the creation of use values is probably interesting, but redundant for the extended theoretical argument.
2. To the extent that these unproductive outputs can be considered as inputs that are recurring in the process of reproduction ($\mathbf{Q'} > \mathbf{0}$), they reduce the total surplus value that can be distributed as profits in all sectors, i.e. the total profits

in the economy. Well-known historical attempts to quantify the Marxist rate of profits[23] are mostly incorrect: assuming that $\boldsymbol{\lambda}' = \mathbf{0}$ and $\mathbf{Q}' > \mathbf{0}$, but using statistical data that are based on *prices* instead of labour values, no unproductive expenses at all need to be deducted from total *profits*. However, if we want to calculate total surplus value using data based on prices, we should add the relevant supplies of the productive to the unproductive sectors, to the total profits.

* * *

What if Marx's axiom that $\boldsymbol{\lambda}' = \mathbf{0}$ is ignored and the unproductive activities relating to the production of cost-increasing inputs are considered to be value creating ($\boldsymbol{\lambda}' > \mathbf{0}$)? As before, we will first investigate the situation when $\mathbf{Q}' > \mathbf{0}$, after which the $\mathbf{Q}' = \mathbf{0}$ case will be considered. The labour value part of equation (12) applies, from which the rate of profits can be derived:[24]

$$r = \frac{\sigma\,(\mathbf{Q}\,\mathbf{l} + \mathbf{Q}'\,\mathbf{l}')\,\mathbf{c}\,\boldsymbol{\lambda}}{(\mathbf{Q}\,\mathbf{B} + \mathbf{Q}'\,\mathbf{B}_{12})\,\boldsymbol{\lambda} + (\mathbf{Q}\,\mathbf{B}_{21} + \mathbf{Q}'\,\mathbf{B}_{22})\,\boldsymbol{\lambda}'} \tag{17}$$

In contrast, if $\mathbf{Q}' = \mathbf{0}$ and $\boldsymbol{\lambda}' > \mathbf{0}$, the rate of profits formula is reduced to:

$$r = \frac{\sigma\,\mathbf{Q}\,\mathbf{l}\,\mathbf{c}\,\boldsymbol{\lambda}}{\mathbf{Q}\,(\mathbf{B}\,\boldsymbol{\lambda} + \mathbf{B}_{21}\,\boldsymbol{\lambda}')} \tag{18}$$

What should be remembered for our final evaluation of Marx's views on unproductive labour?

1. To the extent that the outputs of the unproductive sectors by their nature cannot be stored and cannot be accumulated as capital ($\mathbf{Q}' = \mathbf{0}$), but are the result of activities that are considered as value creating ($\boldsymbol{\lambda}' > \mathbf{0}$, in deviation of Marx's axiom), the value of their supplies to the productive sectors influences the determination of the rate of profits. However, only the surplus value, produced in the productive sectors, is of relevance.
2. To the extent that the unproductive outputs recur in the next cycle of the reproduction process ($\mathbf{Q}' > \mathbf{0}$) and are value creating ($\boldsymbol{\lambda}' > \mathbf{0}$), the rate of profits is the ratio of the total surplus value of the productive and unproductive sectors, and the total constant and variable capital of both types of sectors.

In conclusion, the general rate of profits of an economy is therefore equal to:

$$r = \frac{\sigma\,(\mathbf{Q}\,\mathbf{l} + \mathbf{Q}'\,\mathbf{l}')\,\mathbf{c}\,\boldsymbol{\lambda}}{(\mathbf{Q}\,\mathbf{B} + \mathbf{Q}'\,\mathbf{B}_{12})\,\boldsymbol{\lambda} + (\mathbf{Q}\,\mathbf{B}_{21} + \mathbf{Q}'\,\mathbf{B}_{22})\,\boldsymbol{\lambda}'} \tag{17}$$

According to their nature, some cost-increasing inputs (e.g. the components of commerce and finance that can be considered to be pure circulation costs) *will enter the formula with zero labour values. Other cost-increasing inputs will enter the formula with zero outputs, as not usable in the next cycle of expanded reproduction (e.g. bookkeeping). Accordingly, the required inputs of means of production and labour, or the associated supplies of cost-increasing inputs, do not affect the rate of profits.*

What is the importance today of the cost-increasing inputs, the production of which does not give rise to a surplus product and therefore cannot contribute to expanded reproduction? Stated differently: what is the importance of the cost-increasing inputs for which $\mathbf{Q'} = \mathbf{0}$?

Another issue is how the labour theory of value can contribute to an understanding of why these unproductive expenses are made, if they induce the rate of profits to decrease. The point is that these unproductive expenses generate dynamic effects which cause the rate of profits to increase. First of all, commerce, for instance, will shorten the time taken for capital to circulate. The industrial capitalist sells his produce to merchants, who will store it in anticipation of a final sale. The industrial capitalist thus disposes of his advanced capital more quickly and can restart production. The amount of output of goods and services that can be produced with the same amount of capital increases. Consequently, the surplus value produced in the given time period increases, and since the quantity of capital is still the same, the rate of profits rises. In addition, many of these unproductive activities will lead to higher labour productivity due to the increased scale of production that accompanies the higher division of labour and specialisation. Marx develops this argument pertaining to the dynamic effects of commercial capital over many pages in Volume 3 of *Das Kapital* and summarises these effects as follows:

> Commercial capital thus creates neither value nor surplus-value, at least not directly. In so far as it contributes towards shortening the circulation time, it can indirectly help the industrial capitalist to increase the surplus-value he produces. In so far as it helps to extend the market and facilitates the division of labour between capitals, thus enabling capital to operate on a bigger scale, its functioning promotes the productivity of industrial capital and its accumulation. In so far as it cuts down the turnover time, it increases the ratio of surplus-value to the capital advanced, i.e. the rate of profit. And in so far as a smaller part of capital is confined to the circulation sphere as money capital, it increases the portion of capital directly applied in production.
>
> (Marx, 1981, pp. 392–393)

Economies of scale violate the assumption of given input coefficients of Matrix $\mathbf{A}$ and Vector $\mathbf{l}$, which will then change with an expansion of the scale of operations. If labour productivity evolves endogenously in the various sectors, this is good reason to closely study the relationship between the factors that cause labour productivity to rise and their effects, and to integrate this into the model that we have used until now. This field of research is still in its infancy.[25] A clue might be supplied by the so-called “Verdoorn law” on the relationship between the growth rate of industrial production and that of the productivity of labour (Kaldor, 1966).[26] This "law", however, applies to the entire manufacturing industry, not to individual sectors. One reason for this is that economies of scale are essentially a macro-phenomenon. Between the sectors "spill-over effects" are playing out, with technological knowledge (and thus increasing labour productivity) accumulated in one sector flowing to other sectors. An alternative to the "Verdoorn law" is to assume that productivity increases are generated by new technological knowledge. However, if we do not want to assume that this new knowledge falls from the skies like manna but rather generates itself endogenously in the dynamic process of capital accumulation, we must return to Marx's views about technological change; yet we also find ourselves in the middle of the present-day discussions on endogenous economic growth.[27]

The impact of economies of scale, learning curve effects and spill-over effects on the coefficients of Matrix **A** and Vector **l** has to be built endogenously into the input-output structure of the linear model. This brings us to dynamic input-output analysis.

4.6 Unproductive labour today and further arguments

On the previous pages we were confronted by the assumptions needed to deal with Marx's concepts of productive and unproductive labour within the framework of the linear production model à la Leontief–von Neumann–Sraffa. We stressed that Marx considers as productive inputs only those that create use value or contribute to maintaining use value. According to this view, commerce and financial services take place within the sphere of circulation. They create no use value and are therefore unproductive. The importance of this view for our purpose is that the activities involved, although indirectly productive,[28] reduce, in Marx's view, the surplus value that is distributed over the economic sectors.

In abstracto, this reasoning makes sense. But does it also make sense in reality? As a reader, and even as a researcher, it is easy to get carried away by an abstract argument. This seems to be the case here. Whereas in Marxism there was, with some further refinement, still a connection between value and price theory the integration and evaluation of productive and unproductive labour now becomes problematic, for both theoretical and empirical reasons.

The systems of equations enabling the determination of outputs, labour values and prices of production could also be solved when adding unproductive activities, albeit in some cases the transformation problem reappeared. It is not the mathematics that is the problem, but rather what is included in the system of price- and output-determining equations. If Marx's axiom that cost-increasing inputs are not value creating is abandoned, the relevant unproductive sectors can be added, leading to outputs, labour values and prices of production. This evidently implies that what is considered as "productive" and "unproductive" is often based on the researcher's judgement (and pre-judgement), which also implies that much of the occasionally heated debate among Marxists as to how to theoretically explain the expansion of the service sector is largely misguided as a result of misconceptions and a large dose of metaphysics.

With respect to the value-creation process, David Laibman, a leading Marxist economist, refuses to distinguish between the sphere of production and the sphere of circulation. He concludes that all attempts have failed to establish a solid basis for the analytical distinction between productive and unproductive labour (Laibman, 1992, Chapter 4). In a comment on a paper, he writes:

> In either its analytic or descriptive guises, the distinction [between productive and unproductive labour] plays no useful role, and should be dropped. The clear positive implication of this conclusion is that *all waged labor employed by capitalists creates value;* there is no secondary redistribution from productive to unproductive sectors.
>
> (Laibman, 1999, p. 64; italics are Laibman's)

Moreover, the discussion about unproductive labour has important consequences for the logic behind Marx's value theory. It was argued in Chapter 2 that labour values receive the same status as prices of production if we assume – along with Marx – that the workers in the various sectors pursue the same (average or normal) wages–profits ratio. When capital mobility between the sectors is absent, prices will emerge that are equal or proportionate to labour values. When both tendencies prevail, prices in the economy can be considered as oscillating between labour values and prices of production, and that, from a theoretical point of view, labour values and prices of production are two sides of the same coin.[29]

If we assume, for the sake of the argument, that there is no active tendency towards the equalisation of the rate of profits and we apply this line of reasoning to an a-capitalist economy with Marx's unproductive sectors, it will be seen that:

1. Equilibrium prices will be found in the sectors producing cost-increasing inputs that are equal (or proportional) to $\boldsymbol{\lambda}'$ the corresponding labour values. This is evidently impossible when $\boldsymbol{\lambda}' = \mathbf{0}$.
2. If Marx's axiom $\boldsymbol{\lambda}' = \mathbf{0}$ is followed, equilibrium prices will be found in the productive sectors, which are not equal (or proportional) to $\boldsymbol{\lambda}$, the labour values of the respective outputs produced in the productive sectors. The reason for this is that the proportion in which cost-increasing inputs are used differs between the sectors, as does the surplus value that remains as profits.

It can therefore be concluded that unproductive inputs in Marx's analysis cannot be reconciled with equilibrium prices that are equal (or proportional) to labour values. Only when it is allowed that the production of cost-increasing inputs is value creating can Marx's suggestion of a tendency towards equalisation of the rate of exploitation lead to labour values, $\boldsymbol{\lambda}$ and $\boldsymbol{\lambda}'$, as equilibrium prices.

The question regarding what prevents the production of cost-increasing inputs being considered as value creating is therefore a legitimate one. In an interesting passage in Volume 2 of *Das Kapital*, Marx writes about the transport sector:

> In the general formula, the product of *P* is considered as a material thing different from the elements of the productive capital, an object that has an existence of its own, apart from the production process, possessing a useful form different from that of the elements of production. In so far as the result of the production process does appear as a thing, this is always the case, even when a part of the product enters once more as an element into the renewed production process. Thus grain serves as seed-corn for its own production, but the product consists only of grain, and thus has a different physical shape from the elements applied together with it: labour-power, instruments of labour, fertilizer. There are however particular branches of industry in which the product of the production process is not a new objective product, a commodity. The only one of these that is *economically important* is the communication industry, both the transport industry proper, for moving commodities and people, and the transmission of mere information, letters, telegrams, etc. (. . .) [W]hat the transport industry sells is the actual change of place itself. The useful effect produced is inseparably connected with the transport process, i.e. the production process specific to the transport industry. (. . .) *The useful effect can only be consumed during the production process; it does not exist as a thing of use distinct from this process, a thing which functions as an article of commerce and circulates as a commodity only after its production. However the exchange value of this useful effect is still determined, like that of any other commodity, by the value of the elements of production used up in it (labour power and means of production), plus the surplus-value created by the surplus labour of the workers occupied in the transport industry. In respect of its consumption, too, this useful effect behaves just like other commodities. If it is consumed individually, then its value vanishes with its consumption; if it is consumed productively, so that it is itself a stage of production of the commodity that finds itself transported, then its value is carried over to the commodity as an addition to it.*
>
> (Marx, 1978, pp. 134–135; our italics)

This passage shows clearly that, in spite of the output of the transport sector being not material and not storable, and therefore not part of the surplus product and not suitable for accumulation, the sector nevertheless produces exchange value. It is a logical statement as transportation (like storage) conserves use value, which for Marx is a productive activity. The point that we want to make is that a similar argument can be advanced for the “products” and services of the financial sector, as well as i.e. advertising, designing, accounting, etc., which today, like the transport sector in Marx’s time, are “economically important”. Jacques Nagels argues (1974, pp. 73–75, 92–93, 98–99, 101–102) that as a result of the increasing socialisation of production, industrial research and also a growing component of management should be considered as part of *die Gesamtarbeit*.

We have also seen that Marx, in different places in *Das Kapital*, indicates that such services are indirectly productive: they accelerate the rotation time of capital and allow more to be produced with the same amount of capital. In terms of the mathematics of the linear production model, the input coefficients of the production period become smaller, which has a positive impact on the rate of profits. But it should be clear that the model completely neglects the dynamics of this process. How, and by how much, the input coefficients are changing remains outside of the model, which means that the concept of unproductive labour time being spent on the production of cost-increasing inputs is also based on this purely static linear model. To what extent these so-called unproductive activities are "indirectly productive" (dixit Marx) is not elaborated on in *Das Kapital*; nor can it be based on a model with essentially fixed input proportions. It is a scientific mission to transform the linear production model, which implicitly or explicitly is used in Marxist economics, into a non-linear dynamic model. Much can be learnt from the abundant scholarly literature available in this field, which introduced the time factor (Leontief, 1970), technological change (Pasinetti, 1965, 1993), choice of production techniques as a function of prices and wages (Morishima, 1964, pp. 54ff.), R&D-driven growth (Los, 2000), etc. (see, for example, Duchin and Szyld, 1985). However, these dynamic models lead to unstable rates of growth due to their underlying assumptions, which has reduced their academic appeal over time.[30]

Marx's "productive labour" concept can contribute little to the understanding of today's world and has lost much of its empirical usability. We already know from Marx that most of the logistics activities are productive. However, how are we to view the spectacular expansion of the financial sector, which is not only involved in money circulation and lending, and is "indirectly productive", but also provides financial services (management of financial assets, insurance, etc.) to consumers (and hence to the workers)? What about advertising and publicity, which create consumer needs and thus contribute to the production of new use values – which, in turn, are steadily penetrating socially necessary consumption?

The service sectors have become a large part of the gross national product and we find it appalling that all these service activities are financed by the surplus value of the economy. In the 1960s, the Marxist literature devoted a lot of attention to the interpretation of the service sectors and the implications for the future of capitalism. In *Monopoly Capital*, the ground-breaking book that Paul Baran and Paul Sweezy published in 1966, it was argued that an ever-increasing proportion of the actual as well as the potential economic surplus is wasted on publicity, sales promotion, finance, insurance, etc., which showed the growing irrationality of the capitalist system. Apparently almost seamlessly, their neo-Marxist theory linked Marx's theory of unproductive labour to that of growing surplus value realisation problems and the resulting stagnation of monopoly capitalism.

That what happens in the sphere of production and the sphere of circulation is intertwined and that both spheres are necessary for the creation of the surplus value – in other words that the potential surplus value which is produced needs to be realised and is determined by the capitalists' savings and investment behaviour – is forcefully argued by Harris (1975, 1978), and can clearly be seen by combining our Figures 2.1[31] and 3.4: with a given wage rate ω^* (and a rate of exploitation) corresponds a rate of profits r^*, which, however, requires a rate of accumulation g^* to be realised. Through accelerating the rotation of capital, commerce and banking, for instance, have an increasing effect on the rate of profits. As a result, as long as this effect is working, the wage–profits relationship (Figure 2.1) changes, with a higher rate of profits corresponding to each given wage rate. If the urge to accumulate is high enough, this higher rate of profits will be realised by increasing capital accumulation.

Over the years, the Marxist view that an increasingly important part of the surplus value is realised by unproductive expenditures has become more implausible. Ernest Mandel gives an alternative interpretation by pointing to the existence of surplus capital in "late capitalism", which he attributes to the over-capitalisation of non-invested capital. He writes:

> As long as "capital" was relatively scarce, it normally concentrated on the direct production of surplus-value in the traditional domains of commodity production. But if capital is gradually accumulated in increasingly abundant quantities, and a substantial part of social capital no longer achieves valorization at all, the new mass of capital will penetrate more and more into areas which are non-productive.
>
> (Mandel, 1975, pp. 387–388)

To some extent this reconciles the orthodox Marxist view of unproductive labour as not being value creating with the empirically untenable implication that if these activities are financed by surplus value, the remainder of that surplus value to be distributed as profits will be very small indeed and occasionally even negative! How long can Mandel's surplus capital thesis be upheld? It is true that, given this reasoning, unproductive activities are no longer deducted from the amount of surplus value, but the capital used in these activities still participates in the distribution of the surplus value. Furthermore, this thesis logically leads to a falling rate of profits and to the apocalyptic breakdown of capitalism.

4.7 Capitalist cost-increasing inputs: productive but wasteful . . .

A final remark is needed regarding the position of the financial sector in the capitalist economy, which evidently extends far beyond that of financial intermediation. Rather, it is essentially concerned with ownership of capital and the reproduction of the power relations in the class structure of capitalist society. Evidently, when compared with a hypothetical classless society, the activities performed by the financial sector are hugely inefficient and an enormous waste of economic resources. In this sense, such activities could easily be considered as unproductive[32] – not based on the analytical definition used in this chapter but on an "evaluative definition", to use David Laibman's terminology. Similarly, much of the labour time that is directly or indirectly spent on advertising, on socially unnecessary product changes, etc. can be considered as wasteful and unproductive – which is the result of an "evaluative" approach and in spite of the relation to material use-value production. In contrast, within the capitalist mode of production and reproduction, such activities both create value and often produce a surplus product.

Here, Paul Baran's concept of "potential economic surplus" becomes relevant (Baran, 1957, p. 42). We define the "actual economic surplus" (ES) as the difference between the total value of macroeconomic output Y and the necessary costs to produce it (the total constant and variable capital):[33]

$$ES = Y - (C + V)$$

The constant and variable capital consists of two parts: the costs of production C^0 and V^0 which are technologically given, and the typical capitalist production and circulation costs C^c and V^c. This definition implies that:

$$ES = Y - (C^0 + V^0) - (C^c + V^c)$$

Next, we define the "potential economic surplus" (PES) (Cuyvers, 1972, pp. 208–209) as:

$$PES = PY - (C^0 + V^0) - \Delta(C^0 + V^0)$$

with PY the value of potential output. According to the neo-Marxists, monopoly capitalism is, for various reasons that we will discuss in subsequent chapters, characterised by less output than that which is possible with the technological knowledge and capabilities, and with the existing production capacity. According to the above definition, the potential economic surplus is in fact the surplus in the economy if, from the value of potential output we deduct the technologically/socially necessary costs $(C^0 + V^0)$, as well as the additional technologically/socially necessary cost $\Delta(C^0 + V^0)$ required to produce $(PY - Y)$.

Since from the ES definition it follows that:

$$(C^0 + V^0) = Y - ES - (C^c + V^c)$$

we can also, after rearrangement, rewrite the PES formula as:

$$PES = [(PY - Y) - \Delta(C^0 + V^0)] + ES + (C^c + V^c)$$

The first part $[(PY - Y) - \Delta(C^0 + V^0)]$ represents the potential output that is not produced $(PY - Y)$ after deduction of the required additional technologically/socially necessary costs. The second part of the PES is the actual economic surplus ES, i.e. what is available as actual resources for accumulation. The third part of the PES is made up of the specifically capitalist social costs.

Based on this PES definition, the specifically capitalist social costs are part of the potential economic surplus, irrespective of whether or not they are also included in the Marxian surplus value. In this way, we avoid the discussion on what is "productive" and what is not, and the focus is on what is available for accumulation and largely wasted. This wastage relates to: (1) what is not produced as output/value added, (2) capitalist consumption, the spending on the state apparatus, etc.,[34] and (3) specifically capitalist socially necessary costs. Whether banking and finance or publicity belong to category (2) or (3) is irrelevant, as they are part of the waste component of the PES. In contrast, what is considered as relevant is how the PES is used.

How the PES is used is thus the most relevant issue, not what is productive or unproductive. Is the PES spent in a socially useful way, or is it wasted? In other words, is part of the PES lost on expenditure that does not contribute to economic growth, to material and immaterial welfare, to necessary infrastructure and environment-improving investments, etc.?

According to Baran and Sweezy, the waste components of the PES consist of:

- The consumption of the ruling class
- The spending of the state apparatus, the army, the church, etc.
- Irrational investments and "conspicuous consumption" (luxurious offices, private jets for the top management, etc.)
- Banking and finance, publicity, sales promotion, etc.
- The not-produced surplus.

A crucial question relates to how to determine potential output. In the past, we followed two approaches: an empirical approach and a theoretical approach (Cuyvers, 1972, 1982). The former approach calculates potential output, using macroeconomic data on the capital stock and the available labour power. This approach is easy, but from a theoretical perspective debatable. Furthermore, alternative (possibly non-capitalist) production methods are not taken into account. The effect of alternative production methods can be analysed using the theoretical approach of the linear production model, but cannot be empirically applied because of the lack of data.

4.8 Conclusions

In this chapter, we investigated Marx's concept of unproductive labour. Most of our attention was devoted to unproductive labour in the production of cost-increasing inputs for the productive spheres of production (the so-called "*faux frais*" of production). Marx defines as unproductive those activities and inputs that do not create, conserve or contribute to the creation of use value. This definition forced us to think carefully about the spectacular expansion seen in the service sector over the last century. We found, however, that whether or not the outputs of such activities create a surplus product depends on these outputs being storable, such that they can be used in the following cycle of expanded reproduction. In fact, the same can be said of a number of services that are used in production processes, such as management in its many guises, consulting, design, R&D, etc., or the many consumer services that can be considered today as being part of the socially necessary consumption required for the reproduction of labour.

In Marx's highly theoretical and abstract model of capitalism, unproductive labour is, no doubt, an interesting and compelling concept. However, it should be reconsidered as a concept that is useful for understanding the laws of motion in capitalism. At first sight it looks that from a purely theoretical point of view it makes sense to distinguish, as Marx does, between the spheres of production and circulation, but this distinction becomes untenable if we consider the impact of changes in the sphere of circulation on the input-output relations in the sphere of circulation.

Moreover, the distinction has become increasingly blurred in the real world. It is worthwhile looking at an alternative form of treatment of the activities leading to cost-increasing inputs, particularly as the list of services – the productive or unproductive character of which one can argue about – has become inexhaustible. How should publicity or the services of the financial sector be dealt with in this context? According to Marx, both should be seen to be related to circulation, not production. However, often these services are inextricably intertwined with productive activities. Publicity, for instance, also leads – albeit artificially – to the production of new use values, and the financial sector provides services to blue and white collar workers in the productive sectors. The aim of this chapter was not to be conclusive, but rather to reveal and analyse the theoretical problems relating to Marx's concept of unproductive labour.

Having translated Marx's productive and unproductive labour concept in terms of the matrix algebra of the linear production model pioneered by Sraffa, Morishima and others, we found that a crucial characteristic of unproductive, cost-increasing inputs is their inability to be stored and therefore their unsuitability for accumulation. Stated differently: outputs are not part of the surplus product of the economy; they are neither part of the outputs along Morishima's balanced growth path nor the outputs of Sraffa's standard system. They have no labour value and they do not have any effect on the rate of profits.

However, it can be argued that cost-increasing inputs, such as advertising, banking products and services, or even bookkeeping, are either value creating or lead to a "surplus product" of some kind, or both. In this case, their status in the linear production model is entirely different. We have shown that the rate of profits is only reduced by such "unproductive activities" if they are not value creating but rather allow expanded reproduction. This is, however, one of the four possible combinations of value creation and/or of leading to a surplus product. In an economy with only one type of cost-increasing input, the situation would be clear. Yet if all four types of unproductive activities exist simultaneously, some do not contribute to surplus value production, while others do. Furthermore, for some, only the supplies from the productive sectors are relevant, while for others, all capital use should be taken into account, making the impact of the so-called unproductive sectors on total surplus value much more complicated than Marx and orthodox Marxists envisaged. We argued, however, that the dominant form of these inputs is value creating.

We also argued that if some cost-increasing inputs can be "stored" for expanded reproduction, to consider them as not value creating would re-introduce the old "transformation problem" that we had hoped to avoid by analysing output along a balanced growth path or using a Sraffa-like standard system. This "problem" can only be avoided if the production of cost-increasing inputs is taken as value creating. Moreover, as most (if not all) cost-increasing inputs of commerce and finance are indistinguishable from the pure costs of circulation, they should all be considered as value creating. This, in turn, implies that in the end all sectors producing cost-increasing inputs contribute to surplus value creation and their capital has an impact on the rate of profit in the economy.

Finally, we argued that, following Marx's suggestion about a tendency towards the equalisation of the rate of exploitation between the sectors of production, labour values are equilibrium prices that have the same status as prices of production. However, if some cost-increasing inputs are not value creating, it becomes hard to imagine how such a tendency would work. Furthermore, in this case, the inconsistencies can only be avoided by considering cost-increasing inputs as value creating which, however, cannot be reconciled with the views of Marx and those of many (though not all) Marxists.

One possible way out of a sterile discussion on which activities are productive and which are not is to use Paul Baran's "potential economic surplus" rather than total surplus value. The crucial issue becomes whether or not activities are a waste of the potential economic surplus, and all unproductive activities, in Marx's view, will be part of this surplus.

Notes

1 We should warn the reader about the popular misconception that productive labour is useful labour. For example, the labour of the accountant can be useful, but is considered by Marx as unproductive for reasons that will become clear in due course.

2 "Monopoly capitalism" is "capitalism in its monopoly stage", as Baran and Sweezy put it (1966, p. 6), with the proviso that the term "monopoly" includes the situation of a single seller, but also, more importantly, that of oligopoly. For a review of the Marxist and neo-Marxist theories of monopoly capitalism from Lenin, over Kalecki and Steindl, to Baran and Sweezy, see Sawyer (1988).

3 An excellent overview of the literature is to be found in Hunt (1979). For some more recent contributions to the debate, we refer to, for example, Laibman (1992, Ch.4), Mohun (1996, 2002), Houston (1997) and Cockshott and Zachariah (2006).

4 The accumulation in economic infrastructure by the state is assumed to take place in sector 1, whereas the consumption of this infrastructure is counted as used-up constant capital of the respective sectors. The usage of this infrastructure by the workers is part of the variable capital of each sector.

5 As mentioned before, our definition of the standard system is not Sraffa's, since we include the necessary consumption of the workers as inputs and advanced as variable capital. Our "standard system" is the system of equations used by Michio Morishima, one of the solutions of which is the vector of outputs that, in the absence of capitalist consumption, allows balanced growth of the system at $R \times 100\%$ (see Morishima, 1973).

6 For the sake of clarity, we must stress that the iron sector is identical to that in the numerical example of Chapter 2, but unlike the wheat sector which shows different technological input coefficients.

7 We are using $\mathbf{q^*}$ to denote the output vector of the standard system, as distinct from $\mathbf{q}$, an output vector that is not linked to the standard system.

8 The calculation rules for matrices and vectors that are relevant for us are summarised in Chapter 1, section 1.4.

9 The reader is referred to Chapter 1, section 1.4 for the relevant calculation rules for matrices and vectors.

10 Here and in the next section, we follow the arguments presented by Cuyvers (1978), while also building on the insights gained in the previous chapters.

11 At this stage there is no need for **q** to be equal to **q*** of the standard system.

12 As discussed in Chapter 2, this statement is not correct, as Marx's rate of profits based on value aggregates, is only equal to the rate of profits as defined in the price system, if the composition of capital is the same in all sectors of the economy, or if this is not the case, if the structure of outputs is that of Sraffa's standard system.

13 This ensures equality between total surplus value and total profits.

14 We assume that bookkeeping entries (or other unproductive services), by their nature, cannot be transferred to the next production period and therefore cannot be accumulated as capital. Hence, they are not part of the surplus product.

15 It should be stressed that this interpretation of (**Q, Q'**) does not permit capitalist consumption. It is similar, but not identical, to Sraffa's standard system (see Sraffa, 1960). It differs from Sraffa's standard system in that it includes the matrix of necessary consumption, as we assume with Marx and Morishima that variable capital is advanced by the capitalists (see Morishima, 1973).

16 Since it follows from (10b) that $\mathbf{Q}\,\boldsymbol{\lambda} - \mathbf{Q}\,\mathbf{B}\,\boldsymbol{\lambda} = r\,\mathbf{Q}\,\mathbf{B}\,\boldsymbol{\lambda} + (1 + r)\,\mathbf{Q'}\,\mathbf{B}_{12}\,\boldsymbol{\lambda}$.

17 Remember that in the standard system, $R = 115.1\%$, $q^*_1 = 2152$ kg iron and $q^*_2 = 568$ kg of wheat. We have also just seen that $q^*_3 = 0$.

18 As in Chapter 2, the prices of production are expressed in labour hours, not in monetary units. The so-called "New Interpretation" of Marx's theory of value, which makes use of the "Monetary Expression of Labour Time" (MELT), assumes that what is productive and unproductive was defined previously and that the MELT is the ratio between the total nominal value added and the labour time that has been spent productively. See, for example, Foley (2000, p. 21). Consequently, the procedures for the "New Interpretation" are not suited to determining what is productive and what is not.

19 We write "labour values" to distinguish them from the labour values produced in productive sectors.

20 In a linear model of production, positive prices and outputs in all sectors are only possible if all sectors show a surplus above what is required for simple reproduction. In the literature on the subject, this condition is known as the Hawkins-Simon condition (see Hawkins and Simon, 1949, pp. 247–248).

21 The reader is reminded that we are placing no limitation on the surplus product of the unproductive sectors in the form of $\mathbf{S'} = \mathbf{0}$. Therefore, it is possible that $\mathbf{Q'} > \mathbf{0}$.

22 In his impressive empirical research on the evolution of the rate of profits in the USA between 1948 and 1989, Mohun (2002, p. 213) used a rate of profits formula in which only wages of the unproductive workers are deducted from total surplus value and not the value of the supplies of the productive to the unproductive sectors, which is evidently wrong.

23 See, for example, Gillman (1956) or Phillips (1966), and more recently, Moseley (1991, 1997).

24 In the numerator of (17), $\boldsymbol{\lambda'}$ is absent as we previously assumed that no elements of $\mathbf{Q'}$ belong to the socially necessary consumption ($\mathbf{c'} = \mathbf{0}$).

25 Pasinetti (1993, pp. 30ff.) has modelled continuous changes in labour productivity, i.e. in the coefficients of Vector **l**, but not in the input coefficients of Matrix **A**. However, what is causing labour productivity to change at a given rate is not specified.

26 The "Verdoorn law" goes back to P.J. Verdoorn (1949). For a good overview of the literature, see, for example, Bairam (1987).

27 One of Paul Romer's premises on which the theory of endogenous growth is based is that technological change is largely due to intentional actions taken by agents who are reacting to the market situation (see Romer, 1989).

28 Marx (1981, p. 392): "Commercial capital thus creates neither value nor surplus-value, at least not directly. In so far as it contributes towards shortening the circulation time, it can indirectly help the industrial capitalist to increase the surplus-value he produces."

29 See, in this respect, Cuyvers (1986), where this tendency towards an equalisation of the rate of exploitation is used to prove mathematically that positive labour values exist in a system with alternative techniques of production.
30 Among the assumptions that are causing this dynamic instability, we mention investment reversibility, full capacity utilisation and perfect foresight. See Takayama (1985, pp. 503–517).
31 When not using Sraffa's standard system, the wage–profits curve of Figure 2.1 is evidently not linearly shaped.
32 This type of analysis goes back to Paul Baran's concept of potential economic surplus (see Baran, 1957).
33 Baran views the actual economic surplus as the difference between the current output of society and current consumption, i.e. what is immediately available for accumulation.
34 See this chapter, section 4.1.

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5 Laws of motion of capitalism – accumulation, technical change and super-profits

It must be acknowledged that Marx was a master in the art of constructing a theoretical model based on his views and expectations about the dynamics of capitalism, forging interactions between various crucial variables and also establishing links to his theory of value and surplus value. This model was the springboard to Marx's theory of the laws of motion of capitalism.

In the previous chapters we outlined, from a somewhat more static perspective, the relationship between accumulation, the rate of profits and the wage rate, under conditions in which there was no interaction between the wage rate and technical change. In the present chapter we will first discuss the dynamics underlying these relationships.

In Marx's view, the never-ending capitalist urge to expand and accumulate is often frustrated either by the available labour reserves, i.e. the size of the working population and the number of hours that this population can work, or by high wage levels. To the extent that wages are the constraining factor, it should be noted that this is only temporary since the capitalists will create unemployment by accumulating less than before, which will induce the wage rate to fall. To the extent that the available labour-power is the constraining factor, Marx expects the capitalists to introduce labour-saving technologies, which will reduce this constraint, if not eliminate it completely.

At least two laws of motion can be derived from the capitalist urge to accumulate: (1) the cyclical replenishment of the "industrial reserve army" and (2) the permanently felt tendency to introduce new production methods that increase labour productivity. In this chapter we will limit ourselves to the latter.[1] This law of motion will be discussed in great detail in the sections that follow because of the theoretical complexities involved and the importance of understanding how technical change influences the rate of surplus value and the degree of mechanisation in an economy. We will follow Marx's reasoning that changes in the degree of mechanisation will also change the ratio between the constant capital and the variable capital. This ratio, in turn (in Marx's words, "the value composition of capital"), is crucial for understanding the impact of technical change on the average rate of profits in the long run. The degree of mechanisation of production (in Marx's words, the "technical composition of capital") will appear to be better suited to the identification of the complex and (from an analytical perspective) often indeterminate effect of technical change. It is to the extent that the technical composition of capital influences the value composition that Marx introduces his concept of the organic composition of capital.

In a famous passage from Volume 1 of *Das Kapital*, Marx writes:

> The composition of capital is to be understood in a twofold sense. As value, it is determined by the proportion in which it is divided into constant capital, or the value of the means of production, and variable capital, or the value of labour-power, the sum total of wages. As material, as it functions in the process of production, all capital is divided into means of production and living labour-power. This latter composition is determined by the relation between the mass of the means of production employed on the one hand, and the mass of labour necessary for their employment on the other. I call the former the value-composition, the latter the technical composition of capital. There is a close correlation between the two. To express this, I call the value-composition of capital, in so far as it is determined by its technical composition and mirrors the changes in the latter, the organic composition of capital.
>
> (Marx, 1976, p. 762)

Although this distinction has caused much confusion in the past, it is an appropriate instrument of analysis, as we will demonstrate below.

5.1 Marx on the relationship between capital accumulation, the rate of profits and the wage rate, and the so-called "industrial reserve army"[2]

Given the capitalists' strong urge to accumulate, the total surplus value, i.e. the total amount of surplus labour, limits the amount of capital that can be accumulated. As the size of the working population determines the maximum number of hours that can be worked (Marx, 1981, p. 523), it also – at each point in time and for a given rate of surplus value σ – determines the surplus value that can be produced (Marx, 1981, p. 352). The capital stock with which the workers work is evidently the result of accumulation of the past, and at each moment, a datum. However, the size of the working population is continuously adjusting. Marx sees the operation of the mechanism that brings the required labour power into line with the accumulation needs of the capitalists as follows:

> A momentary excess of surplus capital over the working population it commands has a double effect. On the one hand it will gradually increase the working population by raising wages, (. . .) while on the other hand, by using methods that create relative surplus-value (introduction and improvement of machinery), it produces far more quickly an artificial and relative

> over-population (. . .). It thus follows from the very nature of the capitalist accumulation process (. . .), that the increased mass of means of production designed to be turned into capital finds a correspondingly increased and even excessive working population available for exploitation.[3]
>
> (Marx, 1981, p. 325)

Marx also deals extensively with this relationship in Volume 1 of *Das Kapital*. In this regard, for example, he writes: "To put it mathematically: the rate of accumulation is the independent, not the dependent variable; the rate of wages is the dependent, not the independent variable" (Marx, 1976, p. 770).

Following on from this, the creation of a relative reserve of labour power (Marx's "industrial reserve army") through technological change is "the absolute general law of capitalist accumulation" (Marx, 1976, p. 798). It is significant that Marx refers to the size of the accumulation and not to the rate of accumulation, a point that he also stresses elsewhere in *Das Kapital*. The reason for this is that the development of capitalism is accompanied by an increase in the absolute size of the total surplus value and of capital accumulation[4] (Marx, 1976, p. 770).

This argument incorporates, in the absence of technical change, the relationship between the desired rate of accumulation and the change in the rate of profits, which we discussed in Chapter 3. A high desired rate of accumulation will create labour scarcity, which will put upward pressure on the wage rate and will tend to force the rate of profits down. However, so the argument goes, technical changes and innovations will be introduced which will eliminate the pressure on the labour market conditions and, via the growth of the "industrial reserve army", will transform the labour market into a "buyer's market" dominated by the capitalists (Marx, 1981, pp. 771, 780–781; Marx, 1981, p. 364).[5]

5.2 The capitalist hunger for super-profits as the motivation for technical change and innovation

It would be wrong to think that Marx only established the relationship between labour scarcity and technical change. In fact, in Marx's view, production processes and methods are being continuously revolutionised in the wake of technical changes, which are driven by capitalist competition. The capitalists are under continuous pressure to produce at lower cost. Such pressure is also present in times of high unemployment, i.e. when the pressure to cope with labour scarcity is absent, and is therefore unrelated to Marx's labour scarcity argument. During times of massive unemployment, increases in labour productivity will allow cut-throat price competition (Marx, 1976, p. 582; Marx, 1981, pp. 363, 373).

We first discuss how the introduction of new or improved production methods initially generates super-profits, after which the innovations spread to the competitors and induce changing equilibrium prices. We then investigate how new production methods in the sector producing necessary consumer goods affect the total surplus value produced, via an increase in the rate of surplus value to which this at first leads.[6]

Consider the following two-sectoral system of labour values:

$$\mathbf{A}_{11}\boldsymbol{\lambda}_1 + \mathbf{l}_1 = \boldsymbol{\lambda}_1$$
$$\mathbf{A}_{12}\boldsymbol{\lambda}_1 + \mathbf{A}_{22}\boldsymbol{\lambda}_2 + \mathbf{l}_2 = \boldsymbol{\lambda}_2$$

In this system of equations, $\boldsymbol{\lambda}_1$ is the vector of the labour values of the means of production produced in sector 1 and $\boldsymbol{\lambda}_2$ is that of the necessary consumer goods produced in sector 2. The solution of $\boldsymbol{\lambda}_1$ is:

$$(\mathbf{I} - \mathbf{A}_{11})^{-1}\, \mathbf{l}_1 = \boldsymbol{\lambda}_1$$

The second sub-system can then be rewritten as:

$$\mathbf{A}_{12}(\mathbf{I} - \mathbf{A}_{11})^{-1}\, \mathbf{l}_1 + \mathbf{A}_{22}\boldsymbol{\lambda}_2 + \mathbf{l}_2 = \boldsymbol{\lambda}_2$$

or: $$\mathbf{A}_{12}(\mathbf{I} - \mathbf{A}_{11})^{-1}\, \mathbf{l}_1 + \mathbf{l}_2 = (\mathbf{I} - \mathbf{A}_{22})\, \boldsymbol{\lambda}_2$$

which has as a solution:

$$(\mathbf{I} - \mathbf{A}_{22})^{-1}\, \mathbf{A}_{12}(\mathbf{I} - \mathbf{A}_{11})^{-1}\, \mathbf{l}_1 + (\mathbf{I} - \mathbf{A}_{22})^{-1}\, \mathbf{l}_2 = \boldsymbol{\lambda}_2$$

This becomes somewhat more complicated in the system of equations of production prices because we have assumed that the wages are advanced by the capitalists and therefore yield a rate of profits. This means that while Matrix **A** is not relevant, Matrix **B** is; in contrast to **A**, **B** contains no zero partition. The price of production of the means of production can thus not be calculated independently of that of consumer goods. The necessary consumer goods required to produce the means of production contribute to the determination of the price of production of the means of production. The price vector **p** follows as the solution of the system of equations: $(1 + r)\, \mathbf{B}\, \mathbf{p} = (1 + r)(\mathbf{A} + \mathbf{l}\, \mathbf{c})\, \mathbf{p} = \mathbf{p}$. However, for the sake of simplicity, we use labour values in our further analysis. This approach is easier and allows more transparent conclusions.[7]

It is assumed that in all economic sectors, capitalist producers are competing with one another. This competition gives rise to constant pressure to reduce prices. A capitalist who introduces a new production method which, for instance, increases labour productivity in his factory only, will have a lower cost of production than his competitors and will earn *super-profits*. After a period of time, the new method of production will also be adopted by the other capitalists in the sector, which will lead to a fall in the price of production in the sector. Thus, the super-profits of the first introducers will have disappeared by then.

We again consider our imaginary economy of Chapter 2, which we assume to be on its maximum balanced growth path. By implication we are assuming that the sectoral output proportions in the economy are those of its standard system. If, in total, 100 labour hours are performed, the standard outputs are: $q_1{}^* = 123.8$ kg of iron and $q_2{}^* = 118.5$ kg of wheat (assuming, as before, that the socially necessary consumption of the workers per working hour amounts to 0.5 kg of wheat). The scheme of reproduction is therefore as follows:

$$\begin{array}{l} 41.3\,\text{kg iron} + 20.6\,\text{kg wheat} \rightarrow 123.8\,\text{kg iron} \\ \underline{59.3\,\text{kg iron}} + \underline{77.0\,\text{kg wheat}} \rightarrow 118.5\,\text{kg wheat} \\ 100.6\,\text{kg iron} \quad 97.6\,\text{kg wheat} \end{array}$$

This leads us to the system of price of production equations:

$$(0.333\, p_1 + 0.167\, p_2)\,(1 + r) = p_1$$
$$(0.500\, p_1 + 0.650\, p_2)\,(1 + r) = p_2$$

with the solutions: $r = 21.8\%$, $p_1 = 0.437$ hours of labour time and $p_2 = 1.302$ hours of labour time.

Now assume that sector 1 comprises ten identical producers who are each producing 12.4 kg of iron and that one of them succeeds in increasing labour productivity in his factory by 20% through the introduction of more efficient organisational methods or a newly invented labour-saving production method. He then produces 20% more (14.9 kg of iron) during the same time as before (3.33 hours). Whereas the average labour productivity in the sector amounts to 3 kg of iron per hour, it rises to 3.6 kg in the factory of our innovator. We can safely assume that since only one capitalist introduced the innovation, p_1 is unchanged. The cost of production per kilogram of iron output, measured in hours of labour time, is:

$$0.333\, p_1 + 0.167\, p_2 = 0.362 \text{ hours}$$ [8]

and the profits per kilogram of iron: 0.075 hours.[9]

Following the introduction of the new production method, the cost of production in the factory of the innovating capitalist will be:[10]

$$0.333\, p_1 + 0.139\, p_2 = 0.327$$ [11]

and his profits per kilogram of output will be: 0.110, which is the normal profit per unit of output in the sector but augmented by a super-profit of 0.036.

The super-profits will be eroded when the other capitalists also start to introduce the new production method. When the new method has become the norm in the sector, the economy at large will be proportioned according to the scheme of reproduction:

$$\begin{array}{l} 49.6\,\text{kg iron} + 20.6\,\text{kg wheat} \rightarrow 148.6\,\text{kg iron} \\ \underline{59.3\,\text{kg iron}} + \underline{77.0\,\text{kg wheat}} \rightarrow 118.5\,\text{kg wheat} \\ 108.9\,\text{kg iron} \quad 97.6\,\text{kg wheat} \end{array}$$

The net product of the economy now consists of 39.7 kg of iron and 70.9 kg of wheat, with a remaining surplus of 39.7 kg of iron and 20.9 kg of wheat.[12] Ultimately, new prices of production will take hold, based on the following system:

$$(0.333\, p_1 + 0.139\, p_2)\,(1 + r) = p_1$$
$$(0.500\, p_1 + 0.650\, p_2)\,(1 + r) = p_2$$

Making (as in Chapter 2) the value of the net product equal to the directly spent labour time in the economy:

$$100 \text{ hours} = 70\, p_1 + 60\, p_2$$

the equations can be solved, which gives: r = 25.14%, p_1 = 0.407 hours and p_2 = 1.191 hours.

It can thus be concluded that the quest of each individual capitalist for new and better production methods is fuelled by his hunger for super-profits. Given, though, that the new methods will also penetrate the factories of the other capitalist producers, the super-profits will evaporate and a new set of prices of production and another general rate of profits will emerge (leading, in our example, to a higher rate of profits than before).

5.3 The effect of technical change on the surplus value and the rate of surplus value

We have seen that, according to Joan Robinson's model, ever-recurring labour scarcity tends to keep the actual rate of accumulation below the desired rate of accumulation. The permanent introduction of labour-saving technology and production methods will either fully or partly eliminate this constraint. In all sectors, more output will be produced where the same amount of labour is used, which will make the surplus product expand, thus stimulating the accumulation of capital and the rate of expansion of the economy.

We must first investigate the impact of the introduction of labour-saving technical change on the actual and possible rate of accumulation in the economy, starting with Figure 3.5 in Chapter 3 where the maximum attainable rate of growth g' is lower than the equilibrium rate of growth g^*. Ignoring capitalist consumption, such that the rate of profits coincides with the maximum attainable rate of growth, the slope of the realisation curve will be 45 degrees. An increase in labour productivity in the consumer goods sector and/or in the capital goods sector will bring about an increase in the rate of profits and the maximum rate of accumulation. This is illustrated in Figure 5.1 below.

The rate of growth thus moves towards the desired rate of accumulation g^*. As long as this desired rate is not reached, new labour-saving (labour productivity-enhancing) production techniques will be introduced.

The outcome described in section 5.2 illustrates the effect of an increase in labour productivity in the capital goods sector. However, what will be the impact

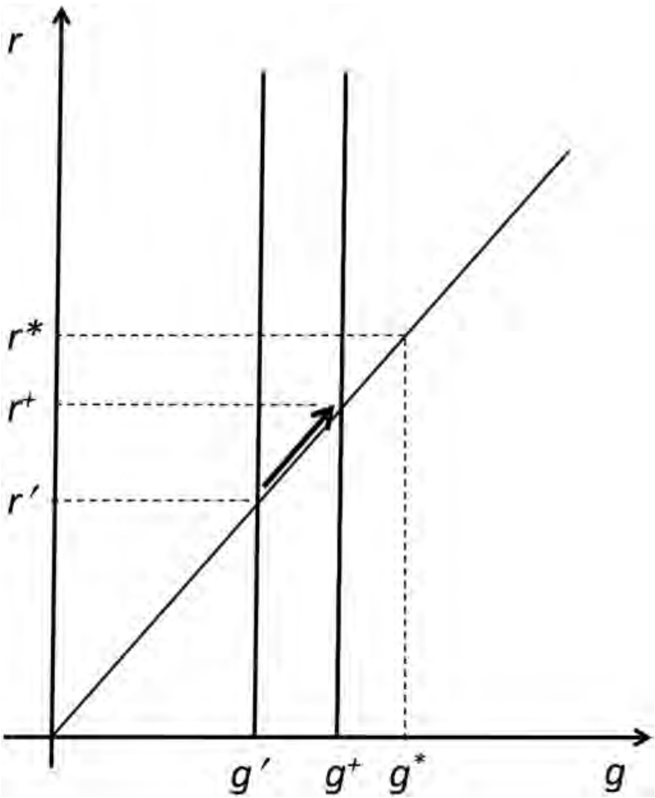

Figure 5.1 Labour-saving production techniques in the consumer goods sector lead to an increase in the maximum attainable rate of accumulation from g' to g^+

of an increase in labour productivity in the consumer goods sector? We must first consider the general case, with the help of a numerical example. For the sake of simplicity, we assume that labour productivity in both sectors is rising at a rate of $\alpha \times 100\%$.[13] This means that after such a rise, the labour input coefficients of Vectors $\mathbf{l}_1$ and $\mathbf{l}_2$ will be:

$$\mathbf{l'}_1 = \mathbf{l}_1/(1 + \alpha)$$
$$\mathbf{l'}_2 = \mathbf{l}_2/(1 + \alpha)$$

The rise in labour productivity has the following effect on $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$:

$$(\mathbf{I} - \mathbf{A}_{11})^{-1}\, \mathbf{l}_1/(1 + \alpha) = \boldsymbol{\lambda}_1' = \boldsymbol{\lambda}_1/(1 + \alpha)$$
$$(\mathbf{I} - \mathbf{A}_{22})^{-1}\, \mathbf{A}_{12}(\mathbf{I} - \mathbf{A}_{11})^{-1}\, \mathbf{l}_1/(1 + \alpha) + (\mathbf{I} - \mathbf{A}_{22})^{-1}\, \mathbf{l}_2/(1 + \alpha) = \boldsymbol{\lambda}_2' = \boldsymbol{\lambda}_2/(1 + \alpha)$$

It can be seen that a general rise in labour productivity of $\alpha \times 100\%$ leads to a fall in the labour values of $\frac{\alpha}{1+\alpha} \times 100\%$.[14]

We assume that the necessary consumption of the workers per working hour, Vector **c**, is unchanged and that **q** = **q***, i.e. the outputs with balanced expansion of the economy without capitalist consumption (our standard system in previous chapters). Before the increase in labour productivity in sector 2, the situation was as follows:[15]

$$\mathbf{q}_1\,\mathbf{A}_{11}\boldsymbol{\lambda}_1 + \mathbf{q}_1\,\mathbf{l}_1\,\mathbf{c}_2\,\boldsymbol{\lambda}_2 + \sigma\,\mathbf{q}_1\,\mathbf{l}_1\,\mathbf{c}_2\,\boldsymbol{\lambda}_2 = \mathbf{q}_1\,\boldsymbol{\lambda}_1$$

$$\mathbf{q}_2\,\mathbf{A}_{12}\boldsymbol{\lambda}_1 + \mathbf{q}_2\,\mathbf{A}_{22}\boldsymbol{\lambda}_2 + \mathbf{q}_2\,\mathbf{l}_2\,\mathbf{c}_2\,\boldsymbol{\lambda}_2 + \sigma\,\mathbf{q}_2\,\mathbf{l}_2\,\mathbf{c}_2\,\boldsymbol{\lambda}_2 = \mathbf{q}_2\,\boldsymbol{\lambda}_2$$

The total variable capital V was equal to:

$$V = \mathbf{q}_1\,\mathbf{l}_1\,\mathbf{c}_2\,\boldsymbol{\lambda}_2 + \mathbf{q}_2\,\mathbf{l}_2\,\mathbf{c}_2\,\boldsymbol{\lambda}_2$$

The total surplus value M and the total value added equalled (respectively):

$$\sigma\,\mathbf{q}_1\,\mathbf{l}_1\,\mathbf{c}_2\,\boldsymbol{\lambda}_2 + \sigma\,\mathbf{q}_2\,\mathbf{l}_2\,\mathbf{c}_2\,\boldsymbol{\lambda}_2 = M = (\mathbf{q}_1\boldsymbol{\lambda}_1 - \mathbf{q}_1\mathbf{A}_{11}\boldsymbol{\lambda}_1 - \mathbf{q}_2\mathbf{A}_{12}\boldsymbol{\lambda}_1 - \mathbf{q}_1\,\mathbf{l}_1\mathbf{c}_2\boldsymbol{\lambda}_2) + (\mathbf{q}_2\boldsymbol{\lambda}_2 - \mathbf{q}_2\mathbf{A}_{22}\boldsymbol{\lambda}_2 - \mathbf{q}_2\,\mathbf{l}_2\mathbf{c}_2\boldsymbol{\lambda}_2)$$

and: $M + V = (1+\sigma)(\mathbf{q}_1\,\mathbf{l}_1\,\mathbf{c}_2\,\boldsymbol{\lambda}_2 + \mathbf{q}_2\,\mathbf{l}_2\,\mathbf{c}_2\,\boldsymbol{\lambda}_2) = \mathbf{q}_1\,\mathbf{l}_1 + \mathbf{q}_2\,\mathbf{l}_2$

After the introduction of the technical change that caused the general rise in labour productivity by $\alpha \times 100\%$, M, V and M + V become (respectively):[16]

$$V' = \mathbf{q}_1\,\mathbf{l}_1'\,\mathbf{c}_2\,\boldsymbol{\lambda}_2' + \mathbf{q}_2\,\mathbf{a}\,\mathbf{l}_2'\,\mathbf{c}_2\,\boldsymbol{\lambda}_2' = V/(1+\alpha)^2$$

$$M' = (\mathbf{q}_1\boldsymbol{\lambda}_1' - \mathbf{q}_1\mathbf{A}_{11}\boldsymbol{\lambda}_1' - \mathbf{q}_2\mathbf{A}_{12}\boldsymbol{\lambda}_1' - \mathbf{q}_1\,\mathbf{l}_1'\,\mathbf{c}_2\boldsymbol{\lambda}_2') + (\mathbf{q}_2\boldsymbol{\lambda}_2' - \mathbf{q}_2\mathbf{A}_{22}\boldsymbol{\lambda}_2' - \mathbf{q}_2\,\mathbf{l}_2'\,\mathbf{c}_2\boldsymbol{\lambda}_2')$$

$$= \frac{1}{1+\alpha}\left[\mathbf{q}_1\,\mathbf{l}_1 + \mathbf{q}_2\,\mathbf{l}_2 - \frac{V}{1+\alpha}\right]$$

$$M' + V' = \mathbf{q}_1\,\mathbf{l}_1' + \mathbf{q}_2\,\mathbf{l}_2' = (M+V)/(1+\alpha).$$

After some further manipulation (see Appendix 1), this leads to:

$$\frac{M'}{V'} = \frac{M}{V}(1+\alpha) + \alpha = \sigma' = \sigma\,(1+\alpha) + \alpha$$

and: $\frac{\Delta\sigma}{\sigma} = \frac{\alpha(1+\sigma)}{\sigma} > \alpha$

The relentless quest for capitalist super-profits prompts a general rise in labour productivity in all spheres of production in the economy. Assuming that the physical input coefficients remain unchanged, a given percentage increase in labour productivity will lead to a higher rate of increase in the rate of surplus value.

The following numerical example illustrates this. Starting with the labour values system in our imaginary economy:

$$0.333\,\lambda_1 + 0.333 = \lambda_1$$

$$0.500\,\lambda_1 + 0.400\,\lambda_2 + 0.500 = \lambda_2$$

or, when the economy is on its balanced expansion path (the outputs **q*** are rounded off):

$$41\,\lambda_1 + 41 = 123\,\lambda_1$$

$$59\,\lambda_1 + 47.2\,\lambda_2 + 59 = 118\,\lambda_2$$

The labour values are, respectively, $\lambda_1 = 0.5$ and $\lambda_2 = 1.25$. A rise in output of 20% ($\alpha = 0.2$), with the amount of labour spent remaining the same but the physical inputs of means of production also rising at a rate of 20%, will change the outputs on the balanced expansion path. Knowing that:

$$\mathbf{A} = \begin{pmatrix} 0.333 & 0 \\ 0.500 & 0.400 \end{pmatrix} \text{ and } \mathbf{l}\,\mathbf{c} = \begin{pmatrix} 0 & 0.167 \\ 0 & 0.250 \end{pmatrix}$$

the introduction of the technical change will make these, respectively:

$$\mathbf{A}' = \mathbf{A} = \begin{pmatrix} 0.333 & 0 \\ 0.500 & 0.400 \end{pmatrix} \text{ and } \mathbf{l}'\,\mathbf{c} = \begin{pmatrix} 0 & 0.139 \\ 0 & 0.208 \end{pmatrix}$$

which, in turn, will transform the outputs **q*'** along the new balanced expansion path into:

$$\mathbf{q}^{*\prime}\,(\mathbf{A} + \mathbf{l}'\,\mathbf{c})\,(1 + R') = \mathbf{q}^{*\prime}$$

or, in our example:

$$(q_1{}^{*\prime}\; q_2{}^{*\prime}) \begin{pmatrix} 0.333 & 0.139 \\ 0.500 & 0.608 \end{pmatrix} (1 + R') = (q_1{}^{*\prime}\; q_2{}^{*\prime})$$

At the same time, $\mathbf{q}^{*\prime}\,\mathbf{l}' = 100$ hours of labour time, or written in full:

$$(q_1{}^{*\prime}\; q_2{}^{*\prime}) \begin{pmatrix} 0.278 \\ 0.417 \end{pmatrix} = 100$$

This leads to the solution: $R' = 30.24\%$, $q_1{}^{*\prime} = 156.2$ kg of iron and $q_2{}^{*\prime} = 135.7$ kg of wheat, and to the new labour values system:

$$52.1\,\lambda_1' + 41 = 156.2\,\lambda_1'$$

$$67.9\,\lambda_1' + 54.3\,\lambda_2' + 59 = 135.7\,\lambda_2'$$

The new output vectors clearly change nothing of importance for determining the labour values, and so we can equally use the equations with the input coefficients in our calculations. In the next chapter, we will return to the R', which is equal to the general rate of profits in the new production prices system.[17]

Per unit of output, the new labour values system is as follows:

$$0.333\,\lambda_1' + 0.277 = \lambda_1'$$

$$0.500\,\lambda_1' + 0.400\,\lambda_2' + 0.417 = \lambda_2'$$

Solving this system of equations shows that the labour values decreased to $\lambda_1' = 0.4167$ and $\lambda_2' = 1.0418$. A quick inspection indicates that:

$$\Delta\lambda_1/\lambda_1 = \Delta\lambda_2/\lambda_2 = -\frac{\alpha}{1-\alpha} = -16.7\%.$$

Per unit of output, the old and new labour values system can be split into the required constant and variable capital and the produced surplus value, when duly taking into account the workers' necessary consumption of 0.5 kg of wheat per working hour spent:

$$0.333\,\lambda_1 + 0.167\,\lambda_2 + m_1 = \lambda_1$$

$$(0.500\,\lambda_1 + 0.400\,\lambda_2) + 0.139\,\lambda_2 + m_2 = \lambda_2$$

and:

$$0.333\,\lambda_1' + 0.139\,\lambda_2' + m_1' = \lambda_1'$$

$$(0.500\,\lambda_1' + 0.400\,\lambda_2') + 0.208\,\lambda_2' + m_2' = \lambda_2'$$

or, using λ_1 and λ_2, and λ_1' and λ_2', found previously:

$$0.167\,(c_1) + 0.208\,(v_1) + 0.125\,(m_1) = 0.500\,(\lambda_1)$$

$$0.750\,(c_2) + 0.313\,(v_2) + 0.187\,(m_2) = 1.25\,(\lambda_2)$$

and:

$$0.139\,(c_1') + 0.144\,(v_1') + 0.134\,(m_1') = 0.4167\,(\lambda_1')$$

$$0.625\,(c_2') + 0.217\,(v_2') + 0.200\,(m_2') = 1.0418\,(\lambda_2')$$

We next calculate the respective original sectoral rates of surplus value and find that:

$$\sigma_1 = m_1/v_1 = 0.6 \qquad \sigma_2 = m_2/v_2 = 0.6$$

which, however, after the introduction of the new technique of production, become:

$$\sigma_1' = m_1'/v_1' = 0.92 \qquad \sigma_2' = m_2'/v_2' = 0.92$$

thus confirming our previous algebraic result:

$$\frac{\Delta(m_1/v_1)}{\left(\frac{m_1}{v_1}\right)} = \frac{\Delta(m_2/v_2)}{\left(\frac{m_2}{v_2}\right)} = \frac{0.32}{0.6} = \frac{\Delta\sigma}{\sigma} = \frac{\alpha(1+\sigma)}{\sigma} = \frac{0.2\times 1.6}{0.6} = 0.533$$

The rate of surplus value has increased by 53.3% because the value of the necessary consumption (i.e. the value of labour power) has become cheaper. To use Marx's terminology: the increase in labour productivity has created relative surplus value. The expression of the percentage change in the rate of surplus value σ will serve our purpose in Chapters 6 and 7, where the evolution of the general rate of profits and the rate of surplus value will be investigated.

5.4 The effect of technical change on the organic composition of capital

In the face of labour-saving technical change, the degree of mechanisation in the spheres of production will also change. The same amount of labour will produce more output. If the use of raw materials and means of production increases in the same proportion as output, the technical input coefficients of Matrix **A** remain constant. However, since the labour input coefficients of Vector $\mathbf{l}$ drop by $\alpha/(1 + \alpha) \times 100\%$, the physical inputs used as means of production per hour of labour spent will increase. This is an illustration of how Marx's *technical composition of capital* increases as new production methods are introduced.

Of course, it is not possible to depict this technical composition of capital as a simple ratio between means of production and labour used. In the wheat production sector just considered, for example, the quantity of iron and wheat per hour of labour spent cannot simply be added up. A measure of value is needed, such as the labour value of wheat and iron. Evidently, the ratio thus derived will have an immediate link to the technical composition of capital, but since it consists of value aggregates, Marx calls it the *value composition of capital*. Insofar as a change in the technical composition also leads to a change in the value composition of capital, Marx calls the ratio the *organic composition of capital*: "I call the value-composition of capital, in so far as it is determined by its technical composition and mirrors the changes in the latter, the organic composition of capital" (Marx, 1976, p. 762).

However, there is confusion as to how exactly the technical composition of capital should be represented. In a passage from Volume 3 of *Das Kapital*, Marx indicates that the amount of means of production can be considered to be materialised labour. He writes:

> The first relationship depends on technical conditions and is to be taken as given, at any particular stage of development of productivity. A certain quantity of labour-power, represented by a certain number of workers, is required to

> produce a certain volume of products in a day, for example, and this involves putting a certain definite mass of means of production in motion and consuming them productively – machines, raw materials, etc. A definite number of workers corresponds to a definite quantity of means of production, and thus a definite amount of living labour to a definite amount of labour already objectified in means of production. (. . .) This proportion constitutes the technical composition of capital, and is the actual basis of its organic composition.
>
> (Marx, 1981, p. 244)

The evolution of the value composition of capital as a reflection of the changing technical composition helps to determine the character of technical change. It also influences the evolution of the rate of profits, as will be seen in Chapter 6.

5.4.1 An increase in labour productivity with unchanged proportional inputs of means of production

Measured in labour values,[18] the value composition κ_i in a given sector i is:

$$\kappa_i = \frac{C_i}{V_i} = \sum_j \frac{\lambda_j A_{ij}}{\lambda_j c_j l_i}$$

Owing to the change of l_i into $l_i/(1+\alpha)$ and the resulting change of the λs in $\lambda/(1+\alpha)$, the value composition becomes, after the introduction:

$$\kappa_i' = \frac{C_i'}{V_i'} = \sum_{\mathbf{j}} \frac{\frac{\lambda_j}{1+\alpha} A_{ij}}{\frac{\lambda_j}{1+\alpha} c_j \frac{l_i}{1+\alpha}} = (1+\alpha)\sum_{\mathbf{j}} \frac{\lambda_j A_{ij}}{\lambda_j c_j l_i}$$

such that: $\Delta\kappa_i = \kappa_i' - \kappa_i = \alpha \sum_j \frac{\lambda_j A_{ij}}{\lambda_j c_j l_i}$

and: $\frac{\Delta\kappa_i}{\kappa_i} = \alpha$

The same holds for every other sector, such that:

$$\frac{\Delta\kappa}{\kappa} = \frac{\Delta\kappa_i}{\kappa_i} = \alpha$$

It can thus be seen that an increase in labour productivity of $\alpha \times 100\%$ and the constant use of the means of production per unit of output also lead to the value composition of capital increasing by $\alpha \times 100\%$. It does not make any difference if $(C + V)/V$ or C/V is used as the definition of the value composition.[19]

Translating Marx's reasoning into mathematics, the change in time of each κ_i can be written as:[20]

$$\frac{\partial}{\partial t}\kappa_i = \Sigma_j \frac{\lambda_j}{\lambda_j\, c_j} \frac{\partial}{\partial t}\left(\frac{A_{ijt}}{l_{it}}\right) + \Sigma_j \frac{\partial}{\partial t}\left(\frac{\lambda_{jt}}{\lambda_{jt}\, c_j}\right)\frac{A_{ij}}{l_i}$$

Based on this expression, changes in time of κ_i can be viewed as consisting of two parts. The first part aggregates the changes in the ratio's A_{ij}/l_i on the basis of the given price (here, labour value) ratio's $\lambda_j/\Sigma\lambda_j\, c_j$.[21] These changes reflect changes in the technical composition of capital. The second part aggregates the changes in the respective labour values for a given and constant degree of mechanisation in the production process in the sector in question.

For a $\alpha \times 100\%$ increase in labour productivity, the constant use of means of production per unit of output changes the technical composition of capital by:

$$\Sigma_j \frac{\lambda_j}{\lambda_j\, c_j} \frac{\partial}{\partial t}\left(\frac{A_{ijt}}{l_{it}}\right)$$

with, for each arbitrarily chosen sector, *i* holding that:

$$\frac{\partial}{\partial t}\left(\frac{A_{ijt}}{l_{it}}\right) = \frac{A_{ij}}{l_i/(1+\alpha)} - \frac{A_{ij}}{l_i} = \alpha\frac{A_{ij}}{l_i}$$

from which it follows that in each sector the technical composition also increases by $\alpha \times 100\%$, i.e. the same percentage rate of change as that of labour productivity.

To illustrate this for our imaginary economy, the value composition of capital is in the following two respective sectors:

$$\kappa_1 = \frac{C_1}{V_1} = \Sigma_j \frac{\lambda_j\, A_{1j}}{\lambda_j\, c_j\;\; l_1} = \frac{0.5\times 0.333}{1.25\times 0.5\times 0.333} = 0.8$$

$$\kappa_2 = \frac{C_2}{V_2} = \Sigma_j \frac{\lambda_j A_{2j}}{\lambda_j\, c_j\;\; l_2} = \frac{(0.5\times 0.5)+(1.25\times 0.4)}{(1.25\times 0.5\times 0.5)} = 2.4$$

After the introduction of the new production technique, with only l_1 and l_2 changing, these value compositions become:

$$\kappa_1{}' = \frac{C_1{}'}{V_1{}'} = \Sigma_j \frac{\lambda_j{}'\, A_{1j}}{\lambda_j{}'\, c_j\;\; l_1{}'} = \frac{0.4167\times 0.333}{1.0418\times 0.5\times 0.278} = 0.96$$

$$\kappa_2{}'\ \frac{C_2{}'}{V_2{}'} = \Sigma_j \frac{\lambda_j{}' A_{2j}}{\lambda_j{}' c_j\; l_2{}'} = \frac{(0.4167\times 0.5)+(1.0418\times 0.4)}{(1.0418\times 0.5\times 0.416)} = 2.88$$

It then follows that:

$$\frac{\Delta\kappa}{\kappa} = \frac{\Delta\kappa_1}{\kappa_1} = \frac{\Delta\kappa_2}{\kappa_2} = 0.2 = \alpha$$

i.e. the percentage rate of increase of labour productivity. Since each element A_{ij}/l_i increases at the same percentage rate of $\alpha \times 100\%$ and the ratio λ_1/λ_2 remains unchanged, this is also the percentage rate of increase of the value composition of capital.[22] It is in this way (as Marx points out) that changes in the technical composition determine the value composition.

5.4.2 An increase in labour productivity with decreasing proportional inputs of the means of production

When the new production technique does not merely increase labour productivity (i.e. decrease the use of direct labour per unit of output) but also decreases the use of the means of production per unit of output, the changes in the technical and value composition of capital will evidently differ from that in the previous situation. When both the elements of Matrix **A** and Vector **l** decrease by $\alpha/(1 + \alpha) \times 100\%$,

$$\mathbf{A'} = \mathbf{A}/(1 + \alpha)$$

$$\mathbf{l'} = \mathbf{l}/(1 + \alpha)$$

As a result, the labour values system becomes:

$$\mathbf{A'}\boldsymbol{\lambda'} + \mathbf{l'} = \boldsymbol{\lambda'}, \text{ or}$$

$$\frac{\mathbf{A}}{1+\alpha}\boldsymbol{\lambda'} + \frac{l}{1+\alpha} = \boldsymbol{\lambda'}$$

The solution of this system of equations is:

$$\lambda' = [\mathrm{I} - \frac{\mathrm{A}}{(1+\alpha)}]^{-1}\frac{\mathbf{l}}{1+\alpha}$$

It was seen in Chapter 2, $\boldsymbol{\lambda} = (\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{l} = (\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \ldots)\,\mathbf{l}$. Similarly, $\boldsymbol{\lambda}' = [\mathbf{I} + \mathbf{A}/(1 + \alpha) + (\mathbf{A}/(1 + \alpha))^2 + (\mathbf{A}/(1 + \alpha))^3 + \ldots]\ \mathrm{l}/(1 + \alpha)$. This equality is reminiscent of Sraffa's having proved that prices of production are the sum of dated quantities of labour, with the labour time spent n stages back in time weighted by a factor $(1/(1 + r))^n$ (r being the general rate of profits and n being an arbitrarily chosen moment in the past). The same can now be said of the labour values, in that the labour time spent n stages back in time is weighted by the factor $(1/(1 + \alpha))^n$.

It will be clear that the labour values are affected differently by the proportional decreases in the various physical inputs. The value composition is now:

$$\kappa_i' = \frac{C_i'}{V_i'} = \Sigma_j \frac{\lambda_j' \frac{A_{ij}}{1+\alpha}}{\lambda_j' c_j \frac{l_i}{1+\alpha}}$$

Owing to the disproportionate changes in the labour values, it is not clear how the value composition will evolve, but the change in the technical composition of capital can be determined algebraically as being equal to:

$$\sum_j \frac{\lambda_j}{\lambda_j \, c_j} \frac{\partial}{\partial t}\left(\frac{A_{ijt}}{l_{it}}\right) = \sum_j \frac{\lambda_j}{\lambda_j \, c_j}\left(\frac{\frac{A_{ijt}}{1+\alpha}}{\frac{l_{it}}{1+\alpha}}\right) - \sum_j \frac{\lambda_j}{\lambda_j \, c_j} \frac{A_{ijt}}{l_{it}}) = 0$$

The technical composition of capital does not change. The same quantity of physical inputs is used per man hour as before, since both the labour productivity and the output per unit of physical inputs are increasing at the same percentage rate. We know that labour values will drop fairly dramatically, but due to the complexity of the induced changes, there is no way to analytically determine the percentage rates of change. Therefore, no simple formula exists into which the evolution of the value composition can be cast.

As far as our imaginary economy is concerned, we can illustrate the effects of technical change (which alters both the labour input coefficients l_i and the physical input coefficients A_{ij} by the same percentage rate of change on labour values and the composition of capital) as follows.

After the introduction of the technical change that decreases both input coefficients in the same proportion, the labour values system is:

$$0.2778\,\lambda_1' + 0.2778 = \lambda_1'$$

$$0.4167\,\lambda_1' + 0.3333\,\lambda_2' + 0.4167 = \lambda_2'$$

This system of equations produces the solution: $\lambda_1' = 0.3846$ and $\lambda_2' = 0.8654$. Compared with the initial $\lambda_1 = 0.500$ and $\lambda_2 = 0.125$, it is now found that $\Delta\lambda_1/\lambda_1 = -0.2308$ and $\Delta\lambda_2/\lambda_2 = -0.3076$. This is because a still-identical proportional change in the input coefficients A_{ij} has a larger impact on the value of wheat than on the value of iron, due to the larger inputs of iron and wheat in the former sector. In contrast, with the previous type of technical change the A_{ij}'s did not change.

As in the previous case, we can rewrite the labour values system as follows:

$$0.2778\,\lambda_1' + 0.139\,\lambda_2' + m_1' = \lambda_1'$$

$$(0.4167\,\lambda_1' + 0.3333\,\lambda_2') + 0.208\,\lambda_2' + m_2' = \lambda_2'$$

or:

$$0.107\,(c_1') + 0.120\,(v_1') + 0.158\,(m_1') = 0.3846$$

$$0.449\,(c_2') + 0.180\,(v_2') + 0.236\,(m_1') = 0.8654$$

such that: $m_1'/v_1' = 1.31 \qquad m_2'/v_2' = 1.31$

and: $$\frac{\Delta(m_1/v_1)}{\left(\frac{m_1}{v_1}\right)} = \frac{\Delta(m_2/v_2)}{\left(\frac{m_2}{v_2}\right)} = \frac{0.71}{0.6} = 118.3\%$$

However, it can be seen that:

$$\kappa_1' = \frac{C_1'}{V_1'} = 0.892 \qquad \kappa_2' = \frac{C_2'}{V_2'} = 2.5$$

$$\text{and: } \frac{\Delta\kappa_1}{\kappa_1} = 11.5\% \neq \frac{\Delta\kappa}{\kappa} \qquad \frac{\Delta\kappa_2}{\kappa_2} = 4.2\% \neq \frac{\Delta\kappa}{\kappa}$$

The technical change that we consider here increases the rate of surplus value by around 120% but increases the value composition by only 4–12%. It will be remembered that when the new production technique impacted only the labour input coefficients, this increased both the value composition *and* the technical composition of capital by α = 20%. The technical change that is now being considered is therefore not only labour-saving but also capital-saving. Since the value composition of capital is different in the two sectors, the changes in the labour values have a very different effect on the sectors. The value composition increases at a lower rate than that at which the labour productivity is increasing. However, the technical composition remains the same as before. Thus, the increase in the value composition of capital is the result of a stronger decrease in the labour value of the necessary consumer goods than in the means of production. Moreover, this stronger rate of decrease in the labour value of the necessary consumption lays the foundation for Marx's views on the creation of relative surplus value and the increasing rate of surplus value.

Although with labour-saving and capital-saving technical change the different variables are changing in a complex way, it is important to stress that when both the A_{ij}'s and the l_i's are decreasing in the same proportion, the technical composition of capital will remain unchanged. This is a situation of Harrod-(and Robinson-) neutral technological progress. We will return to this finding in section 5.6 below, as well as in Chapter 6.

5.4.3 An increase in labour productivity with increasing proportional inputs of the means of production

The last situation that we investigate is when the newly introduced labour-saving technology is capital-using – which Marx assumes to be the dominant case. It is mathematically easiest to formulate this as the input coefficients A_{ij} changing into $A_{ij}' = \gamma\, A_{ij} = ((1 + \beta)/(1 + \alpha))\, A_{ij}$, with $\gamma > 1$. This implies that the rate of change of each A_{ij}, $\gamma - 1 = ((\beta)/(1 + \alpha) - (\alpha)/(1 + \alpha))$. In this expression $\beta/(1 + \alpha)$ is the rate of change of **A** over and above its assumed $\alpha/(1 + \alpha) \times 100\%$ decrease, because of the increase in labour productivity and in the sectoral outputs of $\alpha \times 100\%$. It will be shown below that β plays an important role in the argument insofar as it is the rate of growth of the technical composition of capital.

In matrix notation, the labour values system is now:

$$\gamma\, \mathbf{A}\, \boldsymbol{\lambda}' + \frac{\mathbf{l}}{1+\alpha} = \boldsymbol{\lambda}'$$

which provides the following solution for $\boldsymbol{\lambda}'$:

$$\boldsymbol{\lambda}' = (\mathbf{I} - \gamma\,\mathbf{A})^{-1}\,\frac{\mathbf{l}}{1+\alpha} = \frac{\mathbf{l}}{1+\alpha} + \frac{\mathbf{l}}{1+\alpha}\left(\gamma\mathbf{A}\right) + \frac{\mathbf{l}}{1+\alpha}\left(\gamma\mathbf{A}\right)^{\mathbf{2}} + \frac{\mathbf{l}}{1+\alpha}\left(\gamma\mathbf{A}\right)^{\mathbf{3}}\ \ldots$$

The labour values are changing in a complex way due to the differing time structure of production across sectors and the fact that use is being made of the various means of production. Again, it is impossible to determine analytically by how much, as a consequence, the rate of surplus value and the value composition will change. It can be shown, however, that the technical composition of capital will increase at a rate of $\beta \times 100\%$.[23]

The impact of this type of labour-saving technical change can be illustrated as before by starting with our imaginary economy. Since $\gamma > 1$, we have to look into the phenomenon of the technical composition of capital increasing proportionately more than output ($\beta > \alpha$), e.g. $\beta = 32\%$, while the rate of increase of output in each sector is $\alpha = 20\%$. Taking into account that $\gamma = (1 + \beta)/(1 + \alpha) = 1.32/1.20 = 1.10$, i.e. that the inputs of the means of production are increasing by 10%, the labour values system is now:

$$0.3667\ \lambda_1' + 0.2778 = \lambda_1'$$

$$0.5500\ \lambda_1' + 0.4400\ \lambda_2' + 0.4167 = \lambda_2'$$

such that: $\lambda_1' = 0.438$ and $\lambda_2' = 1.136$. Therefore, $\Delta\lambda_1/\lambda_1 = -0.124$ and $\Delta\lambda_2/\lambda_2 = -0.09$.

As before, the unit labour values can be split into their components c, v and m, with the necessary consumption per working hour remaining the same:

$$0.3667\ \lambda_1' + 0.1389\ \lambda_2' + m_1' = \lambda_1'$$

$$(0.5500\ \lambda_1' + 0.4400\ \lambda_2') + 0.2084\ \lambda_2' + m_2' = \lambda_2'$$

or:

$$0.161\ (c_1') + 0.158\ (v_1') + 0.119\ (m_1') = 0.438$$

$$0.741\ (c_2') + 0.237\ (v_2') + 0.158\ (m_2') = 1.136$$

It then follows that:

$m_1'/v_1' = 0.753$ $\qquad$ $m_2'/v_2' = 0.667$

and: $\dfrac{\Delta\left(m_1 / v_1\right)}{\left(\dfrac{m_1}{v_1}\right)} = 25.5\%$ $\qquad$ $\dfrac{\Delta\left(m_2 / v_2\right)}{\left(\dfrac{m_2}{v_2}\right)} = 11.1\%$

This is the first time in our narrative that the introduction of technical change influences the rate of surplus value differently across sectors. This is so because the labour value of the necessary consumption has decreased by a smaller amount than that of the means of production. On the other hand, it is found that:

$$\kappa_1' = \frac{C_1'}{V_1'} = 1.019 \qquad \kappa_2' = \frac{C_2'}{V_2'} = 3.127$$

$$\text{and: } \frac{\Delta\kappa_1}{\kappa_1} = 27.4\% \neq \frac{\Delta\kappa}{\kappa} \qquad \frac{\Delta\kappa_2}{\kappa_2} = 30.3\% \neq \frac{\Delta\kappa}{\kappa}$$

The conclusion that can be drawn from this simple numerical example is that when all physical inputs of means of production per unit of output increase by 10% and labour productivity increases by 20%, it leads in the first instance to a proportionate increase of 32% in the technical composition of capital. However, in our example, the impact of the technical change is highly concentrated in the capital goods sector where the unit labour values show a more pronounced decline than they do for the necessary consumer goods.[24] This illustrates the situation that Marx took for granted, i.e. that the value composition of capital, via a decrease in labour values, is increasing at a slower pace than the technical composition. Also clear from our example is that the rate of surplus value in both sectors increased by a proportionately smaller amount than the value composition of capital. We will return to this situation in Chapter 6.

5.5 The scope for labour-saving but also capital-using technical change in the pursuit of super-profits

The increase in the degree of mechanisation of production, i.e. the substitution of living labour for dead labour, as Marx describes the process, has its economic limitations. The introduction of labour-saving but capital-using technical changes, which Marx considers to be the dominant type of technical change, has to deliver a temporary cost advantage for the introducing capitalist. It is also the source of super-profits. It was shown how these super-profits are generated in section 5.2 of this chapter.

Marx seems to be totally convinced that labour-saving technology goes together with more mechanisation. Following on from this, Marx introduces another "law of motion" in his model of capitalist development which states that, apart from the increase in labour productivity, the physical quantity of means of production per unit of output will also increase on average. It is a "law of motion" of capitalism, since the higher degree of mechanisation stems from the pursuit of super-profits by the capitalists.

Evidently, per unit value of output new capital-using methods of production will create smaller super-profits than methods of production that are purely labour-saving. This is because of the higher value of the means of production per unit value of output that is transferred. Moreover, the mechanisation of production should not increase to such an extent that the unit cost price of the introducing capitalist exceeds the price of producing the product (as dictated by the market), which would lead to his facing a competitive disadvantage.

Regarding the degree of mechanisation (or the capital-using character) of the new labour-saving technology that is permitted, given competitive conditions, it can be stated that the percentage increase in the use of means of production ($\gamma - 1$) – which is combined with the given percentage increase in labour productivity (α) – should, after being introduced by the individual capitalist, result in a price that is at best equal to the price of production in the sector:

$$(1 + r)\ [(\hat{\gamma})\mathbf{A} + \frac{\mathbf{l\,c}}{1+\alpha}\ ]\ \mathbf{p} \leq \mathbf{p} \tag{19}$$

where r and $\mathbf{p}$ derive as solutions of the system of production prices $(1 + r)$ $\mathbf{(A + l\ c)\ p} = \mathbf{p}$ and:

$$(\hat{\gamma}) = \begin{pmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{pmatrix}$$

the diagonal matrix of sectoral γ's.

It will be remembered from Chapter 2 that provided the correct "numéraire" of the p's is chosen, i.e. $\mathbf{c\ p} = 1$, the solution of the production prices system is $[\mathbf{I} - (1 + r)\ \mathbf{A}]^{-1}\ \mathbf{l}\ (1 + r) = \mathbf{p}$. After substituting this solution in condition (19) and doing the necessary rearranging, expression (20) is derived:

$$(1 + r)\ \mathbf{I} - (1 + r)\ [(\hat{\gamma})\mathbf{A} + \frac{\mathbf{l\,c}}{1+\alpha}\ ]^{-1}\ [\mathbf{I} - (1 + r)\ \mathbf{A}]^{-1}\ \mathbf{l} \geq 0 \tag{20}$$

With reference to the numerical example introduced in the previous section, we now investigate the situation in which a capitalist from the capital goods sector introduces a new production technique. Because of the super-profits that he will receive, there is room for an increase in the degree of mechanisation – to the extent, however, that his unit cost price, including the normal profits on his invested capital, will not exceed the price of production in the market, i.e. the price applied by his competitors. The theoretical problem is as follows: given the introduction of a new production method, what percentage rate of increase in the capitalist's physical means of production (iron) is he prepared to accept if his cost price (taking into account the increased labour productivity, plus normal profits) cannot exceed the price of production of his competitors? The price of production in the capital goods sector (sector 1: iron) is made up as follows:

$$(0.333\ p_1 + 0.167\ p_2)(1 + r) = 0.437$$

with $r = 21.8\%$, $p_1 = 0.437$ and $p_2 = 1.302$

With $(\gamma - 1) \times 100\%$ being the allowed percentage rate of increase in means of production inputs (iron) in the capital goods sector, and with due regard for the 20% increase in labour productivity that is induced by the new production technique, the condition becomes:

$$(0.333\ \gamma\ p_1 + 0.139\ p_2)(1 + r) = 0.437$$ [25]

Substituting in this equation: $r = 21.8\%$, $p_1 = 0.437$ and $p_2 = 1.302$, we find that $(\gamma - 1) \times 100\% = 22\%$. When the available technology requires this percentage rate of increase in the physical input of means of production per unit of output in the sector, then in the introducing capitalist's factory the input of iron will become 0.407 kg, while the labour input per unit of output will fall to 0.139 hours. The scope for an increase in the technical composition of capital in the iron-producing sector through the introduction of the new production technique is up to 2.93 kg of iron/kg of necessary consumption (wheat), whereas before the introduction, the scope was 2 kg of iron/kg of necessary consumption, pointing to an increase of 46.5%. *Ceteris paribus*, this implies an increase in the value composition of capital at the same percentage rate of increase.[26]

Evidently, in their quest for super-profits, the capitalists in the wheat sector (sector 2) can introduce new labour-saving technology as well. If this technology uses more mechanisation, we can calculate how far the introducing capitalist will be prepared to go in mechanising production. With a rate of increase in labour productivity of 20% and a normal rate of profits of 21.8%, the more mechanised new technology should not push the price of the introducing capitalist above the price of production of his competitors, which in the consumer goods sector (wheat) is made up as follows:

$$(0.500\, p_1 + 0.650\, p_2)\,(1 + r) = 1.302$$

with $r = 21.8\%$, $p_1 = 0.437$ and $p_2 = 1.302$

We assume that the increased mechanisation only relates to the use of iron in the wheat sector, and not to the use of wheat (as seed for sowing).[27] After the introduction of the new technology by one of the capitalists in the sector, the competitiveness condition is:

$$(0.500\, \gamma\, p_1 + 0.608\, p_2)(1 + r) = 1.302$$

For the given r, p_1 and p_2, the allowed percentage rate of increase in iron input use per unit of output in the wheat sector is $(\gamma - 1) \times 100\% = 38.3\%$, whereas labour productivity increases by 20%.

In the spirit of industrialisation and mechanisation at the time, Marx took it for granted that in their relentless pursuit of super-profits, the capitalists would introduce innovative, labour-saving production methods that would clear the way for the heightened use of means of production per worker – as would be evidenced in the technical composition of capital. Based on the above illustrative example, it can be concluded that Marx saw the possibility of such a scenario materialising. This "law of motion" of capitalism which Marx introduced in his model is crucial for his additional views on the to-be-expected secular fall of the general rate of profits. This, though, is the subject of Chapter 6.

The question now is, to what extent does this conclusion depend on the assumption of Marx's linear production model and, as elaborated on in previous pages, the fact that labour is homogeneous or abstract? Recent theoretical and empirical research has, in fact, provided evidence that capital/means of production and skilled labour are complementary – in contrast to capital/means of production and less-skilled or unskilled labour, which are substitutes (see Bentolila and Saint-Paul, 2003; Arpaia, Pérez and Pichelmann, 2009; and Bassanini and Manfredi, 2012). The implication is that the substitution of labour for capital essentially hurts the less-skilled or unskilled blue collar workers, while more skilled labour is used with the rising stock of means of production. This has both a fundamental theoretical and empirical impact on the extent to which labour-saving and capital-using technology is introduced, and therefore on the analysis of skilled and unskilled labour in the linear production model. More is at stake here than the mere "reduction" of skilled labour to abstract labour. However, such an analysis falls outside the scope of this book and is on the agenda for future research.

5.6 How did the early post-Keynesians look at mechanisation and technical change? The case of Joan Robinson's "real-capital ratio"

There can be little doubt that at an analytical level the way that the early post-Keynesians looked at mechanisation and technical change was closely aligned to Marx's views. Joan Robinson is probably the most important post-Keynesian neo-Marxist scholar to investigate this subject in great detail and with a high level of sophistication. In order to conduct a theoretical analysis of movements in the real wage rate and the rate of profits during the capital accumulation process, while also considering technological progress, she introduced her ex ante production function which shows the relationship, for an individual capitalist producer, between output per worker and the "real-capital ratio". In contrast to the neo-classical production function, Robinson's ex ante production function shows such a relationship for a given rate of profits, indicating that the "quantity of capital" depends on it.

To simplify things, we do not follow Robinson's approach of drawing the ex ante production function as showing discontinuities; rather, for a given rate of profits r, we show it as a continuous convex-shaped curve (Robinson, 1956, pp. 416–420, but see also p. 108).[28] In Figure 5.2, the right-hand quadrant depicts two ex ante production functions, corresponding with the same spectrum of techniques but for two different rates of profits.

The value of output of the individual capitalist is assumed to consist exclusively of wages and profits, or: $p\ \mathrm{O} = w\ \mathrm{L} + r\ \mathrm{K}$, with p being the unit price of output, O the quantity of output produced, w the nominal wage rate, L the number of workers, r the rate of profits and K the value of the capital stock invested. This equality leads after rearrangement to:

$$\frac{\mathrm{O}}{\mathrm{L}} = \frac{w}{p} + r\frac{w}{p}\frac{\mathrm{C}}{\mathrm{L}}$$

In this equation, $w/p = \omega$, the real wage rate, and C/L is Robinson's "real-capital ratio", with: $\mathrm{C} = \mathrm{K}/w$, i.e. the value of the capital stock (which is a function of r)

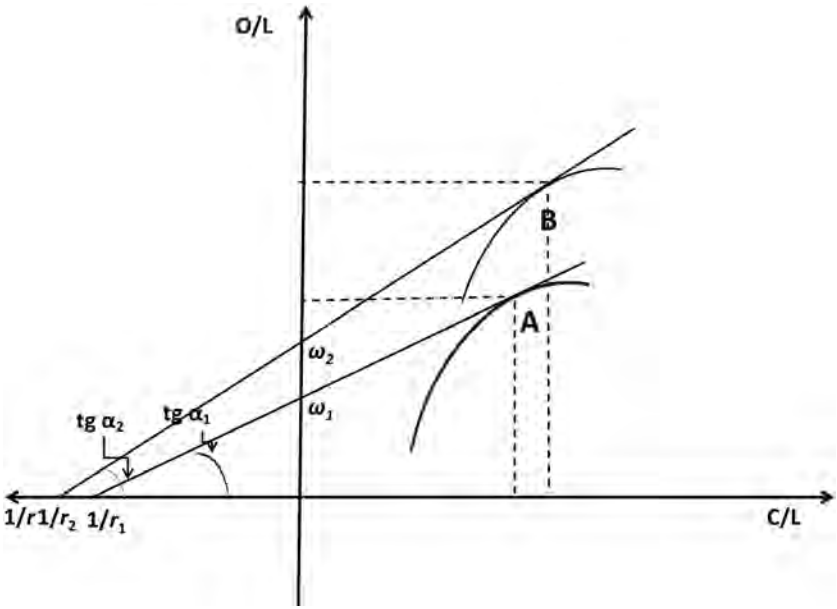

Figure 5.2 Two Robinsonian ex ante production functions showing the same spectrum of techniques for two rates of profits

measured in units of labour time. This "real-capital ratio" can easily be transformed into a Sraffian sum of dated quantities of labour (Robinson, 1956, pp. 122–123). With L_t being the number of working hours spent indirectly t production stages ago ($t = 1, 2, 3, \ldots$) in producing the given capital stock, the nominal value of the capital stock is:

$$K = (1 + r)\, w\, L_1 + (1 + r)^2\, w\, L_2 + (1 + r)^3\, w\, L_3 \ldots$$

and, therefore (with wages advanced):

$$C = (1 + r)\, L_1 + (1 + r)^2\, L_2 + (1 + r)^3\, L_3 \ldots$$

Robinson's "real-capital ratio" is derived from a suggestion in Sraffa's "Introduction" to David Ricardo's *Principles* (Robinson, 1972, p. 234). It will become clear that it bears a close resemblance to Marx's value composition of capital, although the "dead labour" that the means of production contain is "dated" à la Sraffa since Robinson does not measure in labour values but in "normal prices" (essentially, Marx's prices of production). Robinson states (1956, p. 121):

> This is in some way the most significant way of measuring capital, for the essence of the productive process is the expenditure of labour time, and labour

> time expended at one date can be carried forward to a later date by using it to produce physical objects (or to store up knowledge) which will make future labour more productive, so that capital goods in existence to-day can be regarded as an embodiment of past labour time to be used up in the future.

The intimate relationship between Marx's value composition of capital and the Robinsonian "real-capital ratio" rests on three closely related concepts. First of all, Marx considers capital to be "dead labour" (Marx, 1976, pp. 322, 342; Marx, 1981, p. 524) and Robinson views it as "an embodiment of past labour time" (1956, p. 121). Second, the "real-capital ratio" is nothing but the value composition of capital, at least when the capital stock is aggregated in Robinson's "normal prices", which in the long run are identical to Marx's prices of production: since C = K/*w*, C/L = K/*w* L, the ratio of the value of the capital stock in "normal prices" and the wage bill. Third, in Robinson's analysis, she also uses the average "real-capital ratio", which over a sufficient range of rates of profits leaves the normal prices largely unchanged in the sectors with average C/L (Robinson, 1956, p. 353). The latter plays the same role as the average composition of capital in Marx's analysis (Marx, 1981, p. 273, pp. 302 ff.), i.e. as the dividing line between sectors, such that with an increase in the rate of profits, the production prices in the sectors above the dividing line fall and those in the sectors below the line increase.

Moreover, as was highlighted in Chapter 2, it holds that for an economy that is expanding along its maximum balanced growth path (no capitalist consumption) the total value of output and the value of the workers' necessary consumption (in both labour values and in prices of production) are identical. It follows, too, that the value of the inputs of physical means of production is the same in labour values and in prices of production, and that the value composition of capital in labour values and in prices of production is exactly the same.

Similarly, Robinson's degree of mechanisation concept coincides with Marx's technical composition of capital concept, with the result that they perform the same function. We indicated above how changes in the technical composition of capital should be regarded as equal to $\sum_j \frac{\lambda_j}{\lambda_j \, \mathrm{c}_j} \frac{\partial}{\partial \mathrm{t}}\left(\frac{\mathrm{A_{ijt}}}{\mathbf{l}_{\mathrm{it}}}\right)$, i.e. the changes in the individual $\mathrm{A_{ijt}}/\mathrm{l_{it}}$ ratios, aggregated using the given $\lambda_j/\lambda_j c_j$'s. This means that changes in the technical composition solely reflect changes in the physical inputs of means of production per direct labour expenditure. The identical function performed by Robinson's degree of mechanisation concept is most easily highlighted when the "real-capital ratio" of two economies is compared. Differences between the two "real-capital ratios" are then partly the result of differences in the rates of profits. Yet, most importantly, they reflect differences in the degree of mechanisation: "a marked difference in the real-capital ratio should be possible to detect, and a higher real-capital ratio is likely to mean a higher degree of mechanisation" (Robinson, 1956, p. 136).

As already discussed in Chapter 3, Robinson also shares Marx's view that long-period labour scarcity, which frustrates the desired rate of accumulation, is the major cause of induced technical change. Robinson writes (1975b, p. 105): "The chief driving force behind technical progress is the scarcity of labour in relation to capital which is produced by a rate of accumulation in excess of the rate of growth of population."

She calls this insight "the most interesting and important Marxian idea" in her model (Robinson, 1963, p. 410). In addition, she stresses the importance of "competitive technological progress", which is triggered by the capitalist "rules of the game" and the cut-throat price competition that these entail (Robinson, 1962, p. 52). She writes in this respect:

> The capitalist rules of the game foster large-scale production and the use of elaborate techniques (. . .). Entrepreneurs can organize large masses of workers, bringing about economies of division of labour; their command of finance makes it possible for each to provide the workers whom he employs with complicated equipment, and he is impelled to do so by the competitive struggle to undersell others.
>
> (Robinson, 1956, p. 6)

Competitive technological progress is evidently the result of the relentless capitalist pursuit of super-profits, which Marx emphasised as well (Marx, 1976, p. 582; Marx, 1981, pp. 363, 373) and which we analysed above. Its existence, so Robinson states, is most clearly evidenced in periods of high unemployment when "induced technological progress", i.e. the technological change triggered by long-run labour scarcity, is absent and competitive technological progress fuels cut-throat price competition during such periods (Robinson, 1956, p. 86).

Where Robinson's and Marx's views on technological change differ the most – and in fact are irreconcilable – is in relation to the long-term neutrality versus the capital-using bias of technological progress. We have seen that Marx attempts to show that capitalism has a tendency to develop and implement labour-saving, but also capital-using, techniques of production. In contrast, Robinson has always insisted that technical change tends to be neutral in the long run.

In the given spectrum of techniques, the technique chosen will be the one that delivers the highest rate of profits. For the real wage rate $w/p = \omega_1$, this is r_1, leading to production technique A in Figure 5.2. Since the tg $\alpha_1 = \omega_1/(1/r_1)$, the slope of the tangent through ω_1 to the relevant ex ante production function equals $\omega_1 r_1$. When the real wage increases to ω_2, the rate of profits will be lower and the ex ante production function shifts to a new position. The most efficient technique now adopted is production technique B, which combines a higher labour productivity with a higher C/L. It is therefore a more mechanised production method. Robinson also stresses that with a desired rate of accumulation that is higher than the one attainable due to the lower rate of population growth, labour scarcity will once again develop and will trigger a further slide in the rate of profits.

A completely different situation emerges when induced, competitive or autonomous technological progress takes place. It will shift the ex ante production function upwards, which in turn will have an impact on both the real wage rate and the rate of profits. In order to distinguish between the choice of technique from a given spectrum and from a new spectrum, Robinson considers the new spectrum at the given rate of profits r_1.[29] The new production technique chosen will then combine a higher labour productivity with either a lower or higher C/L than before, in which case the technological change will have been capital-saving or capital-using. In Figure 5.2, that new production technique is A', with the same C/L as A; technological change, in turn, has remained neutral.

Although technological progress is often induced by labour scarcity, it is not necessarily capital-using. Robinson states that there is no reason to assume that the increase in labour productivity is higher in the consumer goods sector than in the capital goods sector (Robinson, 1962, pp. 51, 111). Two spectrums of techniques are neutral vis-à-vis each other if for all techniques of the superior spectrum the quantity of labour per unit of output and the quantity of "real capital" per unit of output declined by the same percentage rate – in other words, the extent of labour-saving and capital-saving in the technological progress is the same (Robinson, 1956, p. 133).

With neutral technological progress, says Robinson, the rate of growth in labour productivity in the capital goods sector and in the consumer goods sector is the same, which implies that the distribution of the labour force between the two sectors is unchanged. With a capital-using bias, labour productivity increases at a more rapid rate in the consumer goods sector, while with a capital-saving bias, the increase is more rapid in the capital goods sector (Robinson, 1956, pp. 159–160).

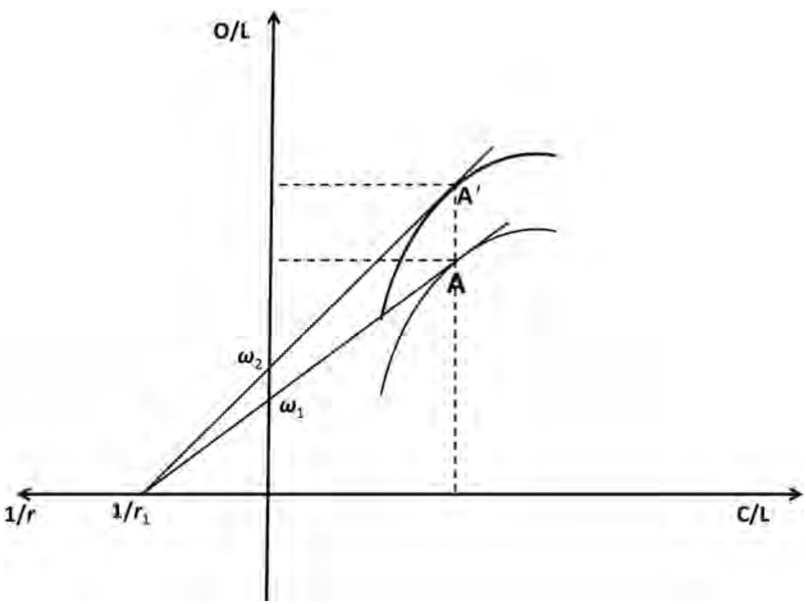

Figure 5.3 Neutral technical change as a shift in the Robinsonian ex ante production function for a given rate of profits

With neutral technological progress, the share of wages and the share of profits from income will remain constant. Output per worker will increase at the same percentage rate of change as the real wage rate. In the case of a capital-using bias, the share of profits will increase. Clearly, so Robinson points out, technological progress can prevent a fall in the rate of profits, which would occur as a result of the scarcity of labour (Robinson, 1956, p. 96). However, since the shift in the ex ante production function is a unique event, labour scarcity will manifest again if the desired rate of accumulation stays above the population growth rate, causing the rate of profits to fall. For the rate of profits to remain constant, technological progress has to be sufficiently fast and steady, while for the income shares of capital and labour to remain unchanged this technological progress should also be neutral.

We came across this situation of neutral technological progress in section 5.4.2 in this chapter. We found that when both A_{ij} and l_i change in the same proportion, Marx's technical composition of capital remains unchanged (as one would expect) but the value composition changes. However, with differences in the value compositions of the sectors, the labour values of output of sectors 1 and 2 will decrease at a different percentage rate. In our numerical example, the unit labour value of wheat decreased by a higher proportion than the unit labour value of iron, due to the larger inputs of iron and wheat in the wheat sector compared to the iron sector. Moreover, the rate of surplus value increased dramatically, unlike what one would expect with neutral technological progress. However, the reason for this can be found in the falling unit labour values of the workers' necessary consumer goods, together with the assumption that in spite of the increasing labour productivity (both in the consumer goods and the capital goods sectors), the real consumption of the workers is not increasing at all. At the same time, it needs to be stressed that Robinson's statement of equal rates of change in labour productivity in the sectors producing capital goods and consumer goods is a necessary but not a sufficient condition. We have seen that when the inputs of means of production per unit of output in the two sectors also change, it all depends – given the past time pattern of the spending of indirect labour – on how the labour values of the outputs, and therefore also their "normal prices", will react. It then remains to be seen by which percentage rate the real wage rate, or Marx's value of labour power, will change as a result of the increase in the productivity of labour. The other necessary condition for Robinsonian neutral technological progress is therefore, as Asimakopulos and Weldon (1963, pp. 377, 381) have pointed out, that the "time pattern of investment" is not changing, an assumption that Robinson is making (Robinson, 1956, p.73).

Not surprisingly, the spectrum of techniques as a theoretical concept has no counterpart in *Das Kapital*. In fact, Joan Robinson mentions her indebtedness to Knut Wicksell's 1893 volume, *Value, Capital and Rent* (Robinson, 1956, p. 411), but also remarks that the concept is based on an artificial assumption: "The spectrum of techniques retains its form for long periods only in stagnant economies where accumulation is very sluggish" (Robinson, 1956, p. 151).

In addition, knowledge about alternative production techniques to the one that is used must be considered vague and general (Robinson, 1956, p. 156). Even so, in Marx's view, a more mechanised production technique is the result of the development of "productive forces" in society, i.e. technological development and innovation. Any notion of a truly alternative production technique is in contrast to Marx's opinion about technological change. Moreover, it is appropriate to mention that Joan Robinson in later years abandoned the spectrum of techniques concept and seemed to adopt Marx's view (see also Harcourt and Kenyon, 1976, pp. 459–461). She states in her controversial 1975 article in the *Quarterly Journal of Economics*:

> There are no ready-blueprinted techniques to choose from. When the technique to be installed is designed to give a higher output per man than that formerly in use, it must be a recent innovation or an adaptation from one already known in the broad, though not in detail.
>
> (Robinson, 1975c, pp. 38–39)

The same opinion is advanced by Nicholas Kaldor in his seminal 1957 paper in which he developed his post-Keynesian model of economic growth: "any sharp or clear-cut distinction between the movement along a 'production function' with a given state of knowledge and a shift in the 'production function' caused by a change in the state of knowledge is arbitrary and artificial" (Kaldor, 1957, p. 596).

Kaldor's model is built on a "technical progress function" which shows the relationship between the rate of growth of capital per worker and the growth of output per man, and presumes a constant flow of new ideas and inventions and a constant degree of technical dynamism over time (Kaldor, 1957, p. 596).[30] In Kaldor's (and also in Robinson's) analysis of present-day capitalism, the share of wages is a residual, with the share of profits determined by the propensities to save and invest (Kaldor, 1957, p. 620). However, Kaldor's post-Keynesian model of economic growth, unlike that of Joan Robinson, cannot be considered neo-Marxist. The position and slope of his "technical progress function" at each point in time are given and there is no feedback from labour scarcity towards labour-saving technological change (whether neutral, capital-using or capital-saving). Furthermore, his model of "Stage II" of capitalism rejects the possibility of a profit/surplus value realisation problem, due to under-spending – a result which is hardly surprising given his assumption that a state of Keynesian "under-employment equilibrium" cannot persist for an extended period of time.

5.7 To conclude

The relationship between the capitalists' urge to accumulate and labour scarcity in the short and longer term creates important links to Marx's economic model which, in turn, produces a number of dynamic changes during the period under consideration. We have seen that, according to Marx, in the shorter term – particularly during an economic boom – the increasing wage rates will also increasingly hamper accumulation. The capitalists will react to this by reducing their investments, resulting in reduced economic activity and higher unemployment. In this way, the wage rates will be reduced to a level that the capitalists consider to be reconcilable with their planned accumulation. Labour scarcity during boom times thus seems to play an important role in Marx's theory of the laws of motion in the shorter term in that it acts as the detonator for a downturn in the business cycle via its effect on the accumulation of capital and the resulting economic expansion. It will be shown in Chapter 8 how labour scarcity, via a route other than an increase in wages, will produce a downturn in the business cycle. This other theoretical route involves limiting the production of surplus value, and therefore the total amount of profits, which (with reference to Marx's theory of value) labour scarcity would cause.

Of course, also during an economic boom the introduction of labour-saving production techniques will be beneficial for the capitalists, and it can be expected that not only will labour intensity increase but many new innovations in terms of production methods will be introduced. Competition and the quest for super-profits are a persistent reality in capitalism, although the intensity thereof will fluctuate at different times. In their quest for super-profits, the capitalists will introduce and apply systematically new or improved labour-saving production methods. In this way (as has been pointed out), what Marx calls the relative surplus value will, due to an increase in the rate of surplus value, increase proportionately more than the increase in labour productivity. Unfortunately, to the extent that new labour-saving production methods are also capital-using, their exact effect is not always clear. In fact, the degree of mechanisation – Marx's technical composition of capital – will increase or decrease, respectively, in the case of capital-using or capital-saving innovations.

By studying the impact of new labour-saving methods of production on the technical composition of capital, Marx uses an ingenious trick that isolates the consequences that these methods have for the unit labour value of output from those that they have for the degree of mechanisation. However, in the end, it should not be forgotten that it is the value composition of capital that is relevant when it comes to the behaviour of the rate of profits.

Marx's particular concern here is with the capital-using bias of new labour-saving technology, from which he derives the "law of motion", that during the process of capitalist development, the value composition of capital secularly increases together with the mechanisation of production. It seems to be impossible to theoretically prove this hypothesis conclusively, due to the complexities of the sectoral production and input relations that are involved. However, our illustrative numerical examples have shown that with the relentless capitalist pursuit of super-profits, there is some scope for the technical composition of capital in the industrial sectors to increase. But there is no "law" that dictates to the capitalists per se, forcing them to use this scope in all circumstances for increased mechanisation. In any case, it remains to be seen to what extent the value composition of capital will follow this mechanisation, and to what extent the less-skilled or unskilled labour force, rather than the skilled, will be replaced during the process of increasing mechanisation.

Finally, Marx's law of motion of secular increase in the degree of mechanisation was compared with the post-Keynesian views on the relationship between increasing output per man and mechanisation, best achieved by looking into Joan Robinson's concept of the spectrum of techniques and her theory about induced and competitive technological progress. It was shown that the Robinsonian "real-capital ratio" closely resembles Marx's value composition of capital which, like his organic composition of capital, is considered to essentially reflect the degree of mechanisation. Robinson's spectrum of techniques in her 1956 magnum opus, *The Accumulation of Capital*, is an alien concept to Marx for whom each increase in the mechanisation of production represents an increase in the productive forces of society, i.e. technological progress. Actually, in the 1970s, Joan Robinson abandoned the concept of spectrum of techniques and apparently became a convert to Marx's view.

Appendix: The effect of a general increase in the productivity of labour on the rate of surplus value

Consider the system of labour values with two sectors:

$$A_{11}\lambda_1 + l_1 = \lambda_1$$

$$A_{12}\lambda_1 + A_{22}\lambda_2 + l_2 = \lambda_2$$

The solution of this system is:

$$(I - A_{11})^{-1}\, l_1 = \lambda_1$$

$$(I - A_{22})^{-1}\, A_{12}(I - A_{11})^{-1}\, l_1 + (I - A_{22})^{-1} l_2 = \lambda_2$$

On the other hand, the outputs system shows the following picture:

$$q_1\, A_{11} + q_2\, A_{12} + S_1 = q_1$$

$$q_1\, l_1\, c_2 + q_2\, A_{22} + q_2\, l_2\, c_2 + S_2 = q_2$$

After combination, it follows that:

$$V = q_1\, l_1\, c_2\, \lambda_2 + q_2\, l_2\, c_2\, \lambda_2$$

$$M = S_1\lambda_1 + S_2\lambda_2 = (q_1\lambda_1 - q_1A_{11}\lambda_1 - q_2A_{12}\lambda_1 - q_1\, l_1\, c_2\lambda_2) + (q_2\lambda_2 - q_2A_{22}\lambda_2 - q_2\, l_2\, c_2\lambda_2)$$

An increase in labour productivity will change the labour input coefficients into:

$$l_1' = l_2/(1 + \alpha)$$

$$l_2' = l_2/(1 + \alpha)$$

This transforms the system of labour values into:

$$A_{11}\lambda_1' + l_1/(1+\alpha) = \lambda_1'$$
$$A_{12}\lambda_1' + A_{22}\lambda_2' + l_2/(1+\alpha) = \lambda_2'$$

for which the solution is:

$$(I - A_{11})^{-1}\, l_1/(1+\alpha) = \lambda_1' = \lambda_1/(1+\alpha)$$
$$(I - A_{22})^{-1}\, A_{12}(I - A_{11})^{-1}\, l_1/(1+\alpha) + (I - A_{22})^{-1}\, l_2/(1+\alpha) = \lambda_2' = \lambda_2/(1+\alpha)$$

We now expand the expressions for V' and M' as follows:

$$V' = q_1\, l_1'\, c_2\, \lambda_2' + q_2\, l_2'\, c_2\, \lambda_2' = \frac{1}{(1+\alpha)^2}\, q_1\, l_1\, c_2\, \lambda_2 + \frac{1}{(1+\alpha)^2}\, q_2\, l_2\, c_2\, \lambda_2$$
$$= \frac{V}{(1+\alpha)^2}$$

$$M' = (q_1\, \lambda_1' - q_1 A_{11}\lambda_1' - q_2 A_{12}\lambda_1' - q_1\, l_1'\, c_2\, \lambda_2) + (q_2\, \lambda_2' - q_2 A_{22}\lambda_2' - q_2\, l_2'\, c_2\lambda_2')$$

$$= \frac{1}{1+\alpha}\, q_1\lambda_1 - \frac{1}{1+\alpha}\, q_1 A_{11}\lambda_1 - \frac{1}{1+\alpha}\, q_2 A_{12}\lambda_1 - \frac{1}{(1+\alpha)^2}\, q_1\, l_1\, c_2\lambda_2 +$$
$$\frac{1}{1+\alpha}\, q_2\lambda_2 - \frac{1}{1+\alpha}\, q_2 A_{22}\lambda_2 - \frac{1}{(1+\alpha)^2}\, q_2\, l_2\, c_2\lambda_2$$

Rearranging the terms of M' gives:

$$M' = \frac{1}{1+\alpha}\, q_1\lambda_1 - \frac{1}{1+\alpha}\, q_1 A_{11}\lambda_1 - \frac{1}{1+\alpha}\, q_2 A_{12}\lambda_1 + \frac{1}{1+\alpha}\, q_2\lambda_2 - \frac{1}{1+\alpha}\, q_2 A_{22}\lambda_2$$
$$- \frac{1}{(1+\alpha)^2}\, q_1\, l_1\, c_2\lambda_2 - \frac{1}{(1+\alpha)^2}\, q_2\, l_2\, c_2\lambda_2$$

or:

$$M' = \frac{1}{1+\alpha}\, [q_1\lambda_1 - q_1 A_{11}\lambda_1 - q_2 A_{12}\lambda_1 + q_2\lambda_2 - q_2 A_{22}\lambda_2 - \frac{1}{1+\alpha}\, q_1\, l_1\, c_2\lambda_2$$
$$- \frac{1}{1+\alpha}\, q_2\, l_2\, c_2\lambda_2]$$

We add to and simultaneously deduct from the part of M' between [ ] the following term:

$$q_1\, l_1\, c_2\, \lambda_2 + q_2\, l_2\, c_2\, \lambda_2 - q_1\, l_1\, c_2\, \lambda_2 - q_2\, l_2\, c_2\, \lambda_2$$

As a result, M' becomes:

$$\mathrm{M}' = \frac{1}{1+\alpha}\ [\boldsymbol{q_1\lambda_1} - \boldsymbol{q_1\mathrm{A}_{11}\lambda_1} - \boldsymbol{q_2\mathrm{A}_{12}\lambda_1} + \boldsymbol{q_2\lambda_2} - \boldsymbol{q_2\mathrm{A}_{22}\lambda_2} - \frac{1}{1+\alpha}\ q_1\,\mathrm{l}_1\,\mathrm{c}_2\lambda_2$$
$$- \frac{1}{1+\alpha}\ q_2\,\mathrm{l}_2\,\mathrm{c}_2\lambda_2 + q_1\,\mathrm{l}_1\,\mathrm{c}_2\,\lambda_2 + q_2\,\mathrm{l}_2\,\mathrm{c}_2\,\lambda_2 - \boldsymbol{q_1\,\mathrm{l}_1\,\mathrm{c}_2\,\lambda_2} - \boldsymbol{q_2\,\mathrm{l}_2\,\mathrm{c}_2\,\lambda_2}]$$

Since $q_1\lambda_1 - q_1\mathrm{A}_{11}\lambda_1 - q_2\mathrm{A}_{12}\lambda_1 + q_2\lambda_2 - q_2\mathrm{A}_{22}\lambda_2 - q_1\,\mathrm{l}_1\,\mathrm{c}_2\,\lambda_2 - q_2\,\mathrm{l}_2\,\mathrm{c}_2\,\lambda_2 = \mathrm{M}$ (the part in bold in the above equation) and $q_1\,\mathrm{l}_1\,\mathrm{c}_2\lambda_2 + q_2\,\mathrm{l}_2\,\mathrm{c}_2\lambda_2 = \mathrm{V}$, it holds that:

$$\mathrm{M}' = \frac{1}{1+\alpha}\ [\mathrm{M} + \mathrm{V} - \frac{1}{1+\alpha}\ \mathrm{V}] = \frac{1}{1+\alpha}\ [q_1\,\mathrm{l}_1 + q_2\,\mathrm{l}_2 - \frac{\mathrm{V}}{(1+\alpha)}] = \frac{1}{1+\alpha}$$
$$[q_1\,\mathrm{l}_1 + \mathrm{q}_2\,\mathrm{l}_2] - \frac{\mathrm{V}}{(1+\alpha)^2}$$

$\frac{\mathrm{M}'}{\mathrm{V}'}$ can now be found to be:

$$\frac{\mathrm{M}'}{\mathrm{V}'} = \left[\frac{1}{1+\alpha}\,[q_1\;\mathrm{l}_1 + q_2\;\mathrm{l}_2] - \frac{\mathrm{V}}{(1+\alpha)^2}\right] \Bigg/ \frac{\mathrm{V}}{(1+\alpha)^2}$$

$$= \left[\frac{1+\alpha}{(1+\alpha)^2}\,[q_1\;\mathrm{l}_1 + q_2\;\mathrm{l}_2] - \frac{\mathrm{V}}{(1+\alpha)^2}\right] \Bigg/ \frac{\mathrm{V}}{(1+\alpha)^2}$$

$$= \left[(1+\alpha)(q_1\;\mathrm{l}_1 + q_2\;\mathrm{l}_2) - \mathrm{V}\right] / \mathrm{V}$$

The numerator of this expression can also be written as:

$$(q_1\,\mathrm{l}_1 + q_2\,\mathrm{l}_2) - (q_1\,\mathrm{l}_1\,\mathrm{c}_2\,\lambda_2 + q_2\,\mathrm{l}_2\,\mathrm{c}_2\,\lambda_2) + \alpha\,(q_1\,\mathrm{l}_1 + q_2\,\mathrm{l}_2) = \mathrm{M} + \alpha\,(q_1\,\mathrm{l}_1 + q_2\,\mathrm{l}_2)$$

As a result, we find that:

$$\frac{\mathrm{M}'}{\mathrm{V}'} = \frac{\mathrm{M}}{\mathrm{V}} + \frac{\alpha(q_1\;\mathrm{l}_1 + q_2\;\mathrm{l}_2)}{\mathrm{V}}$$

As $q_1\,\mathrm{l}_1 + q_2\,\mathrm{l}_2 = \mathrm{M} + \mathrm{V}$:

$$\frac{\mathrm{M}'}{\mathrm{V}'} = \frac{\mathrm{M}}{\mathrm{V}} + \alpha\,\frac{\mathrm{M}}{\mathrm{V}} + \alpha = (1+\alpha)\,\frac{\mathrm{M}}{\mathrm{V}} + \alpha = \sigma' = \sigma\,(1+\alpha) + \alpha$$

From this it follows that:

$$\frac{\mathrm{M'}}{\mathrm{V'}} - \frac{\mathrm{M}}{\mathrm{V}} = \alpha\frac{\mathrm{M}}{\mathrm{V}} + \alpha = \Delta\sigma$$

$$\text{and: } \frac{\Delta\sigma}{\sigma} = \left.\alpha\frac{\mathrm{M}}{\mathrm{V}} + \alpha \middle/ \frac{\mathrm{M}}{\mathrm{V}}\right. = \alpha + \frac{\alpha}{\sigma} = \frac{\alpha(1+\sigma)}{\sigma}$$

Notes

1 For a discussion on Marx's theory of the economic cycle, see Chapter 8.
2 This section draws on Cuyvers (1977, pp. 20ff).
3 The creation of relative surplus value will be discussed shortly.
4 The development of capitalism is accompanied by an increase in the absolute size of the total surplus value (Marx, 1976, p. 324) and of the capital accumulation (Marx, 1976, pp. 353–354).
5 Joan Robinson calls this "the most interesting and important Marxian idea" in her model (Robinson, 1963, p. 410), which is discussed in more detail in Chapter 6.
6 This holds only insofar as labour productivity increases are not accompanied by wage increases. We will return to this later.
7 Dividing p_1 and p_2 by c_2p_2 can lead to expressions for these normalised p_1 and p_2, which allows us to analyse, for instance, the impact of an increase in labour productivity on a given rate of profits. This, however, will only be a first approximation since the changes of l_1 and l_2 will alter Matrix **B** and therefore the rate of profits.
8 $0.333 \times p_1 = 0.333 \times 0.437$ hours $= 0.146$ hours and $0.167 \times p_2 = 0.167 \times 1.302$ hours $= 0.217$ hours. Thus, $0.333\, p_1 + 0.167\, p_2 = 0.362$.
9 This is $p_1 - 0.363 = 0.437 - 0.362$.
10 Owing to the increase in labour productivity of 20%, the capitalist will now produce 12 kg of iron by using the same amount of labour as before (3.33 hours of labour). His $l_1 = 0.278$ hours, which he is paying at a rate of 0.5 kg of wheat per working hour, i.e. $l_1 c_2 = 0.139$ kg of wheat.
11 $0.333 \times p_1 = 0.333 \times 0.437$ hours $= 0.146$ hours and $0.139\, p_2 = 0.139 \times 1.302$ hours $= 0.181$. Therefore, $0.333\, p_1 + 0.139\, p_2 = 0.327$ hours.
12 The number of working hours remains 100, such that the necessary consumption of wheat will be 50 kg while the surplus product of wheat will be 20.9 kg.
13 The mathematical argument is somewhat more complicated if only an increase in labour productivity in the consumer goods sector is assumed to take place, and nothing else of essence changes.
14 Since each $\boldsymbol{\lambda}' = \boldsymbol{\lambda}/(1+\alpha)$, the change in $\boldsymbol{\lambda}$, $\Delta\boldsymbol{\lambda} = \boldsymbol{\lambda}' - \boldsymbol{\lambda} = -\alpha\boldsymbol{\lambda}/(1+\alpha)$ and thus $\Delta\boldsymbol{\lambda}/\boldsymbol{\lambda} = (\boldsymbol{\lambda}' - \boldsymbol{\lambda})/\boldsymbol{\lambda} = -\alpha/(1+\alpha)$. This is the same percentage decline as that of the labour input coefficients of Vector $\mathbf{l}$.
15 It should be borne in mind that, by definition, capital goods (means of production) cannot belong to the set of necessary consumer goods, i.e. $\mathbf{c}_1 = 0$.
16 We have already established that $\mathbf{l}_1' = \mathbf{l}_1/(1+\alpha)$ and $\mathbf{l}_2' = \mathbf{l}_2/(1+\alpha)$, and that $\boldsymbol{\lambda}_1' = \boldsymbol{\lambda}_1/(1+\alpha)$ and $\boldsymbol{\lambda}_2' = \boldsymbol{\lambda}_2/(1+\alpha)$. These values are introduced in the first term of M', V' and M' + V', after which the expressions are worked out further.
17 In Chapter 6, we will need the total surplus value, variable capital and constant capital, and we will then use the outputs in the respective balanced expansion paths.
18 In the standard system or in an economy undergoing balanced expansion without capitalist consumption, the level of the value composition calculated with labour values and prices of production are the same.

19 The rate of change of (C + V)/V is also $\alpha \times 100\%$.
20 We differentiate κ_i in terms of time. A t after the other subscripts means that the variable is considered to change over time.
21 It will be noticed that the **c** vector is also taken as given and constant in both parts of the expression for $\frac{\partial}{\partial t}\kappa_i$.
22 Again, this assumes that the **c** vector is given and remains the same.
23 The technical composition changes from $\sum_j \frac{\lambda_j}{\lambda_j c_j}\left(\frac{A_{ijt}}{l_{it}}\right)$ into $\sum_j \frac{\lambda_j}{\lambda_j c_j}\left[\frac{\gamma A_{ijt}(1+\alpha)}{l_{it}}\right]$. Each A_{ij} is multiplied by the factor $\gamma(1 + \alpha)$ and increases by

$$\gamma(1+\alpha) - 1 = \frac{1+\beta}{1+\alpha}(1+\alpha) - 1 = \beta.$$

24 This is not the only possible outcome; it is the result of the initial numerical example having been chosen.
25 Since $(\gamma - 1) \times 100\%$ is the permitted percentage rate of increase in the inputs of means of production in the capital goods sector, the value of the permitted iron inputs in iron production becomes $0.333\,\gamma\,p_1$ per unit of output.
26 Evidently, the prices of production will change and nothing will remain the same. We will return to this in Chapter 6.
27 This means that for each kilogram of wheat, 0.4 kg of seed is required for sowing, but only $0.25/(1 + \alpha) = 0.208$ kg of wheat is necessary consumption for the workers because of the fall in the labour input.
28 These discontinuities, however, contribute to her analysis in an important way, from which flows a fundamental criticism of the neo-classical theory of distribution: with an infinitesimal increase in the wage rate, there is no corresponding increase in the marginal product of labour, as pointed out by, for example, Robinson (1975a, p. 189).
29 This is because the new spectrum for the new rate of profits would make it impossible to distinguish between the shift due to the technological progress and that due to the changing rate of profits.
30 Kaldor states that it is futile to consider movements of the capital–output ratio as being dependent on the character of technological progress. In his model the capital–output ratio will rise (fall) when the rate at which new ideas are developed and introduced (characterised by the shape and position of his “technical progress function”) is slower than the rate of capital accumulation.

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6 Long-term developments – the tendency of the rate of profits to fall

According to Marx, one of the most significant contradictions to emerge during the development of capitalism over the long term is the secular movement in the average rate of profits – about which Marx, in his economic model of capitalism, has ventured to make some "risky predictions". In this chapter, the so-called "law" of the tendency of the rate of profits to fall will be discussed, based on what is said about this topic in *Das Kapital*, after which it will be evaluated using the linear model of production which featured in previous chapters.

As a start to the present chapter, the following somewhat provocative question could be posed: if Marx's law of the tendential fall in the rate of profits is correct, why did capitalism not collapse? As in the previous chapters, we will not attempt to provide an overview of the many statistical verifications of the law, which in any case often show mixed results; rather, we will concern ourselves mainly with the logic of Marx's theory.

6.1 Marx's theory of the falling rate of profits – in a nutshell

The general (or average) rate of profits is nothing but the ratio of total profits to total invested capital. For example, a general rate of profits of 12% indicates that in the capitalist economy in question the investor earned an average of 12% on his investment. Evidently, at each moment in time this is the average across different sectors and individual capitalist enterprises. Owing to sectoral differences in the rate of profits, capital moves from sectors where the rate of profits is lower, to sectors where it is higher, than the average. Denoting total profits in the economy by P and total invested capital by K, the rate of profits equals $r = P/K$.

On a day-to-day basis, it is generally not contested what K is.[1] As invested capital, it is the sum of the respective values of commercial buildings, machines and equipment, stocks of raw materials and the capital that is used to pay the wages of labour. What P is at the macroeconomic level is, however, less clear. With reference to Marx's theory of the origin of profits, the working class produces more goods and services than are required to continue production on the same scale in the next production period. The value of this surplus expressed in labour values is equal to the total surplus value M. Assume that the wage bill paid to the workers during the course of the year amounts to €500 million.[2] Using their wages, the workers will purchase the necessary consumer goods. Also assume that the total value added[3] during the same year amounts to €1.2 billion, implying that the workers produced a surplus value of €700 million. If – following Marx's reasoning – no labour was spent on unproductive activities, i.e. activities that are not increasing or contributing to maintaining the use value of the output produced (commerce, bookkeeping, etc.), then total profits *P* are exactly equal to the total surplus value M. Based on our arguments in Chapter 4, it is assumed that no unproductive labour is spent.

It will be remembered that Marx splits total capital into constant capital C and variable capital V, and that the general or average rate of profits can be written as:

$$r = \frac{\mathrm{M}}{\mathrm{C}+\mathrm{V}}$$

For a further analysis of the factors determining the evolution of the rate of profits, both numerator and denominator are divided by V, such that:

$$r = \frac{\dfrac{\mathrm{M}}{\mathrm{V}}}{1+\dfrac{\mathrm{C}}{\mathrm{V}}}$$

In this formula, M/V is what Marx calls the rate of surplus value σ (or the rate of exploitation, when expressed in more ideological terms) and C/V is the value composition of capital κ.

Thus:

$$r = \frac{\sigma}{1+\kappa}$$

In the above example, M = €700 million and V = €500 billion. The rate of surplus value $\sigma = 7/5 = 140\%$, which means that a worker putting in an average of 48 hours per week is in fact working 28 hours for the capitalist (surplus labour) and 20 hours to produce value, which is equal to his weekly wages. Assume that $\kappa = 4$, i.e. for each euro that is paid as wages, an average of €4 of constant capital (means of production) has to be employed. Thus, the rate of profits is:

$$r = \frac{\dfrac{7}{5}}{1+4} = 28\%$$

Marx argues that κ will show a tendency to rise, since more and more constant capital per worker employed is required (Marx, 1981, pp. 318, 326, 330). We discussed this thesis in Chapter 5 and will return to it in the next section. Such an increase in κ will *ceteris paribus* lead to a decline in the rate of profits. If, for instance, κ increases from 4 to 6, with σ remaining the same, the rate of profits becomes:

$$r = \frac{\dfrac{7}{5}}{1+6} = 20\%$$

According to Marx, σ will increase in the course of capitalist development, but less than κ (Marx, 1976, p. 702; Marx, 1981, p. 322):

> [I]f the constant capital set in motion by a worker increases ten-fold, the surplus labour-time would have to increase ten-fold as well, and very soon the total labour-time, or even the full twenty-four hours of the day, would not be sufficient, even if it were entirely appropriated by capital.
>
> (Marx, 1981, p. 523)

However, according to Marx, the law of the falling rate of profits is not absolute. Many factors can inhibit the fall in the rate of profits or may even trigger its temporary rise. Increasing labour productivity in the capital goods sector will result in reduced prices for the various parts of the constant capital (Marx, 1981, p. 343). In addition, the development of credit as well as transport and communication will permit the same invested capital as before to finance a larger turnover or, put differently: with the same amount of capital, more surplus value will be produced (Marx, 1981, pp. 424, 427–428).

Other factors that counteract the fall in the rate of profits are capital exports to the colonies (Marx, 1981, p. 345) and the development of a system of limited liability companies, based on share capital, with the majority of the shareholders being content with a return on their investment that is lower than the average rate of profits (Marx, 1981, pp. 347–348).

If one reads about the internal contradictions surrounding the law of the tendential fall in the rate of profits in Chapter 15 of Volume 3 of *Das Kapital,* one is struck by the fact that Marx regards this "law" as confirmation of the temporary nature of capitalism. Once the rate of profits has fallen below a certain limit, the capitalists will no longer invest, the surplus value will no longer be sufficiently realised and the capitalist system will break down. He writes:

> [I]n view of the fact that the rate at which the total capital is valorized, i.e. the rate of profit, is the spur to capitalist production (in the same way as the valorization of capital is its sole purpose), a fall in this rate slows down the formation of new, independent capitals and thus appears as a threat to the development of the capitalist production process; it promotes overproduction, speculation and crises, and leads to the existence of excess capital alongside a surplus population.
>
> (Marx, 1981, pp. 349–350)

The scenario that Marx describes here explains why many Marxists consider Marx's law of the tendential fall in the rate of profits to be of paramount importance. They view the "law" as "scientific proof" that capitalism is inherently doomed, since the capitalists at some future time will stop accumulating capital when the rate of profits drops too low.

Before Marx, David Ricardo, who – in the history of economic thought – was a major proponent of the classical school, pointed out that with the ongoing accumulation of capital and the employment of a growing labour force, agricultural land of lower quality would be cultivated, thus making it increasingly difficult to feed the labour force according to the usual norms. In the face of declining agricultural returns, ensuring that the necessary consumption needs of the workers are met would put a squeeze on profits. The only way out of this impasse, which would become more aggressively evident, is through technological improvements and innovations, which would prompt output to increase again (Ricardo, 1821, p. 80). Marx polemicises with this Ricardian view, but adopts the "classical belief" in a secular fall in the rate of profits. However, Marx substitutes Ricardo's law of diminishing returns in agriculture with his "law" of the increasing organic composition of capital. Maurice Dobb who, nearly 40 years after his death is still regarded as an authoritative Marxist economist and *maître à penser*, was right to point out (Dobb, 1973, pp. 157–158):

> It seems probable that Marx, in common with other economists of the early and mid-nineteenth century, assumed that this [the falling rate of profits] was an actual trend for which an explanation was called for; and treated it as such rather than as a dogmatic forecast for the future.

For many years, economists have stated that C/V per se does not need to increase when new and better methods of production are introduced and applied. In Chapter 5, we found that the introduction of labour-saving technological innovations can be both capital-using and capital-saving. We therefore return to the value composition and the organic composition of capital, and investigate their role in Marx's "law" of the tendential fall in the rate of profits. Like before, the value composition of capital is expressed as the ratio between the constant and variable capital used, both aggregated in labour values. For the sake of simplicity, however, it is assumed that in the process of production only machines and labour are used and that all machines, like all labour, are alike and can be added up, giving a total number of machines. The unit value of a machine is λ_1 and A_1 represents the number of machines in use (based on the matrix notation of the previous chapters, $A_1 = A_{11}q_1 + A_{12}q_2$, the machines used in sectors 1 and 2, respectively). The total value of the constant capital is thus $\lambda_1 A_1$. If ω is the average wage rate per worker and L is the total number of workers employed, then we can write the total variable capital as $V = \omega\ L$.

Like Marx, we assume that the wage rate allows the worker to purchase on average a bundle of consumer goods of a given composition, which are necessary for reproducing his labour force. Again for simplicity's sake, it is assumed that there is only one consumer good (wheat), of which c units (e.g. c kg) are purchased with the wage ω. With λ_2 being the value of one unit of this consumer good, C/V can be written as:

$$C/V = \lambda_1 A_1 / \lambda_2\ c\ L$$

The ratio A_1/L represents the number of machines per worker, or what Marx calls the technical composition of capital. To the extent that, feeling pressure from the competition, the individual capitalists introduce technical innovations that reduce their prices, they will substitute machines for labour or, as Marx puts it: dead labour for living labour, which will increase A_1/L. As a result of the increased labour productivity, the prices (labour values) λ_1 and λ_2 will decrease.

Since there is no reason why labour productivity would increase faster in the production of machinery than in the production of consumer goods, it can be assumed that both λ_1 and λ_2 are, in the long run, changing in the same proportion, leaving the rate of profits unaffected. This is in contrast to the C/V increasing effect of the rise in A_1/L. This, we think, is the core of Marx's argument: to the extent that the evolution of the value composition of capital C/V is determined by the technical composition of capital (here, A_1/L), Marx says that the organic composition of capital determines the rate of profits.

The crucial issue is not what happens to c, the average quantity of the consumer good that is paid for by the wage rate. True, the new techniques of production introduced reduce, via cheaper consumer goods, the share of wages in value added. We will return to this in Chapter 7. When the increase in labour productivity in the consumer goods sector is increasing real workers' consumption by the same proportion, then λ_2 is decreasing in the same proportion as that in which c is increasing, such that λ_2 c (Marx's value of labour power) and the rate of surplus value M/V remain constant. For the rest of this section, it is assumed that in the long run M/V remains roughly constant.[4]

Assume that three machines are on average manned by one worker. Thus, $A_1/L = 3$. Assume further that the labour value of a machine $\lambda_1 = 1200$ hours and that the labour value of a consumer good $\lambda_2 = 2$ hours. If an average worker consumes 360 units (e.g. kg) of the consumer good, then the value of labour power is $\lambda_2 \, c = 2 \times 360 = 720$ hours and $C/V = [\lambda_1 \, A_1/ \lambda_2 \, c \, L] = (1200/720) \times 3 = 5$.

With the introduction of innovations, labour productivity in both sectors increases by, for example, 100% and the labour value is halved. Thus, λ_1 becomes 800 hours and λ_2 becomes one hour of labour time. Furthermore, if c increases by 100% (to become 720 units of the consumer good), then the value of labour power remains at its original level of 720 hours. Assume now that A_1/L increases from 3 to 3.6; then, after the innovations, C/V is: $(800/720) \times 3.6 = 4$. In spite of the rise in the technical composition of capital, the value composition has decreased, and with a constant rate of surplus value, the rate of profits has increased. Evidently, the reason for the increasing rate of profits is that the rate of change in labour productivity (100%) was larger than the rate of change in the number of machines per worker (20%).[5]

Bear in mind that this numerical example can only provide a simple illustration. As was found in Chapter 5, it is not only the increase in labour productivity that influences λ_1 and λ_2, but also the changes in the technical composition or the number of machines per unit of output. How the rate of profits is affected by technical change is better analysed using the linear model of production. This approach also provides for a clearer analysis of prices of production as opposed to labour values.

6.2 The rate of profits in the linear production model

We will now "translate" the rate of profits into the language of the linear production model. As before, it is assumed that there is no fixed capital, in that all capital is circulating. In addition, we assume that capital circulates once during a particular production period. In other words, in all sectors capital is earned back during that period.

It appears from the linear production model that the general rate of profits, irrespective of what the prices of production or the labour values are, is a number that is determined by the necessary physical inputs of the means of production and the necessary consumer goods of the workers involved. This implies that the rate of profits is not based on the prices of production but, conversely, that the prices of production depend on the rate of profits.

This is an obvious deduction if we consider an economy in which only one good is produced, e.g. wheat. If 10 kg of wheat are needed as seed for sowing, which will be combined with 100 hours of labour time to cultivate 100 kg of wheat, and if we take as given that 0.5 kg of wheat is required as necessary consumption, or 50 kg of wheat for 100 working hours, then the price of production p_2 of 1 kg of wheat should obey the following equality:

$$(10\,p_2 + 50\,p_2)\,(1 + r) = 60\,p_2\,(1 + r) = 100\,p_2$$

or, per unit of output:

$$0.60\,p_2\,(1 + r) = p_2$$

p_2 can now be found by solving:

$$[1 - (1 + r)\,0.60]\,p_2 = 0$$

which is only possible if $[1 - (1 + r)\,0.60] = 0$, or $1 = (1 + r)\,0.60$, from which it follows that $(1 + r) = 1.667$ and $r = 66.7\%$. Using this value for the rate of profits, p_2 will be found if a "scaling equation" is added to equation $[1 - (1.667)\,0.60]\,p_2 = 0$. This factor, for instance, states that the total value added is equal to the total labour time spent, or:

$$100\,p_2 - 10\,p_2 = 100 \text{ hours of labour}$$

such that $p_2 = 1.11$ hours. Again, this simple example shows that the rate of profits is determined by the necessary physical inputs (in this case, wheat) of means of production and of consumer goods, and not by the price. Where the rate of profits is higher than 66.7% (e.g. 80%), the production price of wheat, which is used as an input and increased by the profit amount based on 80%, will generate a higher wheat price as an output. Put differently, $r = 0.80$, $1 < (1 + r)\,0.60$[6] means that at the rate of profits of 80%, the value of the outputs is lower than the value of the inputs (gross of profits). Only $r = 66.7\%$ will lead to equality between the value of the inputs, including profits, and the value of the outputs.

We now return to the two sectors in our imaginary economy of Chapter 2, with the system of prices of production:

$$(50\,p_1 + 25\,p_2)\,(1 + r) = 150\,p_1$$
$$(50\,p_1 + 65\,p_2)\,(1 + r) = 100\,p_2$$

or, per unit of output in both sectors:

$$(0.333\,p_1 + 0.167\,p_2)\,(1 + r) = p_1$$
$$(0.500\,p_1 + 0.650\,p_2)\,(1 + r) = p_2$$

The solution was: $r = 21.8\%$, $p_1 = 0.437$ hours of labour time and $p_2 = 1.302$ hours of labour time.

However, there is a second solution for r which fulfils the equality condition between the value of the inputs, including profits, and the value of the outputs, which is $r = 517.6\%$. When this rate of profits is applied, $p_1 = -8.7$ hours (and $p_2 = 9$ hours), which is evidently nonsensical. The reason for this result is that at this rate of profits, the cost of production, inclusive of the profit, in both sectors is so high that it can only be reconciled with a production price that in at least one of the sectors is negative (in our example, sector 1). With $p_2 = 9$, the part of the constant capital in sector 1 that per unit of output is supplied by sector 2, amounts to 1.5 ($= 0.167 \times 9$). Furthermore, if the profit margin at $(1 + r) \times 100\%$ is taken into account, the unit cost of production becomes $[0.333\,p_1 \times (1 + r)] + [1.5 \times (1 + r)] = p_1$. With $r = 517.6\% = 5.176$, this becomes $2.06\,p_1 + 9.26 = p_1$, leading to $p_1 = -8.7$.

In the general case of an economy with n sectors, n different solutions for r can be derived from the inputs structure. Only one r (that with the lowest value) leads to prices of production that are strictly positive. For all other r's, at least one of the prices of production will be negative.[7] Thus, only the lowest r permits price conditions that entail the lowest unit costs, including the "cost of capital" at the applicable rate of profits.

We can now investigate the effect of the various types of technical change that we distinguished in Chapter 5. However, we realise that these lead to very complex inter-sectoral interactions, making it largely impossible to analyse the effect in a transparent, mathematical way. Therefore, we opt to simply calculate the rate of profits of our imaginary economy, based on the assumptions made regarding the changes in the A_{ij}'s and the l_i's.

Some of the results of this exercise are listed in Table 6.1. In each row of the table, another assumption is made about the percentage rate of change in labour productivity, which is combined with a certain rate of change in the A_{ij}'s, as indicated in the second column of the table. Two further situations are considered: (1) the workers' necessary consumption remains constant, and (2) the workers' necessary consumption increases at the same percentage rate as labour productivity.

The various combinations produce the results for the rate of profits in the third and fourth columns of the table. It will be remembered that these are different scenarios of labour-saving technical change.

In Table 6.1 two things are striking. If the necessary consumption of the workers is increasing at the same percentage rate as labour productivity, this has important consequences for the evolution of the rate of profits when the use of means of production per unit of output is increasing. The increase in the components of the necessary consumption at the same rate of change as that of labour productivity will change c and $\mathfrak{l}$ into $c' = (1 + \alpha)\, c$ and $\mathfrak{l}' = \mathfrak{l}/(1 + \alpha)$, such that $c\, \mathfrak{l} = c'\, \mathfrak{l}'$. The necessary consumption per unit of output thus remains the same. It seems reasonable to assume that the workers, through their organisations, have sufficient negotiating power to realise a percentage change in real wages that is equal to that of labour productivity. It is also possible, however, that in the capitalist enterprises, this could lead to the introduction of more labour-saving innovations designed to substitute capital for labour. This is all the more so since, with reference to Table 6.1, a percentage increase in the inputs of means of production per unit of output that is smaller than the percentage increase in labour productivity (e.g. 10% compared to $\alpha \times 100\% = 20\%$) will ultimately cause a substantial fall in the rate of profits (from 21.8% to 14.7%). Each further increase in capital intensity of production, together with the concomitant expulsion of labour, will weaken the position of the workers in the wage negotiations. As David Laibman shows while attempting to arrive at a generalisation of the Okishio theorem (see below) using a constant rate of surplus value (instead of a constant c), it is possible – but not necessarily the case – that the rate of profits will fall after the introduction of labour-saving technological change.

Table 6.1 Rate of profits in an imaginary economy with a 20% increase in labour productivity and alternative changes in the use of the means of production

	Rate of change in A_{ij} (%)	*Rate of profits r (%) with necessary consumption constant*	*Rate of profits r (%) with necessary consumption increasing at $\alpha \times 100\%$*
Initial labour productivity ($\alpha \times 100\% = 0\%$)	0	21.8	21.8
Increase in labour productivity in both sectors ($\alpha \times 100\% = 20\%$)	0	30.2	21.8
Increase in labour productivity in both sectors ($\alpha \times 100\% = 20\%$)	−16.7	46.3	36.1
Increase in labour productivity in both sectors ($\alpha \times 100\% = 20\%$)	+10	22.3	14.7
Increase in labour productivity in sector 1 ($\alpha \times 100\% = 20\%$)	+22	22.1	18.8
Increase in labour productivity in sector 2 ($\alpha \times 100\% = 20\%$)	+38.3	19.5	15.9

The other extreme is when the workers' necessary consumption per working hour, i.e. the real wage rate, remains constant. Using our numerical example, the rate of profits will rise while the extent to which capitalists are prepared to introduce technical change that will yield them super-profits allows a rate of increase in the inputs of means of production per unit of output that is larger than the rate by which the input of labour per unit of output is decreasing (increase in labour productivity). What ultimately happens to the rate of profits? When combined with a constant real wage rate, it could well remain constant, or otherwise increase with the introduction of labour-saving production technology, as the Okishio theorem demonstrates.

It is intuitively clear that, for the economy as a whole, we can assume that technological change is, on average, neutral. This means that the value of the capital stock per unit of value added is unchanged. A given and constant share of profits in total value added will also lead to a constant rate of profits. In the next section, it is shown that with constant real wages, rational decisions by the capitalists about technological innovation in the production process will lead to a rise in the rate of profits, not a fall.

6.3 The relationship between labour-saving technological change and the rate of profits: generalisations in Okishio's theorem

In Chapter 5 it was shown how labour-saving technology can also be capital-using, and that the rational introduction of new production methods by individual capitalists offers scope for increasing both labour productivity and the capital intensity of production. The latter has an immediate connection to the technical and capital composition of capital, and therefore how the rate of profits will evolve in the long run as labour-saving technological innovations are relentlessly introduced.

As early as 1961, Nobuo Okishio (Okishio, 1961) showed that, assuming a constant wage rate (value of labour power), the rational introduction of new labour-saving technology by a capitalist will increase the rate of profits rather than reduce it, as alleged by Marx and the orthodox Marxists.[8]

Okishio starts from a linear two-sector model of production, similar to that used in the previous chapters, but assumes that the consumer goods sector is neither supplying any direct inputs to the capital goods sector, nor to itself. Okishio further assumes, as asserted by Sraffa, that the wages are paid at the end of the production period, such that no variable capital is advanced by the capitalists and the variable capital earns no profit.[9]

The unit price of the means of production is p, while that of the consumer goods is equal to 1, which means that it is the unit in which p is expressed. The wage rate is denoted by w and $\xi = 1 + r$ is the profit factor (r being the rate of profits). The linear system of prices is then:

Sector 1 (means of production): $p = p\,\mathrm{A}_1\,\xi + w\,\mathrm{l}_1$

Sector 2 (consumer goods): $1 = p\,\mathrm{A}_2\,\xi + w\,\mathrm{l}_2$

with A_1 and A_2 the necessary inputs of means of production per unit of output in sector 1 and sector 2, and l_1 and l_2 the necessary labour inputs per unit of output in the respective sectors.

From the equation for sector 2, p can be derived:

$$\frac{1 - w\,l_2}{A_2\,\xi} = p \tag{21}$$

whereas from the equation for sector 1, it follows that:

$$p(1 - A_1\,\xi) = w\,l_1$$

or, taking (21) into account:

$$\frac{(1 - w\,l_2)(1 - A_1\,\xi)}{A_2\,\xi} = w\,l_1$$

This expression can be rewritten as:

$$(1 - w\,l_2)(1 - A_1\,\xi) = w\,l_1\,A_2\,\xi$$

which enables an expression for ξ after some further elaboration:

$$\xi = \frac{1 - w\,l_2}{A_1 + w(l_1A_2 - l_2A_1)} \tag{22}$$

Combining (22) and (21), we find that:

$$p = \frac{A_1 + w(l_1A_2 - l_2\,A_1)}{A_2} \tag{23}$$

Okishio's argument proceeds further in three steps:

1 First, it is determined which technological changes are rational from the perspective of the individual capitalist.

What happens when a new technology is introduced in sector 2, which leads to A_2' and l_2'? For this new technology to be introduced by an individual capitalist, the cost of production of consumer goods after the introduction, including profits based on the factor ξ, has to be lower than the price before the introduction. Since the price before the introduction was equal to 1, it holds that before the introduction:

$$p\,A_2\,\xi + w\,l_2 = 1$$

or:

$$\mathrm{l}_2 = \frac{1}{w} - \frac{p}{w}\, \xi\, A_2$$

Given p and w, many technologies (i.e. combinations of inputs of means of production and of labour) are possible, leading to the same profits factor ξ. All these combinations are situated on the straight line in Figure 6.1. One of the combinations is that when only labour is used, $A_2 = 0$ and $\mathrm{l}_2 = 1/w$. The currently used technology with $A_2 > 0$ and $\mathrm{l}_2 > 0$ is another combination on that line.[10]

From the perspective of the individual capitalist of sector 2, the condition for a rational introduction of a new technology is:

$$p\, A_2'\, \xi + w\, \mathrm{l}_2' < 1$$

or:

$$\mathrm{l}_2' < \frac{1}{w} - \frac{p}{w}\, \xi\, A_2' \qquad (24)$$

If, in the consumer goods sector 2, a given technology using A_2 of means of production and l_2 of labour per unit of output is replaced by a new technology with A_2' and l_2', then the new combination must be situated under the straight line in Figure 6.1.

2 Next, it is investigated which technologies (combinations of A_2^* and l_2^*) will lead to a lower general rate of profits.

For r^* to be lower after the introduction than r before the introduction, $\xi^* < \xi$. Since it is assumed that w is constant and also that A_1 and a_1 remain unchanged (we are limiting ourselves to analysing the introduction of new technology in sector 2), this implies that:

$$\xi^* = \frac{1 - w\, \mathrm{l}_2^*}{A_1 + w(\mathrm{l}_1\, A_2^* - \mathrm{l}_2^*\, A_1)} < \xi$$

Rearrangement gives:

$$1 - w\, \mathrm{l}_2^* < \xi\, A_1 + w\, \xi\, \mathrm{l}_1\, A_2^* - w\, \xi\, \mathrm{l}_2^*\, A_1$$

or:

$$1 - \xi\, A_1 - w\, \xi\, \mathrm{l}_1\, A_2^* < w\, \mathrm{l}_2^* - w\, \xi\, \mathrm{l}_2^*\, A_1$$

or else:

$$1 - \xi\, A_1 - w\, \xi\, \mathrm{l}_1\, A_2^* < w\, \mathrm{l}_2^*(1 - \xi\, A_1),$$

such that:

$$\frac{1 - \xi A_1}{w(1 - \xi A_1)} - \frac{\xi l_1 A_2^*}{(1 - \xi A_1)} < l_2^*$$

This expression can be further simplified to:

$$\frac{1}{w} - \frac{\xi l_1 A_2^*}{1 - \xi A_1} < l_2^*$$

or, taking into account that $p = p A_1 \xi + w l_1$, to:[11]

$$\frac{1}{w} - \frac{p}{w} \xi A_2^* < l_2^* \tag{25}$$

This equality indicates that each technology with a combination of A_2^* and l_2^* that is not fulfilling this inequality will lead to a lower rate of profits than before.

3 Finally, it is shown that a new technology that is rational for the capitalists will raise the rate of profits.

Comparing (24) and (25), it can be seen that no single combination of A_2' and l_2' that satisfies (24) can also satisfy (25). A choice of technology that is rational for the individual capitalist is therefore a technology that will raise the rate of profits. Each technology that results in a fall in the general rate of profits is a combination of A_2 and l_2, which is outside the shaded area in Figure 6.1. Therefore, the capitalists' choices of technology that are rational from a microeconomic perspective will not lead to a fall in the rate of profits at the macroeconomic level.

An identical argument applies when a rational technology choice in the sector of the means of production is considered.

The Okishio theorem thus proves that, given the assumptions of the linear production model and the assumption of a constant wage rate, Marx's tendential fall in the rate of profits is impossible.

The Okishio theorem and the assumptions made are further clarified by a numerical example, which goes back to the labour value system of Chapter 2:

$$\frac{50}{150}\lambda_1 + \frac{50}{150} = \lambda_1$$

$$\frac{50}{100}\lambda_1 + \frac{40}{100}\lambda_2 + \frac{50}{100} = \lambda_2$$

In our exposé of the Okishio theorem, the consumer goods sector does not supply itself. Therefore, the following equivalences hold:[12]

$$A_1 = \frac{50}{150}$$

$$A_2 = \frac{50}{100}$$

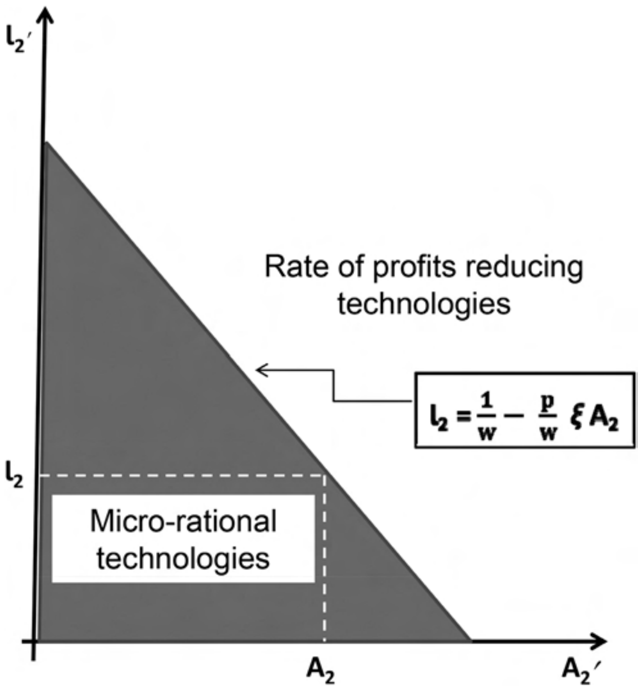

Figure 6.1 The Okishio theorem

$$\mathrm{l}_1 = \frac{50}{150}$$

$$\mathrm{l}_2 = \frac{50}{100}$$

$p = p_1$, the prices of the means of production

$1 = p_2$, the prices of the consumer goods

$\xi = 1 + r$, the profits factor

$w = 0.5$ (kg of wheat)

Moreover, it will be remembered that, according to Okishio's reasoning, no rate of profits applies to the wages (they are not advanced but rather paid at the end of the production period from the sales proceeds).

Okishio's price system for our imaginary economy is:

Sector 1 (means of production): $p = p\ \mathrm{A}_1\ \xi + w\ \mathrm{l}_1 = \dfrac{50}{150}\ p\ (1 + r) + \dfrac{50}{150}$ $0.5 = 0.333\ p\ (1 + r) + 0.167$

Sector 2 (consumer goods): $1 = p\ \mathrm{A}_2\ \xi + w\ \mathrm{l}_2 = \dfrac{50}{100}\ p\ (1 + r) + \dfrac{50}{100}$ $0.5 = 0.500\ p\ (1 + r) + 0.250$

The solution of this system is:

$$\xi = \frac{1 - w\,l_2}{A_1 + w\left(l_1\,A_2 - l_2\,A_1\right)} = \frac{1-0.250}{0.333+0.5\left[\left(0.167\times0.5\right)-\left(0.250\times0.333\right)\right]}$$

$$= \frac{0.750}{0.333} = 2.25$$

or: $r = 1.25 = 125\%$

and:

$$p = \frac{A_1 + w\left(l_1 A_2 - l_2 A_1\right)}{A_2} = \frac{0.333+0.5\left[\left(0.167\times0.5\right)-\left(0.250\times0.333\right)\right]}{0.500}$$

$$= \frac{0.333}{0.500} = 0.667$$

After putting the numerical values of *p*, *w* and ξ in (24), we get:

$$l_2' < \frac{1}{0.5} - \frac{0.667}{0.5}\ 2.25\ A_2'$$

or: $l_2' < 2 - 3\ A_2'$

or else: $l_2' + 3\ A_2' < 2$

Each new technology that entails a combination of l_2' and A_2' that fulfils this condition conforms to the individual capitalist's rationality. In order to confirm Marx's thesis that technological change will lead to a lower general rate of profits, it should also fulfil condition (25), which in our numerical example means:

$$\frac{1}{0.5} - \frac{0.667}{0.5}\ 2.25\ A_2^* < l_2^*$$

or: $2 - 3\ A_2^* < l_2^*$

or else: $2 < l_2^* + 3\ A_2^*$

Combining both conditions:

$$l_2' + 3\ A_2' < 2 < l_2^* + 3\ A_2^*$$

this inequality shows that *not a single combination* of l_2' and A_2' exists that is both rational *and* leads to a fall in the general rate of profits.

In contrast to Okishio, David Laibman uses a model in which the effect, on the rate of profits, of changes in the balance of power in the class struggle does not lead to a constant real wage rate but rather to a constant rate of exploitation. This means that in Laibman's analysis the real wage rate changes at the same rate as labour productivity, but also that the rate of profits fully depends on what happens with the composition of capital. In that case, technical changes that are introduced by the individual capitalists can (but *do not necessarily*, as we showed in Chapter 5) lead to an increase in the value composition of capital and to a fall in the rate of profits. Specific and complex conditions have to be fulfilled (Laibman, 1992, p. 123). Laibman shows that certain new methods of production will be rational at a microeconomic level (and are thus situated under the straight line in Figure 6.1) and yield a super-profit for the introducing capitalist, and will also lead to a higher general rate of profits. When the rate of exploitation is constant, there are techniques of production that are increasing the rate of profits, as well as techniques of production that are decreasing the rate of profits, located in the area of the micro-rational technologies. Given the assumptions made, Okishio's theorem is not false, but the assumptions are too restrictive and need to be adapted.

However, Laibman (1997, p. 32) has advanced another argument against the law of the tendential fall in the rate of profits, pointing out that if capitalists switch from production technique A to the new technique B, anticipating a higher rate of profits, they would immediately switch back from B to A, and thus restore the original rate of profits, if the first switch in fact lowered the rate of profits.

Okishio's theorem has been subjected to sharp, although often unfounded, criticism. Whereas Laibman assesses Okishio's theorem in terms of its own logic and nuances it by building his argument on the linear model of production, many other Marxist economists have attacked the theorem and the underlying model. One of the most notorious opponents is Andrew Kliman, who rejects the linear model and advocates a "temporal single-system" approach to the theory of value (see Kliman and McGlone, 1999; Freeman and Carchedi, 2000). In the linear model approach, the prices of the inputs and the outputs are the same. Before the introduction of an innovation, they are equal to Vector $\mathbf{p}$, and after such introduction, to $\mathbf{p'}$, with $\mathbf{p} = (1 + r)\, \mathbf{p}\, \mathbf{B}$ and $\mathbf{p'} = (1 + r')\, \mathbf{p'}\, \mathbf{B'}$. In Kliman's reasoning, the prices do not reach their final "equilibrium values" because of new technological innovations occurring during the adjustment process:

> What happens (. . .) if the innovation is followed by subsequent innovations before prices have time to approach the stationary state? Because labor-saving innovation reduces commodities' prices (given a constant monetary expression of value), continuous labor-saving innovation leads to continuous reductions in prices (on average). And if prices are continually falling, output prices are always lower than input prices. The general rate of profit will thus be lower than r* [the equilibrium rate of profits in a linear production model], and labor-saving technical changes may thus lower the profit rate, even if uniform profitability is assumed.
>
> (Kliman, 1997, p. 46)

Surely, the attentive reader will have witnessed this criticism with mounting astonishment. After all, the rise of the organic composition of capital – the cornerstone of Marx's theory of the secular movement in the rate of profits – has disappeared completely. According to Kliman, it is purely the labour-saving character of technological innovations (i.e. detached from the issue of whether or not it is capital-using) that explains the falling rate of profits, via its dynamic effect on prices. Even without any innovation and with price deflation only, the rate of profits will fall, according to the "temporal single-system" approach.[13]

6.4 The rate of profits in the long run: some statistical data

In this book, we have opted for a theoretical analysis of Marx's economic views. Nevertheless, we will briefly look at some statistical material that is relevant for an assessment of the theory of the tendential fall in the rate of profits. A multitude of statistical studies, analysing the rate of profits in the long run, have been published. Already in 1977 we came to the conclusion that these studies showed conflicting results (Cuyvers, 1977, 1978). In the economic literature that is well-disposed towards Marx, starting with Gillman (1956), consideration is often given to unproductive activities. However, we argued in Chapter 4 that which activities should be considered as productive and which not, remains inconclusive, to say the least, and shrouds Marx's labour theory of value in more inconsistencies. Moreover, the statistical analysis of the long-run macroeconomic income share of profits (or the profits–wages ratio) and even more so that of the capital–output ratio (and hence the value composition of capital), has proved to be a difficult exercise. For instance, it is often not clear how the incomes of independent workers and professions are dealt with, while the estimation of the value of the capital stock is a perilous affair which crucially depends on a number of simplifying assumptions.

An extensive statistical research study on the secular behaviour of the profits share and the average rate of profits in the United Kingdom was conducted by Matthews, Feinstein and Odling-Smee (1982). Their analysis covered the period 1856–1973. Although we will deal with the profits share in Chapter 7, it is appropriate to mention here that the authors found evidence of a fall in the profits share in Gross National Product in the period 1914–1918 (although this was derived from the calculated 1913 profits share compared with that of 1919, there being no data for the intervening years) and a declining trend in the post-Second World War period, accompanied by an increase in the share of wages and salaries in these periods (Matthews *et al.*, 1982, Figure 6.1 and pp. 177–178). The average (gross) rate of profits behaved similarly, although it is remarkable that the gross capital–output ratio – while having increased between 1938 and 1947[14] – apparently showed a decreasing trend from 1947 to 1967, only to rise again in the period up to 1973, which marked the end of the authors' investigation (Matthews *et al.*, 1982, Figure 6.3 and p. 184). The decline in the net rate of profits was, however, more pronounced in the period 1947–1973 (Matthews *et al.*, 1982, Figure 6.4 and pp. 185ff.), which we think is a less than reliable result considering the difficulty in determining fixed capital depreciation and in isolating it from gross profits.

One the most recent studies into the evolution of the general rate of profits in the long run was conducted by Basu and Manolakos (2013), in which a thorough and "state of the art" econometric analysis was carried out for the USA during the period 1948–2007. The authors used the time series created by Gérard Duménil and Dominique Lévy from CEPREMAP in Paris, which is considered to be among the best and most relevant statistical time series.[15] Figure 6.2 depicts the general rate of profits in the USA during the period 1869–2009.

First of all, Basu and Manolakos found that the general rate of profits was non-stationary during the period 1869–2007 and followed a long wave pattern. It showed a declining trend from the mid-1860s until the mid-1910s, after which it showed an upward trend until the early 1960s and then another decline until 2007 (Basu and Manolakos, 2013, pp. 81–82). Next, the authors tested the hypothesis of a falling rate of profits during the period 1948–2007, taking into account the counteracting tendencies. They found that, when corrected for the counteracting tendencies, the general rate of profits in the United States after 1948 declined by an average annual rate of 0.2%. Basu and Manolakos concluded that this result:

> draws attention to the existence of possible mechanisms, like the inexorable mechanization under capitalist production or the long-run labor-saving bias of technological change, that drive the rate of profit to *conditionally* decline over time. When the counteracting tendencies are strong enough to nullify or even reverse this mechanism, the rate of profit might display an upward movement (as in the period 1982–2000).
>
> (Basu and Manolakos, 2013, p. 93)

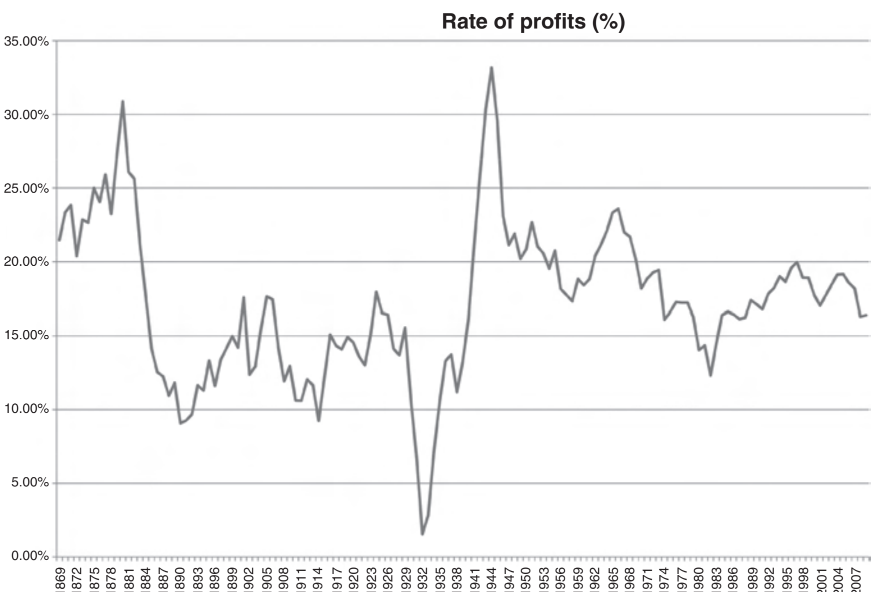

Figure 6.2 The evolution of the general rate of profits in the USA, 1869–2009

Source: www.jourdan.ens.fr/levy/uslt4x.txt (accessed on 30 October 2013)

This interesting result is not entirely convincing, since the authors previously pointed out that the rate of profits in the USA during the 1869–2007 period showed a rising trend until the early 1960s. Moreover, they failed to consider some important counteracting tendencies, among which was the impact of American capital exports to countries where the rate of profits was higher. It could well be that this counteracting tendency, mentioned by Marx, which was still predominant in the period 1947–1970, weakened afterwards with the rise of Japan and the Asian "Tigers".

Worth noting for the USA, as well as the OECD countries, is the comprehensive study by Hill (1979) and, based on this, the analysis by Hargreaves Heap (1980). According to this study, the rate of profits in the manufacturing industries during the period 1955–1973 showed a declining trend at an annual rate of −0.13% in the USA, −0.86% in Japan, −4.03% in Germany, −1.99% in Italy, −1.39% in Sweden and −4.37% in the UK. The decline intensified during the 1970s, dominated mainly by the decline in the share of profits in value added (Hargreaves Heap, 1980, p. 68). However, the observed downward trend was heavily dependent on assumptions made about the duration of the economic life of the capital goods in question.

Recent pioneering statistical research into the long-run behaviour of the rate of profits in the industrial countries and the world at large is provided by Thomas Piketty, author of the controversial bestseller, *Capital in the 21st Century* (2014). The time series, collected by Piketty and an international research team, is based on national wealth and income data. Following a theoretical neoclassical growth approach,[16] Piketty also forecasts the future expected average savings ratio and the rate of growth of the economy. As to the past, he estimates that the rate of return to capital in the world rose from 4.5% to 5.1% in the period 1700–1820, but decreased slightly to 5% in the period 1820–1913. Some decades later, in the period 1950–2012, the rate of return to capital increased to about 5.3% (Piketty, 2014, p. 354, Figure 10.9).[17]

* * *

We do not intend to bother the reader with a long review of the many studies of the long-run behaviour of the rate of profits. Since we have shown that the determining factor par excellence of a hypothetical tendentially falling rate of profits is the rise in the value composition of capital, we will limit ourselves to the "stylised facts".

In their pioneering 1961 study, Klein and Kosobud (1961) showed that the capital–output ratio in the USA declined during the period 1900–1953. The capital–output ratio is not identical to the value composition of capital but, together with the share of wages in national income, it determines the general rate of profits of an economy.[18] Over the years, the Klein-Kosobud series and calculations have been analysed again, using the newest econometric techniques and tests. These have indicated that the capital–output ratio has followed a non-stationary trend and that, for the period investigated, no evidence has been found of a long-term relationship (Mills, 2009, pp. 19, 24). This conclusion is, however, disputed by Madsen and Smyth (2008) and Madsen, Mishra and Smyth (2012), who take into account structural breaks and find that over a period of 135 years, the capital–output ratio in 16 OECD countries has been constant and stationary.

It is interesting to look at the evolution of the value composition of capital, again based on the time series of Duménil and Lévy.[19] Figure 6.3 shows the variation of C/V in the USA during the period 1869–2009.

A visual inspection of Figure 6.3 reveals that C/V in the USA shows a rising trend between 1870 and 1914, after which it declines until the outbreak of the economic crisis in 1929. As to the era after the Second World War, it appears that the C/V ratio tends to rise between 1947 and 1980. Over the next 20 years it declines, but then starts to rise again from the turn of the century. It is not permissible to conclude from Figure 6.3 that the value composition of capital in the USA has increased during the last 140 years.

More recent research than that of Klein and Kosobud (1961), in which the capital–output ratio of other countries was also analysed, provides a nuanced picture. Romer (1989) reports that during the periods 1870–1913, 1913–1950 and 1950–1979, the rates of growth of the capital stock and output were remarkably similar. D'Adda and Scorcu (2003) find, in contrast, a slightly decreasing capital–output ratio for the periods 1890–1992 and 1935–1991 in the United States and Germany, respectively, but an increasing ratio in the United Kingdom, Italy, Japan, the Netherlands and France. The average annual growth rates of the capital–output ratios are, however, nothing compared to the much stronger growth rates of income per capita (a measure of average labour productivity). Yet in a study by two World Bank economists, rising capital–output ratios were reported for most countries. For the USA and Canada, they found a constant capital–output ratio during the period 1960–1990, but a rising ratio for Japan, Germany, Great Britain, Italy and France (Nehru and Dhareshwar, 1993, pp. 52–53).[20]

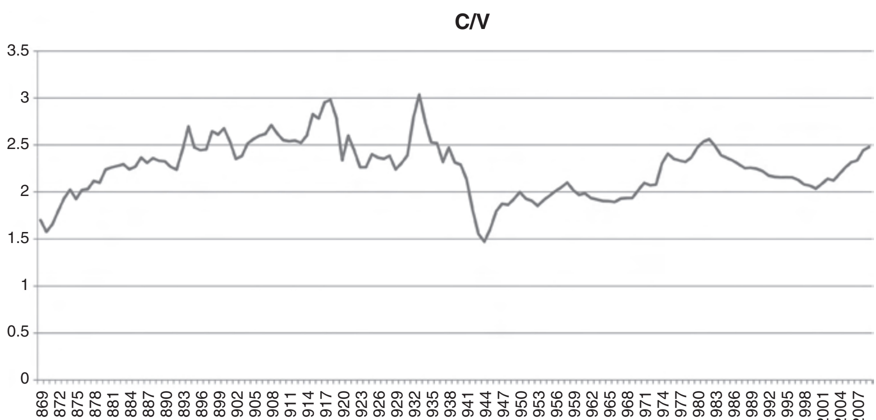

Figure 6.3 Evolution of the value composition of capital in the USA, 1869–2009

Source: Calculated using the data in www.jourdan.ens.fr/levy/uslt4x.txt, accessed 30 October 2013

These results, in turn, are contradicted by Hargreaves Heap (1980, p. 74) who, for the period 1955–1973, provides evidence of a rising capital–output ratio in the UK and Germany, a fluctuating ratio in Japan and the USA, and a decreasing ratio in Italy. These are related to the available labour reserves and the technological opportunities, which would account for the rapid increase in accumulation in Japan, Germany and Italy until the mid-1960s. Since the early 1960s, soaring raw material prices were also responsible for the rise in the capital–output ratio in the USA, the UK, Germany, Japan and Italy (Hargreaves Heap, 1980, pp. 77–78). There was a further increase in raw material prices during the 1970s.

In this respect it is also interesting to look at the series that Thomas Piketty constructed of the wealth–income ratio in the world, which should be a reasonable approximation of the capital–output ratio. During the period 1870–1910, this ratio increased from around 4.5 to 5, only to decline to about 2.7 in 1950.[21] During the post-war era until 2010, the ratio increased systematically (although not always at the same rate) to 4.3 (Piketty, 2014, Figures 10.9 and 12.4).[22]

As mentioned, the calculated size of the capital stock depends on the assumptions made about the economic life of various types of capital goods and which components are taken into account. Houses are often added to the fixed capital stock of the producing companies, a specific depreciation of investment is assumed, the prices of the various components of the fixed capital and of output are held constant, etc. All these assumptions and rules of thumb greatly complicate a comparison of the main findings in the literature.

In an attempt to avoid these assumptions, Figure 6.4 shows the evolution of the ICOR, the incremental capital–output ratio in the world. The ICOR is the ratio between the *increase* in the capital stock ΔK (in Figure 6.4, gross fixed capital formation) and the *increase* in national income ΔY (here, at the global level), or ΔK/ΔY. The advantage of the ICOR is that we avoid calculating the capital stock. And, in spite of the ICOR not being the capital–output ratio, a declining trend in the ICOR indicates that the capital–output ratio will show a similar trend. Again, we are confronted by an undulatory movement during the period. In the period 1961–1980, there seems to be a declining trend, after which the trend is unclear due to marked instability and some spectacular drops in the ICOR in the period 1997–2001.

All the above is, we think, sufficient evidence that Marx's law of the tendential fall in the rate of profits and the alleged underlying increasing capital intensity of production, is inconclusive, if not plainly false. From the available statistical data, there appears to be more reason to accept as a stylised fact that in the long run capital intensity is more or less constant.

* * *

Another approach to investigating statistically how the rate of profits secularly evolved is to follow the return on equity (ROE). However, also based on expectations, the ROE is what investors are greatly interested in and what also, to some extent, determines their investment behaviour.

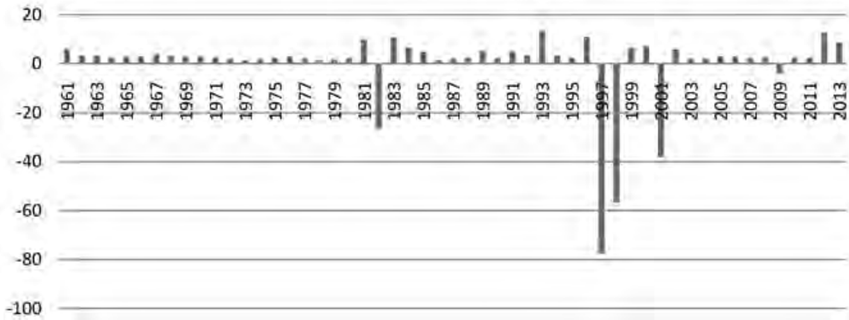

Figure 6.4 Evolution of the incremental capital–output ratio in the world, 1961–2013

Source: http://data.worldbank.org/topic/economy-and-growth, Gross fixed capital formation (current US$) as a percentage of the change in GDP (current US$), accessed 9 June 2015

In a recent first study (Buelens, Cuyvers, Deloof and de Smedt, 2016), the ROE was calculated for two samples of Belgian companies, listed on the Brussels Stock Exchange, for five periods during the twentieth century,[23] using the extensive historical database of SCOB *(Studiecentrum voor Onderneming en Beurs*) at the University of Antwerp. The first sample comprises the ten largest listed companies in Belgium and the second comprises the largest companies in the various industries, including the financial and transportation sectors. The results of these calculations are shown in Table 6.2.

Since the data from the samples are not normally distributed, the median ROE is of most interest to us. The median ROE is rather volatile in the sample of the ten largest listed companies, but much less so in the sample of the largest companies per industry. In any case, in the course of the twentieth century, there is no observable decline in the ROE.[24]

In another recent research study (Pierenkemper, de Smedt, Buelens, Cuyvers and Deloof, 2016), the ROE of the largest companies in the European old industries was investigated. The relevant results for our purposes are shown in Table 6.3.

Table 6.2 Return on equity of the largest companies in Belgium, 1911–2000

Period	*ROE (ten largest companies) (%)*		*ROE (largest companies in the industrial sectors) (%)*	
	Average	*Median*	*Average*	*Median*
1911–1913	8.71	8.55	17.95	12.61
1927–1929	18.45	17.54	22.10	16.42
1954–1956	17.85	11.31	15.81	12.05
1970–1972	10.25	8.53	9.07	7.98
1998–2000	18.02	17.61	20.55	14.22

Source: Buelens *et al*. (2016); reprinted with the permission of the publisher

Table 6.3 Return on equity in Europe's "old industries", 1911–2000

Period	*"Return on equity" (ROE) (%)*	
	Average	*Median*
1911–1913	11.12	9.12
1927–1929	10.87	8.86
1954–1956	10.31	7.43
1970–1972	4.15	4.99
1998–2000	11.69	10.31

Source: Pierenkemper et al. (2016); reprinted with the permission of the publisher

The "old industries" comprise the food, textile, iron and steel, and iron-ore and coal industries.

The ROE in the European old industries has remained stable across the subperiods of the twentieth century, with an exception being 1970–1972.

Evidently it is possible that the large companies are maintaining a dominant position in their respective industries, which allows them to realise a higher ROE than smaller companies. On the other hand, "old industries" might experience lower than average, as well as declining profitability. In fact, compared with Table 6.2 this seems to be the case, but the remarkable stability of the ROE is established once more. It can be concluded that there is evidence of stability in the average profitability of the large companies. While this is not sufficient to statistically disprove Marx's law of the falling rate of profits, it at least provides additional "circumstantial evidence".

Notes

1 The statistical determination of K is far from straightforward. Often K is calculated by adding up the investments in fixed capital over an extended period of time, but how far back into the past one should go in order to do this calculation is the subject of debate. Another complication is how to treat the used-up capital when taking into account depreciation allowances, which are often influenced by taxation considerations.
2 In contrast to our approach in the previous chapters, here we follow Marx's argument which is cast in monetary values. This, in turn, assumes that a bridge is created between labour value in labour time and that in monetary units – hence, the relevance of the present-day Marxist theory of money (which will be discussed in Chapter 8) and the theory of the "monetary equivalent of labour time", which has been the subject of much debate over the past 20 years. For references in this regard, see Chapter 2.
3 In modern macroeconomic terminology, this is the net national product (NNP).
4 Importantly, according to this reasoning, M/V is determined by the balance of power between workers and capitalists in the wage negotiation process. Other factors, such as the degree of monopoly, also play their role. In this regard, see Chapter 7.
5 Before the innovations, $A_1/L = 3$; after the innovations, $A_1/L = 3.6$, i.e. there has been an increase of 20%.
6 Since at $r = 66.7\%$, $1 = (1 + r)\,0.60$, a higher r (e.g. 80%) leads to $1 < (1 + r)\,0.60$.
7 In the matrix algebra of linear systems of equations, this is known as the Perron-Frobenius theorem (see Meyer, 2000, p. 667).

8 We are following Laibman's version of Okishio's theorem in Laibman (1992, pp. 104–109). For an alternative proof in a single-good capitalist economy, see Laibman (1997, pp. 36–38).

9 In the previous chapters, we assumed that the consumer goods sector supplies inputs to itself and that wages are advanced. Okishio's assumptions make the mathematical proof easier.

10 This also highlights the importance of the assumption of a given and constant *w*. Without this assumption, the straight line in Figure 6.1 would shift when *w* changes.

11 We can also write $p = p\,A_1\,\xi + w\,l_1$ as $p\,(1 - A_1\,\xi) = w\,l_1$. This can, in turn, be transformed into $(1 - A_1\,\xi)/\xi = w\,l_1/p\,\xi$. As a result, the second term in our inequality $\xi\,l_1\,A_2'/(1 - A_1\,\xi)$ becomes $(p\,\xi/w\,l_1)\,(l_1\,A_2')$, or, after deleting l_1, $(p\,\xi/w)\,A_2'$.

12 Based on Okishio's assumptions, there is no consumer goods input in the production of consumer goods.

13 In terms of the approach adopted in this book, price deflation is excluded since values and prices are measured in labour time and no "monetary expression of labour time" is introduced. For a thorough, critical assessment of the "temporal single-system" approach, including its alleged refutation of the Okishio theorem, see, for example, Veneziani (2005). See also Laibman (1997, pp. 77–79) who criticises the non-equilibrium argument of the "temporal single-system" approach for confusing "ontological equilibrium" and "methodological equilibrium". For further refutations of Marxists' theoretical arguments against the Okishio theorem, including these derived from a falling maximal rate of profits, the reader is referred to Roemer (1981).

14 Here, too, the time series was interrupted between 1938 and 1947.

15 These time series data can be consulted at www.jourdan.ens.fr/levy/uslt4x.txt (accessed on 30 October 2013).

16 The central building block of the neo-classical model of economic growth is the production function, a concept that is at odds with both the Marxist and the post-Keynesian models. In a neo-classical model, the factor of production "capital" generates a marginal product which is equal to the long-run average rate of profits. As the capital stock rises, the marginal product of capital (and thus the rate of profits) will *ceteris paribus* fall.

17 However, for the UK, he notes a decline between 1940 and 1990 (although a relatively stable period between 1770 and 1940) and for France, a decline between 1950 and 2010 (and more volatility than in the UK during the period 1820–1920, though no specific trend). See Piketty (2014, p. 202, Graphs 6.3 and 6.4).

18 Since the rate of profits $r = P/K$, both numerator and denominator can be divided by the value of national income Y, which then indicates that *r* is the ratio between the share of profits in national income P/Y and the capital–output ratio K/Y.

19 See above: www.jourdan.ens.fr/levy/uslt4x.txt. Strictly speaking, the ratio depicted is not the value composition of capital but the ratio of the net capital stock in current dollars, on the one hand, and the product of the number of working hours (in millions of hours) and the hourly wage rate in current dollars, on the other.

20 This increase is attributed to the capital destruction during the Second World War and the reconstruction of infrastructure and industry in the subsequent period.

21 The reason for this decline was the capital depreciation during the Great Depression of the 1930s and the capital destruction during the Second World War.

22 See also Piketty and Zucman (2014). For a comparison between the USA and Europe, see Piketty (2014, Chapter 5) and Piketty and Saez (2014, Figure 3). It is important to stress that the increase in the wealth–income ratio since the late 1950s is largely due to the rise of the housing component in total wealth, not capital used in the production processes.

23 1911–1913; 1927–1929; 1954–1956; 1970–1972; and 1998–2000.

24 The "holding return" ratio is more volatile but takes into account the evolution of the stock prices and therefore the investors' expectations.

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7 Long-term developments – changes in the rate of surplus value, the distribution of income and the class struggle

It will be remembered from Chapter 2 that a positive rate of exploitation, according to Marx's definition, is a necessary condition for a positive rate of profits (Morishima, 1973, pp. 63–68). It was also argued that to the extent that workers aim to achieve a uniform rate of exploitation in the various professions and industries, long-term prices will evolve which will tend to coincide with, if not become identical to, labour values.

In Chapter 6 we found that the so-called tendential fall in the general rate of profits not only depends on the alleged increase in the composition of capital but it is counteracted by an increase in the rate of surplus value σ – a factor that we had, for the sake of the argument, kept constant. In this chapter we will examine the determinants and dynamics of Marx's rate of exploitation/rate of surplus value, thereby making good our deliberate oversight in Chapter 6.

Yet we established in Chapter 5 that the rate of surplus value will increase with the introduction of technical change, which will reduce the direct and indirect costs required to produce the necessary consumption of the workers. Marx calls this an increase in relative surplus value, in contrast to an increase in absolute surplus value. We will illustrate this first.

Next we will consider the ways in which the workers in capitalist enterprises can undo an increase in the rate of surplus value. We will show that this largely depends on workers' bargaining power and the feasibility of counteracting growing exploitation through demands for wage increases. We will also discuss how the capitalists' reaction to changes in the bargaining power of the workers can generate an economic cycle, in line with Richard M. Goodwin's analysis. Having arrived at that point, we will investigate the relationship between the pricing determined by individual capitalist enterprises (i.e. at the microeconomic level) and the rate of surplus value. This will also shed light on the neo-Marxist and post-Keynesian contributions.

In addition, we will review Michał Kalecki's theory of the "political business cycle", as well as the theory of the "profits squeeze" which attempts to explain the origin of the economic crisis of the 1970s and early 1980s. Finally, we will provide and discuss some statistical data pertaining to the United States and the United Kingdom with a view to illustrating how the rate of surplus value has evolved in the long run.

7.1 Absolute and relative surplus value from Marx's standpoint

As before, we consider our imaginary economy:

$$\begin{array}{llll} 50\,\text{kg iron} + & & 50 \text{ hours of labour} \rightarrow & 150\,\text{kg iron} \\ \underline{50\,\text{kg iron}} + & \underline{40\,\text{kg wheat}} + & 50 \text{ hours of labour} \rightarrow & 100\,\text{kg wheat} \\ 100\,\text{kg iron} & 40\,\text{kg wheat} & & \end{array}$$

The "net product" consists of 50 kg of iron (150 kg output – 100 kg inputs) and 60 kg of wheat (100 kg output – 40 kg inputs). As before, we assume that half a kilo of wheat is paid to each worker per working hour, i.e. 25 kg of wheat to the workers of sector 1 and 25 kg of wheat to the workers of sector 2. *Total inputs* (means of production and necessary consumption together) are then as follows:

$$50\,\text{kg iron} + 25\,\text{kg wheat} \rightarrow 150\,\text{kg iron}$$

$$50\,\text{kg iron} + 40\,\text{kg wheat} + 25\,\text{kg wheat} \rightarrow 100\,\text{kg wheat}$$

It will be remembered from Chapter 2 that the respective labour values are: $\lambda_1 = 0.5$ hours of labour time (= 1/2) and $\lambda_2 = 1.25$ hours of labour time (= 5/4), from which the sectoral equalities followed:

$$50\,\lambda_1 + 25\,\lambda_2 + M_1 = 150\,\lambda_1$$

$$50\,\lambda_1 + 65\,\lambda_2 + M_2 = 100\,\lambda_2$$

or, expressed in labour values:

$$25\,(C_1) + 31.25\,(V_1) + M_1 = 75\,(W_1)$$

$$25 + 50\,(C_2) + 31.25\,(V_2) + M_2 = 125\,(W_2)$$

Therefore, the surplus value of sector 1 and sector 2 equals:

$$M_1 = 18.75$$

$$M_2 = 18.75$$

and in both sectors, $M_1/V_1 = M_2/V_2 = \sigma = 0.6$.

The rates of surplus value of both sectors are thus identical. In each sector, 50 hours of labour are added to the constant capital, of which 31.25 hours are paid out to the workers and 18.75 hours to the capitalists. The share of profits (surplus value) in value added is 18.75/50 = 37.5%. Marx calls V_1 and V_2 the necessary labour time spent in sectors 1 and 2, respectively, and M_1 and M_2 the surplus labour time. We also see that the surplus product consists of: (150 – 100 =) 50 kg of iron and (100 – 90 =) 10 kg of wheat, of which the value equals $50\,\lambda_1 + 10\,\lambda_2 = 25 + 12.5 = 37.5 = M_1 + M_2$, the total surplus value.

To satisfy their greed for profits, the capitalists, so Marx explains, can use two methods to extract surplus value: the method of absolute surplus value creation and/or the method of relative surplus value creation.

Suppose that in both sectors the capitalists succeed in making the workers work for 60 hours instead of 50 hours, while their wages are still equal to 31.25 hours. Since 60 hours of labour time is spent, proportionally more grain and iron inputs are necessary in the two sectors, such that:

$$\begin{array}{llll} 60\,\text{kg iron} & & + 60 \text{ hours of labour} \rightarrow & 180\,\text{kg iron} \\ \underline{60\,\text{kg iron}} & + \underline{48\,\text{kg wheat}} & + 60 \text{ hours of labour} \rightarrow & 120\,\text{kg wheat} \\ 120\,\text{kg iron} & 48\,\text{kg wheat} & & \end{array}$$

After payment of the wages at 25 kg of wheat, it implies that:

$$60\,\text{kg iron} + 25\,\text{kg wheat} \rightarrow 180\,\text{kg iron}$$

$$60\,\text{kg iron} + 48\,\text{kg wheat} + 25\,\text{kg wheat} \rightarrow 120\,\text{kg wheat}$$

The surplus product is now: (180 − 120 =) 60 kg of iron and (120 − 98 =) 22 kg of wheat. With the labour values of iron and wheat remaining the same, $60\ \lambda_1 + 22\ \lambda_2 = 30 + 27.5 = 57.5 = M_1' + M_2'$, the new total surplus value.

Per sector, the total surplus value is as follows:

$$60\ \lambda_1 + 25\ \lambda_2 + M_1' = 180\ \lambda_1$$

$$60\ \lambda_1 + 73\ \lambda_2 + M_2' = 120\ \lambda_2$$

or:

$$30\ (C_1') + 31.25\ (V_1') + M_1' = 90\ (W_1')$$

$$30{+}60\ (C_2') + 31.25\ (V_2') + M_2' = 150\ (W_2')$$

from which it follows that:

$$M_1' = 28.75$$

$$M_2' = 28.75$$

and that $M_1' + M_2' = 57.5$, which is the value of the surplus product. The surplus value has increased in absolute value. *Absolute surplus value* has been created, while the variable capital has remained the same. By making the workers work 10 hours longer for the same wage, the rate of surplus value has increased in both sectors to reach $M_1'/V_1' = M_2'/V_2' = 28.75/31.25 = \sigma = 0.92$.

Also, the total value added has increased from 100 hours to 120 hours. The share of profits in value added is now: 28.75/60 = 47.9% (previously 37.5%).

What if the workers in both sectors still spend 50 hours working, but labour productivity increases by 20%? Assuming that in both sectors the quantity of means of production per unit of output remains constant, this means that a new method of organising the production process is applied, which increases labour productivity without changing the proportional use of means of production. The output equation is then:

$$\begin{array}{l} 60\,\text{kg iron} \qquad\qquad\qquad\quad + 50 \text{ hours of labour} \rightarrow 180\,\text{kg iron} \\ \underline{60\,\text{kg iron}} + \underline{48\,\text{kg wheat}} + 50 \text{ hours of labour} \rightarrow 120\,\text{kg wheat} \\ 120\,\text{kg iron} \;\; 48\,\text{kg wheat} \end{array}$$

or, if wages at 0.5 kg of wheat per working hour are taken into account:

$$\begin{array}{l} 60\,\text{kg iron} + 25\,\text{kg wheat} \rightarrow 180\,\text{kg iron} \\ 60\,\text{kg iron} + 48\,\text{kg wheat} + 25\,\text{kg wheat} \rightarrow 120\,\text{kg wheat} \end{array}$$

The surplus product consists of (180 – 120 =) 60 kg of iron and (120 – 98 =) 22 kg of wheat. Assuming the labour values of iron and wheat are unchanged,[1] we find that $60\,\lambda_1 + 22\,\lambda_2 = 30 + 27.5 = 57.5 = M_1' + M_2'$, the new total surplus value.

The surplus value in the sectors can be derived as M_1' and M_2', based on the equalities:

$$\begin{array}{l} 60\,\lambda_1 + 25\,\lambda_2 + M_1' = 180\,\lambda_1 \\ 60\,\lambda_1 + 73\,\lambda_2 + M_2' = 120\,\lambda_2 \end{array}$$

or:

$$\begin{array}{l} 30\,(C_1') + 31.25\,(V_1') + M_1' = 90\,(W_1') \\ 30{+}60\,(C_2') + 31.25\,(V_2') + M_2' = 150\,(W_2') \end{array}$$

It then follows that:

$$\begin{array}{l} M_1' = 28.75 \\ M_2' = 28.75 \end{array}$$

The total surplus value now amounts to 57.5 hours of labour time. As a ratio of total variable capital of 62.5 hours, this gives a rate of surplus value of $\sigma = 0.92$. The difference with absolute surplus value creation is that in each sector, the workers still work – as they did initially – for 50 hours and are paid the original hourly wage rate. In the respective sectors, the value added equals 50 hours, of which the surplus value (28.75 hours) represents 57.5%. Thus, the *distribution* of the same value added as before has changed. *Relative surplus value* has been created.

We are thus confronted by two interventions at the micro level of the capitalist enterprises that affect the distribution of income at the macro level. Marx devotes an important part of Volume 1 of *Das Kapital* (Marx, 1976, Parts 3, 4 and 5) to the ways that absolute and relative surplus value is created. Both forms of surplus value creation are of paramount importance in his arguments and reasoning, although the production of absolute surplus value through the lengthening of the working day was more relevant during the rise of industrial capitalism than it is today.[2] At the global level, exploitation of the working class by way of low wages, long working days and a systematic rise in the intensity of labour is particularly rife among multinational enterprises and their subcontractors in the developing countries, but also in some production processes in the mature capitalist countries where organised labour organisations are weak or non-existent. The creation of relative surplus value involves an intervention in the production process itself which involves increasing the intensity of labour or introducing a technological or organisational innovation. See Chapter 5 in this regard.

If we want to understand the evolution of the rate of surplus value in relation to Marx's "laws of motion" of capitalism, the class struggle as a determining factor needs to be analysed. After all, the strength of the working class, especially the organised labour movement, relative to the strength of the capitalist class determines the extent to which wages and labour intensity will change. On the other hand, capitalist entrepreneurs have a formidable weapon: when it is impossible to lower the wages of the workers, layoffs follow which undermine the bargaining power of the working class.

In advanced capitalist economies, relative surplus value creation through innovations in the processes of production is of particular interest. Marx points out that the mechanisation of the production process and the associated expulsion of labour, although increasing the rate of surplus value, reduces the number of workers who produce surplus value:

> Hence there is an imminent contradiction in the application of machinery to the production of surplus-value, since, of the two factors of the surplus-value created by a given amount of capital, one, the rate of surplus-value, cannot be increased except by diminishing the other, the number of workers.
>
> (Marx, 1976, p. 531)

Thus, an "industrial reserve army" is replenished regularly:

> The section of the working class thus rendered superfluous by machinery, i.e. converted into a part of the population no longer directly necessary for the self-valorisation of capital, either goes under in the unequal contest between the old handicraft and manufacturing production and the new machine production, or else floods all the more easily accessible branches of industry, swamps the labour-market, and makes the price of labour-power fall below its value.
>
> (Marx, 1976, p. 557)

However, during periods of strong capital accumulation, labour will be drawn from this "industrial reserve army", which after some time will lead to a labour shortage. As a result, wages will increase and an economic crisis will set in. Marx states this as follows:

> [A]s soon as capital has grown in such proportion to the working population that neither the absolute labour-time that this working population supplies nor its relative surplus labour-time can be extended (the latter would not be possible in any case in a situation where the demand for labour was so strong, and there was thus a tendency for wages to rise) (. . .) there will be an absolute overproduction of capital (. . .). In both cases there would even be a sharper and more sudden fall in the general rate of profit, but this time on account of a change in the composition of capital which would not be due to a development in productivity, but rather to a rise in the money value of the variable capital on account of higher wages and to a corresponding decline in the proportion of surplus labour to necessary labour.
>
> (Marx, 1981, p. 360)

We also need to consider the extent to which the workers succeed, through their class struggle, in countering an increase in the rate of surplus value (or in the share of profits in value added). This issue is taken up in the next section.

Pioneering scientific research has been conducted into the causes, in many countries, of the declining share of wages in income since the 1970s, although mostly in the neo-classical tradition. Using a neo-classical production function and without technological progress, it can be derived mathematically that the shares of wages and profits in value added remain constant. Hence, changes in the wages share should be attributed to technological change. Based on a standard neo-classical production function, Bentolila and Saint-Paul (2003) estimated the impact of technological progress on the wages share. This allows an assessment of the impact of technological progress on "total factor productivity", as well as the impact of capital-augmenting and/or labour-augmenting technological progress.[3] They found that the increase in capital intensity and "total factor productivity" reduced the share of wages and suggested that the substitution of labour for capital and capital-augmenting technological change were the driving force. This was later confirmed for the EU-15 by Arpaia, Pérez and Pichelmann (2009). In a more recent study, Karabarbounis and Neiman (2014) argued that the increase in capital intensity since the mid-1970s originated in the relative decline in the price of means of production/investment goods, which seems to explain about half the decline in the share of wages in value added in 59 countries.[4]

7.2 Class struggle and the share of labour in value added

We saw earlier that an increase in labour productivity in the sector where necessary consumer goods are produced will reduce the unit value of these goods and, therefore, the value of labour power. The quantities of necessary consumer goods remain the same, but the value of the respective goods decreases at a rate that is inversely proportionate to the increase in labour productivity.

In our imaginary economy, the expenditure of 50 hours of labour resulted in the production of 150 kg of iron and the cultivation of 100 kg of wheat, implying an average labour productivity of 3 kg of iron per working hour in sector 1 and 2 kg of wheat per working hour in sector 2. We again consider the initial situation where 0.5 kg of wheat per working hour is necessary consumption and $\lambda_1 = 0.5$ and $\lambda_2 = 1.25$, the labour value of 1 kg of iron and 1 kg of wheat, respectively:

$$50\,\lambda_1 + 50 \text{ hours of labour} = 150\,\lambda_1$$

$$50\,\lambda_1 + 40\,\lambda_2 + 50 \text{ hours of labour} = 100\,\lambda_2$$

The value of labour power is $1.25 \times 0.5 = 0.625$.

Suppose labour productivity increases at a rate of 10%, which brings average labour productivity to 3.3 kg of iron and 2.2 kg of wheat per working hour, and the labour value equations are transformed into:[5]

$$55\,\lambda_1' + 50 \text{ hours of labour} \rightarrow 165\,\lambda_1'$$

$$55\,\lambda_1' + 44\,\lambda_2' + 50 \text{ hours of labour} \rightarrow 110\,\lambda_2'$$

It will be noticed that with the same employment as before, 10% more output is produced in sectors 1 and 2 (165 kg of iron and 110 kg of wheat). The labour value λ_1' becomes 0.455 and $\lambda_2' = 1.136$.[6] If the necessary consumption is still 0.5 kg of wheat per hour of labour spent, the value of labour power is now $0.5\text{kg} \times 1.136 = 0.568$ hours of labour, dropping from the previous level of 0.625.[7]

Marx states that the reduced value of labour power represents a *lower limit* to the real decrease, which depends on the relative bargaining power of the workers vis-à-vis the capitalists in the wage negotiations. He writes:

> For example, if, as a result of an increase in the productivity of labour, the value of labour-power falls from 4 shillings to 3, or the necessary labour-time from 8 hours to 6, the price of labour-power might well fall only to 3s. 8d.,[8] 3s. 6d. or 3s. 2d., thus allowing the amount of surplus-value to rise only to 3s. 4d., 3s. 6d. or 3s. 10d. The amount of this fall, the lowest limit of which is 3 shillings (the new value of labour-power), depends on the relative weight thrown into the scale by the pressure of capital on the one side, and the resistance of the worker on the other.
>
> (Marx, 1976, pp. 658–659)

The relative weight of the workers in the wage negotiations with the capitalists evidently also determines the extent of the wage increases in relation to labour productivity. If the workers succeed in making real wages (the bundle of necessary consumer goods per working hour) increase *pari passu* with labour productivity, then the share of wages will remain constant. In the struggle for wage increases at the company level, the share of wages is a benchmark.

If, in our example, real wages are increasing by the same rate of 10% as labour productivity, the workers' necessary consumption per working hour will increase from 0.5 kg of wheat to 0.55 kg. With the corresponding labour value now being $\lambda_2' = 1.136$, the value of labour power becomes: $0.55\ \lambda_2' = 0.625$, which is the same as $0.50\ \lambda_2 = 0.625$, as before. The value added of 50 hours in both sectors thus remains $50 = 31.25\ (V_1) + 18.75\ (M_1) = 31.25\ (V_2) + 18.75\ (M_2)$. In both sectors, the rate of surplus value is also unchanged at $M_1/V_1 = M_2/V_2 = \sigma = 0.6$. The wages share in value added is the same as before: $V_1/(V_1+M_1) = V_2/(V_2 + M_2) = 62.5\%$, to which a constant share of profits also corresponds: $M_1/(V_1+M_1) = M_2/(V_2 + M_2) = 37.5\%$.

One can conclude that when the balance of power between labour and capital at the factory level is unchanged, and the workers enforce a rise in their consumption which is in line with the rise in labour productivity, then the share of wages in value added – and therefore also the share of profits – will remain constant. When the workers enforce a rise in real wages that is proportionately higher than the rise in labour productivity, the share of profits will fall, which ceteris paribus will also lead to a fall in the rate of profits. In Chapter 5 we learned that this will cause a decline in investment, which will trigger unemployment and adversely affect the negotiating position of the workers. Conversely, if the real wages do not follow the rise in labour productivity, the profits share will increase as will – *ceteris paribus* – the rate of profits. The accumulation of capital will accelerate and more workers will be hired, thus strengthening the position of labour in the wage negotiations. It can thus be expected that with the balance of power between labour and capital remaining the same, the profits share will show an undulatory movement in the short run, but it will not show an increasing trend in the long run, contrary to what Marx seems to have expected.

This conclusion brings us to the pioneering theoretical work of Richard M. Goodwin on the endogenous cyclical movement of the wage and profits shares. In Goodwin's model (1967), it is assumed that the capital stock is always used at full capacity. In contrast, workers can be unemployed and wages are negatively affected by the degree of unemployment. By assuming that the capital–output ratio remains constant, the rate of growth of the capital stock is equal to that of output. After some mathematical manipulation, Goodwin derives two non-linear differential equations: one for the rate of growth of employment (the complement of the degree of unemployment) and one for the rate of growth of the wage share in value added. The Goodwin model shows that these rates of growth will evolve cyclically over time. This is reminiscent of what is observed in nature where the growth of the prey population leads to an increase in the predator population, which in turn annihilates the prey population growth rate and therefore also reduces the growth rate of the predator population. The Goodwin model functions similarly: an increase in the rate of employment brings about a rise in the share of wages, which causes the rate of growth of employment, and therefore the rate of growth in the share of wages, to drop. In the model, employment and the wages share show a cyclical movement.[9]

Of course, in the long run the balance of power between labour and capital does not remain the same, but changes as the rate of unemployment changes. Laibman (1997, pp. 93–97) formulated a comprehensive Marxist model of accumulation in which the capital–output ratio is not constant, but changes due to micro-rational decisions being made by the capitalists to introduce labour-saving technologies. In Laibman's model, changes in the unemployment of labour, which the capitalists use as an indicator of future market prospects and sales opportunities, lead to drastic changes in the savings ratio from profits. This causes a capitalist "stampede into liquidity", which also triggers and creates the expected problems of effective demand. His model generates cyclical liquidation and realisation crises.

In the world we live in today, the balance of power between labour and capital has also been disrupted by the increasingly internationalising, capitalist nature of production processes. Given the wave of globalisation that has been sweeping through the world economy and which has been picking up momentum since the early 1980s, the workers and their organisations are under growing pressure to limit their demands for higher pay, accept more flexible work, and also accept the higher profits of the capitalists and bonuses for the management.[10] We will investigate these long-run movements of the wages and profits shares in the next section.

7.3 Exploitation and class struggle: game-theoretical insights

In the previous section, we showed that Marx established that the class struggle influences the share of wages in value added. According to Marx, the workers' militancy prevents a situation in which, given rising labour productivity, wages will fall to the new value of labour power. That the workers succeed in preventing this "depends on the relative weight thrown into the scale by the pressure of capital, on the one side, and the resistance of the worker, on the other" (Marx, 1976, p. 659)

Although the value of labour power does not equate to the bare necessities of life but is rather historically and socially determined,[11] the question remains: how far was Marx willing to go in accepting that, over an extended period of time (i.e. beyond the duration of the business cycle), the value of labour power can increase? We have seen that it is Marx's thesis that the capitalists dispose of the powerful weapon of dismissing workers when their accumulation of capital is likely to be compromised. Moreover, so Marx's reasoning continues, the proverbial sword of Damocles, i.e. the threat of being fired, is also hanging over the workers' heads due to increasing mechanisation which goes together with the introduction of labour-saving production techniques. The introduction of such techniques is apparently not just dictated by competition and the need to reduce costs; it is also a weapon in the class struggle, the more so as it boosts both the relative surplus value created and the rate of exploitation.

In addition, during times of labour scarcity, the capitalists will endeavour to compel the workers to work longer and/or to accept increased labour intensity.

The extent that they achieve this, without providing any compensating wage increase, is in each case a function of the aforementioned balance of power between labour and capital on the factory floor.

Marx makes it abundantly clear that the source of the capitalists' profits is unpaid labour. Why do the workers agree to such exploitation? They do so because they do not own any means of production, which are instead possessed by the capitalists whose wishes the workers must abide by if they are to make a living.

Marx stresses that the capitalist ownership of the means of production is based on an *historical process of appropriation, the primitive accumulation* (Marx, 1976, Part 8, pp. 871ff.). Upon the dissolution of feudalism, common land and buildings fell into private hands, while those who previously cultivated the common land and worked in these buildings became "free". These "free" workers were, in order to provide a livelihood for themselves and their family, forced to sell their labour power to the new owners. Analogously, in continental Europe in the aftermath of the French Revolution, much church land and the buildings thereon, as well as the possessions of the nobility, fell into the hands of greedy capitalists.

Marx writes:

> In the history of primitive accumulation, all revolutions are epoch-making that act as levers for the capitalist class in the course of its formation; but this is true above all for those moments when great masses of men are suddenly and forcibly torn from their means of subsistence, and hurled onto the labour market as free, unprotected and rightless proletarians. The expropriation of the agricultural producer, of the peasant, from the soil is the basis of the whole process. The history of this expropriation assumes different aspects in different countries, and runs through its various phases in different orders of succession, and at different historical epochs.
>
> (Marx, 1976, p. 876)

There is a growing body of scholarly literature devoted to the exploitation of the working class and the position of workers and capitalists in the class struggle, starting with the capitalists' monopoly of the means of production. A pioneer in this regard is John Roemer, a prominent representative of "analytical Marxism". In his approach, the historical context and the conditions surrounding the formation of economic classes is lacking, but he does give an *endogenous* theory of such a formation: economic classes arise within a given "institutional framework", with "haves", the owners of the capital goods, and "have nots". This situation is then compared with an institutional setting in which all producers are owners of their means of production.

In an initial institutional framework that Roemer analyses, there is only a market for goods, not for labour (Roemer, 1986, pp. 84ff.). For each producer v – with an output vector $\mathbf{q}^v$, his available means of production $\boldsymbol{\mu}^v$ and his subsistence needs $\mathbf{c}$ – the question of what will be produced, and how, can be formulated as the following programming problem: minimise the expenditure of labour time

$\mathbf{q}^v \mathbf{l}$, given the outputs $\mathbf{q}^v$, such that for all outputs a non-negative surplus remains. Mathematically, this programming problem can be written as:[12]

$$\text{Min } \mathbf{q}^v \mathbf{l}$$

subject to the limiting conditions that:

$$\mathbf{q}^v (\mathbf{I} - \mathbf{A})\, \mathbf{p} \geq \mathbf{c}\, \mathbf{p}$$

$$\mathbf{q}^v \mathbf{A}\, \mathbf{p} \leq \boldsymbol{\mu}^v \mathbf{p}$$

$$\mathbf{q}^v \mathbf{l} \leq \mathbf{1}$$

$$\mathbf{q}^v \geq \mathbf{0}$$

The first condition states that for each output (and given the prices of the outputs) a non-negative surplus above subsistence must remain, while the second condition states that the means of production used cannot be larger than the stock available to the producer. The last two limiting conditions state that the expenditure of labour $\mathbf{q}^v \mathbf{l}$ cannot be larger than 100% (or 1) of the time that the producer can spend, and that outputs must be non-negative.

In this model, a division of labour will develop between the individual producers, leading to the exchange of the outputs produced. The producers will choose different production processes, implying different expenditures of labour. If, based on the labour values $\boldsymbol{\lambda}$, all producers just spend $\mathbf{c}\, \boldsymbol{\lambda}$ of labour time, then $\mathbf{c}\, \boldsymbol{\lambda}$ is the average socially necessary labour time. Roemer calls this an egalitarian solution. If some producers work longer than is required to produce their means of subsistence (i.e. they work longer than $\mathbf{c}\, \boldsymbol{\lambda}$), and some others less, this is a non-egalitarian solution.

We next consider the institutional framework in which there is a market for both goods and labour. In the labour market, the hourly wage rate ω equilibrates the demand for and supply of labour. Producers can now sell or buy labour time. There are also producers with no or insufficient means of production. There are three possibilities: (1) producers employ their means of production, while working and producing $\mathbf{q}^v$, (2) producers employ their means of production with labour power that they bought, and produce $\mathbf{y}^v$, (3) producers sell their labour power, which is then employed by the buyers together with the buyers' means of production, and work $\mathbf{z}^v$. The programming problem of the randomly chosen producer v now adopts the following general form:

$$\text{Min } (\mathbf{q}^v \mathbf{l} + \mathbf{z}^v)$$

subject to the limiting conditions:

$$\mathbf{q}^v (\mathbf{I} - \mathbf{A})\, \mathbf{p} + \mathbf{y}^v [(\mathbf{I} - \mathbf{A})\, \mathbf{p} - \omega\, \mathbf{l}] + \omega\, \mathbf{z}^v \geq \mathbf{c}\, \mathbf{p}$$

$$\mathbf{q}^v \mathbf{A}\, \mathbf{p} + \mathbf{y}^v \mathbf{A}\, \mathbf{p} \leq \boldsymbol{\mu}^v \mathbf{p}$$

$$\mathbf{q}^v \mathbf{l} + \mathbf{z}^v \leq \mathbf{1}$$

The first condition indicates that after the employment of the own means of production $\mathbf{q}^{v}\,(\mathbf{I}-\mathbf{A})\,\mathbf{p}+\mathbf{y}^{v}\,(\mathbf{I}-\mathbf{A})\,\mathbf{p}$ and the payment of the wages $\mathbf{y}^{v}\,\omega\,\mathbf{l}$, augmented by the possible receipt of wages for the sale of the own labour power $\omega\,\mathbf{z}^{v}$, a non-negative surplus of the various outputs must remain, above that required for subsistence. The second condition states that the employment of the available means of production for the production of $\mathbf{q}^{v}$ and $\mathbf{y}^{v}$ may not be higher than the proportion of the means of production that v owns. The last condition states that each producer cannot employ more labour than his available labour supply.

Suppose that there are n producers. If the total socially necessary labour time to produce n × **c** equals n $\mathbf{c}\,\boldsymbol{\lambda}$, then an egalitarian solution for this institutional framework is that each producer works $\mathbf{c}\,\boldsymbol{\lambda}$ hours. In the case of a non-egalitarian solution, some producers work more than is socially necessary for the production of their subsistence, and are exploiters. Other producers work less and are exploited. Roemer then shows that, based on initial wealth, the richest exploiters become "pure" capitalists while the poorest exploited become proletarians. In this manner, five classes emerge: (1) the capitalists who do not work, but own means of production and buy labour power, (2) the small capitalists who themselves work, using their means of production, but also buy labour power, (3) the petty bourgeoisie who work, using their means of production, and do not buy labour power from others, (4) the mixed proletarians who work for part of the time, using their own means of production, but also sell some of their labour time, and (5) the proletarians who only sell their labour power because they own no means of production.[13]

Next, Roemer proves his "class exploitation correspondence principle": anyone belonging to a class of buyers of labour power is an exploiter, and anyone belonging to a class of sellers of labour power is exploited. To the exploiting classes belong the capitalists and the small capitalists; to the exploited classes belong the mixed proletarians and the proletarians. The petty bourgeoisie belong to neither the exploiters nor the exploited.

In any event, the conclusion is that it can be logically and mathematically demonstrated how an institutional framework that entails the sale of labour – at least when associated with an unequal ownership of the means of production – leads independently to the creation of socio-economic classes. The fundamental question then is how the unequal ownership of the means of production is established. Roemer's analysis conceals this, or gives the impression that it would result from the "surplus labour" of the diligent producers in his first institutional framework. Although important for understanding the logic of capitalist exploitation, Roemer's reasoning is a-historical and assumes that it is not necessary to have a theory about how the direct producers become detached from their means of production, such as Marx's theory of primitive accumulation.

Furthermore, when the change in the institutional framework from a pure exchange economy to an economy with a labour market is conceived as an historical process, then Roemer's conclusions about the formation of the capitalist class and the proletariat conceal the real historical process. Marx is very scornful of this:

> This primitive accumulation plays approximately the same role in political economy as original sin does in theology. Adam bit the apple, and thereupon sin fell on the human race. Its origin is supposed to be explained when it is told as an anecdote about the past. Long, long ago there were two sorts of people; one, the diligent, intelligent and above all frugal elite; the other, lazy rascals, spending their substance, and more, in riotous living.
>
> (Marx, 1976, p. 873)

Nevertheless, Roemer's analysis is important for Marx, since it shows that, given that the process of primitive accumulation is in motion, the formation of the capitalist class and the working class takes place by itself and is *explained by the rational behaviour of individuals, not classes*. That this process leads to five classes instead of two, as in Marx's case, is less relevant since it can be explained by the technological dynamics involved, i.e. that large-scale capitalist production leads to superior economic results and, as a result, capitalists' competitive edge destroys the small capitalists and the mixed proletarians.[14]

7.4 The degree of monopoly and "mark-up" pricing

Marx emphasises the exploitation of the workers at the factory floor. This form of exploitation belongs to the "micro-foundations" of Marxist economic theory. It is at the company level that the workers produce surplus value and set wage demands in motion, i.e. that real wages should increase with labour productivity. Whether or not this happens depends on the balance of power between workers and capitalists. These "micro-foundations" are clearly rooted in Marx's labour theory of value and allow him to elaborate on the reality of the capitalist mode of production and to integrate it in a logically constructed economic theory.

Until now we, too, have assumed, like Marx, that the rates of profits between the economic sectors will be equalised in the longer run through the inter- and intra-sectoral mobility of capital. This was the essence of the theory of the general (or average) rate of profits which leads to a system of prices of production. In today's reality, however, the rates of profits differ between the economic sectors, which is often due to the so-called "entry barriers" which can be very varied indeed. These barriers can relate to the nature of the product that is produced in a sector. If a producer succeeds in making his product unique, or at least sufficiently distinct from that of his competitors (be it a car, computer, soft drink, fashion clothing, etc.), he can practise price setting – in other words, set a price that is higher than that when the products of the various producers were identical (and therefore not distinct). This is a situation of oligopolistic competition.

Oligopolistic competition differs from "free competition" in that in the latter case the producers are confronted by a market price above which they should not attempt to sell their products as they would risk pricing themselves out of the market. With "free competition", the market price is given exogenously for each producer. Each of these producers will increase his output as long as the cost price of the last unit or kilogram he produces is below the market price. In this situation, each additional unit of output he produces will yield a profit. He will stop increasing his output when the last produced unit has a unit cost that is equal to or higher than the price he receives in the market.[15] The price that the capitalist receives for the last produced and sold unit of output is the marginal revenue, while the cost price of this last unit is the marginal cost.

The situation of free competition that confronts the individual producer is illustrated in Figure 7.1. The unit price, and thus the marginal revenue, is given and constant: each additional output produced yields – at least when sold – the same price/marginal revenue for the producer. The unit cost price in Figure 7.1 remains constant over a considerable range of output levels, with each additional unit of output produced leading to an identical increase in the total cost price. When full capacity utilisation is reached, the total cost price climbs rapidly and the rising average and marginal costs deviate.

Figure 7.1 shows that under conditions of free competition, profits maximisation will take place when the producer produces output *Q*. Before *Q* is reached, the producer receives for each additional unit of output a price (marginal revenue = market price) that is higher than the marginal cost and he will expand production. When the output level *Q* is reached, his total costs will rise, with a marginal cost that is exactly equal to the price received in the market. At that point he has maximal profits and each additional unit of output produced and sold will reduce profits.

Figure 7.1 can also be interpreted differently. The horizontal line that shows the average cost = marginal cost is the demand curve with which the individual producer is confronted: at the given market price he can sell any quantity of output produced. The marginal cost curve is the supply curve of the individual producer.

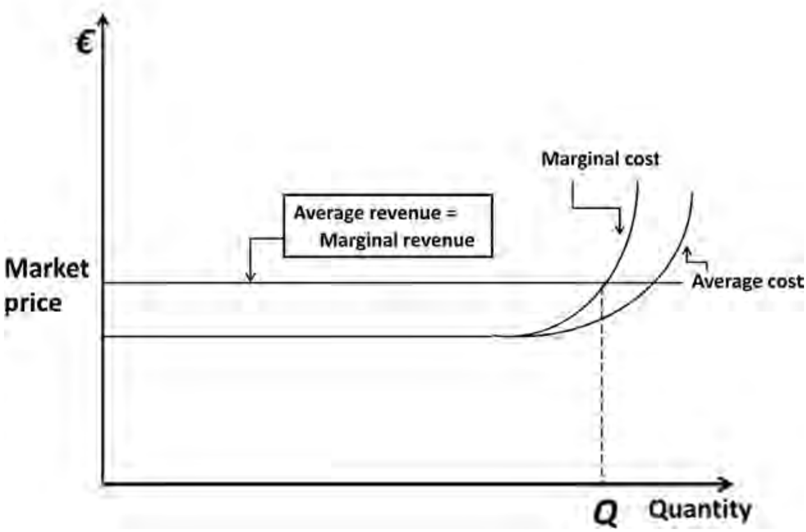

Figure 7.1 Profits maximisation under free competition

Under conditions of oligopolistic competition, the situation is fundamentally different. What each individual producer in the same sector is producing is distinct from what his competitor is producing under another brand name (which triggers a certain type of customer awareness and loyalty) or a different model, a different flavour, a different quality, etc. Each producer is confronted with the usual downward sloping demand curve for his product. There is no market price and when the producer increases his output, he will receive a lower unit price. Consumers will only be prepared to buy the quantity produced at a lower price.

Price setting under oligopolistic competition was first investigated in 1933 by Joan Robinson (Robinson, 1933) and Edward H. Chamberlin (1933).[16] Because of Kalecki's close relationship with neo-Marxist economic theory, we will shortly discuss his price theory.[17]

First of all, regarding price formation, Kalecki states that there are two types of goods, the price of which are "demand-determined" and "cost-determined", respectively. The price of raw materials and that of primary food is "demand-determined", since in the short run the supply of these goods can vary only a little. Consequently, when demand changes, the price of these goods also changes. In contrast, the price of many manufactured goods is "cost-determined". Due to continuing unused production capacity, supply can easily expand when the price increases (Kalecki, 1952, p. 11).

When the capitalist producer sets the price of a "cost-determined" manufactured product, he will first take into account the price of his competitors. These competing products can easily replace his, and thus his price should not be too much higher than that of his competitors. In addition, the capitalist will make sure that his price is not too low compared with his cost price.[18] Therefore, Kalecki writes that for an individual entrepreneur i the price of his product is determined by the following formula:

$$p = \mathrm{m_i}\, u + \mathrm{n_i}\, p^*$$

with p being the unit price, u the unit cost price, p^* the average price of the competitors, and $\mathrm{m_i}$ and $\mathrm{n_i}$ positive parameters which depend on the entrepreneur's decisions. When unit costs increase, the price of the final product will increase as well, but only if the competitors are also adjusting their prices upwards. If this adjustment is not (or only insufficiently) taking place, the entrepreneur in question can only amend his price accordingly (Kalecki, 1952, p. 12).

Considering the "average" enterprise in the sector, it will hold that its price $p = p^*$, which simplifies the price-setting formula to:

$$p = \mathrm{m}\, u + \mathrm{n}\, p$$

For this representative enterprise of the sector, it then follows that:

$$p = \kappa\, u$$

with: $$\kappa = \frac{\mathrm{m}}{1 - \mathrm{n}}$$

This means that the price of the representative enterprise shows a certain "mark-up" κ ($\kappa > 1$) to the unit costs. The coefficients m and n are characteristic of the price setting of the enterprise. They reflect the average degree of monopoly in the sector: i.e. the higher $m/(1 - n)$ is, the higher is the degree of monopoly in the sector, which finds its expression in the higher "mark-up" above the unit cost price (Kalecki, 1952, pp. 13–15).

It is interesting to compare Kalecki and Marx in this respect. Halevi and Kriesler (1991, p. 81) point out that with vertically integrated sectors and a closed economy, all marginal costs are labour costs and Kalecki's mark-up is conceptually similar to Ricardo's rate of profits; without fixed capital and overheads, Kalecki's average degree of monopoly would be identical to it. In fact, this observation evidently also applies to Marx's rate of profits when derived from the linear model of production.

Although, in his *Theory of Economic Dynamics*, Kalecki was not inclined to adopt marginalist reasoning, he did so in earlier versions (see Kriesler, 1987, pp. 29–34; J. Lopez G. and Assous, 2010, pp. 78ff.).

Figure 7.2 translates the situation of oligopolistic competition into a marginalist explanatory framework. We assume the same cost curve as in Figure 7.1.

The oligopolistic entrepreneur is not confronted with a given market price. From the entrepreneur's perspective, the demand curve for his product is not a horizontal curve, as in Figure 7.1, but the downward sloping curve of Figure 7.2, which indicates the average revenue. Since the average revenue decreases with larger quantities demanded and supplied, the marginal revenue falls twice as fast.[19]

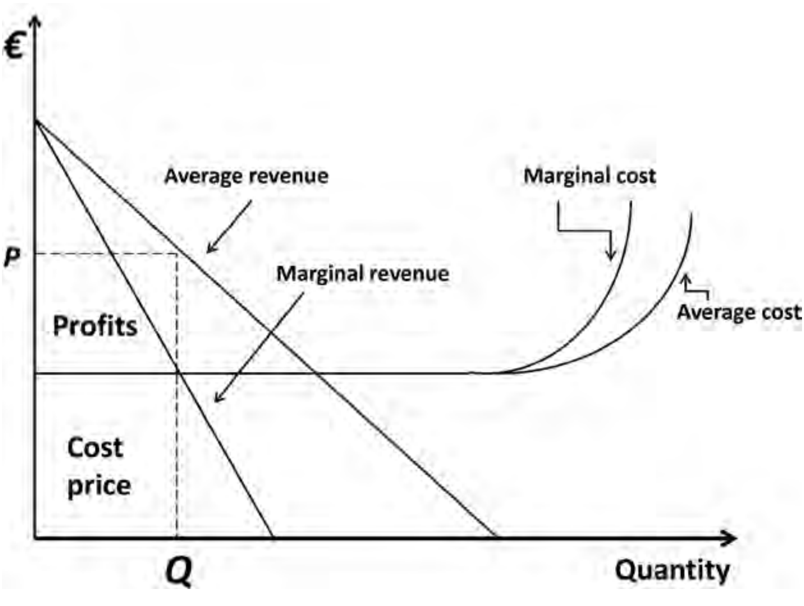

Figure 7.2 Profits maximisation under oligopolistic competition

The oligopolist maximises his profits by producing Q. Each additional unit of output produced before Q is reached has a marginal cost that is lower than the marginal revenue. When Q is reached, the last produced unit of output will yield marginal revenue that is exactly equal to the marginal cost. This approach of modelling the pricing behaviour of the oligopolist is essentially also the one adopted as early as 1936 in his review, in Polish, of Keynes's *General Theory* (Targetti and Kinda-Hass, 1982, pp. 246–247).[20]

It will be remembered that the cost price in Figure 7.2 included the "normal profit", which enters as the so-called cost of capital. What in Figure 7.2 is shown as profits, are the profits that the oligopolist experiences over and above the cost of capital. The price that the oligopolist sets equals the average cost, increased by the "mark-up", which makes the price equal to p, following Kalecki's p-formula.[21]

It is interesting to draw a comparison with the market situation of free competition, where all producers of the same product are considered. In this situation, the average costs curve and the average revenue curve indicate the total supply curve and the total demand curve, respectively. In Figure 7.1, the market clears at an output level far to the right of Q, which is the output level in a situation of oligopolistic competition. Oligopolists clearly produce below full capacity utilisation, which allows them to set a higher price.

It should be stressed that the way Q is determined in Figure 7.2 is questionable in the highly prevalent situation of a duopoly or an oligopolistic market situation, when the "players" involved are not only aiming for profit maximisation but also have a desire for secure profits. Their price strategies will depend on the expected reaction of the others. As a result, the AR curve in Figure 7.2 is not defined if oligopolists take into account the expected reactions of their competitors.[22]

What are the implications for the profits share in the economy at large, assuming that in all sectors oligopolistic price setting is applied?[23] This can easily be inferred from the situation of overcapacity, as Figure 7.2 shows. When the average costs are constant with increasing output, average costs and marginal costs per unit of output coincide. The sales value of total output is then:

$$p\,Q = \kappa\, u\, Q$$

$p\,Q$ is evidently national income Y (national product) and $u\,Q$ are the total production costs, which consist of wages W and the cost of materials PA, or $Y = \kappa\,(W + PA)$. It follows that the total gross profits P (including fixed costs) are equal to:

$$P = \kappa\,(W + PA) - (W + PA)$$

or: $$P = (\kappa - 1)\,(W + PA)$$

Since national income is assumed to be exhaustively split up between wages and profits, the wages share ζ in national income is:

$$\zeta = \frac{W}{W + (\kappa - 1)\,(W + PA)}$$

or, dividing numerator and denominator by W and representing PA/W by θ:

$$\zeta = \frac{1}{1+(\kappa-1)(1+\theta)}$$

Knowing that $\kappa > 1$, it will be noticed that the wages share ζ in national income decreases when the profits mark-up κ, or/and θ, the ratio of materials costs/wages, increases (Kalecki, 1952, pp. 28–29). An important implication is that the oligopolists can protect themselves against rising cost prices. For instance, when materials costs, unlike wages, rise and the mark-up is kept constant, the rise in costs is simply passed on in the prices. Consequently, the wages share falls.

Of course, the economy does not only function on the basis of oligopolistic price setting and Kalecki is not suggesting that it is only the degree of monopoly that influences the macroeconomic income shares. However, in line with Kalecki's thinking, in present-day oligopolistic capitalism, both the wages share ζ and the complementary profits share $(1 - \zeta)$ in national income are in an important way determined by the decisions of the monopolists and oligopolists about the mark-up when they set their prices. Whereas in Marx's view the profits–wages ratio, the profits share and the wages share are determined at the factory floor level by factors such as the wages struggle, the length of the working day or working week, labour intensity, labour productivity, etc., Kalecki and the post-Keynesians stress the mark-up decisions of the oligopolistic entrepreneurs. This evidently reflects the concrete market situation in contemporary capitalism, and it can be argued that this insight complements Marx's exploitation theory and should best be integrated into it (see also Harris, 1975, 1978; Harcourt and Kenyon, 1976).

However, there is an inherent warning that no incompatibilities should be integrated. It can be argued that Kalecki and the neo-Marxist authors who followed in his wake assumed that the production prices approach of Marx is no longer valid for today's capitalism, which is based on monopolistic pricing and "barriers to entry" (e.g. Halevi and Kriesler, 1991). If monopolistic price formation is general, an attempt by the capitalist entrepreneurs to increase the average profit margin will lead to a general price increase, including an increase in the price of labour power. General price inflation will then cause nominal profits to rise, but not in real terms. A higher average profits margin is only attainable when nominal wages remain constant, but real wages fall. This shows that the theoretical "link" between the degree of monopoly and income distribution runs via the real wages and not via the price-setting behaviour of the enterprises (Rugitsky, 2013, p. 448).

Conversely, according to Kalecki's theory, prices are not determined by demand and nor are profits a residue, and can thus be reconciled with Marx's value theory. There are actually two ways in which, according to Kalecki's theory, a tendency towards equalisation of the rate of profits occurs, namely that the individual oligopolist, in his pricing, takes into account the average price of his competitors (the p^* in the above oligopolistic price equation), and investments are driven by the rate of profits (Kerr, 1997, pp. 39–43).[24]

7.5 The working of the "profits squeeze": some neo-Keynesian and neo-Marxist interpretations

An interesting discussion has been taking place between prominent Keynesian and "neo-Kaleckian" economists about how the rise in the wages share has been responsible for the outbreak of the economic crisis of the 1970s and how the first signs of this appeared in the second half of the 1960s – which were nonetheless called the "golden sixties".

Already in the late 1960s, the Keynesian-inspired "profits squeeze" thesis made its entrance. According to this thesis, which starts from statistical data on wages, labour productivity, profits, investments, etc., a general scarcity of labour had developed in some major capitalist countries during the 1950s and (especially) the 1960s, causing wages to rise faster than labour productivity and affecting the share of profits in value added. This eventually led to an economic crisis.[25]

Stephen Marglin made an impressive attempt to combine neo-Marxist and post-Keynesian insights in a model that allows specific developments in capitalism to be explained, particularly the evolution of wages and price inflation (Marglin, 1984). His model uses the key concept of the conventional real wage w^+ (see also Chapter 3, section 3.4), as determined by class struggle and the balance of power between labour and capital. This conventional real wage can change and with each w^+ there is a corresponding rate of profits r^+. To this the profits realisation curve and the Robinsonian investment curve of Figure 3.4 are added. Only by happy fluke will r^+ be equal to the desired and realised rate of profits r^* (see Figure 3.4) or will the demand for investment be equal to the supply of savings. Next, Marglin introduces in his model a mechanism that leads to the equilibrium rate of growth g^* and rate of profits r^*, which essentially relates to the required money wage and price inflation rate. If the actual rates of profits and growth are below the equilibrium values, the capitalists will increase their prices to reduce the real wage rate. If, on the other hand, the actual rate of profits is above the rate r^+, which corresponds with the conventional real wage w^+, the workers will struggle to achieve a higher money wage. As Geoffrey Harcourt summarises:

> The inequality between investment demand and the supply of saving is a permanent feature, as are the departure of the real wage from the conventional real wage and the existence of permanent inflation (. . .). The dynamic equilibrium may be described in terms of a balance, an uneasy truce, between the pressure of aggregate demand on aggregate supply and the pressure of wage-earners on money-wages, so that the sustained rate of inflation measures – is an index of – both the frustration of the wage-earners trying to maintain a conventional real wage and the frustration of the capitalists trying to carry out their investment (accumulation) intentions.
>
> (Harcourt, 2006, p. 80)

Using this model, Harcourt has beautifully explained the periods of "higher growth–higher price inflation" of the post-war era until the late 1960s, the stagflation of the

1970s and the “lower growth–lower inflation” of later decades (Harcourt, 2006, pp. 78–83).[26]

A stimulating “neo-Kaleckian” interpretation of the evolution of the profits share, investments and rate of profits was supplied by Marglin and Bhaduri (1990). They start from the rate of profits r, which can also be written as:

$$r = \frac{P}{K} = \frac{P}{Y} \times \frac{Y}{Z} \times \frac{Z}{K} = \frac{(1-\zeta)z}{K/Z}$$

Here $(1 - \zeta)$ is the profits share P/Y, z is the ratio between the actual output level Y and the potential output level Z (in other words, an indicator of the degree of utilisation of production capacity), and K/Z is the capital–output ratio. Next, the interrelationship between the profits share – in our notation, $(1 - \zeta)$ – and capacity utilisation z is modelled as follows:

The savings function out of profits:[27] $g^s = \mathrm{s}\,\dfrac{P}{K} = \mathrm{s}\,\dfrac{(1-\zeta)z}{K/Z}$

The investment function: $g^i = \dfrac{I}{K}$ = a function of the expected rate of profits[28]

Equilibrium IS: $g^s = g^i$

From this system, the combinations of the profits shares and rates of capacity utilisation are derived for all possible equilibrium positions $g^s = g^i$, as indicated by the IS curve in Figure 7.3. The IS curve is downward sloping.[29]

On the other hand, the curve showing all feasible producers’ equilibria PE can be written as the function:

$$(1 - \zeta) = \mathrm{a_o} + b\,(\mathrm{z}),$$

with $b(z)$ an increasing function of z. It is assumed here that the profits margin used by the price setters is flexible and increases (albeit less than proportionately) with capacity utilisation.

The $(1 - \zeta)$ and z combinations that these feasible producers’ equilibria allow are depicted by the PE curve in Figure 7.3. This curve consists of the combinations when the producers are satisfied with the wage and price levels. It indicates that the profits share that the producers desire depends on the degree of capacity utilisation. Although the PE and IS curves do not need to be linear, Figures 7.3–7.6 show these as linear. The interaction of IS and PE leads to the equilibrium A.

After the Second World War, so Marglin and Bhaduri argue, there was much fear that the economic crisis and stagnation of the 1930s would resurface. It was expected that there would be little, if any, private investment because of the lack of effective demand. Therefore, government spending and wage increases were inflated. During the 1950s and the early 1960s, capitalism was characterised by a “stagnationist cooperative” situation: stagnationist, because of the increased profits margins, which created an under-consumption tendency; cooperative, because rising

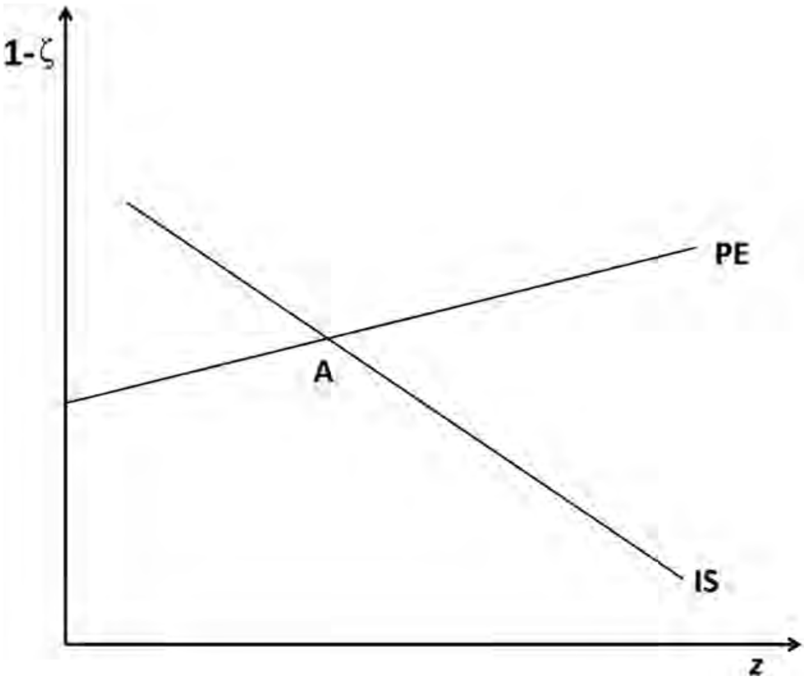

Figure 7.3 Marglin-Bhaduri equilibrium between profits share and rate of capacity utilisation

wages could generate compensatory, effective demand. During this phase, a higher wage share went hand in hand with a higher rate of accumulation. Until the first half of the 1950s, the IS curve was relatively flat since the entrepreneurs believed that economic stagnation would reappear. As a result, investment hardly reacted to the higher profitability. When during the course of the 1950s this belief was increasingly contradicted by reality, the higher profitability began to translate into increased investment as well as rising profit expectations. The IS curve shifted to IS', but because of the higher sensitivity of investment to profit expectations, the curve was also steeper, as depicted in Figure 7.4. A new equilibrium B was reached with a higher wage share going together with higher capacity utilisation, which also brought about a rise in the rate of profits.

During the 1960s, a huge labour shortage developed and wage increases did not keep pace with labour productivity. As a result, the PE curve shifted downwards to PE'. A new equilibrium point C was established. Capacity utilisation increased only slightly and the profits share came under pressure. This created the "profits squeeze" situation. The light increase in capacity utilisation did not compensate the lower profits share and led to a falling rate of profits.

The early 1970s witnessed increasingly intense international monetary crises, which led to the collapse of the Bretton Woods System of fixed exchange rates

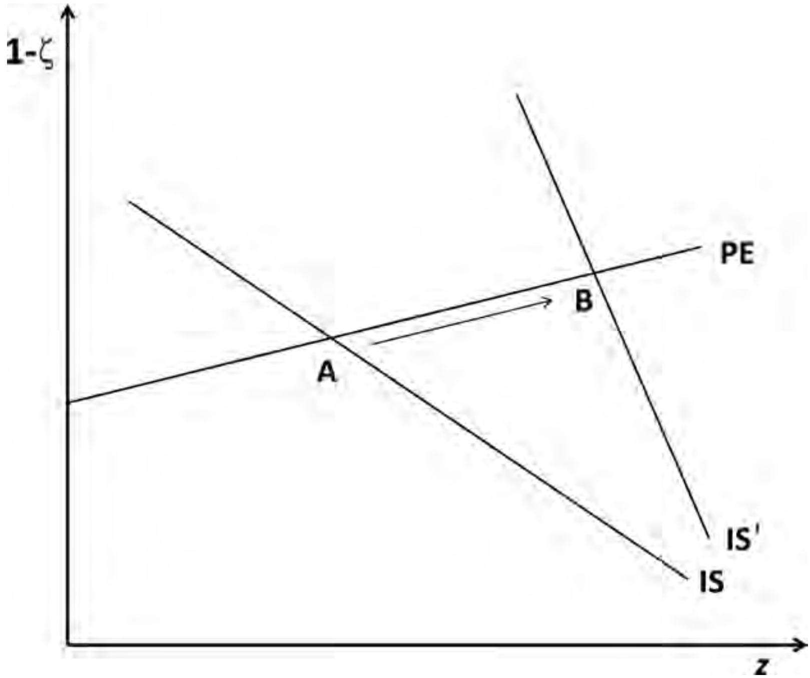

Figure 7.4 Profits share and capacity utilisation in the 1950s and early 1960s

pegged to the dollar. At the end of the decade, this gave rise to dangerous financial and monetary instability. During the 1970s, Keynesian macroeconomic stabilisation policies were adopted less and less. Capacity utilisation and expected profits declined. In Figure 7.6, the IS curve now shifts downwards towards the left and a new equilibrium D is reached, with a lower profits share and rate of capacity utilisation, while the rate of profits plummets.

The "neo-Kaleckian" interpretation of the "profits squeeze" of the 1960s and its aftermath focuses on the evolution of effective demand, paired with a changing sensitivity of the accumulation to profit expectations. Translated into Marxist terminology, full employment in the 1960s led to a rise in wages which reduced the accumulation propensity of monopoly capital. But even without the decline in the readiness to accumulate, according to the Marxist interpretation, the situation of full employment gave way to an economic crisis. Full employment means that the surplus value production bumps against its limits, such that sooner or later newly accumulated capital does not deliver the necessary surplus value (profits). The "profits squeeze" thesis, when translated into the terminology of Marx, implies a situation of "absolute overproduction of capital".

Undoubtedly, this interpretation might also contribute to a fact-based theory about the economic crisis. However, It should take due account of the increase in

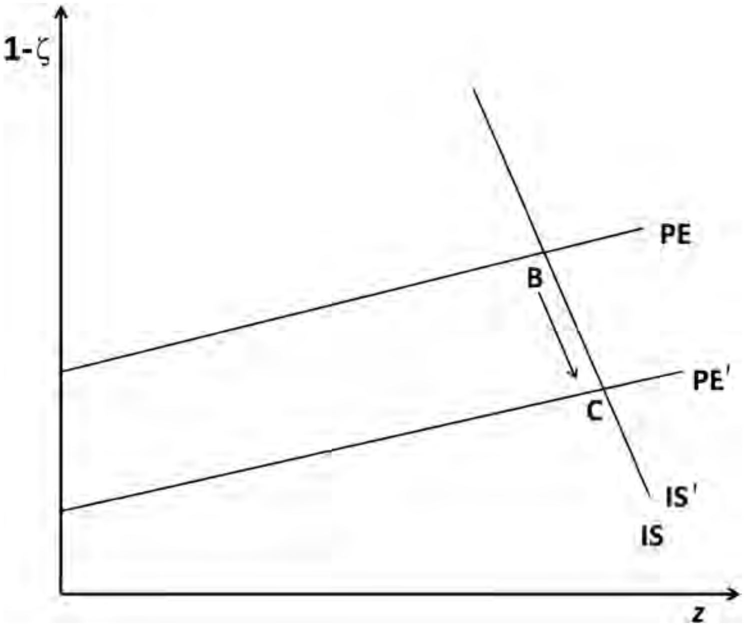

Figure 7.5 Profits share and capacity utilisation in the later 1960s

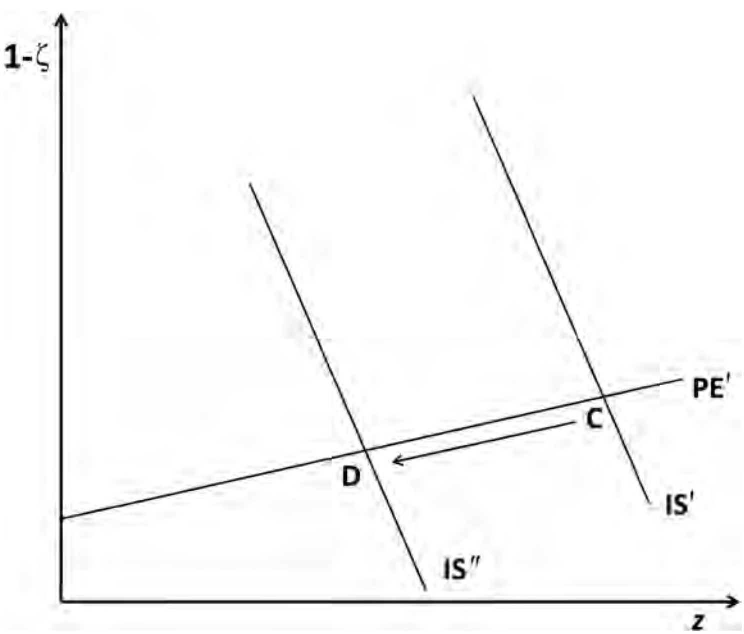

Figure 7.6 Profits share and capacity utilisation in the 1970s

the share of profits in value added in most industrialised countries over the past few decades. Therefore, one should also look at how the composition of capital or the capital–output ratio and capacity utilisation change. James Devine has pointed out how the rate of profits in the capitalist countries was restored in the 1980s, not through wage cuts but through politically motivated and officially imposed "austerity programmes". The reason why the "profits unsqueeze", which began in about 1972, did not adequately contribute to a recovery in the rate of profits is because its effect was counteracted by the increase in the composition of capital and the declining capacity utilisation (Devine, 1994). In this way, Devine throws his weight behind the interpretation of Marglin and Bhaduri.

In Chapter 9 we will again be reminded of the importance of the cyclical movement in the rate of surplus value and the composition of capital when we discuss the long wave theory of Ernest Mandel. On the other hand, regarding the economic situation of today, Joseph Stiglitz (holder of the 2011 Nobel Prize for Economics) has argued that increasing inequality has for some time been seriously hampering economic recovery.

Others have launched into a class struggle explanation for the "profits squeeze" in the second half of the 1960s and the crisis years of the 1970s. This interpretation is indebted to the theory of Kalecki about the "political business cycle", which highlights how a macroeconomic stimulus at full employment, under pressure from the bosses, will be abandoned. In 1943, after having analysed the political opposition of the capitalists and the bourgeoisie to a government policy aimed at creating jobs, Kalecki wrote the prophetic words:

> This state of affairs is perhaps symptomatic of the future economic regime of capitalist democracies. In the slump, either under the pressure of the masses, or even without it, public investment financed by borrowing will be undertaken to prevent large scale unemployment. But if attempts are made to apply this method in order to maintain the high level of employment reached in the subsequent boom a strong opposition of "business leaders" is likely to be encountered. As has already been argued, lasting full employment is not at all to their liking. The workers would "get out of hand" and the "captains of industry" would be anxious to "teach them a lesson". Moreover, the price increase in the up-swing is to the disadvantage of small and big *rentiers* and makes them "boom tired". In this situation a powerful block is likely to be formed between big business and the *rentier* interests, and they would probably find more than one economist to declare that the situation was manifestly unsound. The pressure of all these forces, and in particular of big business – as a rule influential in Government departments – would most probably induce the Government to return to the orthodox policy of cutting down the budget deficit. A slump would follow in which Government spending policy would come again into its own.
>
> (Kalecki, 1943, pp. 329–330)

Again, Kalecki has been the inspiration behind an impressive series of subsequent articles and studies on such a "political business cycle".[30]

7.6 Some statistical evidence

Figures 7.7 and 7.8 below show the evolution of surplus value in the USA in the period 1869–2011 and in the United Kingdom in the period after the Second World War.[31]

Between 1869 and the end of the Second World War, the USA series shows some outliers – a dramatic increase in the rate of surplus value σ in the period 1874–1880 (up from 45.9% in 1874 to 69.1% in 1880), after which there is an even more dramatic fall until 1892 ($\sigma = 21.7\%$). The rate varies between 25% and 45% in the period up to 1930, and shows a dramatic decline, which coincides with that of the rate of profits, during the Great Depression. By 1940, the rate of surplus value reaches a "normal" level again ($\sigma = 37.1\%$ in 1940) and increases during the war to 48.8% in 1944. From 1947 until 2011, the rate of surplus value in the USA hovers around 40%, with peaks in 1965–1966 of 44.5%, and downward outliers in 1982 of 31.6% and in 2000 of 36.1%. Its post-war relative stability is remarkable, which might be attributable to the relative stability in the balance of power between labour and capital until about 1978. It is also noteworthy that the rate of surplus value increases in the first half of the 1960s and declines in the second half of the 1960s. This confirms the "profits squeeze" thesis for the latter sub-period.

For the UK, we notice in the post-war period greater fluctuations in the rate of surplus value around 50%, reaching a low in 1975 and highs in the mid-1980s and in 1996 (Figure 7.8). Also, there is evidence for the period of "profits squeeze" during the 1960s. Cuestas and Philp (2012), who constructed this statistical series, also carried out econometric tests to the extent that indicators of the intensity of the class struggle explain the evolution of the rate of surplus value in the period 1955–2010. They use the number of days of strikes and unemployment, as well

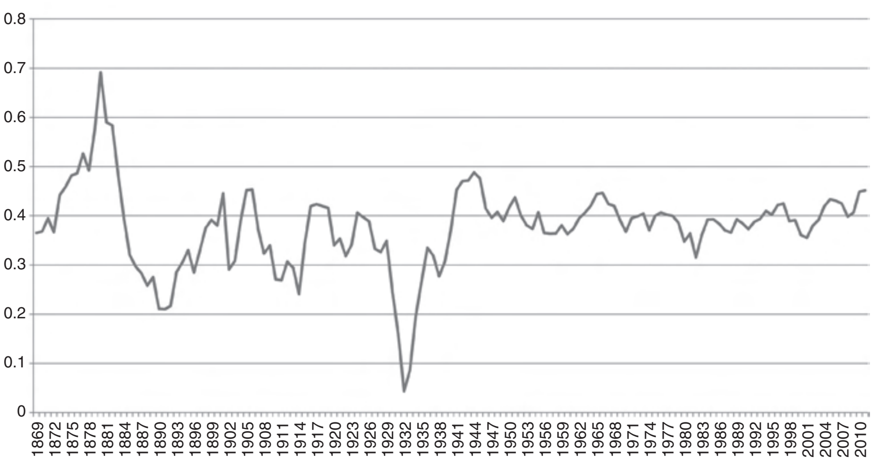

Figure 7.7 Rate of surplus value M/V

Source: Duménil and Lévy data: http://www.jourdan.ens.fr/levy/uslt4x.txt (accessed on 17.02.2014)

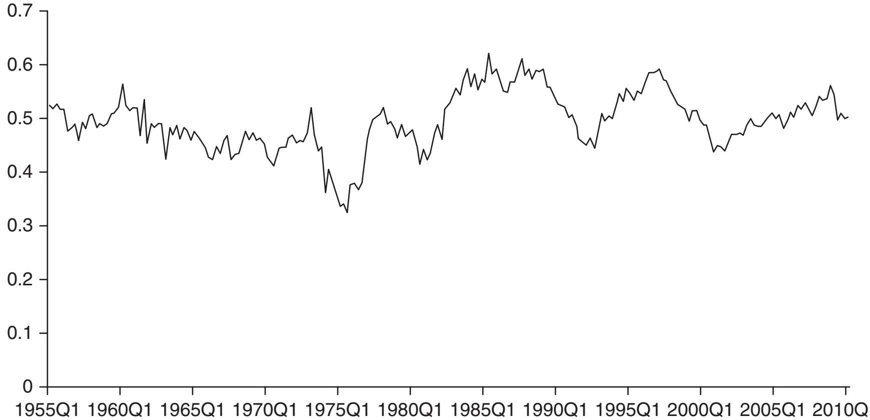

Figure 7.8 The rate of surplus value in the United Kingdom, 1955–2010

Source: Cuestas and Philp (2012, pp. 565–578, published by Taylor & Francis Ltd, www.tandfonline.com, and reprinted with the permission of the publisher)

as an indicator of the "political colour" of the British government in power at any time. They confirm the Marxist expectations, i.e. the intensity of the class struggle, measured by the number of strike days, tends to reduce the rate of surplus value, while an increase in the "reserve army" of the unemployed increases the rate of surplus value. They also confirm the "simple" variant of Kalecki's "political business cycle", i.e. under a Labour government, the rate of surplus value decreases and under the Conservatives it rises.

Finally, we mention the recent study by Thomas Piketty and his team, based on many years of long-term data series collection on wealth and income distribution in various countries. Although this research aims to thoroughly study the evolution of income and wealth inequality in the industrialised countries, it also contains interesting data about the evolution of the capital income share in the USA, Japan, Germany, France, UK, Canada, Australia and Italy, which all show a clear upward trend in the period 1970–2010, albeit with significant fluctuations (Piketty and Zucman, 2014, Figure 6.4). The share in total wealth of the top 10% of richest people in Europe and the USA has clearly risen again since the 1970s. It is particularly interesting that today – and in contrast to the situation in Europe – rising income inequality in the USA is more the result of the rise of the "top labour incomes" than of the possession of wealth, as was previously the case. Undoubtedly, the high and rising executive salaries and bonuses are responsible for this (Piketty, 2014, Ch. 5; Piketty and Saez, 2014, p. 839).

7.7 What to conclude?

The rate of surplus value is a key concept in Marx's theory. In Chapter 2 we mentioned how a positive rate of surplus value is a necessary condition for a positive rate of profits – that is to say, exploitation of labour is necessary for the existence of capitalist profits – and in Chapter 5 we examined the relationship between the rate of surplus value and the capitalist pursuit of super-profits. We also came across the rate of surplus value in Chapter 6 in that it determines the evolution of the average rate of profits. In that chapter, we still assumed that the rate of surplus value is given.

In this chapter, we abandoned the assumption that the rate of surplus value is given and instead investigated the determining factors according to Marx. It was shown that Marx points to the balance of power between labour and capital at the factory floor level as highly relevant, evident in the struggle over wage levels but also over the number of working hours and the degree of labour intensity. Unless the workers organise themselves, these power relations will remain highly unequal. As a result, the capitalists' hunger for profits will prevail over the hopes of the workers to acquire a proportionate share of the increase in productivity. The capitalists introduce labour-saving production techniques, which allow a regular replenishment of the "industrial reserve army" of the unemployed and keep wages low.

Since Marx's time, the balance of power between labour and capital at the factory floor level in the mature capitalist countries has been modified by the countervailing power of the labour unions. Although today, after nearly four decades of increasing globalisation, labour union power has been compromised by "outsourcing" and relocation of production to "low-wage countries", we are not back in the nineteenth century. The rate of surplus value at the enterprise level, being determined by the ratio between the total labour time spent and the "necessary labour time" will reflect changes in the balance of power between labour and capital and determine the evolution of the share of wages and profits in the value added at the macro level. The view that the class struggle is the determinant of the evolution of exploitation has led to an impressive list of scientific publications. The point is to assess their inherent value and see whether, and if so how, to integrate their insights with Marx, even if often against the wishes of the authors of these publications.

Considering the insights of "analytical Marxism" in relation to exploitation, we reviewed in particular the work of John Roemer, who examined its institutional determinants. Roemer's mathematical and game-theoretical approach to exploitation confirms the crucial role of the capitalists' monopoly of the means of production. Roemer also shows that there are forms of exploitation other than the capitalist exploitation of labour. This is not entirely new, but is now right under the noses of the Marxists. The criticism voiced by many of them that Roemer's "institutionalist" view is a-historical is mostly justified, but it is up to the Marxists to thoroughly investigate the many cases of so-called "primitive accumulation", using the same mathematical tools as those applied by Roemer if both views are to become integrated.

On the other hand, following Michał Kalecki, we looked into how in monopoly capitalism the oligopolists' mark-up pricing strategies have an impact on the profits share in income. "Monopolistic competition" is an essential element in the neo-Marxist analysis of contemporary capitalism. It demonstrates that it is not only the "production sphere" that determines the distribution of the national income, as Marx emphasises, but also the "sphere of circulation" of the economy. In addition, the class struggle can manifest itself via different macroeconomic and political mechanisms in the evolution of the income share of wages and profits, which has become especially relevant since governments have learned how to handle the Keynesian tools of macroeconomic fiscal and monetary policies. This influence was first indicated by Kalecki in his prophetic 1943 article, "Political Aspects of Full Employment", which became the source of inspiration for thousands of scientific publications.

The determining role of the class struggle in the evolution of income distribution is the basis of the theory of the "profits squeeze". This theory explains the decline in the profits share of national income in the dominant capitalist countries, roughly during the period 1965–1973, by referring to the prolonged full employment that caused a marked rise in wages. Although we stated at the beginning of this book that we would only evaluate Marx's economic theory and not the Marxist theories after Marx, we found it appropriate to look (albeit in a rudimentary way) at some post-Keynesian and neo-Marxist interpretations of this evolution; particularly as this also throws a different light on the factual evolution of the average rate of profits.

In order to provide some empirical evidence supporting the theoretical story, we finally considered the evolution of the rate of surplus value, using statistical data for the United States and the United Kingdom. As to the post-war period, a relatively stable rate of surplus value was observed which, however, has shown an upward trend since the late 1970s (although of a shorter duration in the UK than in the United States).

Notes

1 If a labour productivity-enhancing method of production is introduced by one or an insignificant number of capitalist producers, the labour values will remain unchanged. In Chapter 5, we saw that the introduction of these methods of production gave rise to super-profits. It is only when the new method is applied by a sufficient number of producers that the socially necessary costs of production – and therefore the labour values – will change.

2 However, it remains to be seen what the impact would be on the number of working hours in an internet-directed working environment.

3 In neo-classical models, total labour productivity increases when output increases due to new technology being introduced but the same labour and capital being used as before. Labour-augmenting technological change improves labour productivity in the form of, for example, a more efficient working environment. Capital-augmenting technological change improves the efficiency in how the means of production are used. An electronic signalling system, for instance, increases the efficiency of the railway infrastructure and the transport services provided by the railway system. In these models, technological progress is exogenous, i.e. its cause is not explained.

4 The findings of Karabarbounis and Neiman will be discussed further in Chapter 9.

5 We assume that the use of means of production also increases by 10%.

6 These solutions are found to be $\lambda_1' = 50/(165 - 55) = 0.455$ and $\lambda_2' = ((55 \times 0.455) + 50)/(110 - 44) = 75/66 = 1.136$.

7 See Chapter 5 for the relationship between increases in labour productivity, labour values and the rate of surplus value.

8 s. stands for shilling, d. for pence.

9 Laibman (1992, Ch. 9) presents a similar model cast in a two-sector Marxist reproduction scheme. His model also leads to a growth cycle that flows from under-consumption, which is accompanied by an increasing rate of exploitation. As first shown by Shah and Desai (1981), the cycle that the Goodwin model generates disappears when induced technological change is introduced in the model.

10 That globalisation has undermined the relative bargaining power of the workers with respect to capital is demonstrated by, for example, Rodrik (1997). See also Harcourt (2007) and Onaran (2011). Dumont, Rayp and Willemé (2006) show, in the case of Belgium, Germany, France, Italy and the United Kingdom, that during the period 1994–1998 the process of internationalisation weakened the trade unions, which thereafter impacted the evolution of wages. Dumont, Rayp and Willemé (2012) found further evidence in respect of Belgium that the bargaining position of lower skilled workers, in particular, has been undermined and that, in contrast, increased R&D activities have improved the bargaining position of highly skilled workers. The impact of the weakened balance of power and the dismantling of the welfare state on the declining wage share during the period 1970–2007 has been estimated by Stockhammer (2013).

11 This is most clearly stated in Marx (1981, p. 999): "The actual value of his labour-power diverges from this physical minimum; it differs according to climate and the level of social development; it depends not only on physical needs but also on historically developed social needs, which become second nature. In each country, however, this governing average wage is a given quantity at a given time."

12 We are, as far as possible, using the symbols from the previous chapters: $\mathbf{p}$ is the $n \times 1$ prices vector; $\mathbf{q}^v$ is the given $1 \times n$ output vector of producer v; $\mathbf{A}$ is the $n \times n$ matrix of input coefficients; $\mathbf{l}$ is the $n \times 1$ vector of labour input coefficients. In Roemer's analysis, $\mathbf{p}$ and $\mathbf{q}^v$ are row vectors, which, of course, is purely a matter of convention.

13 Since labour is assumed to be homogeneous, nobody will simultaneously buy and sell labour time. Consequently, there is a class that complements its own labour time in the production processes with that of others (the small capitalists), or that sells its labour time for use in others' production processes or in their own (the mixed proletarians).

14 Roemer also investigates a third institutional framework with a credit market instead of a labour market. He shows that in this institutional framework, using the same mechanism as that in his second institutional framework, classes of lenders and borrowers are created, with the lenders exploiting the borrowers. Therefore, Roemer (1982, p. 104) alleges that applying the concept of exploitation to the labour process only is misleading, and the Marxist definition of exploitation in terms of the extraction of surplus labour in the production process has to be substituted for a definition based on property relations. Marxists, however, have rightly pointed out that their focus is on labour, since this is relevant as far as the rest of the Marxist theory goes and not because they are denying the existence of other forms of exploitation. See, for example, Laibman (1992, p. 57). For a thorough and recent discussion on Roemer's theory of exploitation, including "socialist exploitation", see Philp (2005, pp. 74ff.)

15 This is based on the assumption that unit costs are increasing with output.

16 The analyses of Chamberlin and Robinson are different. In Chamberlin's analysis, the situation of imperfect competition is characterised by product differentiation, which is welfare increasing. See, for example, Bellante (2004).

17 We follow Kalecki's *Theory of Economic Dynamics* (1952), which was reprinted in 1965. As Kriesler (1987, pp. 80–82) points out, Kalecki became dissatisfied with his approach and modified his measure of the degree of monopoly substantially in Kalecki (1971).

18 This "securing" oligopolistic price strategy was investigated in the seminal 1947 paper by Kurt Rothschild (1947, pp. 310–311).

19 Assume that the demand curve in Figure 7.2 takes the functional form $Q = 150 - p$, which can also be written as $p = 150 - Q$. Total revenue for each demand price is equal to pQ and thus given by the function $pQ = 150\,Q - Q^2$. Average revenue is then $pQ/Q = p = 150 - Q$, which is the functional form of the demand curve. Marginal revenue is $\Delta pQ/\Delta Q = 150 - 2Q$, which shows that the slope of the marginal revenue curve is double that of the average revenue curve.

20 Kalecki deducts, from both the marginal revenue curve and the marginal cost curve, raw material costs and the cost of used-up equipment and machinery, and calls the resulting curves the value-added curve and the labour costs curve, respectively.

21 In Kalecki's *p*-formula, m and n are parameters that depend on the oligopolist's decisions. There is no assumption made that the oligopolist is maximising his profits above the capital cost, but is applying a mark-up on the variable costs. In the marginalist version of oligopolistic price setting, the oligopolist is, however, a profit-maximising entrepreneur. It can be shown that in that case the optimal price is $p^* = (\varepsilon/(1 + \varepsilon))MC$, with MC being the given marginal cost and ε the price elasticity of demand, i.e. the percentage rate of change in the quantity demanded for a small proportional change in the price. For instance, if $\varepsilon = -2$, the mark-up will equal $\varepsilon/(1 + \varepsilon) = 2$, but for $\varepsilon = -10$, the mark-up is 1.11. In the marginalist version, it should rather be stressed that the ability to apply "mark-up" pricing is determined by the degree of elasticity of demand for a particular product. The more elastic demand is (or the less inclined the demand curve), the lower the opportunity for mark-up pricing and the more the market is approaching free competition. See Lerner (1934).

22 These theoretical problems, relating to an analysis of duopoly and oligopoly along the lines of mainstream neo-classical microeconomic theory, were first listed in Rothschild (1947). Rothschild stated: "Any theory (. . .), which tries to explain price behaviour in terms of marginal curves derived from long-term demand and cost curves really bypasses the problem of uncertainty, and thus the very factor which gives rise to that desire for security which the theory tries to explain" (p. 308).

23 We will focus further on the implications of Kalecki's price theory. Evidently, later variants exist, such as Asimakopulos (1975), or Cowling and Waterson (1976), and Cowling (1982). If we were to dwell on this too much, we would stray too far from our review of and re-thinking on Marx's economics. For this and further developments, the reader is therefore referred to Arestis (1992, pp. 142ff.).

24 In a critical review of Kalecki's income distribution theory, Duménil and Lévy state that the rate of profits maximisation by the capitalist entrepreneurs cannot be reconciled with profits maximisation. See the Appendix in Duménil and Lévy (1999, pp. 73–94). This seems to be beside the point since Kalecki's pricing theory is not based on profits maximisation, which he rejected, as shown by J. Lopez G. and Assous (2010, p. 72), and also does not describe a situation in which all output in the economy is produced by oligopolists. It is true that the fundamental difference between Marx and the classical economists and Kalecki is that with the later post-Keynesian neo-Marxists, long-term spare capacity is available, which is not the case with Marx and the classical economists.

25 Another, contested version of the "profits squeeze" thesis is by Brenner (1998), who explains the fall of the profits share by reference to the competition between the US, Japan and Germany during the period 1965–1973, which prompted a price war. This price war, in turn, caused a decline in mark-ups, from which the salaried benefitted. Also, according to Brenner, the enterprises during the 1970s and 1980s were unable to pass on the increased costs through their prices.

26 Another version of this type of model was developed in Roemer (1981, pp. 190ff.), with employment plotted against the after tax rate of profits as real wage-profits frontier, by which the interaction of the Robinsonian realisation curve and the investment function leads to various types of crisis, depending on the after tax rate of profits and on whether the achieved rate of growth is lower or higher than the planned rate of growth.

27 We encountered this function in Chapter 3 as the "realisation function".
28 The expected rate of profits is, in turn, a function of $(1 - \zeta)$ and z, as shown in the g^s formula. This feature adds to the Robinsonian investment function from Chapter 3, the capacity utilisation z being an explanatory factor because it affects the expectations about profits.
29 Marglin and Bhaduri investigate mathematically the reasons for the declining IS curve, which relate to the higher sensitivity of savings than of investment to changes in profitability.
30 A Google search on 11 July 2015, "Kalecki political aspects of full employment", gave 34,100 results. The paper is cited 1,385 times. For a good overview of the most important studies, although pertaining to the United States only, see, for example, Drazen (2001); for many other papers, see Franzese (2002). It should be pointed out that the class struggle element, which is highlighted by Kalecki, has been mostly replaced in the subsequent academic literature by changes in macroeconomic policy in the run-up to or following democratic elections. See also Harcourt (2006, pp. 148–149).
31 The way the rate of surplus value is calculated in both series is similar, albeit not identical. Neither the Duménil-Lévy series for the USA nor the Cuestas-Philp series for the UK takes into account the "unproductive" activities which, following orthodox Marxist reasoning, are to be added to M and the wages spent on these activities to be deducted from V.

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8 The economic cycle and monetary theory of *Das Kapital*

In this chapter we will temporarily leave behind Marx's laws of motion of capitalism in the long run and instead focus on the business cycle, i.e. the economic cycle that manifests not over an extended period of time but rather spans some 8 to 11 years. Why is there a business cycle? Why does economic expansion come to a stop and a general contraction take hold? Such a change in economic activity is not characteristic of a long-term trend – not, for example, a "long wave", which is dealt with in Chapter 9.

In the analysis of short-run economic activity – more than in the analysis of long-term economic developments – there is a need to include monetary phenomena. What is the relationship between changes in output and accumulation in the course of the business cycle and what happens in the monetary sphere, such as changes in the money supply, the rate of interest, and so on?

We will first discuss the economic cycle theory that is provided in *Das Kapital* and assess it against the macroeconomic theory of the business cycle since Keynes and Kalecki. We will then dwell on Marxian monetary theory. The "New Interpretation" of Marx's theory of value has attempted to integrate money in terms of Marx's value and price theory (without having recourse to the value of a metallic monetary unit such as gold) by introducing the concept of the "monetary expression of labour time" (MELT). This concept will be discussed in the second section, after which the monetary theory found in the pages of *Das Kapital* will be reviewed.

Furthermore, we will point out that Marx considers economic crises to be caused by disproportionate developments in the financial sector. His views are interesting for those attempting to interpret and understand the financial crises that have taken place since the early 1980s as well as the economic crisis of 2008–2010. These momentous events can also be linked to the "financialisation" of monopoly capitalism, a topic that is explored in Chapter 9.

8.1 The economic cycle in *Das Kapital*

The classical economists devoted little attention to the economic and business cycles. They considered a general economic crisis to be an impossibility. Only sectoral crises fitted in with their views since, according to Say's law ("*la loi des débouchés*"), supply at the macroeconomic level, via the income that production generates, creates its demand.[1] General overproduction is therefore impossible, unlike at the sectoral level where it manifests due to insufficient demand for the sectoral output. However, since general overproduction is deemed impossible, the sectoral overproduction will necessarily be compensated for by excess demand for goods and services produced in other sectors.

Before Marx, the Swiss economist, Jean Charles Léonard de Sismondi (1773–1842), investigated the periodic economic crises that used to erupt, which he attributed to general overproduction and underconsumption. However, it was Marx who linked the evolution of the business cycle to the way capitalist accumulation occurs, and who integrated these insights into his economic theory.

The economic cycle theory that Marx develops in *Das Kapital* is as follows: At first, real wages are low and invested capital is relatively small, whereas the degree of exploitation is high as a result of high unemployment. Therefore, the average rate of profits shows a strong tendency to rise, leading to higher capital accumulation and more output. The economy thus moves into an upward phase in the cycle of economic activity, which will, however, at some future stage lead to a scarcity of labour. When this happens, real wages will rise (Marx, 1978, pp. 391, 486; Marx, 1981, p. 360) and the rate of exploitation will fall below a certain threshold (Marx, 1981, p. 364). Consequently, the produced and realised surplus value will rise at a rate that is lower than that of the capital stock. The rate of profits will first stagnate and then fall. Furthermore, a surplus of capital will develop since the newly accumulated capital cannot earn additional surplus value (Marx, 1981, p. 360). The production of capital goods will decline, unemployment will rise and real wages will fall. Businesses will go bankrupt or be taken over, leading to the centralisation of capital (Marx, 1981, pp. 361–362). At the same time, part of the capital stock will be destroyed and/or will devalue (Marx, 1981, p. 362). The economy will now go into a total downswing, which will persist until the conditions for a new start have been properly brought about.

The use of “Ockham’s razor” allows us to eliminate some of the determining factors that Marx advances in his theory of the economic cycle as being logically redundant. In fact, from his reasoning it follows that neither the increase in real wages nor the fall in the rate of exploitation of labour is essential for the downturn in the economic cycle. Movements in real wages and the rate of exploitation might accompany the cycle, but they do not generate it. It is rather the excess of capital and the scarcity of labour that produce the downturn. Based on the labour theory of value, scarcity of labour surely implies that even at a constant rate of exploitation, the surplus value produced will not increase at the same rate as before. With the rate of accumulation remaining the same, the rate of profits will decline. In Marx’s model, a downturn in the economic cycle will inevitably follow, not because of an increase in real wages but rather because of the capitalists’ hunger for profits, which pushes the rate of accumulation above the attainable rate of increase in the surplus value. Hence, based on the essentials of Marx’s model, it is not underconsumption that is responsible for the downturn in the economic cycle, as Sismondi stated, but the over-accumulation of capital, which will lead to a decline in the average rate of profits. Likewise, the upswing will set in as soon as sufficient capital has disappeared during the economic crisis. Because of high unemployment, there is at that point labour available to start capital accumulation again.

The economic cycle has a duration of approximately 11 years. Its periodicity is caused by the periodicity of the over-accumulation of capital. Marx writes:

> Just as the heavenly bodies always repeat a certain movement, once they have been flung into it, so also does social production, once it has been flung into this movement of alternate expansion and contraction. Effects become causes in their turn, and the various vicissitudes of the whole process, which always reproduces its own conditions, take on the form of periodicity.
>
> (Marx, 1976, p. 786)

In the French edition he adds that the periodicity is affected by the spread of capitalist production and that the duration of the economic cycle is likely to diminish:

> But only after mechanical industry had struck root so deeply that it exerted a preponderant influence on the whole of national production; only after foreign trade began to predominate over internal trade, thanks to mechanical industry; only after the world market had successively annexed extensive areas of the New World, Asia and Australia; and finally, only after a sufficient number of industrial nations had entered the arena – only after all this had happened can one date the repeated self-perpetuating cycles, whose successive phases embrace years, and always culminate in a general crisis, which is the end of one cycle and the starting point of another. Until now the duration of these cycles has been ten or eleven years, but there is no reason to consider this duration as constant. On the contrary, we ought to conclude, on the basis of the laws of capitalist production as we have just expounded them, that the duration is variable, and that the length of the cycles will gradually diminish.
>
> (Marx, 1976, p. 786n)

Based on the free competition assumptions of Marx's model, it should be expected that the phase of economic crisis is followed by a phase of recovery and expansion. Under conditions of free competition, capitalists accumulate and produce as much as possible, and therefore fully use their production capacity. Mainly due to the limited capacity of the workers to consume, Sardoni (2011, Ch. 3) argues, this leads to periodic general overproduction, which, in turn, creates expectations of falling prices and prompts a rise in capitalists "hoarding demand for money". As a result, the economic reproduction process falters. In Chapter 9, this situation of underconsumption will be explored further. The next point that Sardoni (2011) makes is that under free competition, no Keynesian prolonged "underemployment equilibrium" is possible and, therefore, in Marx's model, the economic reproduction process will pick up again.

It is tempting to accept the thesis that Marx's theory of the economic cycle is based on underconsumption and insufficient effective demand. However, Marx makes it abundantly clear that he does not consider the lack of effective demand to be the cause of the downturn in the business cycle:

> It is a pure tautology to say that crises are provoked by a lack of effective demand or effective consumption. The capitalist system does not recognize any forms of consumer other than those who can pay, if we exclude the consumption of paupers and swindlers. The fact that commodities are unsaleable means no more than that no effective buyers have been found for them, i.e. no consumers (no matter whether the commodities are ultimately sold to meet the needs of productive or individual consumption). If the attempt is made to give this tautology the semblance of greater profundity, by the statement that the working class receives too small a portion of its own product, and that the evil would be remedied if it received a bigger share, i.e. if its wages rose, we need only note that crises are always prepared by a period in which wages generally rise, and the working class actually does receive a greater share in the part of the annual product destined for consumption.
>
> (Marx, 1978, pp. 486–487)

Rather, in Marx's view, the downturn in the economic cycle is caused by the excess of capital accumulated compared to the available and increasingly scarce labour force. As a result, the average rate of profits plummets and capital accumulation comes to a standstill. This will evidently be accompanied by reduced effective demand – but as a consequence, not as a cause, of the downturn.

8.2 The economic cycle and the post-Keynesian neo-Marxists

The evolution of the average rate of profits during the economic cycle thus assumes a central position in Marx's theory and, at first sight, money and credit are not assigned a role. An analogous view is found in the work of Michał Kalecki, one of the pioneers of neo-Marxist and post-Keynesian economic theory.

It can be shown that economic cycles are also generated by the interaction between the wages share in value added and the rate of employment, as explored by Richard M. Goodwin and mentioned in Chapter 7. While being different from the post-Keynesian neo-Marxist approach of Kalecki who, like Marx, stresses the role of over-accumulation, Goodwin's model is very interesting. In defiance of the dominant opinion, we consider this model to be also relevant in the context of the theory of the "long waves" in economic activity, not only the theory of the business cycle, and refer to Chapter 9 in this regard. However, giving consideration to the Goodwin type of interactions between the long-run income shares and long-run unemployment rates as providing a theoretical foundation for such long waves is difficult to swallow from the post-Keynesian standpoint that: "the long-run trend is but a slowly changing component of a chain of short-period situations; it has no independent identity" (Kalecki, 1968, p. 165).[2]

Kalecki developed an essentially Keynesian model of the business cycle at the same time as, but independently of, John Maynard Keynes. In Kalecki's theory of the business cycle, the rate of profits plays the same role as in Marx's thinking. The declining rate of profits after the peak in the cycle is the result of economic activity stabilising and stagnating, without capital accumulation stagnating yet (Kalecki, 1952, p. 125). Like with Marx, at the root of the falling rate of profits and the reduced investment decisions lies the increased capital stock, which in Kalecki's view is a decreasing function of the net increase in capital equipment (Kalecki, 1952, p. 98).

In *The Theory of Economic Dynamics* (1952), the economic model builder Kalecki further analyses the determining factors of investment in the short and long run. He states that in the short run D, the value of the investment decisions at time t is determined by the capitalist savings S (non-distributed profits, capital depreciation and personal savings), the change in profits ΔP/Δt and the change in the capital stock ΔK/Δt. Capitalist savings and the change in profits influence D positively, but the change in the capital stock has a negative effect on D. In other words, if the capitalists have more savings and if profits grow, they will decide to invest more. The investment decisions will decline, however, with increasing capital stock. When D is a linear function of these variables, it can be written as:

$$D = a\,S + b\,\frac{\Delta P}{\Delta t} - c\,\frac{\Delta K}{\Delta t} + d = F_{t+\tau}$$

$F_{t+\tau}$ denotes the investments in fixed capital that take place τ time periods after they are decided, and a, b, c and d are parameters for this linear equation. Parameter d changes in the long run (Kalecki, 1952, pp. 96–97). The equation plays an important role in Kalecki's explanation of the business cycle. It can be further elaborated on since the change in the fixed capital stock:

$$\frac{\Delta K}{\Delta t} = F - \delta$$

consists of the investment F, net of depreciation of the fixed capital stock in the period considered. Substituting this equality in the above equation for $F_{t+\tau}$ and elaborating further will lead to a new equation for fixed investment in period $t + \theta$:

$$F_{t+\theta} = a'\,S + b'\,\frac{\Delta P}{\Delta t} + d'$$

Investments in inventory J in $t + \theta$ can be added, being a proportion e of the changes in output O in the private sector:

$$J_{t+\theta} = e\,\frac{\Delta O}{\Delta t}$$

in order to find total investment in period $t + \theta$:

$$I_{t+\theta} = F_{t+\theta} + J_{t+\theta} = a'\,S + b'\,\frac{\Delta P}{\Delta t} + e\,\frac{\Delta O}{\Delta t} + d'$$

Investments in $t + \theta$ thus depend on S, which in turn depends on the *level* of economic activity in t and on $\frac{\Delta P}{\Delta t}$ and $\frac{\Delta O}{\Delta t}$, which are determined by the *change* in economic activity in t (Kalecki, 1952, pp. 107–108). After combining this with $P = I/s_p$ and some transformations, we arrive at the equation (Kalecki, 1952, pp. 121–122):[3]

$$I_{t+\theta} = \alpha I_t + \beta \frac{\Delta I}{\Delta t} + d'$$

Yet if it is assumed that the investments are just sufficient to replace the depreciated capital (or, as expressed in present-day economic jargon: if total investments are equal to replacement investments), it holds that:

$$I_{t+\theta} = I_t = \delta$$

$$\frac{\Delta I}{\Delta t} = 0$$

As a result, the equation for $I_{t+\theta}$ can be written as:

$$\delta = \alpha \delta + d'$$

This is the condition for the "static equilibrium" of the economy.

During the upswing in the business cycle net investments are made above replacement investments, but during the downswing net investment is negative since less is invested than is required for replacement. Net investment equals $I_{t+\theta} - \delta = i_{t+\theta}$ and equals:[4]

$$i_{t+\theta} = \alpha i_t + \beta \frac{\Delta i}{\Delta t}$$

Kalecki explains the fall in the rate of profits that accompanies the increasing capital stock and adversely affects investment decisions by referring to the effect of "newcomers" on the attractiveness of the incumbents' investment plans (Kalecki, 1952, p. 98). This effect can best be likened to Marx's views on the excess of capital and the resulting capital competition, although in Kalecki's system profit expectations play an important role via the decline in investment decisions. Completely lacking in Kalecki's model, however, is the exhaustion of the available labour reserves during the economic boom, which is crucial in Marx's explanation of the start of the decline in the rate of profits and the emerging surplus capital. With Kalecki, the downturn in the business cycle is the result, both directly and indirectly, of insufficient effective demand that is fuelled by the net investments.

According to Kalecki, since new investments have to generate a normal or standard rate of profits (see Kalecki, 1968, pp. 266–268), a fall in the rate of profits will depress new investments. Viewed from a mathematical angle, the cyclical nature of investments and of economic activity follows from the parameter values in Kalecki's investment function, particularly those of the present investments (Kalecki, 1952, pp. 122ff.). On another occasion, he states this as follows:[5]

> When investment reaches its top level during the boom the following situation arises. Profits and national income, whose changes are directly related to those of investment, cease to grow as well, but capital equipment continues to expand because net investment is positive. The increase in productive capacity is thus not matched by the rise in effective demand. As a result, investment declines, and this causes in turn a fall in profits and national income.
>
> (Kalecki, 1962, p. 139)

Kalecki also differs from Marx in terms of the behaviour of real wages in the course of the business cycle: for Marx, real wages rise during the boom as a result of the depletion of the "industrial reserve army" (the army of the unemployed) but for Kalecki, nominal wages and prices change in the same proportion, at least in the competitive phase of capitalism, and real wages change little, if at all (Kalecki, 1971, pp. 3, 5–6). Kalecki's view is the same as that adopted by the neo-Marxists of his time, i.e. Josef Steindl, Paul Baran, Paul Sweezy and others.[6]

With the necessary reserve and nuance, it can be stated that the views of Marx *and* Kalecki are found in present-day post-Keynesian business cycle theory, in which the multiplier and accelerator mechanism cause the cyclical movement in economic activity. The multiplier mechanism is analogous to Kalecki's view, i.e. an increase in investments brings about a larger increase in national income. The accelerator mechanism indicates that each increase in national income (production) will generate growing demand for investment goods; it is a kind of investment decisions function à la Kalecki.[7]

Kalecki differs, in turn, from Keynes in terms of the absence of a monetary factor in his (Kalecki's) business cycle theory (see, however, Sawyer, 2006). The upswing in the business cycle in Kalecki's model is based on the assumption that the financial sector always comes forward with the credits that are required to finance the investments and that the speculative demand for money can be neglected (J. Lopez G. and Assous, 2010, p. 141). Somewhat surprisingly, we will see later in this chapter that, although he hardly integrated them into his theory of the economic cycle, Marx took financial crises and crises caused by monetary developments into account. But first, something else needs to be considered . . .

8.3 Labour values and prices of production in money terms

Orthodox Marxist economists commonly stress that Marx's theory of money is based on a commodity functioning as money – i.e. a general equivalent – such as gold. This theoretical presumption allows one to relate the value of all other commodities to their nominal (money) prices, since they all are expressed in the value of the general equivalent.[8] What becomes the general equivalent is not an arbitrary choice of a "numéraire" in which prices are expressed; rather, it is the outcome of a real historical and spontaneously unfolding process of acceptance of a specific commodity as the medium of exchange by society (de Brunhoff and Foley, 2006, p. 190).

Since the general equivalent is produced like all other commodities, it has a value, which allows it to be used as a measure of value or, stated somewhat imprudently, to "translate" the labour values into the value of a standard unit of the general equivalent. In this way, the labour theory of value and the monetary phenomenon, at least the metallic variety, are inextricably connected but lose much of their relevance in the context of a modern monetary economy, and even more so in the globalised capitalist system.[9] It is deplorable that so many Marxists are repeating a mantra on which almost unreadable exegetic papers are frequently based, and are not taking any notice of the evolution of monetary economic theory since 1867, the year Volume 1 of *Das Kapital* was published. Suzanne de Brunhoff, who thoroughly studied Marx's monetary theory, pays full attention to the relationship between the general equivalent, credit money, money as a measure of value and money prices. She writes:

> The problem of prices is not, for Marx, a monetary problem, once the origin of the general equivalent has been established. The maintenance of the *principle* of convertibility serves to preserve the primitive role of money as measure of value. But except in a credit crisis, that principle is not *applied*, because the economically important variations of prices under capitalism do not depend on the variations in the value of gold (which is postulated), but on the contrary the circulation of all money is itself dependent on prices.
>
> (de Brunhoff, 1976, p. 84; italics by de Brunhoff)

Governments can replace gold as a general equivalent with paper money or debt certificates. This does not change anything, so Marx and the Marxists allege, provided these substitutes are convertible into gold (de Brunhoff and Foley, 2006, p. 191). Gold convertibility, however, is an historical notion. It is possible, even likely, that the constraint that the general equivalent imposes on the determination of money prices and on the value of the money stock is in operation in special and abnormal times of general international economic unrest and widespread monetary panic. Yet the relationship, in normal times, between the value of gold and the labour value of the produced goods and services has never been clarified. For this, we have to find the answer in present-day economic theory, particularly – though probably not exclusively – in post-Keynesian monetary theory.

The link between labour values and money prices is, however, not dependent on the labour value of one gram of gold when the MELT concept ("monetary expression of labour time") is introduced.[10] Such an introduction has led to the so-called "New Interpretation" (Foley, 2000, pp. 20ff.).[11]

The MELT indicates that in an economy, the total labour time spent in one period of production equals the money value of the total value added produced in that period, i.e. what we previously called the net product or what is commonly called the net national product. We clarify this by returning to our imaginary economy of Chapter 2:[12]

$$
\begin{array}{llll}
50\,\text{kg iron} & & +\,50 \text{ hours labour} \rightarrow 150\,\text{kg iron} \\
\underline{50\,\text{kg iron}} & +\,\underline{40\,\text{kg wheat}} & +\,50 \text{ hours labour} \rightarrow 100\,\text{kg wheat} \\
100\,\text{kg iron} & \;\;40\,\text{kg wheat} &
\end{array}
$$

The total labour time spent is 100 hours, i.e. 50 hours in the iron sector and 50 hours in the wheat sector. However, in our imaginary economy value is not measured or expressed in labour time but in, for example, francs, marks, dollars or pounds. If the total value added – the net national product – is equal to 200,000 francs, we know that:

$$100 \text{ hours of labour} = 200{,}000 \text{ francs}$$

and that:

$$\frac{200{,}000 \text{ francs}}{100 \text{ hours of labour}} = 2000 \text{ francs/hour of labour} = \text{MELT}$$

Using this equivalence allows us to express the labour values and the production prices in the commonly used monetary unit.

In Chapter 2 it was shown that the labour value of 1 kg of iron and 1 kg of wheat equals $\lambda_1 = 1/2$ hour of labour time and $\lambda_2 = 5/4$ hours of labour time, respectively. Based on the MELT, this is equivalent to 1,000 francs and 2,500 francs. Similarly, for the prices of production of iron and wheat: $p_1 = 0.437$ hours of labour and $p_2 = 1.302$ hours of labour, or 874 francs and 2,604 francs, respectively.

So far, so good. The real discussion starts when we attempt to define the value of labour power. In our imaginary economy, the workers receive half a kilo of wheat per working hour. The labour time that is "congealed" in this half kilo of wheat is: $0.50\ \lambda_2 = 0.625$ hours or 1,250 francs. However, expressed in the labour time of the prices of production: $0.50\ p_2 = 0.651$ or 1,302 francs. Foley (2000) states that the value of labour power is nothing but the hourly money wage rate, divided by the MELT. Starting with the production prices, this is 0.651 hours of labour. However, we saw in Chapter 2 that labour values are equal to the corresponding prices of production in our imaginary economy if the workers are pursuing equality in the profits–wages ratio in all sectors. In that case, the value of labour power is: $0.50\ \lambda_2 = 0.625$ hours or 1,250 francs. The value of labour power can then be expected to oscillate between the two, depending on the system of prices.[13]

The advantage of using the MELT is that it expresses labour values and prices of production in monetary units, without the nature of the underlying monetary system being of importance. Whether gold is coined as money, whether the money in circulation is based on the quantity of gold in the vaults of the central bank, whether the money supply depends on the international reserves of the country or on the credit money created by the banking sector, are irrelevant. What is relevant is the value, expressed in money, of the net national product and the total amount of (productive) labour time spent during the period in question (Foley, 2000, pp. 21–22).

8.4 Money and credit in *Das Kapital*

Marx's views on money and credit are spread throughout his magnum opus, since he aims to integrate these while developing the various layers of his model (Geoff Harcourt likened Marx's method of analysis to an onion, with overlapping "layers of skin") (Harcourt, 2006b, p. 131). However, at the same time, due to *Das Kapital* being unfinished, many of Marx's views on credit and speculative "bubbles", which might also help to explain the world of today, are rudimentary and fragmentary. It is nevertheless revelatory that Marx devotes a due amount of attention to such issues.

In Volume 1 of *Das Kapital*, Marx discusses the role of paper money in terms of simple commodity circulation and states that credit money will be dealt with later. This happens in parts of Volumes 2 and 3 which are devoted to extended reproduction, but Marx's treatment of paper money itself is not entirely clear (de Brunhoff, 1976, p. 35). Pending his views on credit money, he writes:

> (The) minimum mass (of the circulating medium) can therefore be replaced by paper symbols. If however all the channels of circulation were today filled with paper money to the full extent of their capacity for absorbing money, they might the next day be over-full owing to the fluctuations in the circulation of commodities. There would no longer be any standard. If the paper money exceeds its proper limit, i.e. the amount in gold coins of the same denomination which could have been in circulation, then, quite apart from the danger of becoming universally discredited, it will still represent within the world of commodities only that quantity of gold which is fixed by its immanent laws.
>
> (Marx, 1976, p. 225)

Das Kapital tells us that Marx was very conscious of the role that money plays in the functioning of the capitalist economy. In his analysis of the capitalist reproduction process, he indicates how money capital makes way for the continuation and expansion of production. His analysis, when viewed through a contemporary lens, is not earth-shattering, but in retrospect it should be borne in mind that it was, after all, 1867 and the analysis reflects Marx's views on the "real economy". Because of this integration, his theoretical model shows how an increase in the circulation of money in the process of expanded reproduction of capitalist production promotes a greater accumulation of capital and surplus value realisation. It should be emphasised, however, that Marx did not use this opportunity and so omitted to mention the role of monetary policy. Possible reasons for this are as follows: first, the opportunities offered by monetary policy were generally challenged in Marx's time, but more importantly, Marx disputes the economic role that the government can play because money creation takes place in the "sphere of circulation" and the government is unable to determine the value of money (de Brunhoff, 1976, pp. 46–47). Also, from the perspective of Marx, the revolutionary, it might have seemed futile to devote serious attention to economic intervention in the "sphere of circulation" to preserve capitalism. Instead we had to wait for John Maynard Keynes who, in the 1920s and 1930s, led the reformist socialist movement in the advanced capitalist countries, using various weapons from his arsenal of macroeconomics policies.

Reviewing Marx's other views on money by concentrating on Volumes 2 and 3 of *Das Kapital* is very revealing.[14] Marx distinguishes himself from the economists of his time by rejecting the quantity theory of money.[15] This theory, which can be traced back to William Petty (1623–1687) and David Hume (1711–1776), has a number of variants, but it emphasises that in the long run a proportional relationship exists between the quantity of money in circulation in an economy and the level of nominal prices. Since the liberal economists of the nineteenth century subscribed to Say's law, i.e. that the supply of goods and services determines the demand for these goods and services, they considered general economic crises to be impossible. Consequently, in their view, an increase in the money supply has no impact whatsoever on the level of output and will only lead to a proportional increase in nominal prices.[16] Crucial to this liberal orthodoxy is the separation of the real economy from the monetary sphere of the economy. The demand for money by the public is a transaction motive demand, i.e. money is demanded because it is required to effectuate the desired purchases of goods and services. There is also a demand for money to keep as wealth, as an asset, but this demand is also assumed to be a function of the quantity of goods and services that are produced.[17] The velocity of circulation of money is for liberal economists a datum, and if it is not considered to be constant, it only changes in the long run following a given trend.[18]

There is nothing of this kind with Marx! Although Marx occasionally assumes a constant velocity of money circulation (Marx, 1978, pp. 192, 407, 576), and on other occasions states that it changes in the longer run with the rotation time of capital (Marx, 1981, pp. 389–390; Marx, 1978, p. 494), he stresses that money is held by the public (Marx only mentions the capitalist enterprises) as a reserve fund for future purchases and as temporary idle money capital (Marx, 1978, p. 333; Marx, 1981, pp. 435–436).

Marx puts it as follows:

> The capitalist production process, and trade in general, even on the basis of pre-capitalist modes of production, lead to (. . .) the accumulation of money as a hoard, in this case as the section of capital that must always exist in the money form, as a reserve fund of means of purchase and payment. This is the first form of the hoard, as it reappears in the capitalist mode of production and generally comes into being with the development of commercial capital, at least for the use of this capital. In both cases this applies as much to

> international circulation as to domestic. This hoard is in constant flux, constantly spilling out into circulation and returning from it. The second form of the hoard is that of idle capital temporarily unoccupied in the money form, together with newly accumulated money capital that has not yet been invested.
> (Marx, 1981, p. 435)

John Maynard Keynes (1883–1946) – who in his 1930 *Treatise on Money* considered for the first time the velocity of circulation of "business deposits" as indeterminate and unstable (Keynes, 1930, Vol. I, pp. 47–48) – rejects the quantity theory of money completely in *The General Theory of Employment, Interest and Money* (1936). We would therefore rather call this the transaction and precautionary demand and the speculative demand for money, respectively,[19] although Marx does not devote any attention to the factors that determine speculative demand. We will return to this.

At the same time, Marx sheds light in *Das Kapital* on the development of the credit system and the channels through which it influences capitalist production and accumulation. He points out, for instance, that this development allows production over longer production periods as well as inventory production, which causes a relative increase in the productive power of labour (due to the relative decline in the labour required for gold production) (Marx, 1978, p. 420) and a decline in the share of money capital used in (unproductive) trade (Marx, 1981, pp. 390–391), thus freeing money capital and increasing its circulation (Marx, 1978, pp. 357–358). Again, these are long-run developments that determine the so-called laws of motion of capitalism.

At this stage it is relevant to revert to Marx's thesis on paper money from Volume 1 of *Das Kapital* (see above). The question is: what exactly happens to prices when more paper money is brought into circulation than what is required for the "commodity circulation"? Since each bank note then represents a smaller quantity of gold than before, and hence also represents a lower labour value, it seems logical that the money prices of goods and services expressed in this paper money will be higher. This then means that money depreciation and price inflation will set in. However, such reasoning, in as far as it comes from Marx, is linked to the quantity theory of money which Marx, as we have pointed out, rejects. A logical way out of this apparent contradiction might be that the relationship between the excess paper money in circulation and prices is much more tenuous than the quantity theory dictates and that Marx prescribes the "hoarding" of excess paper money. Not only with simple money circulation but also with fiduciary money circulation, it would hold, following Marx's logic, that the money that is really in circulation is determined by the prices of goods and services, as well as the number of transactions – not the other way around (Marx, 1981, p. 655). To state this in terms of the transaction/precautionary motive and the speculative motive: the transaction and precautionary demand for money – not the speculative demand for money! – is determined by the transactional volume as expressed in prices, a view that is shared by Keynes and which Marx traces back to Sir James Steuart (1713–1780).

When too much paper money is brought into circulation compared with transactional needs, the excess supply will be left idle by the public, “hoarded” and/or deposited in a bank account. In Chapter 33 of Volume 3 of *Das Kapital*, Marx enthusiastically quotes the various financial experts of his time who approve of this (Marx, 1981, pp. 657–659). Credit creation does not change anything, since the amount of these credits is determined by the needs of the capitalist businesses (Marx, 1981, p. 674).[20] It is true that nowhere does Marx describe the mechanism that incites the public to put aside the excess money. It can be argued, however, that in this regard he probably followed the views of the financial expert, John Fullarton (1780–1849), and considered the rate of interest as the determining factor.[21]

Whereas profit accrues to the capitalist as a capital user, the capital owner receives interest (Marx, 1981, pp. 496–498). Marx views the average rate of interest as nothing but the price of money capital (Marx, 1981, pp. 517, 549–550, 553), which moves between a maximum level – the average rate of profits – and a minimum level which is difficult to determine (Marx, 1981, p. 480). If an abnormally large proportion of the capitalists convert their capital into money capital, this could provoke the rate of interest to plummet and a devalorisation of the money capital (Marx, 1981, p. 500). A “natural” rate of interest does not exist. Marx writes:

> There is no natural rate of interest (. . .) in the sense that economists speak of a natural rate of profit and a natural rate of wages. (. . .) There is no reason at all why the average conditions of competition, of equilibrium between lender and borrower, should give the lender an interest of 3, 4, 5 per cent, etc. on his capital, or alternatively a certain percentage, 20 per cent or 50 per cent, of the gross profit. Where, as here, it is competition as such that decides, the determination is inherently accidental, purely empirical.
>
> (Marx, 1981, pp. 484–485)

In most mainstream macroeconomic models of the so-called “neo-classical-Keynesian synthesis” type (see, for example, Davidson, 2006), the rate of interest is an endogenous variable that is determined by well-specified interactions of other (exogenous) variables. In Keynes’s model, the interest rate is a purely monetary phenomenon (like with Marx) (Visser, 1977, p. 279).[22] In Chapter 13 of the *General Theory*, Keynes writes:

> [T]he rate of interest at any time, being the reward for parting with liquidity, is a measure of the unwillingness of those who possess money to part with their liquid control over it. The rate of interest is not the “price” which brings into equilibrium the demand for resources to invest with the readiness to abstain from present consumption. It is the “price” which equilibrates the desire to hold wealth in the form of cash with the available quantity of cash; which implies that if the rate of interest were lower, i.e. if the reward for parting with cash were diminished, the aggregate amount of cash which

> the public would wish to hold would exceed the available supply, and that if the rate of interest were raised, there would be a surplus of cash which no one would be willing to hold. If this explanation is correct, the quantity of money is the other factor, which, in conjunction with liquidity-preference, determines the actual rate of interest in given circumstances.
>
> (Keynes, 1936, pp. 167–168)

For Keynes, the rate of interest is the price that has to be paid to the potential hoarder of money for not hoarding it. Given the portion of the money supply that is not held for the transaction and precautionary motive, it depends on the liquidity preference due to the speculation motive of the demand for money. It is the reward that should incite the hoarder to abandon his liquid position. Marx's point of view seems to be more in line with the "loanable funds" theory, like that developed by Dennis Robertson (1890–1963) in a number of papers in the 1920s and 1930s (Robertson, 1940, pp. 2–3) – in spite of Marx's "hoarding demand for money".[23]

In Marx's economic model the rate of interest is not a key element, whereas in the Keynesian model it is an essential element that links the "monetary sphere" to the "real sphere" of the economy, i.e. a given liquidity preference and an available liquidity supply (Dillard, 1984, p. 430)[24] lead to the rate of interest which, in turn, influences enterprises' investment decisions and thus the level of national income and employment.[25]. If we want to integrate the real sphere of production and the monetary sphere into Marx's model, than we should best leave behind the views on the interest rate of *Das Kapital* which, by the way, are not even essential for Marx's model.

8.5 The money supply in post-Keynesian economic theory

The anti-quantity theory of money of *Das Kapital* stood (in its time) in opposition to what orthodox economic theory defended, i.e. that given the velocity of money circulation, the supply of money was directly related to money prices. It will also be remembered from Chapter 1 that, based on Marx's reproduction schemes, a smooth and simple reproduction of the economic system implies that the capitalists advance the required money capital. Normally, such advances are made by using bank credits that are paid back at the end of the time period considered, such that the amount of money in circulation does not increase. In expanded reproduction, when additional means of production and labour have to be financed, the amount of money must increase with the volume of transactions. The money in circulation is therefore endogenous.

In Keynes's *General Theory* and in neo-classical economics, including the so-called "neo-classical-Keynesian synthesis", the money supply is given exogenously and controlled by the monetary authorities.[26] In the usual diagram with the rate of interest on the vertical axis and the relevant money variable on the horizontal axis, the money supply curve is a vertical line. The short-term rate of interest will then depend on the demand for money, i.e. where its curve intersects the supply curve. For all relevant rates of interest, the money supply is the same. In contrast, we have just pointed out that in Marx's monetary theory, the supply of money in circulation is a function of the need for money capital to finance the business transactions and that the demand for money determines the supply of money in circulation. Supply adapts to demand. The supply of money is thus endogenous.

Importantly, Marx's view on the essential endogeneity of the supply of money is also shared with the post-Keynesians, in spite of there being no unanimity on the degree of endogeneity. This evidently is not to say that Marx's monetary views can replace those of the post-Keynesians; rather, it is the other way around, i.e. much of post-Keynesian monetary theory can help to expand the mostly underdeveloped Marxist monetary theory. We will therefore review the arguments of the post-Keynesian authors.

Among the post-Keynesian economists, there is consensus that the money supply is not completely under the control of the monetary authorities and that it changes in an important way through the creation of credit (see, for example, Dow, 2006, p. 37). However, an important group of post-Keynesian economists considers that the supply of money is fully endogenous and the rate of interest is determined by the central bank, and thus the exogenous factor. This view goes back to Kaldor (1970) and Moore (1988), and can be traced to Keynes's *Treatise on Money* (1930). The creation of bank deposits goes together with the lending operations of the banks, with the central bank as "lender of last resort" playing an accommodating role by providing the necessary reserves to the banks. This view is denoted as "horizontalist", i.e. for a given rate of interest (determined by the monetary authorities), the supply of money can be any amount and therefore the money supply curve is horizontal. The endogeneity of the money supply refers to the banking sector being able, on the basis of the exogenous stock of monetary reserves, to create all bank deposits required. Apart from the opinion of notable early post-Keynesians, such as Kalecki, Richard Kahn, Joan Robinson and Nicholas Kaldor, the reason for the "horizontalist" view is mostly found in central bank practice (Lavoie, 2006). It is also denied that an excess supply of money is possible – not even in the case of a government budget deficit (Moore, 2003, p. 118). An excess supply of money is considered to be used to pay back loans. The "horizontalist" view on the full endogeneity of the money supply is evidently at odds with Keynes's liquidity preference theory.

In spite of the (what seems at first sight to be) apparent correspondence between the views of the "horizontalists" and those of Marx (Wray, 2003, p. 264; de Brunhoff and Foley, 2006), it should not be forgotten that with Marx it is the supply of *money in circulation* that is endogenous (because it is determined by the need for money capital arising out of a "transaction-type" motive), and not the total money supply; the money supply created by the monetary authorities in excess of the business requirements will be hoarded.

The "structuralist" post-Keynesians, in turn, are more nuanced than the "horizontalists" (Harcourt, 2006a, pp. 66ff.). They emphasise that the monetary authorities and the banks can influence the volume of credit and therefore also the rate of interest, and that the money supply is not entirely endogenous. The "structuralists" argue that liquidity preference has an important influence on the way credit is created by the banks.[27] The endogeneity of the money supply is due to the structure of the financial system, with the central bank being "lender of last resort", and the financial innovations introduced by the banks to avoid the control of the central bank (Dow, 2006, pp. 36–40). The central bank's control is via the rate of interest on the reserves that it is prepared to lend to the banks. The banks, in turn, lend to the economy at an interest rate that is hardly related to the interest rate fixed by the central bank. This is because of the banks' "degree of monopoly" and the fact that their risk assessment is based on non-quantifiable risk and uncertainty. Given the monetary policy of the central bank, credit creation by the banks depends on their liquidity preference and their strategies.

It should be clear that the "structuralist" post-Keynesian contribution offers important clues for the development and updating of Marx's incomplete monetary theory. That the money and credit supply, in Marx's view, adjust to the money and credit needs, and therefore to the demand for money and credit, is evidently the point of departure that Marx and the post-Keynesians share. The "structuralist" theory of the interest rate as a monetary phenomenon, which depends on the competition between the banks and the way it deals with uncertainty and risk, is in our opinion worth incorporating into Marxist monetary theory. Marx also shares with the "structuralist" post-Keynesians a view on the role of instability in the capitalist monetary economy, an issue that is relevant for the relationship between economic crises and money capital. More on this appears in the next section.

8.6 Economic crisis and the role of money capital

For our purposes, it is important to reflect on how Marx attempts in *Das Kapital* to link the monetary phenomenon to that of an economic crisis – all the more so as it is well known that Marx's theory of the business cycle was ground-breaking and preceded many of the insights of today.

It has been noted that with Marx and Kalecki, the relationship between the sphere of money and credit is lacking. Both developed a pioneering "real" business cycle theory, but did not integrate the monetary sphere of the economy into such a theory. Notwithstanding this, *Das Kapital* contains many indications that it was Marx's intention to do so, but as Volumes 2 and 3 (edited and published by Friedrich Engels) show, as well as the many notebooks that Marx left behind, he did not succeed.

In an additional explanatory footnote to the posthumous third German edition of Volume 1 of *Das Kapital* of 1883, Marx writes:

> The monetary crisis, defined in the text as a particular phase of every general industrial and commercial crisis, must be clearly distinguished from the special sort of crisis, also called a monetary crisis, which may appear independently of the rest, and only affects industry and commerce by its backwash.

> The pivot of these crises is to be found in money capital, and their immediate sphere of impact is therefore banking, the stock exchange and finance.
>
> (Marx, 1976, p. 236 n)[28]

It is interesting that this passage was added between the second edition of 1873 and the third edition of 1883.

In Volumes 2 and 3 of *Das Kapital*, further mention is made of financial and monetary instability which contributes to the downturn in the economic cycle. A boom is often accompanied by a speculative bubble, which bursts and causes the downturn. Such bubbles can develop in various markets: markets for raw materials, money markets, capital markets, and so on. Marx indicates, for instance, that the output of animal and plant origin, which is used as raw material for further processing, is relatively inflexible compared to that of machines and other investment goods (Marx, 1981, p. 213). This will lead during a boom period in the business cycle to scarcity and volatile and rising prices of raw materials, which often (though not automatically) culminates in a deterioration in the business climate and a downturn (Marx, 1981, pp. 213, 216). In Chapters 25 and 26 of Volume 3 which, like Chapter 33, are punctuated by long quotations from British official documents and British financial experts, as well as insertions by Engels (as Marx's notes are too fragmentary), the role of speculation and of increased speculative investments is said to detonate a crisis. Marx stresses the 1847–1848 crisis in a lengthy comment he makes about the views expressed by Lord Overstone:

> What Overstone is trying to prove is that the crisis of 1847, and the high rate of interest that accompanied it, had nothing to do with the "quantity of money" present (. . .); although it actually did have something to do with it, as soon as fear of exhaustion of the Bank's reserve (and this was a creation of Overstone's) added monetary panic to the 1847–8 crisis. But this is not the point here. There was a dearth of money capital brought about by the excessive size of operations, in comparison with the means available, and brought to a head by a disturbance in the reproduction process that resulted from the harvest failure, the over-investment in railways, overproduction particularly in cotton goods, swindling in the Indian and Chinese trade, speculation, excessive imports of sugar, and so on.
>
> (Marx, 1981, p. 550)

Marx adds further on:

> If the rate of interest rose to a very high level, this was simply because the demand for money capital grew still more quickly than the supply, which means that, as industrial production expanded, it was conducted to a greater extent on the basis of credit.
>
> (Marx, 1981, p. 553)

8.7 Notebook B 113

Deviating from our rule that we would only use *Das Kapital* as a source, we must at this point introduce Marx's Notebook B 113 since it contains many important hints and clues about financial crises and instability. The Notebook, which is in the Marx-Engels Archive of the International Institute of Social History in Amsterdam, is filled with notes, excerpts and critical annotations of Marx's reading in January and February 1869 of a number of sources on monetary and financial questions, among which are various references to the 1866 crisis. Although Volume 3 of *Das Kapital* was published in 1894 and dwells on the crisis of 1847, the notes of B 113 are not used; nor is the 1866 crisis mentioned.[29] It is for this reason that they were recently investigated by a team of Brazilian researchers (de Paula *et al*., 2011).

Almost half of the 139-page Notebook B 113 contains literal excerpts from *The Economist* and *The Money Market Review* of 1868, as well as Marx's reading notes in his characteristic densely written, minuscule and hardly legible handwriting. Using Marx's references to specific issues and articles, the researchers identified what Marx found to be the most important themes and consequently what could have been of use for *Das Kapital*. They showed that particular notes in B 113 were used by Marx while editing Volume 2 of *Das Kapital*. It is surprising that there was no trace of these notes in Volume 3, which is probably due to a lack of time and the health problems of its author.[30]

It is clear from Marx's notes that he considers the 1866 crisis to have been caused by speculation and financial fraud, which explains his interest in accounting practices in financial reporting in a world of increasing entanglement between new industries and the banks, of false or misleading company information and of many newly emerging limited companies in the financial sector. It is also interesting to see, based on other parts of Notebook B 113, that Marx familiarised himself with the well-worn argument that, in the event of a banking crisis (as occurred in 1866), the central bank in England should act as the "lender of last resort", a novelty at the time.

Notebook B 113 – but also Notebooks B 108 and B 109 – contain verbatim quotes from sources that deal with the financial crisis of 1866, including extensive commentaries by Marx. De Paula *et al*. (2011) indicate that Notebook B 113 contains a coherent set of notes out of five related sources that are relevant for his handling of the crisis of 1866. In our opinion, the relationship between the notes and Marx's comments in *Das Kapital* on general economic downturns and financial crises is obvious. However, it is debatable whether the notes offer more than rudimentary insights into Marx's method and his interest in the theme.

In the previous section we saw how Marx in Volume 3 of *Das Kapital* addresses the financial crisis of 1847 and apparently emphasises that it is different from a "normal" cyclical downturn, caused by labour shortages, overinvestment and a stagnating or falling rate of profits. Although Engels inserted these passages, they do not offer us a complete theory. It is illusory to think that Notebook B 113 (or B 108 and B 109, for that matter), of which no trace seems to have been found in Volume 3, can provide a detailed or coherent vision of the issue of financial and monetary instability and its role at the onset of an economic crisis.

Nonetheless – or maybe just because – it is also relevant to mention the importance of the post-Keynesian insights, since they offer a much-needed complement to Marx's unfinished thoughts on financial and monetary instability. How, in a world of incalculable uncertainty, the risk assessment of the banks and their liquidity preference over the economic cycle is changing and creating a credit cycle that at times amplifies the economic cycle and at other times generates it (a vision that goes back to Keynes's *General Theory*) is convincingly described by Dow (2006, pp. 41–42, 46–49). This insight is also the starting point of Minsky's theory of endogenous instability of capitalism. We will briefly come back to this in Chapter 9 in the discussion on the destabilising impact of "financialisation".[31]

8.8 To conclude

The plausibility of the monetary theory of *Das Kapital* in the economic theory of Marx is on the whole debatable. Attempts by Marxists and Marxist-inspired authors to link it to the labour value of gold as a measure of value, thereby creating a direct link between labour values and money prices, are for the educated economist these days unconvincing. This link can only be relevant to a money economy where money circulation is determined by the gold reserves of its central bank.

Today, gold plays a very limited role as a monetary reserve and its use by central banks of countries that are members of the International Monetary Fund is even forbidden.[32] It is true that in times of global monetary instability, many investors will "dive" back into gold. In such times, which fortunately are rare, the value of money will depreciate sharply. This value is mostly derived not from any precious metal but from confidence that it is accepted as a means of exchange and is used both internally and internationally.[33] As a result, there is no relationship between the conditions under which gold is produced and its labour value, such that the monetary phenomenon ends up outside the Marxist economic model.

Using the so-called MELT, which equates the total money value of output to the total amount of labour time spent in a given period, it is possible to connect the Marxist theory of value to the monetary phenomenon. We are reluctant to go any further in this regard, however, for fear of skating on thin ice.

In Volume 1 of *Das Kapital*, the labour value of the general equivalent (e.g. gold) determines the value of output in money terms. Our analysis has shown that Marx also attempted to formulate a monetary theory in Volumes 2 and 3 of *Das Kapital*, unconnected to that in Volume 1. It is fascinating to read the passages from Volume 3 of *Das Kapital* asserting that monetary crises are essentially different from general economic crises – the business cycle. These are, however, heavily based on Marx's reading notes and are not truly integrated into the rest of his theoretical exposition. In reading these passages, one can ask, somewhat provocatively, whether it could have been Marx's intention (taking into account the stage of his research into these issues) to integrate his monetary theory and his value theory.

Pioneering work has been performed in the past, starting with crucial insights into Marx's economic theory on the development, movement and reproduction of capitalism and leading to the development of a neo-Marxist theory of economic growth. Some of these theories were reviewed and evaluated in previous chapters. From the above analysis of the monetary theory and insights of Marx, it now appears that a reverse movement should be made to integrate Marx's monetary theory with the Keynesian and post-Keynesian monetary theory, with which it shares a number of characteristics.

In Marx's monetary theory, the supply of money by the banks and the financial sector mostly accommodates the business demand for money and credit, which is in an important way, so Marx repeatedly argues, determined by the transaction needs of the capitalist businesses. Like in post-Keynesian economic theory, the money supply in Marx's model is thus largely endogenous, in contrast to what the neo-classical "Keynesians" defend. We have also argued that in Marx's monetary theory, part of the demand for money is driven by what (since Keynes) is called a speculative motive. This implies that both Marx and Keynes reject the quantity theory of money. Finally, in Marx's view, as in Keynes's view 50 years later, the rate of interest is unrelated to the "productivity of capital", as the liberal and neo-classical economists ("vulgar economists", according to Marx) hold, but is a purely monetary phenomenon.[34] Unfortunately, Marx's theory of the rate of interest is not sufficiently well developed, but a neo-Marxist economic model – like the original Keynesian model – could easily connect the "real economy" to the "monetary sphere".

Much work still needs to be done to introduce the factors of uncertainty and expectations in such a model. Again, these can be found in contemporary post-Keynesian economic literature[35] where they are also considered to be at the root of much economic and monetary instability. Although mentioned in *Das Kapital*, the roles of uncertainty, unquantifiable risk and expectations are not systematically analysed and integrated. They are of great importance in a realistic economic model, built on Marx, in which both the "real" sphere and the "monetary" sphere of the capitalist money economy are connected, thus aiming to provide a "general" theory.

Notes

1 The "law" is named after the French economist, Jean-Baptiste Say (1767–1832). The economists of the first half of the nineteenth century strenuously debated Say's law. In a nutshell, the most widely held view is that the income generated in the production process will be spent on the output produced, including the savings on capital goods. Marx was the first to arrive at a thorough criticism of Say's "law of the markets", emphasising the role money plays in breaking the unity between selling and buying. See, for example, Sardoni (2003).

2 It should be stressed that what is considered a "normal" or "conventional" profit share in income and, likewise, the employment rate can change slowly due to sustained short-run deviations in the same direction. Therefore, a case could be made for their long-run interaction if it is assumed that both capitalists and workers react to changes in "conventional" income shares and employment rates.

3 In his model of the short run, Kalecki also takes into account a time lag between $I_{t-\omega}$ (the investments at time $t - \omega$) and the thus realised profits P_t (the profits in t).

4 We deduce $\delta = \alpha\,\delta + d'$ from $I_{t+\theta} = \alpha\, I_t + \beta\,\frac{\Delta I}{\Delta t} + d'$ and get: $I_{t+\theta} - \delta = \alpha\, I_t + \beta\,\frac{\Delta I}{\Delta t} + d' - \alpha\,\delta - d'$. Since $I_{t+\theta} - \delta = i_{t+\theta}$, and δ is constant, simplification will give $I_{t+\theta} = \alpha\, i_t + \beta\,\frac{\Delta i}{\Delta t}$.

5 Kalecki (1968) finally introduces the repercussions of technical progress such that his investment function includes F(t), a factor that incorporates the additional stimulus of innovation and the non-profits-determined part of capitalist consumption, and that changes slowly over time, depending on past social, economic and technological developments. Based on the relevant parameter values, this model yields a trend and a cycle in investment and economic growth, with technical progress affecting the dynamics involved. See also Chapter 9.

6 See Steindl (1952, pp. 236–237), Baran and Sweezy (1966, pp. 85, 145). The British Marxist economist, Maurice Dobb, who greatly admired Kalecki (as gleaned from personal communication with Geoff Harcourt) and who over a number of decades set the tone in the discussions on Marx, stated that with Marx as well as with Ricardo, a nominal wage increase automatically leads to a real wage increase, since they both assume a commodity-money standard (Dobb, 1973, p. 224 n).

7 This is not the place to delve further into this. It suffices to point out that the Keynesian accelerator mechanism goes back to Harrod (1936, 1939), and particularly Samuelson (1939a, 1939b). Kalecki developed his investment decisions function in the same year, if not earlier. An alternative post-Keynesian business cycle theory was developed during the same period by Kaldor (1940), who asserted that savings and investments are an increasing, non-linear function of income, while investments are a decreasing function of the capital stock, which generates an endogenous cycle in economic activity. See Hudson (1957) for an excellent review and further elaboration of the Keynesian trade cycle theories.

8 See, for example, Moseley (2005, pp. 1–5) or Weeks (2011).

9 Williams (2000) makes abundantly clear that a money commodity is redundant in terms of the logic of Marx’s system and is not needed for his theory of capitalist reproduction. For interesting digressions on the money commodity aspect in Marx, the value of inconvertible money and present-day capitalism, see Foley (1983, 2005).

10 This concept was introduced by Gérard Duménil and Duncan Foley, but independently of each other. See Duménil (1980, 1983) and Foley (1982, 1986).

11 The advocates of the “New Interpretation” allege that the labour values system, which we formulated in Chapters 2 and 4, is redundant for the determination of the rate of exploitation, the profits and the wages, the rate of profits, etc. However, the proof that profits are based on unpaid labour and exploitation, as well as the precise relationship between labour values and prices of production, remain theoretically important.

12 This is not the imaginary economy of Chapter 4, in which we analysed Marxian unproductive labour. The total labour time used in the MELT is the productively spent labour time, and thus it is tacitly assumed that the issue of what is productive and what is not has already been settled.

13 For Foley, the value of labour power is simply the money wage per working hour divided by the MELT.

14 It will be recalled that Volumes 2 and 3 were posthumously compiled and edited by Friedrich Engels, based on the manuscripts and notebooks left by Marx. Volume 2 appeared in 1885 and Volume 3 in 1894. Engels struggled in particular with Volume 3. See Engels: “Preface”, in: Marx (1981, pp. 92–94).

15 It is therefore strange that an authority such as Ernest Mandel in his monumental *Late Capitalism* has built up theoretical arguments based on the quantity theory of money. See, for example, Mandel (1975, pp. 421–422). For our critical review of Mandel, see Cuyvers (1978). For an excellent account of Marx's monetary theory of hoarding and its importance for underemployment, see Sardoni (2011, Ch. 2–3).

16 That supply creates its demand is an interpretation of Say's "loi des débouchés" by Keynes. In fact, a distinction should be made between Say's identity and Say's equality. The classical economists were often confused about both versions. The same authors were both advocating the so-called "classical dichotomy" with relative prices determined by supply and demand and absolute prices by the amount of money in circulation, and, at the same time, analysing the impact of the demand for money on the demand for goods and services. Blaug (1968, pp. 146–153) states that Say's equality is the dynamical version of Say's law in the analysis of the short run, whereas Say's identity is the long-term version. Sardoni (2003, p. 311) states, however: "Classical economists, by assuming that saving is investment, accepted the law as an identity (the equality between aggregate supply and demand is always true); neo-classical economists, by concentrating on equilibrating mechanisms, accepted the law as an equality, which is true only in equilibrium."

17 The present-day versions of the quantity theory, including that of Milton Friedman, are a case in point. Marx's criticism of the quantity theory of money as neglecting this "asset motive" no longer applies to these versions.

18 This view is advocated by the monetarists of today and is largely based on Friedman (1956).

19 In his *General Theory*, Keynes distinguishes four motives behind the demand for money: the income motive, the business motive, the precautionary motive and the speculative motive (Keynes, 1936, pp. 195–196). What we call, for the sake of convenience, the transaction and precautionary demand for money is in fact related to the first three motives listed by Keynes. In Keynes (1937), a finance motive for the demand for money by the banking sector is also mentioned.

20 In the original German text: "die Bedürfnisse des Verkehrs".

21 This has been strenuously argued in Likitkijsomboon (2005).

22 While offering a good overview of Marx's monetary theory – in spite of not going into Marx's views about the monetary crisis being unrelated to an economic crisis – Visser (1977) is wrong in stating that Marx has no interest theory.

23 We will see in the next section that post-Keynesian monetary theory contains a more nuanced model of demand and supply of money and bank credit than the monetary theory that Keynes originally developed in his *General Theory*. See Arestis (1992, pp. 186ff.). However, in post-Keynesian monetary theory, the rate of interest is ultimately controlled by the central bank. See Dow (1997).

24 Dillard also points out that, in contrast to the classical and neo-classical economists, Marx and Keynes developed a monetary theory of production (Keynes's terminology). In another paper (Dillard, 1980) he draws a similar parallel between Keynes and Thorstein Veblen and the institutionalists, thus indicating that in the history of economic thought, the integration of Marx, Keynes and institutionalism has laid the foundation for the present-day "post-Keynesian school". Unlike his treatment of the money supply in his *Treatise on Money*, in the *General Theory* Keynes treats the money supply as essentially exogenously determined, in contrast to contemporary post-Keynesian thought. There is more on this in section 8.5. See Arestis (1992, pp. 180ff.).

25 Over the decades, Keynesians, "bastard Keynesians" (according to Joan Robinson) and neo-classicists alike have brought many changes and adaptations – unfortunately, not always for the good – to Keynes's arguments and theses. For an early, influential and readable overview of Keynes's monetary theory and the rate of interest as a monetary phenomenon, and how it relates to investments, employment and the national income, see Dillard (1948, Ch. 8, pp. 161ff.).

26 This, in any case, was originally so. In fact, the endogeneity of the money supply in these theories is limited, e.g. as a result of changes in the international reserves position which depends on the balance of payments. Moreover, it is convincingly argued that Keynes assumes in his *General Theory*, for the sake of his subsequent arguments, that the money supply is given, but not exogenous. See Dow (1997) in this regard.
27 For a thorough review of the points of view on this issue, see Bell (2003).
28 The 1976 Penguin edition of Capital states that the footnote is added by Friedrich Engels. In the 1909 Kerr edition, as in the 1932 German edition by the Marx-Engels-Lenin Institut in Moscow, this footnote is added without any further indication, implying that it is the work of Marx (Marx, 1909, p. 155 n.1).
29 It is likely that Marx, while editing the third edition of 1883, added the explanatory note mentioned above on the difference between a monetary crisis and a general production and commercial crisis (Marx, 1976, p. 236 n), while reflecting on the course of events during the 1866 crisis.
30 Until his death, Karl Marx – together with Friedrich Engels – devoted much time to ideological and organisational aspects of the First International, established in 1864, and of the emerging socialist parties in Europe. In later years, his own health problems and those of his close family members seriously limited his ability to continue his research. He nevertheless carried on working, although sometimes – as in 1881 – on topics for reasons difficult to explain, such as geology (see the quotation from the 1923 Riazanov report to which João Antonio de Paula refers) (de Paula *et al.*, 2011, pp. 7–8). In early December 1881, Marx's wife died after having suffered from liver disease for a long time, and in early January 1883 his daughter Jenny died. Both deaths had an immense impact on Marx. During the last years of his life he was also repeatedly struck down by bronchitis and pneumonia, which on his doctor's advice prompted him to escape damp and chilly England. For this episode in Marx's life see, for example, Fedossejew *et al.* (1975, pp. 558ff., 655ff., 774ff.).
31 Minsky (1975, Ch. 6, pp. 115ff.) builds on Keynes's views on speculation and instability. Minsky considers his view not as a theory but as a hypothesis. For an overview of Minsky's contribution to post-Keynesian monetary theory, see de Antoni (2006).
32 In an otherwise thorough analysis of Marx's theory of money, Lapavitsas (1991, p. 319) states that the international distribution of gold reflects the economic weight of countries. This is only partially correct, as is demonstrated by China, the international reserves of which represent only 1.6% of gold.
33 We follow Bellofiore (2005, p. 138) to the extent that gold retains its key role during monetary crises. Global monetary crises are fortunately rare and therefore a function of the credit system, detached from the "commodity basis" of the monetary system (Bellofiore's terminology). Such crises, being exceptional, are unable to supply a crucial element in a theory of the process of reproduction of the capitalist system.
34 This is – incorrectly, we think – denied by Smithin (2006, p. 276), who states that with Marx high interest is correlated with high profits/surplus value.
35 An impressive and recent "state-of-the-art" collection of post-Keynesian theories is available in Harcourt and Kriesler (2013).

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9 Long-term developments – underconsumption, stagnation, long waves and financialisation

It was seen in Chapter 6 that the so-called tendential fall in the average (or general) rate of profits, attributed to the increasing replacement of labour with capital, provides (to say the least) insufficient and probably no explanation whatsoever for the long-run developments in capitalism. Hence, it also offers an inadequate explanation for the prolonged period of economic stagnation that the capitalist industrialised countries have been experiencing since the first half of the 1970s.

In this chapter we will investigate another economic mechanism, based on important passages in *Das Kapital*, which Marx also considered to be of particular relevance for understanding long-run capitalist development. In turning our attention to this, we come back to the issue of the insufficient realisation of surplus value, which was briefly discussed in Chapter 3. There it was shown how the rate of economic growth in a capitalist economy relates to the rate of capital accumulation, on which the realised rate of profits, in turn, depends. *Das Kapital* refers to a mechanism of stagnation of capitalist development, which is associated with insufficient demand due to limited consumption possibilities compared to what can be produced. In addition, Marx's schemes of reproduction provide insights into the impact of insufficient capital accumulation on the realisation of surplus value. These mechanisms will be reviewed in the next section.

Marx views insufficient surplus value realisation as relevant when one is analysing developments both in the short run (the business cycle) and in the long run. In this chapter we will be focusing on the latter. Although an extension of Marx's views, the long-run relationship between the accumulation of capital and economic activity (particularly the hypothesised tendency towards stagnation and underspending under capitalist conditions) has been analysed in more recent times – first and most prominently by Rosa Luxemburg who emphasised the importance of "external markets". Although Luxemburg regards "external markets" primarily as those geographical territories elsewhere in the world that have not yet been infused with the capitalist mode of production and exchange, military expenditure and technological innovations can play a similar role. This has been the subject of analysis by the post-Keynesian neo-Marxists mainly, such as Kalecki, Steindl, Baran, Sweezy and others, and we will therefore devote attention to such analyses. To the extent that the accumulation of capital gives rise to more or less regular surges in the rate of accumulation, this can lead to "long waves" in economic activity. These Kondratieff cycles allegedly show a wavelength of approximately

50 years. Why, then, after the elapse of nearly five decades since the late 1960s, have the capitalist economies not been swept up in the positive trajectory of the next Kondratieff? Some scholars attribute the prevailing economic instability to the "financialisation" of monopoly capitalism.

Being neo-Marxist and post-Keynesian, these views and analyses are evidently not found in Marx – although they are based on his thesis of the unequal development of forces of production and relations of production, and give the nod to his occasional remarks about the capitalist tendency to underspend. Giving them attention in this chapter therefore seems warranted.

9.1 Underspending and incomplete realisation of surplus value in *Das Kapital*

As outlined before, Marx makes a clear distinction between produced and realised surplus value. In a well-known passage in Volume 3 of *Das Kapital*, he writes:

> As soon as the amount of surplus labour it has proved possible to extort has been objectified in commodities, the surplus-value has been produced. But this production of surplus-value is only the first act in the capitalist production process, and its completion only brings to an end the immediate production process itself. (. . .) Now comes the second act in the process. The total mass of commodities, the total product, must be sold, both that portion which replaces constant and variable capital and that which represents surplus-value. If this does not happen, or happens only partly, or only at prices that are less than the price of production, then although the worker is certainly exploited, his exploitation is not realized as such for the capitalist and may even not involve any realization of the surplus-value extracted, or only a partial realization (. . .). *The conditions for immediate exploitation and for the realization of that exploitation are not identical.* Not only are they separate in time and space, they are also separate in theory. *The former is restricted only by the society's productive forces, the latter by the proportionality between the different branches of production and by the society's power of consumption.* And this is determined neither by the absolute power of production nor by the absolute power of consumption but rather *by the power of consumption within a given framework of antagonistic conditions of distribution, which reduce the consumption of the vast majority of society to a minimum level*, only capable of varying within more or less narrow limits. It is further restricted by the drive for accumulation, the drive to expand capital and produce surplus-value on a larger scale.
>
> (Marx, 1981, pp. 352–253; our italics)

Because of the hunger for profits and enrichment and the accumulation of capital, capitalism tends to be engulfed in a recurring conflict between the increasing output of use values and the limited outlets for this output (Marx, 1981, pp. 358–359; Marx, 1978, p. 391n).[1]

In the same vein is Marx's statement that with the development of capitalism, the scale of production is less determined by demand and more driven by the available capital of the individual capitalists, and that as a result an ever-growing volume of output must find its way to buyers so that the surplus value can be realised (Marx, 1978, p. 221).

Evidently, so Marx points out, the growth of the population will give rise to additional needs for consumer goods but these will only be satisfied if supported by additional solvent demand (Marx, 1981, pp. 289–290; see also Marx, 1981, p. 282). If this conflict between production and realisation possibilities is to be resolved, the market needs to expand, particularly the foreign market (Marx, 1981, pp. 344, 353). This inevitably brings us to the later development of this thesis in Rosa Luxemburg's theory of "external markets".

Pressurised by competition, each capitalist enhances his production capability but does not ensure a corresponding increase in sales and realisation possibilities. This contradiction implies that capitalism is confronted by an inherent tendency towards stagnation, which is different from stagnation arising from the tendentially falling rate of profits. The passage introduced above, in which Marx stresses the difference between produced and realised surplus value, continues:

> This [the drive to expand capital] is the law governing capitalist production, arising from the constant revolutions in methods of production themselves, from the devaluation of the existing capital which is always associated with this, and from the general competitive struggle and the need to improve production and extend its scale, merely as a means of self-preservation, and on pain of going under. *The market, therefore, must be continually extended* (. . .). The internal contradiction seeks resolution by extending the external field of production. *But the more productivity develops, the more it comes into conflict with the narrow basis on which the relations of consumption rest*. It is in no way a contradiction, on this contradictory basis, that excess capital coexists with a growing surplus population; for although the mass of surplus-value produced would rise if these were brought together, yet this would equally heighten the contradiction between the conditions in which this surplus-value was produced and the conditions in which it was realized.
>
> (Marx, 1981, p. 353; our italics.)

As Marx explains, insufficient surplus value realisation stems from a tendency towards underspending. Because of the limited purchasing power of the working class – which, in turn, is due to the capitalist need to limit wage increases – consumer goods remain unsold and the surplus value that they contain is not realised. A growing contradiction between the ever-developing forces of production and the constrained consumption possibilities will then result in a rate of profits that is lower than the desired rate, leading to less capital accumulation and ultimately economic stagnation.

We came across this mechanism in Chapter 1 while reviewing the conditions of expanded reproduction. With reference to Marx, we elaborated on a two-sector scheme of reproduction. Assuming sector 1 produces investment goods and sector 2 consumer goods, we found that, neglecting capitalist consumption, these conditions were:

$$V_1 + M_1 = C_2 + \Delta C_1 + \Delta C_2$$
$$C_2 + M_2 = V_1 + \Delta V_1 + \Delta V_2$$

with C_i, V_i and M_i the constant capital, the variable capital and the surplus value in sector i (i = 1 or 2), and ΔC_i and ΔV_i the additional investment in constant and variable capital in sector i (i = 1 or 2).

The first condition indicates that in the event of a decline in the accumulation of constant capital (a decline of $\Delta C_1 + \Delta C_2$), the surplus value in sector 1 (M_1) will remain unrealised *pari passu*. The reason for a decline in accumulation leading to a decline in the realised surplus value M_1 can be traced to the assumption that C_2 and V_1 are in any case replaced for the sake of simple reproduction.

The second condition states that with a decline in the accumulation of variable capital (a decline of $\Delta V_1 + \Delta V_2$), the surplus value in sector 2 (M_2) is not completely realised. This is based on the same assumption that the condition of simple reproduction is met.

The relationship between insufficient accumulation and insufficient surplus value realisation becomes even clearer if both conditions of expanded reproduction are rewritten after rearrangement of the terms. This effectively leads to the equation:

$$M_1 - (\Delta C_1 + \Delta C_2) = C_2 - V_1 = (\Delta V_1 + \Delta V_2) - M_2$$
$$\text{or:} \quad M_1 + M_2 = (\Delta C_1 + \Delta C_2) + (\Delta V_1 + \Delta V_2)$$

If $(\Delta C_1 + \Delta C_2) + (\Delta V_1 + \Delta V_2)$ plummets, $M_1 + M_2$ (the realised surplus value) will decline *pari passu*.

Alternatively, if $((\Delta C_1 + \Delta C_2) + (\Delta V_1 + \Delta V_2))/(C_1 + C_2 + V_1 + V_2) = g$ declines in the long run, then $(M_1 + M_2)/(C_1 + C_2 + V_1 + V_2) = r$ (the realised rate of profits) will also fall. In fact, we know this from Chapter 3.

In Chapter 1, in our short introduction on Rosa Luxemburg's theory, we pointed out that she emphasised that only averages over the business cycle of the inter-sectoral supplies are of relevance for the study of the conditions of the capitalist reproduction process and that Marx's schemes of reproduction should be regarded in this way (Luxemburg, 1951, pp. 36ff.).[2] This implies that if the above conditions for expanded reproduction are not met, a long-run tendency towards stagnation is introduced in Marx's model.

From Chapter 3 it will be remembered that in the Robinsonian analysis, the interaction between the capitalists' urge to accumulate, plus their profit expectations, and the profit realisation mechanism ultimately explains the realised rate of profits (see Figure 3.4). This also provides a clear explanation for the difference between produced and realised surplus value (profits). In Figure 9.1,

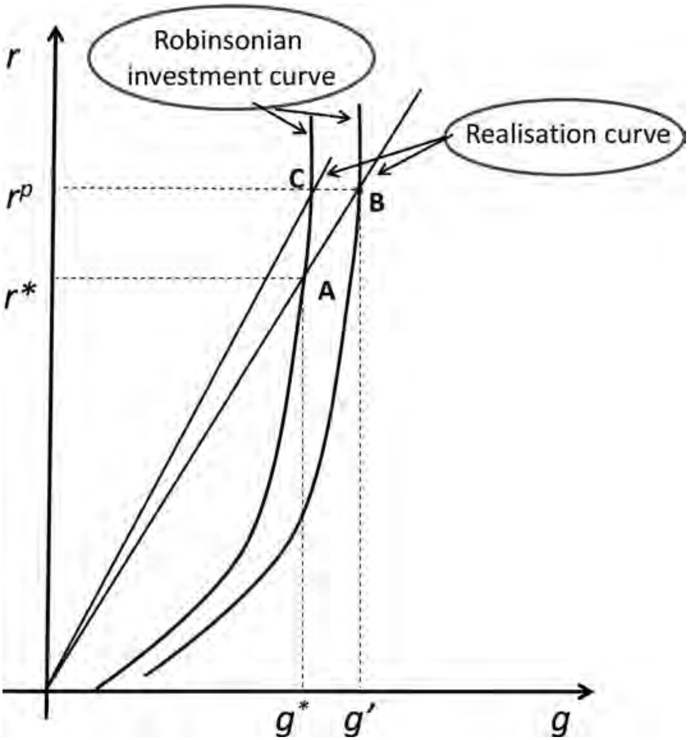

Figure 9.1 Realisation problems with underspending

if an insufficient urge to accumulate pushes r^* below the potential rate of profits r^p (based on the produced surplus value which, in turn, is the outcome of the deployment of labour power and its exploitation), part of the produced surplus value will not be realised. The economy settles at point A and $r^* < r^p$. A realisation crisis then develops. Harris (1975, p. 335) indicates that it is immaterial whether this crisis is attributed to underinvestment or underconsumption. True, if the capitalists were more eager to invest, the position of the Robinsonian investment function would be more to the right and point B would be reached. However, if their consumption rose by a sufficient amount, the Robinsonian realisation curve would turn counterclockwise, as shown, and the economy would reach point C. In both situations, $r^* = r^p$. We would therefore prefer to explain this situation as one of underspending, which leaves open the question of whether underinvestment or underconsumption is at the root of the incomplete realisation of surplus value.

9.2 Underspending based on the schemes of reproduction

How exactly underspending arises can easily be shown by using the schemes of reproduction of *Das Kapital*.

Consider the following case of expanded reproduction:

$$\text{Sector 1 (means of production):} \quad 900(C_1) + 450(V_1) + 450(M_1) = 1800$$
$$\text{Sector 2 (consumer goods):} \quad 450(C_2) + 225(V_2) + 225(M_2) = 900$$

It is evident that the rate of surplus value in both sectors is equal to 100%: $M/V = \sigma = 1$, which corresponds to a 50% share of wages in value added, $M/(V + M)$. It is assumed that the value composition of capital is the same in both sectors: $C/V = 2$, such that the average rate of profits in both sectors amounts to: $M/(C + V) = 33.3\%$.

In addition, it is assumed for the sake of simplicity that the capitalists are not consuming.[3] In sector 1 the capitalists replace and accumulate, respectively, $900(C_1) + 300(\Delta C_1)$, after which a value of means of production of 600 remains, which has to be exchanged with sector 2. In sector 2 the workers are supplied with $225(V_2) + 75(\Delta V_2)$, such that here, too, a value of consumer goods of 600 remains to be exchanged with sector 1. This takes place by sector 2 supplying the workers of sector 1 with $450(V_1) + 150(\Delta V_1)$.

In the next period, the reproduction scheme then becomes:

$$\text{Sector 1:} \quad 1200(C_1) + 600(V_1) + 600(M_1) = 2400$$
$$\text{Sector 2:} \quad 600(C_2) + 300(V_2) + 300(M_2) = 1200$$

This is a case of expanded reproduction which warrants a rate of expansion of 33.3%, unless changes occur in the rate of surplus value and/or the composition of capital.

If, in the next period, the rate of surplus value increases from 100% to 150%, the following situation arises:

$$\text{Sector 1:} \quad 1200(C_1) + 600(V_1) + 900(M_1) = 2700$$
$$\text{Sector 2:} \quad 600(C_2) + 300(V_2) + 450(M_2) = 1350$$

The total value added in our imaginary economy is then $600(V_1) + 900(M_1) + 300(V_2) + 450(M_2) = 2250$ units of value, and if all output produced can be sold, then the realised rate of profits is 50%.

Further assuming, like before, that the rate of accumulation amounts to 33.3%, the replacement and accumulation of capital in sector 1 represent $1200(C_1) + 400(\Delta C_1)$, and means of production with a total value of 1100 remain in that sector to be exchanged with sector 2. However, given the same rate of accumulation, sector 2 will only demand means of production with a value of 800: $600(C_2) + 200(\Delta C_2)$. Moreover, sector 2 supplies its workers with: $300(V_2) + 100(\Delta V_2)$, such that consumer goods with a value of 950 remain to be exchanged with sector 1, of which only $600(V_1) + 200(\Delta V_1)$ can be supplied to the workers of sector 1. Therefore, consumer goods with a total value of 150 remain unsold. Owing to the increase in potential profits (the produced surplus value), which is not matched by additional spending out of those profits, underspending occurs.

What happens if the rate of accumulation becomes 50% instead of 33.3%? In sector 1 the capitalists will then replace and accumulate $1200(C_1) + 600(\Delta C_1)$, and means of production with a total value of 900 will remain to be exchanged with sector 2. Sector 2 supplies its workers with $300(V_2) + 150(\Delta V_2)$, and consumer goods with a value of 900 remain. This value is exactly equal to $600(V_1) + 300(\Delta V_1)$, the value of the consumer goods that can be supplied to the workers of sector 1. *The underspending that we detected in the previous example is thus due to the increased profits relative to the wage bill – a ratio that increases by 50% (from $\sigma = 1$ to $\sigma = 1.5$), and a rate of accumulation that has not increased to the same extent (actually, in our example it remained unchanged at 33.3%). If the rate of accumulation also increases by 50% (from 33.3% to 50%), a case of expanded reproduction arises once more.*

There is no reason to expect this increase in the rate of accumulation, however. The capitalists will only accumulate more if the increased surplus value is also realised. Each capitalist's decision to accumulate his profits as capital is based on his expectations about the future which, in turn, are partly shaped by his past experiences. If, ultimately, the entire surplus value is not fully realised, the accumulation of capital will lag behind. The unsold consumer goods will search for an outlet in an external market, i.e. a market outside of the economy that the scheme of reproduction depicts.

An analogous situation to the above example is encountered if, with the rate of surplus value remaining unchanged, the composition of capital increases during the second period. This is illustrated below, starting with:

$$\text{Sector 1:}\quad 900(C_1) + 450(V_1) + 450(M_1) = 1800$$

$$\text{Sector 2:}\quad 450(C_2) + 225(V_2) + 225(M_2) = 900$$

In the next period, C/V increases, for instance, from 2 to 3, such that the reproduction scheme transforms into:

$$\text{Sector 1:}\quad 1350(C_1) + 450(V_1) + 450(M_1) = 2250$$

$$\text{Sector 2:}\quad 675(C_2) + 225(V_2) + 225(M_2) = 1125$$

After sector 1 has satisfied the needs for intra-sectoral inputs of $1350(C_1) + 450(\Delta C_1)$, a value of means of production of 450 remains for supplying sector 2. In sector 2 the intra-sectoral supplies to its workers amount to $225(V_2) + 75(\Delta V_2)$, such that a value of consumer goods of 825 remains to be exchanged with sector 1. However, the workers of sector 1 only need $450(V_1) + 150(\Delta V_1)$. Again, the underspending phenomenon appears.

Evidently, over an extended period of time, it is also possible for *decreases* in the rate of surplus value or the value composition of capital to occur. We briefly investigate the consequences of this by starting again from the initial situation in the above example and simulating the impact in the next period of a decline in the rate of surplus value from 100 to, for example, 50%. The scheme of reproduction in the next period is:

Sector 1: $1200(C_1) + 600(V_1) + 300(M_1) = 2100$

Sector 2: $600(C_2) + 300(V_2) + 150(M_2) = 1050$

First of all, it can be pointed out that $300(M_1)$ is not sufficient to allow an increase in the constant capital at a rate of 33.3% in sector 1. It therefore has to be assumed that surplus value is flowing from sector 2 to sector 1 (e.g. through the banking sector). In sector 1 the capitalists replace and accumulate $1200(C_1) + 400(\Delta C_1) = 1600$ (assuming a rate of capital accumulation of 33.3%), after which a value of 500 remains in sector 1 to be exchanged inter-sectorally. Sector 2 supplies its workers with $300(V_2) + 100(\Delta V_2)$, such that 650 remains to be exchanged with sector 1; however, sector 1 needs $600(V_1) + 200(\Delta V_1) = 800$ of consumer goods. Thus, consumer goods with a value of 150 are insufficient to enable an adequate number of workers to be employed at the going real wage rate to man the means of production of $1200(C_1) + 400(\Delta C_1) = 1600$. The expanded reproduction process that previously allowed expansion at a rate of 33.3% falters.

An analogous situation arises if the value composition of capital falls in the next period, e.g. from 2 to 1. The scheme of reproduction is then:

Sector 1: $600(C_1) + 600(V_1) + 600(M_1) = 1800$

Sector 2: $300(C_2) + 300(V_2) + 300(M_2) = 900$

In sector 1, $600(C_1) + 200(\Delta C_1) = 800$ means of production are exchanged intra-sectorally, and 1000 means of production remain to be exchanged with sector 2. In sector 2, $300(V_2) + 100(\Delta V_2) = 400$ consumer goods are supplied to its workers, such that 500 of consumer goods remain to be sold to the workers of sector 1. Again, consumer goods are insufficient to drive expanded reproduction at the original rate of accumulation, which results in the reproduction process faltering.

The conclusion is that changes in the timing of the rate of surplus value (or the profit share in total value added) and in the value composition of capital will generate a disproportion between the capital goods sector and the consumer goods sector, causing the process of normal accumulation and reproduction to falter. It will be remembered from Chapter 7 that changes in the rate of surplus value are due to changes in the balance of power between capital and labour, but from Chapters 5 and 6 that the exact manner in which C/V will evolve is difficult to predict. In the capitalist world, the share of profits from income has shown an upward trend over the past 30 years. This partly reflects the weakened position of the working class in the wake of the "second wave of globalisation" and the erosion of trade union influence.[4] This has led to underspending and weakened economic growth, with some scholars regarding these factors as the cause of the economic crisis in the mature capitalist countries.

9.3 The importance of external markets for surplus value realisation in the long run

We mentioned earlier that Marx states that the presence of external markets is a necessary condition for the tendency towards underspending under capitalism to be sufficiently counteracted. This thesis was developed further by Rosa Luxemburg.

In *The Accumulation of Capital*, published in 1913, Luxemburg emphasises the role played by effective demand, pointing out that an expansion of demand is needed to market the additional output that flows directly from the accumulation of capital (Luxemburg, 1951, p. 45; see also p. 461). In addition, solvent demand is a necessary condition for accumulation to take place (Luxemburg, 1951, pp. 136–137). In this regard, she stresses:

> The surplus value must (. . .) shed its form as surplus product before it can re-assume it for the purpose of accumulation; by some means or other it must first pass through the money stage. So the surplus product of Departments I and II [our sectors 1 and 2] must be bought – by whom? (. . .) [T]here will have to be an effective demand outside I and II, merely in order to realise the surplus value of the two departments, just so that the surplus product can be turned to cash. Even then, we should only have got to the stage where the surplus value has become money. If this realised surplus value is further to be employed in the process of enlarging reproduction, in accumulation, an even larger demand must be expected for the future, a demand which is again to come from outside the two departments.[5]
>
> (Luxemburg, 1951, p. 137)

The issue here is not merely underspending, i.e. the inherently limited consumption capacity of the working class compared to the ever-increasing capacity to produce consumer goods. It is also the (by no means least) way in which the enlarged reproduction of the capitalist economy acts on the effective demand of consumer goods and means of production.

For accumulation of capital to take place, Luxemburg explains, additional labour power, as well as additional required means of production and consumer goods (and in the exact proportions), must be available. The tendency towards unbridled accumulation of capital under capitalism thus pushes against its limits. As the natural population growth is insufficient to absorb the jumps and bumps in capitalist accumulation, capitalism must seek additional labour power beyond its borders (Luxemburg, 1951, pp. 370–371), i.e. in the non-capitalist spheres of the economic system or in population strata that belong neither to the working class nor to the capitalist class (Luxemburg, 1951, p. 365).

It is clear that, in Luxemburg's eyes, the way in which the accumulation of capital takes place under capitalist conditions entails a growing dependency on raw materials and means of production from the non-capitalist (external) market. Moreover, the dependency on new outlets implies that in the long run the realisation of surplus value can only be assured if new pre- or non-capitalist social strata or societies are found, to which the growing capitalist output can be sold (Luxemburg, 1951, pp. 351–352).

In connection with the concept of internal and external markets, Luxemburg states:

> They [the internal and external markets] are both vital to capitalist development and yet fundamentally different, though they must be conceived in terms of social economy rather than of political geography. In this light, the internal market is the capitalist market, production itself buying its own products and supplying its own elements of production. The external market is the non-capitalist social environment which absorbs the products of capitalism and supplies producer goods and labour power for capitalist production.
>
> (Luxemburg, 1951, p. 366)

With Marx, the expansion of capitalism depends only on non-capitalist modes of production during its infancy, the period of "primitive accumulation". Luxemburg, on the other hand, points out that this dependency is *permanent* (Luxemburg, 1951, pp. 364–365) and that the disintegration of old non-capitalist structures is a continuous process (Luxemburg, 1951, pp. 416–417).

Finally, the last phase in this process is reached: imperialism, i.e. the phase of capital competition on a global scale (Luxemburg, 1951, pp. 419–420). As Lenin would do (Lenin, 1917),[6] Luxemburg characterises imperialism in *Die Akkumulation des Kapitals* as the period of struggle between the nation states of the capitalist economies for the last non-capitalist territories in the world (Luxemburg, 1951, pp. 367, 446). During this final phase of capitalism, the disintegrated simple-exchange economies are starting their capitalist industrialisation and emancipation process which, however, leads to revolutions and war (Luxemburg, 1951, p. 419) – an interesting thesis that can easily be linked to the rise in military spending in the imperialist nations of the world.

9.4 The importance of military spending for surplus value realisation in the long run

The paradoxical self-destruction tendency of imperialist capitalism, so Luxemburg points out, is clearly evidenced in the contradiction surrounding international loans. For one thing, these open new investment outlets, while prolonging the existence of capitalism, but also bring to life potential new rivals (Luxemburg, 1951, p. 421). Luxemburg stresses, therefore, that in this imperialist phase of capitalism, military spending becomes more important than before, such that militarism is no longer merely an instrument used to subjugate and destroy non-capitalist structures or to fight for the last external markets. Militarism in this phase is at the root of military spending, which is funded by the capitalist governments at the expense of the real purchasing power of the working class and the farmers. *Military spending is thus a new "external market", albeit one created* *by capitalism itself*. Practically at the end of *Die Akkumulation des Kapitals*, Luxemburg writes the prophetic sentences:

> In the form of government contracts for army supplies the scattered purchasing power of the consumers is concentrated in large quantities and, free of the vagaries and subjective fluctuations of personal consumption, it achieves an almost automatic regulatory and rhythmic growth. Capital itself ultimately controls this automatic and rhythmic movement of militarist production through the legislature and a press whose function is to mould so-called "public opinion". That is why *this particular province of capitalist accumulation at first seems capable of infinite expansion*. All other attempts to expand markets and set up operational bases for capital largely depend on historical, social and political factors beyond the control of capital, *whereas production for militarism represents a province whose regular and progressive expansion seems primarily determined by capital itself*.[7]
>
> (Luxemburg, 1951, p. 466; our italics)

Viewed from this perspective, it is strange that Luxemburg formulates a theory of the "collapse of capitalism", which is based on the systematic disappearance of external markets and the liquidation of the non-capitalist "environment" which, in turn, she considers to be the substratum and conditional for the existence of capitalism. In contrast to this view, she builds into her model of monopoly capitalism – along with (as the above quotation shows) her thesis on military spending – a self-regulatory mechanism.

It will come as no surprise that Luxemburg's external markets theory has been advanced as a plausible explanation for structural crises. At the beginning of the 1930s, immediately after the start of the Great Depression, it was one of the views underlying social-democratic programmes of economic revival and reform of capitalism, among which was the Belgian *Plan du Travail* (Cuyvers, 2010, 2011, 2015). The role played by underspending due to the unequal growth of final consumption compared to that of the available means of production and production capacity was revived in the analysis of Paul Sweezy's *The Theory of Capitalist Development*, a book first published in 1942 and still a standard work in (English language) Marxist economic literature (Sweezy, 1942, chs 10–12).[8]

That military spending is an "external market" for capitalism in its imperialist phase is an important element in present-day neo-Marxist theory. Sweezy (1942) emphasised that:

1. It can be taken for granted that the rates of growth of the output of consumer goods and of means of production are stable (pp. 183, 189).[9]
2. Compared to the rate of growth of the use of means of production, the rate of growth of consumption is in a state of decline (p 183, 189).
3. Hence, in capitalism, there is an ever-present tendency towards underspending (p. 183).

4 In the first phase of capitalist development, industrial expansion and the growth of population generate a sufficiently high demand for means of production and consumer goods, and the tendency towards underspending is suppressed (pp. 189, 218–226).
5 This underspending tendency becomes increasingly apparent, leading to stagnation in the later stages of capitalist development (p. 189).
6 During the monopoly capitalist phase, government spending in general (in contrast to income transfers) and military spending in particular counteract the tendency towards underspending (pp. 233–234).

The view that military spending by government counteracts the capitalist tendency towards stagnation was, in the 1950s and 1960s, shared by leading post-Keynesian neo-Marxist economists like Michał Kalecki and Joan Robinson. For instance, Kalecki pointed out that military spending, when financed through government budget deficits, can be a source of "external" profits (Kalecki, 1952, p. 52). In 1967 he wrote about Rosa Luxemburg:

> The "external markets" in the broad sense of Rosa Luxemburg in the form of armament orders and ancillary expenditure – insofar as they are financed by loans and taxation of capitalists – play today a leading role in the functioning of modern capitalism.
>
> (Kalecki, 1967, p. 155)

Joan Robinson, too, clearly influenced by Luxemburg and Kalecki, has emphasised the role played by an increase in armament spending (Robinson, 1956, pp. 93, 273).[10]

Since the 1950s, the strong links between a number of high-technology industries and the US government, military and intelligence agencies has created a "military-industrial complex".[11] The impact of this collusion of economic, military and political power assumed breath-taking proportions during the 1960s and 1970s with the advent of the arms race and the Vietnam War. It is therefore not surprising that the relationship between the emergence and growth of the "military-industrial complex" in the United States and the Marxist theories of underspending has strongly penetrated the American Marxist and neo-Marxist literature.[12]

In Paul Baran's 1957 work, *The Political Economy of Growth*,[13] it is pointed out that to the extent that government spendings are financed by taxes, they are passed on in the form of prices which are then paid by the consumer (Baran, 1957, p. 125), thus leading to both an absolute and relative increase in the economic surplus. Consequently, in the next period, the absorption of the surplus (in Marx's terminology: the realisation of the surplus value) has to increase – as does government spending (Baran, 1957, pp. 128–129). The permanent threat facing capitalism today, i.e. of excess capacity and stagnation due to the mentioned tendency towards underspending, leads to a growing dissipation of the potential economic surplus on publicity and other wasteful expenses by large businesses (Baran, 1957, p. 91).[14] The capitalist underspending tendency is also counteracted by increasing government spending. Since government spending should not interfere or compete with the interests of private business, military spending is ultimately the most important factor, counteracting the threatening economic stagnation (Baran, 1957, p. 119) – a thesis that is developed further in *Monopoly Capital* which Paul Baran published with Paul Sweezy in 1966 (Baran and Sweezy, 1966). There it is also stressed that investments in the arms industry are generating a high rate of return and are almost risk-free, making that particular industry a favourite outlet for investment among the monopolistic oligarchy (Baran and Sweezy, 1966, p. 207).

9.5 Technological innovations as an "external market"

The founders of post-Keynesian neo-Marxist theory also view technological innovations as an "external market", in keeping with Rosa Luxemburg's line of reasoning. In Chapter 3 we mentioned that with Kalecki and Joan Robinson, profits are realised through capitalist spendings, particularly capitalist investment. It will be remembered that with workers' savings being absent ($s_w = 0$):

$$P = I/s_p$$

and: $$\frac{P}{K} = (\frac{I}{K})/s_p$$

or: $$r = g/s_p$$

with r the rate of profits (profits P as a ratio of the capital stock K: P/K), g the rate of accumulation (investment as a ratio of the capital stock: I/K) and s_p the capitalist savings ratio from profits.

In Chapter 8 we pointed out that Michał Kalecki, in *The Theory of Economic Dynamics* (1952), analysed the change in investment mathematically by modelling investment as:

$$I_{t+\theta} = \alpha I_t + \beta \frac{\Delta I}{\Delta t} + d'$$

Without any trend over the economic cycle, this investment equation shows deviations over the economic cycle around zero. However, when the trend in investments is increasing, I_t at each moment will consist of the trend component y_t and the cyclical component i_t, or:

$$I_t = y_t + i_t$$

The y_t equation then becomes (analogously with the reasoning above):

$$y_{t+\theta} = \alpha y_t + \beta \frac{\Delta y}{\Delta t} + d'$$ [15]

This is the general mathematical expression of investment deviations from its long-term trend. The special case is when, over the cycle, only replacement investment is made, such that $y_{t+\theta} = y_t = \delta$. This means that the total replacement investments equal depreciation, or else: to the extent that the capital stock is used up and thus depreciates, it is replaced. The general expression then becomes:

$$y_{t+\theta} = \delta = \alpha\,\delta + d'$$[16]

such that:

$$d' = (1 - \alpha)\,\delta$$

Writing total depreciation δ as a proportion ϕ of the fixed capital stock K, it holds that:

$$d' = (1 - \alpha)\,\phi\,K$$

At this point, Kalecki introduces technological innovations as an example of "development factors" and states that these lift the economy up from its "static equilibrium" because in the long run such innovations will generate additional investment, μ K, transforming the above d' formula into:

$$d' = (1 - \alpha)\,\phi\,K + \mu\,K$$

with μ being positive and a proxy for the intensity of the "development factors" (here the investment intensity of technological innovations) (Kalecki, 1952, p. 150). Kalecki concludes:

> "development factors" such as innovations (. . .) prevent the system from settling to a static position and (. . .) engender a long-run upward trend. The accumulation of capital, which results from the fact that long-run investment is above the depreciation level, increases in turn the scope of the influence of the "development factors" and thus contributes to the maintenance of the long-run trend. The rise in profits and output which occurs as a result of the upward movement of investment makes for a higher rate of growth.
>
> (Kalecki, 1952, p. 151)

In his final model, published in 1968, Kalecki also considers the way technical progress affects profitability. As a result, his investment function there includes F(t), a factor that incorporates the additional stimulus of innovation and the non-profits-determined part of capitalist consumption. F(t) changes slowly over time, depending on past social, economic and technological developments. Based on the relevant parameter values, this model yields a trend and a cycle in investment and economic growth, with technical progress affecting the dynamics involved.

It is important for the purpose of our story, which we will share later in this chapter, to realise that Kalecki's μ is *exogenous* to the development process. In his model, technological development is thus essentially detached from the process of capital accumulation. It is autonomous and seems to drop from heaven like a gift. Neither the invention and development of new production and communication technology nor its introduction are endogenous, i.e. they are unrelated to the accumulation cycle. We will return to this issue when the theories of economic "long waves" are discussed.

Innovations require new investments, but to the extent that "old" capital is replaced, this investment effect peters out (Kalecki, 1944, p. 89). A continuous flow of innovations has the same effect on investments as a continuous increase in profits. When it comes to innovations, not only technological innovations should be considered, but also the introduction of new products, the use of new raw materials, etc. Moreover, Kalecki considers it probable that, among others, due to the tendency towards monopolisation of capitalism, the intensity of the introduction of innovations becomes smaller, which leads to an economic stagnation tendency (Kalecki, 1952, pp. 158–159).

The thesis that a sufficiently high intensity of technological innovations prevents capitalism from crashing into a "stationary state" – or, stated in Marxist jargon, prevents the capitalist system from settling into simple reproduction – is taken up and developed further by later neo-Marxist economists.[17] However, we need to bear in mind that Kalecki's argument leaves no place for an inherent capitalist urge to accumulate, which keeps the growth momentum of capitalism going.[18]

As emphasised in earlier chapters, Joan Robinson (among the pioneers of post-Keynesian neo-Marxism) made an important contribution to the study of the impact of technological change and innovations – or, to be more precise, to the impact of labour-saving technological change and its effect on income distribution. What interests us, however, is the impact of technological innovations on long-run economic growth, which Robinson also analysed in her *Accumulation of Capital*. Here it is pointed out that an acceleration of technological progress is associated with a quicker aging of the existing capital goods, which leads to a higher rate of accumulation. This, in turn, increases the rate of profits, such that producers will have a tendency to choose less mechanised techniques from the more rapidly shifting technique spectra (Robinson, 1956, pp. 91, 136, 161–162). Like Kalecki, she indicates that the speed of technological innovation and its pace of dissemination will nevertheless decrease due to the increasing monopolisation. This will reduce the rate of accumulation and the rate of profits and will lead to stagnant economic activity (Robinson, 1956, pp. 90, 91, 93, 100, 162).

In their analysis of the effect of the monopolisation of capitalism on the speed of technological development and change, Paul Baran and Paul Sweezy go further. In *The Theory of Capitalist Development*, Sweezy stresses that in monopoly capitalism, the capital destruction of technological innovations leads to an inhibition and "timing" of their introduction (Sweezy, 1942, p. 276; see also Baran, 1957, pp. 78–79). In *Monopoly Capital*, Baran and Sweezy assert – in contrast to Kalecki and Steindl – that a strong emphasis on technological innovation is evident in monopoly capitalism (Baran and Sweezy, 1966, pp. 78–79), which is attributed to institutionalised research (pp. 57–58). However, the rate of introduction of technological innovations slows down, since their impact on the overall profitability of the company will be carefully analysed before introduction is considered (pp. 100–101). Compared to competitive capitalism discussed earlier, in contemporary monopoly capitalism the pace of discovery accelerates but the pace of introduction of innovations decreases (pp. 101–102). All in all, technological innovations attract fewer investments than before and they absorb less of the "economic surplus", and therefore also contribute less than previously in the way of (what in Marxist jargon is called) surplus value realisation.

It is also interesting at this point to draw comparisons with how W.E.G. Salter, one of the fathers of the vintage capital model (as opposed to the neo-classical capital theory), investigated delay in the introduction of technical progress. Salter (1966, ch. IV)[19] considers technical progress as being embodied in the latest vintage of the capital equipment of a company or an industry, with the unit labour requirements of the best-practice technique embodied in this latest vintage being lower than those of the earlier vintages. The best-practice technique cannot possibly be applied to the older vintages of capital equipment/plants, and these older vintages will simply be scrapped and/or replaced if they fail to yield a surplus over their operating costs. It thus follows that in a company or industry, the best-practice technique will be applied together with the outdated techniques embodied in the older vintages and, furthermore, there will be a delay in the utilisation of new techniques. This conclusion holds for companies and industries operating under free competition, but the rate of (gross) investment will determine the rate at which new technology can be brought into use (Salter, 1966, pp. 62–64). Under monopoly conditions, however, the monopolist will act differently when investment and replacement decisions affect output. When a new technique becomes available, the profit-maximising monopolist will build new capacity and increase output, but will also replace plants producing output at higher operating costs than those of the new capacity. Compared to the competitive situation, output will be lower and the price higher, but formal analysis shows no difference in the rate of utilisation of new technology (Salter, 1966, pp. 90–92). Salter stresses, however, that the analysis of the profit-maximising monopolist (or oligopolist) can be misleading: "There are many reasons why a monopolist may not exploit his position to the full: a desire not to attract attention, a sense of duty (or fear), or simply inertia" (Salter, 1966, p. 92).

Moreover, combining Salter's model with the post-Keynesian neo-Marxist theory – that the rate of investment under monopolistic and oligopolistic competition will be lower than under free competition – leads to the conclusion that in present-day monopoly capitalism, the rate at which new technologies are introduced is slower than before.

9.6 Long waves in economic activity and accumulation?

Although not traceable to Marx himself, the view that capitalist development proceeds in long waves of economic activity has a long tradition in Marxist economics.

The theory was first launched in 1901 by the Russian Marxist, Alexander Parvus (a pseudonym for Alexander Helphand) (1867–1924) (Parvus, 1901), and was developed further in three 1913 publications by the Dutch Marxist, Jacob Van Gelderen (1891–1940), and later by Salomon de Wolff (1878–1960). Both Parvus and Van Gelderen point to the expansion of the world market, the colonisation and opening of new territories in the world, the introduction of new production technologies, and the development of new industries (the automobile sector, electricity . . .).[20] Long waves in economic activity won fame through the groundbreaking statistical research, conducted over the course of the 1920s, of the Soviet economist Nikolai Kondratieff (1892–1938), who observed 50-year cycles.[21]

Kondratieff remarked that in the downward phase of this long wave, a large number of technological innovations in production and communication were developed which, however, were only significantly applied during the next upward phase of the long wave. He considered these innovations to be exogenous factors and not the cause of the wave movement of economic activity. Under pressure of criticism in Russia (including that from Trotski), Kondratieff later referred to the need for reinvestment in fixed capital (including investments in large infrastructure or in important technological innovations) as a "driving force" of the 50-year cycle, combined with the availability of money and credit. By the end of the 1920s, the major Soviet criticism, however, was that a long wave of economic activity was hard to reconcile with the postulated ongoing "death struggle" of capitalism.

The idea of "Kondratieff waves" was picked up by Joseph Schumpeter,[22] whose theory of the "creative destruction" of capital due to technological progress and new systems of organisation provided the foundation for his theory of the business cycle (Schumpeter, 1934, 1939). According to Schumpeter, important technological changes generate the economic wave that Kondratieff found. As a result, the first wave from 1787 until 1842 was caused by the introduction of the steam engine and the industrial revolution that followed, the second wave from 1843 until 1897 was set in motion by massive railway construction, and the third wave from 1898 until the time Schumpeter published his analysis was related to the introduction of the automobile and electricity.[23] However, in the wake of further studies published by Schumpeter, Kuznets and others, researchers had more and more disagreements over the methodology to use, as well as over the hardly convincing statistical results (the historical turning points in the cycle, if such a cycle existed at all) and their interpretation.

Owing to the Kondratieff cycle's loss of credibility among Marxists following Trotski's criticism and the Stalinist self-censorship, the academic discussion in the capitalist countries about these cycles deteriorated during the 1940s and 1950s into that of the so-called Kuznets cycle of an alleged duration of 15 to 25 years, which seemed to be determined by migration flows from Europe to the USA and the resultant construction activity that took place (in this regard, see Edlund, 1980, p. 400).

Evident interest, in the Marxist economic literature, in the Kondratieff cycle revived in the mid-1970s, largely due to Ernest Mandel's eclectic interpretation, which was also in line with the ongoing worldwide economic crisis and his theory of "late capitalism".[24] For Mandel to have formulated and further advanced the long wave theory using the Marxist analytical framework was an important achievement. In *Late Capitalism*, he goes into detail about the earlier theories and examines their weaknesses. He regards "long waves" as long-run movements in the rate of profits with:

> the diverse combinations of factors that may influence the rate of profit (such as a radical fall in the cost of raw materials; a sudden expansion of the world market or of new fields for investment for capital; a rapid increase or decline in the rate of surplus-value; wars and revolutions) (being related) to the inner logic of the process of long-term accumulation and valorization of capital, based upon spurts of radical renewal or reproduction of fundamental productive technology.
>
> (Mandel, 1975, p. 145)

These are the "triggering factors" (Mandel, 1975, p. 115) that, still in the "trough" of the Kondratieff cycle, will cause the new upward phase. In Mandel's view, the post-war "long wave of expansion" has since 1967 been collapsing into "a long wave of much slower growth" (Mandel, 1975, p. 142). Actually, Mandel's theory is an interesting attempt to reconcile Marx's tendentially falling rate of profits with "long waves", such that the moment (for which Marxist theorists have to wait) that capitalism is plunged into permanent stagnation is postponed. Reduced to its essentials, the argument is as follows:

1 During the upward phase of the cycle, capital-using and labour-shedding technologies are introduced. Consequently, the composition of capital starts to rise, which undermines the rate of profits. The accumulation process sputters and stalls, and the downward phase sets in.
2 During the downward phase, the militancy of the working class is undermined; thus, the rate of surplus value will increase to the point at which the rate of profits has recovered sufficiently to kick-start a new wave of accumulation. In addition, capital devalorises to a growing extent.
3 Moreover, during the downward phase of the long wave, sufficient surplus capital, i.e. capital that remained uninvested because of the low rate of profits, is available to support the process of accumulation in the next upward phase.

Commenting further on Mandel's theory of the long waves would take things too far. We are therefore restricting ourselves to the following three points:

1 Mandel's alleged phasing of the Kondratieff cycle after the Second World War is contradicted by the evolution of the rate of profits and the composition of capital, as reviewed in Chapter 6 (see also Rowthorn, 1976, p. 66).
2 Mandel provides no solution for the cyclical movement in the rate of profits. Evidently, it is far from illogical for the rate of profits to oscillate in the long run,

but why should it move cyclically? And why should the cycle length be approximately 50 years? (See, for example, Edlund, 1980, p. 405).

3 Mandel does not make it clear which "triggering factors" in his theory are endogenous and which are exogenous and/or random events. How are technological "basic" innovations introduced in his model? Are they, or only the time of their introduction, endogenous? How does he view the economic impact of wars on the long wave of economic activity, investment, the rate of profits and the rate of exploitation – unless it is assumed to be endogenous which, however, reduces history to how an economic clock is running?

In reaction to Trotski's criticism, Kondratieff went a long way towards "endogenising" technological progress, wars and resource scarcities in his explanation of the long-wave pattern of economic activity (Day, 1976, pp. 74–75).

At this stage of our discussion, it is interesting to reflect further on the extent to which a long wave of economic activity could be caused by a wave of technological inventions and innovation. It should be noted that Baran and Sweezy's "epoch-making innovations" (Baran and Sweezy, 1966, pp. 216ff.)[25] alone could not have generated the Kondratieff cycle, since they result in one-off shocks, not in a long wave of economic activity. Apart from such innovations, other basic innovations should be applied, the development and implementation of which proceed cyclically. Figure 9.2 depicts the hypothesised Kondratieff cycle, as (allegedly) generated by basic innovations.

Scientific and technological knowledge does not progress linearly, but rather in a jerky fashion, from which a basic innovation arises.[26] According to the well-known definition by Gerhard Mensch: "Technological basic innovations produce new markets and industrial branches, whereas non-technical innovations open

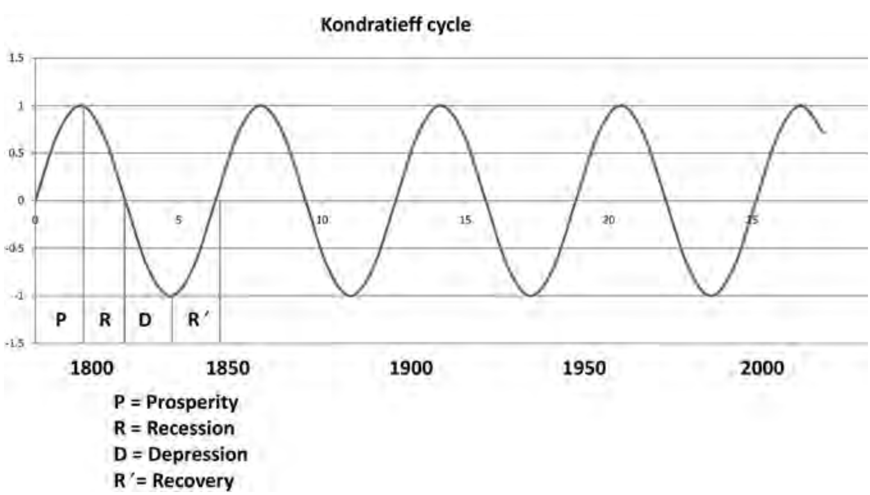

Figure 9.2 Kondratieff cycle and basic innovations

up new realms of activity in the cultural sphere, in public administration and in social services. Basic innovations create a new type of human activity"(Mensch, 1979, p. 47).

Later on, basic innovations give rise to smaller adaptations and improved innovations. According to Mensch, the process of generating and introducing basic innovations is cumulative and self-reinforcing. He writes:

> As long as the basic inventions result from knowledge that is still young, they appear in small numbers. Over the years, they increase in number because more scientists see the new paradigm as correct and therefore accept it as a guideline in choosing problems for research (. . .). Thus the process is self-reinforcing, ultimately producing surges of basic inventions. As the masses of originally and subsequently perceived problems within a given theoretical framework are solved, the numbers of new insights emerging in this area gradually diminish.
>
> (Mensch, 1979, p. 144)

Included among the basic innovations of the first half of the nineteenth century should be: hydraulic coupling, catalytic petroleum "cracking", the diesel locomotive, the helicopter, the jet engine, nylon, penicillin, the radio, radar, silicones, synthetic detergents, plexiglass, the tape recorder, the television, the transistor, etc. (Mensch, 1979, pp. 127–128).

Since basic innovations appear and are introduced in clusters, they give rise to a clustering of capital accumulation which generates an investment cycle.[27] With Kondratieff it can then be assumed that each innovation cluster is introduced during the downward phase of the long wave, an assumption that is also defended by, for example, Graham and Senge (1980, pp. 283–284):

> During a long-wave downturn, basic innovation opportunities gradually improve, as old capital embodying the technologies of the preceding build-up depreciates. Near the trough of the wave, there are great opportunities for creating new capital embodying radical new technologies. The old capital base is obsolescent, bureaucracies that thwarted basic innovation have weakened, many companies committed to producing old types of capital are bankrupt, and traditional methods are no longer sacrosanct.

Viewed from this angle, it should come as no surprise that statistical research into long waves in economic activity – moreover, for time series of a different length and different statistical quality – often failed to produce hard evidence (for example, Solomou, 1987).[28]

One reason is likely: that the "echoes" of subsequent investment waves, depending on their wavelength and intensity, will at times partially annihilate each other and at other times reinforce each other, thus creating a lot of "noise" in the statistical data. A sudden surge in long-run investment activity generates effects that are similar to those stemming from a sudden rise in population, such as the "baby boom" after the Second World War, the impact of which is felt again after 20 or 30 years, and again . . . and again . . . into the future. Let us assume that the massive investments that accompany the introduction of new basic technologies during the downward phase of the long wave are generating "echo effects" in the long run. A case in point is the huge investment that has been made in railways and railway transport equipment since the mid-1850s which, following a period of evident decline, will lead to large replacement investment in railway infrastructure, locomotives and wagons, with the latter being seen to be an "echo" of the initial investment. (See, for example, Semmler, 1986; and di Matteo, Goodwin and Vercelli, 1989. See also the many simulations leading to "chaotic" economic development in Goodwin, 1990b).

It is conceivable that a long wave will arise as depicted in Figure 9.3. The initial wave started with the introduction of the steam engine, which set in motion the first industrial revolution and has continued to propagate in echoes. In the downward phase of this first wave, a new basic innovation is introduced (e.g. the construction of railway infrastructure and railway materials), which sets in motion a second wave which is superimposed on the first. During the downward phase of the second wave, a new investment wave starts with the introduction of the next basic innovation (e.g. electricity) (Cuyvers, 1988a).

Presented in this way, we are confronted by an initial theoretical problem: how is it that in the downward phase of the first wave, which is associated with the steam engine, railway construction is introduced as an innovation? The former can hardly be related to the latter. What Figure 9.3 shows, however, is the evolution of total capital investments. When the first investment wave loses momentum and stagnates, capital will seek new investment opportunities, which happen to be found in the new basic innovation, already fully blueprinted or not.

A second complicating factor is that it cannot possibly be assumed that the introduction of the different basic innovations generates echo effects of the same amplitude and wave length, or that these effects stretch out equally far in the future. For a more than superficial study of such echo patterns, the history of the different basic innovations since the start of the first industrial revolution,

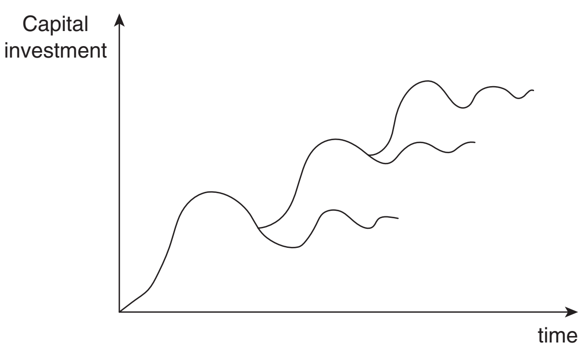

Figure 9.3 Long wave of capital investment with sequential introduction of basic innovations

as well as their consequences for the various industries, has to be painstakingly registered and analysed. These echoes may seriously distort the overall long-term economic growth curve because of interference by the echoes in the successive investment waves generated by the introduction of the basic innovations, such that, ultimately, the actual growth curve hardly resembles a well-defined wave or cycle. In certain periods, "old" echoes may reinforce the newer echoes but counteract them in other periods. Only model simulations can unravel the resulting intricate patterns. However, when such patterns are simulated, it remains questionable whether the model corresponds with reality, a situation not unlike that in pre-Copernican cosmology. The reason for this is that such models have to start from a first wave which is assumed to have generated a dampening sinusoid[29] in economic activity. Next, unless the available statistical data are filtered using "measurement without theory" statistical techniques,[30] tentative and unverifiable assumptions will have to be made about the timing of the implementation of each new basic innovation as well as its impact. This is evidently a major topic for future theoretical and empirical research in the field of economic history, although some tentative first steps have already been made.

Moreover, there is a serious risk of jumping to deterministic conclusions about the past and future from the theory of long waves, which are evidently at odds with the basic principles of both Marxist and post-Keynesian thinking. The *pédigré* of post-Keynesian thinking on long waves in economic activity can be traced to Kalecki and Erwin Rothbarth, who in the late 1930s had long discussions about how to build an integrated model of business cycles, trends and long waves (Cuyvers, 1984, p. 306), which unfortunately remained fragmentary and unfinished. Again, via Kalecki, the thread was picked up some years later by Josef Steindl in his *Maturity and Stagnation in American Capitalism*. Steindl presents the hypothesis that the internal accumulation of capitalist businesses will, over time, generate investment, thus making the growth of capital self-perpetuating (Steindl, 1952, p. 193). He then amends Kalecki's difference-differential investment equation by replacing profits with the rate of profits, and circumventing the insurmountable analytical mathematics by considering deviations in the rate of profits from a "neutral" level (Steindl, 1952, p. 196). Concentrating on the long run, Steindl uses moving averages for investment, internal accumulation[31] and the capital stock, and finds an oscillatory movement of investment with a longer period than that used for establishing the moving averages if the rate at which accumulated savings are transformed into investment – the "reinvestment factor", i.e. the parameter γ in Steindl's equation – lies between two limits, which is determined by v, the (given) ratio between business savings and the capital stock (Steindl, 1952, p. 203). For $v = 0.02$, the duration of the investment cycle will be 45 years or more, i.e. it gives rise to a "long wave". If γ lies outside these limits, there is a trend but no cycle, which, however, may become a cycle due to changes in the structural parameters underlying business savings and investment. Steindl puts it as follows:

> Take the case where there has been a positive trend, and owing to some changes of structural coefficients the theoretical trend solution vanishes

> and there is only the cyclical solution. As there has been a positive growth at the beginning, the "cyclical movement" of the capital stock will start in its ascending phase: the capital stock will continue to grow, but at an ever-decreasing rate it will reach a maximum, and will finally inevitably decline. There will thus sooner or later be a negative rate of growth.
>
> (Steindl, 1952, p. 207)

The next step in Steindl's analysis entails the introduction of long-run variability of capacity utilisation and the impact of changes in the relative indebtedness of the capitalist enterprises. This permits some interesting conclusions to be drawn about capitalist economic development and growth under conditions of monopoly.

It should be stressed that the "long wave" that Steindl's model of self-perpetuating investment and endogenous growth – under specific parametric conditions – generates has no relationship with the Kondratieff cycle. A long wave in Steindl's model corresponds with a situation of endogenous growth in the capital stock within given and appropriate structural parameters. Since these structural parameters change exogenously over time, the characteristics of the long wave in economic activity that follows from his model also change, which leads Steindl to arrive at the following conclusion about this issue:

> Owing to the changes in structural coefficients occurring in the course of time, and the intervention of exogenous factors, the long wave is not likely to be realised in recognisable form. (. . .) Long run development would, then, consist of a succession of parts of "long waves".
>
> (Steindl, 1952, p. 226)

It is not clear how realistic Steindl considers this hypothesis to be,[32] particularly since it depends on the value of specific structural parameters in his long-run investment function (which might not be found in the real world) and the fact that the economic mechanism underlying a "long wave" remains unexplained. Moreover, he points to the role played by what is presently called "institutional changes", thus setting the tone for the post-Keynesian view of long-run economic development which makes the long wave, if it exists at all, "unrecognisable".

For a Kondratieff-type economic cycle to exist, factors must be at work that are generating the cycle. First and foremost, the introduction of important innovations is of relevance here.[33] The issue of how important innovations can generate a cycle in economic activity is researched by some noteworthy post-Keynesians. On the post-Keynesian views on innovation and long waves in economic activity, particularly the conditions that have to be fulfilled for a clustering of innovations leading to long waves, Courvisanos (2003, pp. 192–193) wrote:

> The classic proposition of an investment model with innovation comes from Joseph Schumpeter, who recognized that the investment function responds to waves of optimism and pessimism that create clusters of innovation, and thus "bunching" of investment. This produces susceptibility to unstable

> investment cycles and the development of a trigger mechanism to initiate fundamentally new innovation systems with long-wave implications. Kalecki endorses and reinforces this cycle-trend effect that innovation has on the investment function. (. . .) The cause of clustering of innovation and subsequent bunching of investment ("clust-bun") is in debate. The Kaleckian feature of expanded reproduction has not been recognized by the protagonists in this debate. The prerequisite for clustering is deep depressions or breakthroughs in technology, both reflecting reactions by the private and public sectors to deep problems in the downswing of the previous business cycle. Then, the bunching requires effective demand stimulus through widespread diffusion of the cluster effect that can only be achieved through the availability of a surplus for investment (private profits and public deficit spending). Impediments to this "clust-bun" effect reside in the institutional frameworks of nations, particularly those with still dominant mature industries with older technologies (. . .). Increased uncertainty arising from large investment in the new technology systems also adds a further impediment through increased macroeconomic volatility, slowing down the diffusion process.

This evidently does not prevent post-Keynesian scholars from investigating the relationship between innovations and cycles per se. In his 1968 *Economic Journal* paper, Kalecki (1968) attempted to identify factors affecting the long-run economic growth under capitalist conditions. He did so by introducing in his investment decisions function a "slowly-changing magnitude depending (. . .) on past economic, social and technological developments" (Kalecki, 1968, p. 173) and represented by B(t), reflecting the higher profits of the first introducers of the novel production techniques. He also considered the constant part in the capitalists' consumption function to be slowly changing, depending "on past economic and social developments" (Kalecki, 1968, p. 167) and represented by A(t). After a lengthy period of time, both A(t) and B(t) "will tend to grow for a number of years unless there were some changes in social patterns or in the stream of inventions to upset this tendency" (Kalecki, 1968, p. 178). This, finally, brought him to conclude:

> [I]n our approach the rate of growth at a given time is a phenomenon rooted in past economic, social and technological developments rather than determined fully by the coefficients of our equations as is the case with the business cycle.
>
> (Kalecki, 1968, p. 183)

For our purposes, Kalecki's proviso "unless there were some changes in social patterns or in the stream of inventions to upset this tendency" is important as it indicates that when such societal and technological factors are hypothesised to show a cyclical pattern, this would immediately translate into a cyclical, long-run behaviour of economic activity.

Over the course of the 1960s and 1970s, Joseph Steindl revised his views of *Maturity and Stagnation in American Capitalism*. In 1976, in his Introduction to the Monthly Review edition of *Maturity and Stagnation*, he states that the primary decline in economic activity leading to the economic maturity that he had investigated "might be the result of exhaustion of a long technological wave begun with the Industrial Revolution and reaching its eclipse with the maturity of the railway age" (Steindl, 1976, p. xv). He remained critical of the Schumpeterian view of a long cycle generated by subsequent major technological innovations or its clustering (Steindl, 1976, p. xvi; Steindl, 2005, p. 144).[34]

The relationship between clusters of innovations in the production process and how output changes in the long run is modelled by Richard M. Goodwin,[35] in terms of which investment in innovations of an "innovational swarm" (Goodwin's terminology) follows a logistical path which, in turn, feeds into changing output and labour productivity. Real wages evolve with the gap between the demand for labour (determined by the increase in labour productivity and output) and the supply of labour (assumed as given). Goodwin simulated his model (Goodwin, 1989, 1990a, 1990b) and found that, at first, an important "innovational swarm" leads to almost continuous growth which, however, drops slightly on two occasions before half the Kondratieff cycle has passed. Later, growth recovers, although it is quickly interrupted by a short-lived but deep slump. This, in turn, is followed by a phase of slow growth, ending in a mild cycle.[36] This simulated growth rate served to illustrate the echo effect that is caused by the "innovational swarm" under consideration.

Thompson (1990), in an attempt to detect Kondratieff cycles using available sectoral data instead of macroeconomic statistics, researched the rates of growth of output in the respective "leading industries" in the "leading economies". Evidently, both the "leading industries" and the "leading economies" changed significantly over the course of the last two centuries. By focusing solely on these data, he found a cycle that closely resembles the Kondratieff cycle proposed by Schumpeter, and he identified four waves in the period 1760 until today.

Is it possible to integrate economic cycles in the linear production model? This is a field of research that overlaps significantly with dynamic input-output analysis, to which we referred in a previous chapter. It is a field of research in which progress is made with great difficulty. The introduction of endogenously changing coefficients of the $\mathbf{A}$-matrix and the $\mathbf{l}$-vector from previous chapters is an initial complicating factor, albeit not insurmountable with some additional plausible assumptions (Chander, 1983). This type of research, however, falls within the field of expertise of some highly specialised mathematical economists (in this regard, see Goodwin and Punzo, 1987).[37]

John Craven (1973, 1983) showed that, in a linear production model à la Leontief, there is at each time a subset of Harrod-neutral innovations in the set of innovation possibilities. As a result, the input coefficients of $\mathbf{A}$ can be considered to remain constant, whereas those of the $\mathbf{l}$-vector are decreasing. In each sphere of production, the direct and indirect inputs thus remain unchanged. Craven then proved mathematically that a series of such Harrod-neutral technological changes will cause the outputs per unit of labour to increase by at least (asymptotically) the same rate as that for every other series of innovations. His finding implies that, ultimately, capitalists will be inclined to introduce Harrod-neutral technological change (or will be indifferent to the distinction between this and other innovations when making a choice). It appears, however, that with the introduction of Harrod-neutral innovations, the outputs do not converge with those required for steady, balanced growth. For convergence in a two-sector linear production model, the capital goods sector should be more labour intensive than the consumer goods sector – a weird and highly restrictive condition indeed, which was also found by Nikaido (1983). Subsequently, it was pointed out by Duménil and Lévy (1987, pp. 142–143) that the "convergence problem" is due to a modelling of the dynamic adjustment process, which is different from that of both the classical economists and Marx. After accordingly changing the assumed dynamics in their model and simulating it, they found convergence of prices and the rate of profits for realistic parameter values (Duménil and Lévy, 1987).

The possibility of cycles has also been investigated through the construction and simulation of simple experimental versions of such linear models using two sectors (a capital goods sector and a consumer goods sector) or three sectors. Duménil and Lévy (1986) introduce a three-sector model (sectors in which fixed capital, circulating capital and consumer goods are produced, respectively), incorporating alternative reactions of the capitalists to disequilibria. However, the equilibrium reached is not necessarily stable. They also find situations of "stationary disequilibrium" and their model shows cumulative downward evolutions, leading to crises.

Another interesting example of a two-sector model is that by Jay Forrester, which became the System Dynamics National Model (SDNM).[38] This has a capital goods sector and a consumer goods sector, with the output of the former reacting, after a specific time lag, to output changes in the latter sector, or vice versa. A shock in demand for consumer goods or means of production is thus transmitted to the other sector. Since the capital goods sector is also supplying to itself, the shock is self-reinforcing. On the strengths of expectations of future demand and the required production capacity, the capital goods sector expands further, up to the point that overcapacity is created and the propagation mechanism is reversed. It is in this manner that long waves arise. With the SDNM, Forrester (1976) identified long waves for the US economy spanning 40 to 60 years.

As mentioned earlier, Goodwin's approach (Goodwin, 1967) starts from a fall in the rate of exploitation σ – or, inversely, the wages share in value added – and investigates how an underspending crisis develops. Although this approach is designed to show a business cycle, it also sheds light on the effects of long-run changes of σ, as shown in the changes in the wages share of GNP in the mature capitalist world since the mid-1970s.[39]

It will be clear from the above discussion that the various versions of the economic long waves theory constitute fertile ground for research, and that combining the views of Marx, Schumpeter and others leads to interesting and challenging insights. Such insights are of greater relevance for acquiring an alternative understanding of the worldwide economic recession of the 1970s and 1980s than those offered by mainstream economic theory. Viewed from this perspective, the dollar crisis of the early 1970s and the abolition of the Bretton Woods System, as well as the "oil shocks" of 1973 and 1979, are not causes, but consequences.

Nevertheless, the question remains as to whether the spectacular development of information technology actually started a new upward phase of the Kondratieff cycle. It should be remembered that the first commercial computer applications date back to the 1960s, but we had to wait for the Internet and the World Wide Web to experience the true power and widespread adoption of information and communication technology (ICT). The labour-saving effects of information technology were already being experienced in the 1960s and 1970s, which prompted Nobel Laureate, Robert Solow, to share the paradox: "You can see the computer age everywhere but in the productivity statistics" (Solow, 1987, p. 36). Chris Freeman, in turn, indicates in his momentous study on economic long waves that the absence of necessary social and political change is hampering efforts to assess developments in the ICT field and their associated impact (Freeman and Louçã, 2001, pp. 302–303). Based on a "back of the envelope" kind of calculation, Gordon (2012, p. 9) compares trends in American productivity growth during the period 1972–1996 with those in the period 2004–2012, and concludes that the dot.com "revolution" has produced productivity levels that are hardly 9% higher.[40] Assessing these slow growth rates in the light of the post-war growth record, it can rightly be questioned whether the expectations of the coming upward swing in the Kondratieff wave are justified. And if the delay in the upswing persists, the accuracy of the economic long wave theory becomes highly questionable,[41] unless the long wave runs out due to insufficient basic innovations.

Or are the institutional changes that should accompany the upswing in the Kondratieff wave missing? During the downward phase, a growing "mismatch" develops between the potential of the new technological-economic paradigm and the socioeconomic regulation system, i.e. labour conditions and wages, education, traditions, etc. (Husson and Louçã, 2013, p. 129). However, in the past, other institutional changes have similarly supported the previous upswings in the long wave. During the second industrial revolution, for instance, the legalisation and generalisation of the system of limited liability companies allowed the capitalist producers to both find the large minimum amounts of capital required (such as for railway construction) and shelter their private wealth and that of the other shareholders from the risks involved (see, for example, Lloyd-Jones and Lewis, 1998; Forbes, 1986). This, in turn, was a sine qua non for the later separation of ownership and control of the company (Berle Jr and Means, 1932). The same seems to hold true for the intertwining of industrial and financial companies, which was an institutional change supporting the expansive phase of the period 1885–1900. During the period 1940–1960, contributing to institutional changes was the widespread "managerial revolution" among large corporations (see, for example, Marris, 1964; Chandler Jr, 1977) as well as the Keynesian-inspired macroeconomic policies at the national level and the new international "rules of the game" imposed by, for instance, the General Agreement on Tariffs and Trade (GATT) and the

Bretton Woods System (Cuyvers, 1988a, 1988b, 1994), which John Maynard Keynes had been very influential in shaping.[42]

Or has today's capitalism entered a new phase in its development, which hampers economic expansion and the accumulation of capital in the "real" economy? This question, in turn, is linked to the discussion on the "financialisation" of the capitalist economic system, which will be dealt with in the next section.

9.7 The development of the service sector and "financialisation" of the capitalist economy

The capitalist circulation costs were discussed in Chapter 4. Marx sees these costs primarily as purely commercial costs, i.e. the costs needed to realise the value of output and convert it into money, or vice versa (Marx, 1981, p. 402). Although we concluded in Chapter 4 that many of these costs are "productive" (according to Marx's interpretation of this concept), we also found that some (for instance, the commercial costs) are accelerating the rotation time of capital, and therefore increase the rate of profits. In fact, some advertising expenses have the same effect. Yet even more, argue Baran and Sweezy, are contributing to a reduction in the threat of stagnation in monopoly capitalism, when these and other expenses associated with sales promotion are extremely high. According to Baran and Sweezy: "Price competition has largely receded as a means of attracting the public's custom, and has yielded to new ways of sales promotion: advertising, variation of the products' appearance and packaging, 'planned obsolescence', model changes, credit schemes, and the like" (Baran and Sweezy, 1966, p. 120).

In *Monopoly Capital*, the authors relied heavily on the theory that advertising "convinces" and shifts consumer preferences, which in the meantime needs nuance if we take into account the "informative function" of advertising and the "valuation" of advertising by the consumers. It is far from our intention to discuss the extensive theoretical and empirical literature on advertising here.[43] However, there are good reasons to both share and nuance the Baran and Sweezy thesis. Empirical research shows that, by creating product loyalty, advertising increases profits (Comanor and Wilson, 1967, 1974). From this it follows that publicity expenses, over and above providing an important additional form of surplus absorption, also boost the economic surplus. This is even more so as the demand-creating effect of advertising and sales promotion generates investment opportunities in factories and equipment that would otherwise not exist (Baran and Sweezy, 1966, p. 128).[44]

Neo-Marxists argue that the expansion of commerce and sales promotion in the monopolistic phase of capitalism is the result of a tendency towards stagnation, and that it is also responsible for sizable and growing waste, as mentioned in Chapter 4. The same argument can be advanced about the expansion, since the 1970s, of the banks and the financial sector at large. In *Das Kapital*, Marx pays attention to the role of "money-dealing capital",[45] which he considers to be money capital screened off from industrial capital and commercial capital:

> A definite part of the total capital now separates off and becomes autonomous in the form of money capital, its capitalist function consisting exclusively in that it performs these operations for the entire class of industrial and commercial capitalists. Just as, in the case of commercial capital, a part of the industrial capital present in the circulation process in the form of money capital separates off and performs these operations of the reproduction process for the whole of the remaining capital. The movements of this money capital are thus again simply movements of a now independent part of the industrial capital in the course of its reproduction process.
>
> (Marx, 1981, p. 431)

By the end of the nineteenth century, finance capital had become dominant, as described and analysed by Rudolf Hilferding (1877–1941) in *Das Finanzkapital*, as well as by other authors. In the words of Hilferding:

> I call bank capital, that is, capital in money form which is actually transformed (. . .) into industrial capital, finance capital. (. . .) An ever-increasing proportion of the capital used in industry is finance capital, capital at the disposition of the banks which is used by the industrialists.
>
> (Hilferding, 1981, p. 225)

Hilferding's forecast of increasing monopolisation under the umbrella of ever more powerful banks appeared to be wrong, but the entanglement of industrial capital and bank capital has played a major role in the development of monopoly capitalism.[46]

After the "managerial revolution" of the 1930s and the post-war spread of multinational enterprises, capitalism transformed into present-day monopoly capitalism. Since the late 1960s and early 1970s, it has undergone a further transformation during a process of "financialisation".[47] This process began because of the downward pressure on the rate of profits in the manufacturing sectors, which drove ever-increasing amounts of capital into the financial sector which at that time was still largely protected and sheltered from international competition emanating from the developed capitalist countries.

The demise of the post-war exchange rate system – i.e. the Bretton Woods System – in 1973 and the "non-system" of floating exchange rates that followed have significantly increased the pressure to liberalise international capital flows, including international money flows. Clearly, it is not our intention to reveal these complex institutional changes here; suffice to say that since the end of the 1970s, the financial sector in the major capitalist countries has been "deregulated" under the neo-liberal policies adopted, which were advocated by the re-emerged monetarist economic theories that had once more become fashionable.

Evidently, these developments have not gone unnoticed by Marxist and Marxist-inspired analysts. Costas Lapavitsas recently stressed that the "financialisation" of present-day capitalism is for many neo-Marxist authors a trend that arose *after* that of increasing monopolisation, as analysed in Baran and Sweezy's

Monopoly Capital (Lapavitsas, 2011, pp. 612–613).[48] Under monopoly capitalism, the stagnation tendency, so Paul Sweezy argued in the 1970s and later, leads to an influx of surplus capital in the sphere of circulation, particularly in speculative financial activities (Magdoff and Sweezy, 1987).[49] Other Marxists have linked financialisation to stagnation in the sphere of production, which preceded it since the 1960s and which they considered to be a consequence of Marx's tendentially falling rate of profits.[50]

During the 1970s and later, Harry Magdoff and Paul Sweezy analysed and commented on the explosion of the financial sector in the economy. In this regard they stressed the fundamental contradiction that in the short term "financialisation" promotes economic growth in the stagnating monopoly capitalist system, but in the long term generates greater instability and uncertainty. The fragility of the financial "superstructure" of the stagnant productive "basis" of the capitalist economy would sooner or later give rise to a crisis of the magnitude of the Great Depression in the 1930s (Magdoff and Sweezy, 1982, 1987). From this perspective, John Foster argues: "financialization, while boosting capital accumulation through a process of speculative expansion, ultimately contributes to the corrosion of the entire economic and social order, hastening its decline" (Foster, 2010)

According to Foster, the capitalist system, although modified, has not entered a new phase but has led to a new hybrid form of monopoly capitalism, which he calls "monopoly-finance capitalism" (Foster, 2007). Its epicentre is in the USA and it is caught in a seemingly endless cycle of stagnation and financial explosion. In this way, the neo-Marxist authors of *Monthly Review*, mainly Sweezy, Magdoff and Foster, link the analysis in *Monopoly Capital* to the "financialisation" of contemporary capitalism.

The question, however, is whether enough has been said. "Financialisation" is, after all, still a recently developed concept that receives a lot of attention, but it is actually not yet adequately defined in economic-theoretical terms.[51] It includes a whole range of phenomena, such as the globalisation of financial markets, the so-called "shareholder revolution" and the attendant loss of autonomy of management, and the increase in income from financial investments (Stockhammer, 2004, p. 720).

During the 1980s, the relationship between non-financial companies and the financial markets became more entrenched and more complex. As a result of the international financial deregulation wave,[52] the pressure exerted by financial markets on non-financial businesses has increased dramatically. More than ever before, these businesses – when listed on the Stock Exchange – have to take into account the share price and the wishes of the shareholders, not only in the interests of share price gains but also for fear of a "hostile takeover". Whereas the growth of companies during the 1960s was still largely secured through auto-financing, in more recent years the importance of the financial markets as a source of capital has soared. New financial instruments have been developed and traded. The seizure of interest and dividend payments, as well as the "stock buy-backs" that are levied on the income of non-financial enterprises, have greatly increased. With the financial markets being heavily exposed to short-term expectations and speculative operations, the time horizon of enterprises has shortened considerably.

The “shareholder revolution” – a term referring to the previous “managerial revolution” (Burnham, 1941), to which it stands in contrast – has changed the management strategy of retaining profits and investing them (“retain and invest”) into one of “downsize and distribute” (Lazonick and O’Sullivan, 2000).[53] The large corporations are tied more to the capital market and less to the banks, at least in the United States and the UK.[54] This has prompted post-Keynesian-oriented researchers to use a three-classes model of capitalism, involving workers, capitalists and rentiers/shareholders.[55] Of course, the functions of these classes are interlaced but their interests differ. Important for the thesis of the “shareholder revolution” is that the priorities and interests of the capitalists/managers often conflict with those of the rentiers/shareholders. The former favour mostly investment and growth, whereas the latter tend to look more for dividend payments and an increasing stock price in respect of their shares. During the late 1970s, in the wake of the financial deregulation in the capitalist world, the balance of power switched in favour of the shareholders. At the same time, the rewards system for the managers also changed to a spread of profit-related bonuses and “stock options”, making them more inclined to side with the shareholders (Lazonick and O’Sullivan, 2000). This development has logically brought into vogue the thesis that the managers’ focus on creating “shareholder value” since the 1970s has slowed down the accumulation of capital. Engelbert Stockhammer was one of the first to test this hypothesis for the United States, the UK, France and Germany. He found the hypothesis was supported in the United States and France, while in the UK there was limited supporting evidence and in Germany no evidence at all. The slowing down of accumulation in France is fully explained by the “financialisation” phenomenon, which in the United States accounts for about one-third of the slowdown. That “financialisation” in Germany has no significant influence on investment is due to its still being in its infancy in that country (Stockhammer, 2004, p. 739). These findings have been confirmed for the United States in more recent research by Özgür Orhangazi (2008). This post-Keynesian relationship between the “financialisation” of monopoly capitalism and the slowdown of capital accumulation seems to significantly supplement the neo-Marxist views of Sweezy and Magdoff.

With the dominant role played by the financial sector, “financialisation” has also gone hand in hand with the increasing financial indebtedness of both countries and individuals, leading to a series of consecutive, exploding speculative bubbles and international debt crises, and finally culminating in the financial crisis of 2007–2008 and the euro crisis which began in 2009. Once again, the process is a very complex one, the analysis of which goes far beyond the scope of this book. By drawing attention to these developments we merely wish to illustrate how in the recent past, successive explosive developments in the financial sphere have been followed by economic crisis and stagnation in the sphere of production – as Magdoff and Sweezy argued and predicted. This, in fact, brings us back to the monetary theory of *Das Kapital*, which we discussed in Chapter 8.

9.8 To conclude

Since the underconsumption theory – and underspending theory in general – is little explored in *Das Kapital*, this chapter has largely focused on its development at the hands of Rosa Luxemburg and the later neo-Marxists. The rapid growth of output and the forces of production under capitalist conditions stand in contrast to the absorption capacity of this output, which remains limited, so the thesis goes. One of the main reasons for this, according to Marxist reasoning, is that the consumption capacity of the working population is not increasing by the same degree as the output of consumer goods. Alternatively, the expansion of capitalist production is hitting against the boundaries of the capitalist system and so expansion is only possible by penetrating, exploiting and draining the non-capitalist areas and enclaves in the world economy. Once these external markets disappear, capitalism becomes stagnant. The growth of capitalist investments falters due to insufficient accumulation possibilities and consequently, due to insufficient demand, the produced surplus value is not realised. Or is it?

New "external markets" might well be found and tapped. For Rosa Luxemburg these are armament spending, a thesis the later neo-Marxist economists would agree with. Is there no other government spending that, as an "external market", can generate sufficient demand? Of course there is. Large infrastructural works or massive spending on education and training are striking examples. Kalecki, or Baran and Sweezy for that matter, recognise this, but at the same time they point out that government spending can only generate demand to a limited extent, since in capitalism the government is not allowed to compete with the private sector.[56] This seems to be largely correct and is confirmed in reality, but the argument as such takes insufficient account of the fact that infrastructure and education are also taking an enormous bite out of government budgets in all the developed capitalist countries. These "public goods" not only have a large demand-creating effect, but also make an important contribution to the continuous upgrading of the competitive position of the individual capitalist economies. The development and production of ever better and more sophisticated goods and services, using the newest technology, assumes proper training and education. There is little evidence of integration of "public goods" in the Marxist and neo-Marxist economic analysis. Moreover, government investments that compete with those of the private sector are indeed in evidence, albeit often made under political pressure from the Left. While being the result of accumulated budget deficits in the "mixed economies", such government spending has generally shown a sharp drop since the 1970s, after the major role it played in the past – and, actually, still does. Most likely in the future, too, government spending on the environment and public investment in environmental technology should be viewed as a potential "external market".

This brings us to the role played by technological innovations. In the first analyses by Michał Kalecki, it was seen how these perform as "external markets", albeit as *deus ex machina*. New technology "destroys" earlier investments in old technology, and the introduction of new technology causes a surge in capital accumulation.

A continuous flow of technological innovations thus counteracts the stagnation tendency of capitalism.

Massive investment surges cause, as it were, shock waves in the accumulation of capital. Massive investments made today call for massive replacements later, supposedly generating long waves in the accumulation of capital and in economic activity. The existence of such waves – the Kondratieff cycles (named after the Marxist Soviet economist who postulated these cycles based on his statistical research) – with a wavelength of about 50 years has been repeatedly put forward over the course of the twentieth century by scholars analysing long-run developments, including Marxist authors such as Ernest Mandel in the 1970s. We have argued that such long waves might be attributed to the impact of important innovations, such as the steam engine, railways, the automobile, etc. As such, these do not explain the periodic recurrence, but if modelled as taking place in the downward phase of the long wave, they are contributing meaningfully to the next upswing. The "dynamisation" of the simple linear production model that we used in previous chapters is still in its infancy. Yet there is scholarly literature on the simulation of models with a capital goods sector and a consumer goods sector, showing shocks in one sector being transmitted to the other sector and generating a Kondratieff-like cycle on the computer screen.

The problem with the long wave theory today, as we see it, is that the most important technological innovation of the past 30 years – ICT – does not seem to lead to a new upward phase in the hypothesised long cycle in economic activity. That the coming upward phase fails to start can, however, be explained by the neo-Marxist and post-Keynesian analysis of the "financialisation" of monopoly capitalism, wherein the spectacular development of the financial sector on a global scale leads to speculative "bubbles" and disastrous financial crises, which allegedly stop the economic upswing. This is indeed an interesting argument, which also takes into account the effect of recent changes in the behaviour and attitudes of the managers of large corporations on the short-term investors' expectations of the "value of the company".

Once again, it can be seen that the Marxist analysis of the capitalist "laws of motion" can benefit, and even flourish, when the latest findings of "bourgeois economics" are digested.

Notes

1 The footnote is added by Friedrich Engels, with a note by Marx that the issue of the contradiction between the expanding forces of production and the limited consumption potential of the workers should be given further attention.
2 This implies that, according to Luxemburg, the downturn in the business cycle cannot be explained by inter-sectoral disequilibria.
3 This assumption does not change the argument.
4 Econometric research into the OECD countries as well as the EU has shown that about half the decline in the wage share over the last 30 years is the result of technological progress, which is both capital-using and capital-augmenting and is linked to the

introduction of information technology. See Arpaia, Pérez and Pichelmann (2009), and Bassanini and Manfredi (2012, pp. 15–18).

5 This emphasis on the need for additional effective demand is the cornerstone of her criticism of Marx's schemes of reproduction. See Chapter 1 in this regard.

6 This important pamphlet by Lenin was written in Zürich in the spring of 1916, as indicated in Lenin's Preface (Lenin, 1917, p. 187). Lenin quotes J.A. Hobson and Rudolf Hilferding, but nowhere does he refer to Luxemburg.

7 At this point it is interesting to note that Nikolai Bukharin, "brother in arms" of Lenin and Luxemburg, in various places in his writings pointed to the theoretical possibility that capitalism would transform itself into a "militarist state" and not necessarily into a socialist society. See Cohen (1970).

8 Sweezy (1942) also provides a thorough and critical discussion on underconsumptionist theories, including that of Rosa Luxemburg.

9 Sweezy's argument is a corrected version of that which was developed by Otto Bauer in 1936.

10 This is even more evident in her later, popular scientific writings, such as Robinson (1970, pp. 84–86).

11 This term was introduced by President Dwight Eisenhower on 17 January 1961 in his farewell address as President of the United States of America. It was probably burnt onto the retinas of Western movie-goers by its inclusion at the start of JFK, the blockbuster movie by Oliver Stone about the murder of President Kennedy.

12 As also emphasised by Josef Steindl, Keynesian-inspired public expenditure in Western Europe has been more evident in peaceful projects (see Steindl, 1967, p. 201).

13 From 1949 until his death in 1964, Paul Baran was professor in economics at Stanford University and during this period the only Marxist authority in the USA who held university tenure.

14 In the early 1970s, we statistically tested the thesis of *Monopoly Capital* that the dissipation of the potential economic surplus in the USA is increasing and found evidence of a slightly increasing trend (see Cuyvers, 1972).

15 We ignore the fixed component in capitalist consumption, the overhead salaries and indirect taxes, which Kalecki introduces in his analysis.

16 Since y is constant, $\Delta y/\Delta t = 0$.

17 It is worth noting in this context that for a long time Josef Steindl denied the investment impact of innovations (see Steindl, 1952, p. 133) although this view applies only to endogenously produced innovations. In later years, he adopted a more nuanced position, such as in Steindl (1966).

18 See the introduction of an inherent urge to accumulate in Chapter 3, section 5. In this respect, the opinions of Joan Robinson and Michał Kalecki differ greatly.

19 The first edition of this ground-breaking book dates back to 1960.

20 A number of non-Marxist scholars have also developed theories of long waves in economic activity, although often explained by monetary factors (e.g. the discovery and exploitation of new gold mines). Edlund (1980, p. 385), for instance, refers to Simon Kuznets's references to Lescure, Aftalion, Lenoir, Spiethoff and Cassel.

21 Kondratieff's statistical studies were published in Russian, except Kondratieff (1935).

22 Geoff Harcourt (2006, p. 73n) reminds us that Joan Robinson said of Schumpeter that he was Marx with the adjectives changed.

23 It should be mentioned that for Schumpeter's hypothesis to generate a long wave in economic activity, these important innovations also need to follow a cyclical pattern, which is difficult to demonstrate or explain. See Kuznets (1940, p. 267).

24 For a thorough and extensive review of *Late Capitalism*, see Rowthorn (1976). We critically reviewed Mandel's book on the occasion of its translation into Dutch in Cuyvers (1978).

25 They consider the following to be "epoch-making innovations": the steam engine, railways, the automobile and, to a lesser extent, electricity.

26 Basic innovations are closely related to what in the recent literature on this topic are called “general-purpose technologies” (GPTs). Such GPTs are applied in most economic sectors and therefore have macroeconomic impact. They do not generate immediate productivity growth; only after some time. Also, they give rise to new secondary innovations. See Bresnahan and Trajtenberg (1995, pp. 83–108).
27 An analogous reasoning is followed in the economic models with “general-purpose technologies” “destroying” the fixed capital of various industries, but after some time leading to the introduction of new and suitably adapted means of production, generating waves in accumulation as in, for example, Aghion, Akcigit and Howitt (2013).
28 See, however, van Duijn (1983) and the work conducted at MIT with the System Dynamics National Model, as well as (more recently) Freeman and Louçã (2001), Coccia (2010) or O’Hara (2012).
29 This is a wave that peters out, i.e. the amplitude of the wave diminishes over time. The assumption of a dampening wave needs to be in conformity with the non-explosive character of actual economic growth.
30 See, for example, the results of the Kalman filtering decomposition of the 1870–1913 GDP data of the UK, France, Germany and the USA in Solomou (2001).
31 Steindl also takes “outside saving” into account, which in the long run he considers to be a proportion of national income.
32 In the 1976 Introduction to the Monthly Review reprint of *Maturity and Stagnation*, Steindl stated that he was left “deeply dissatisfied” with his attempt at integrating trend and cycle. See Steindl (1976, p. xvi).
33 Later in life, Steindl considered exogenous stochastic technological change as generating growth and cycles.
34 Steindl’s 2005 paper was published posthumously. It probably dates from 1988 and was considered as preliminary by its author.
35 The reader will remember Goodwin as the author of ground-breaking research on the “growth cycle”, mentioned in Chapter 7, arising from the interaction between workers and capitalists over wages and unemployment.
36 Goodwin’s model is further explored and simulated by other scholars. See, for example, Landesmann and Stehrer (2006).
37 It is argued that the post-war shift in interest towards economic equilibrium and the mathematical tools available for the development of neo-classical models of economic growth, combined with the fact that the task of building a “grand theory of the trend *cum* oscillations” was for many years too ambitious given the mathematics at hand, was responsible for the “historical failure of those who believed in the paradigm of the endogenously sustained oscillations to accomplish the self-assigned task” (Punzo, 2001, p. xvii) of building such a “grand theory”.
38 John Sterman states that the SDNM “shows how fundamental physical processes in the economy can create the long wave without any variation in innovation rates. The bunching of innovations can thus be explained as the result of entrainment of the innovation process by the long wave” (Sterman, 1986, p. 21).
39 See also the downward phase in Mandel’s “long wave”.
40 This opinion was already defended in Gordon (2000). It is reiterated in Gordon (2016). A warning is appropriate here, since Gordon compares US productivity over extended periods of time, not within the framework of the theory of economic long waves.
41 As mentioned in Chapter 7, the growth of computer and information technology since the late 1970s has resulted in a decline in the share of wages in value added. The mechanism responsible for this decline is the increasing substitution of capital for labour (particularly unskilled and low-skilled labour), with capital goods having become cheaper relative to consumer goods. This is said to result from capital-augmenting technological progress in ICT and improved entrepreneurship, the output of R&D initiatives, etc. See Bentolila and Saint-Paul (2003), and Arpaia *et al*. (2009), as well as Bassanini and Manfredi (2012).

42 Post-Keynesians typically argue that the rapid rates of economic growth experienced during the Bretton Woods era resulted to some extent from the restrictions on international capital mobility and the system of fixed, but adjustable, exchange rates. See, for example, Milberg (2003, p. 166). On the importance of post-war institutional change, including the Bretton Woods System and its demise, see e.g. Gilpin (2000) and Lucarelli (2004, ch. 5).
43 For a thorough review, see Bagwell (2007).
44 To be more precise, if advertising expenses are productive, as we argued in Chapter 4, it is the potential economic surplus that is absorbed, and the actual economic surplus that increases with additional investments.
45 In German: "*Geldhandlungskapital*". In the 1909 Kerr edition, this is confusingly translated as "financial capital".
46 In Belgium, for instance, the holding companies, whose influence had however waned with the advent of multinational enterprises, played an important role until the 1980s. How their financial networking and cross-shareholding functioned via interlocking directorships was analysed in Cuyvers and Meeusen (1985). Industrial concentration today, outside the immediate sphere of influence of major banks, is very intense and takes place mainly through the "cascade effect" of mergers and acquisitions of firms participating in the value chains of large multinational enterprises that act as "systems integrators". This process is analysed for the aerospace and beverages sectors in Harcourt and Nolan (2009).
47 It can rightly be asked whether this transformation was such a recent occurrence. We refer to the "financial instability hypothesis", advanced by Hyman Minsky and following Keynes. Minsky views *inherent* financial instability as being a characteristic of modern capitalism with a developed monetary and credit system. See, for example, Minsky (1986, 1993). When viewed from this angle, the consequences of "financialisation" today should be regarded as a recent phase of increased inherent instability.
48 Lapavitsas (2011, p. 612) writes: "It is a measure of Sweezy's brilliance as a political economist that he surmised the future rise of finance so early". In Cuyvers (1977, pp. 160–162) we spoke about the stagnant manufacturing industry in Belgium since 1971 (i.e. half a decade before the financial deregulation started), with outdated production methods and inadequate investment paired with over-accumulation in the tertiary sector (not least of which is the "protected" financial sector), due to a higher rate of profits and a lower composition of capital.
49 Halevi (1985, p. 113) indicates that this builds on Steindl's analysis of the relationship between the development of oligopolies and capital accumulation.
50 Of particular relevance here are the influential publications of Brenner (2002, 2006). We are convinced, however, that linking the expansion of the financial sector and "financialisation" with Marx's theory of the tendentially falling rate of profits prevents a thorough analysis from a Marxist angle.
51 For a good overview of the different theoretical approaches to "financialisation", see, for example, Lapavitsas (2011, pp. 612–613).
52 The international financial deregulation wave started with the fall of the Bretton Woods System in 1973 and the abandonment of the gold exchange standard underlying it. This, in turn, caused macroeconomic monetary policies to become more efficient than fiscal policy (an evolution in line with the Mundell-Fleming theorem of mainstream macroeconomic theory), which drastically reduced the political influence on macroeconomic policies (see Cuyvers, 1988b).
53 This issue, covered in *Economy and Society*, is entirely devoted to "shareholder value".
54 This is much less the case in Germany and Japan.
55 In this regard, see Stockhammer (2004, p. 722). The introduction of rentiers as a separate class in the analysis of the working of capitalism goes back to Keynes (1936). Joan Robinson, in turn, introduced rentiers in her analysis; see Robinson (1956, pp. 68–69, 247).

56 See, for example, Kalecki (1943, p. 140), Baran (1957, pp. 105–108), Baran and Sweezy (1966, pp. 174–175). In 1942, however, probably influenced by the "New Deal" experience in the United States, Sweezy still viewed this issue somewhat differently, stressing the impact of public investment and the growing importance of social security transfers and subsidies (Sweezy, 1942, pp. 232–233).

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10 Reflections, conclusions and an agenda for future research

In previous chapters we examined a number of building blocks supporting Marx's economic theory. While the theory appears to be both logical and consistent, it is possible that some readers remain unconvinced. However, there is no doubt that Marx left behind an impressive theoretical framework which is reinforced by some important pillars. In this final chapter we will endeavour to draw some conclusions, tracing our earlier steps to investigate Marx's economic model.

10.1 The importance of the "no nonsense" approach of *Das Kapital*

Readers who are unfamiliar with the economic theory of Karl Marx might have been surprised to learn that his economic model is based on some well thought-out assumptions, which have (when logical reasoning is applied) given rise to scientific theses that were not only embraced in the nineteenth century, but are also relevant today. Marx formulated his model after studying the classical economists in much depth. However, he did not hesitate to argue forcefully – even offensively at times – against certain of these economists' views if he felt that they were rooted in bourgeois and petty-bourgeois ideology.

Marx did not hold back in being cynical about his opponents if such an approach suited his argument (and also if it did not), and his writings, including *Das Kapital*, are often polemical. Thomas Robert Malthus (1766–1834), John Stuart Mill (1806–1873) and Nassau Senior (1790–1864) were treated badly by Marx. He called Malthus's population theory "a schoolboyish, superficial plagiarism" (Marx, 1976, p. 766, n.6)[1] and he wrote about John Stuart Mill's "usual eclectic logic" (Marx, 1976, p. 221 n.31), which, "half a century after Ricardo, solemnly claims superiority over the Mercantilists by clumsily repeating the wretched evasions of Ricardo's earliest vulgarisers" (Marx, 1976, p. 652). Jeremy Bentham (1748–1832), the major representative of utilitarianism, was buried by Marx as a "soberly pedantic and heavy-footed oracle of the 'common sense' of the nineteenth-century bourgeoisie" (Marx, 1976, p. 758), while Jean-Baptiste Say (1767–1832) was put on trial for plagiarising the (at the time mostly forgotten) writings of the Physiocrats (Marx, 1976, p. 266 n.18). We can only wonder how Alfred Marshall (1842–1924), one of the founding fathers of neo-classical economics, would have fared if he had been born earlier.

The academic community is correct in regarding such treatment of other authors as unscientific. At workshops, seminars or conferences, we might be excused for making fun of the opinions of opponents and even subjecting them to some ridicule, but readers will frown upon such a practice in scholarly writings. In the 1960s and 1970s, for instance, a *Methodenstreit* was raging between the neo-classical and post-Keynesian economists over how to define the "factor of production capital" and its implications for the theory of income distribution. The major post-Keynesian spokesperson in this battle was, no doubt, Joan Robinson, the famous Cambridge economist. Later, Nobel Prize winner, Robert Solow, one of the major neo-classical economists, used to sometimes quip at conferences: "As Mrs Robinson is not in the room, I assume that everybody here knows what we mean by the quantity of capital." But Solow would never write something like that in his academic papers. By painting his real and alleged opponents as filisters, bourgeois or plagiarisers, Marx is, in fact, diminishing the scientific value of his arguments. In this book, we have from the outset opted for a purely scientific approach.

Moreover, in Marx's time, political economy was still closely linked to the other social sciences as well as to philosophy, and its practitioners applied the philosophical argumentation that they had learned. For Marx this was the dialectics, which he had inherited from Hegel, although he turned the Hegelian dialectical philosophy on its head, transforming it into dialectical materialism. Consequently, the economically astute reader of today often has to wade through lengthy passages in *Das Kapital* which, when viewed from a contemporary perspective, are unnecessarily long and tedious and viewed as largely irrelevant or "metaphysical". It is also for this reason that the "no nonsense" approach of Marx's economic theory is appropriate.

We have stressed from the beginning that this book is confined mainly to the Marxist economic theory of Marx himself. Karl Kautsky, Rudolf Hilferding, W.I. Lenin, Henryk Grossman and the evidently present-day Marxists are not, or at least are hardly, discussed. We made an exception, though, when it came to Rosa Luxemburg whose theory and views have provided the impetus for more recent theories that attempt to supplement those of Marx. We have frequently referred to the views of the Marxists of today, although we have generally not attempted to integrate these into our arguments. Moreover, there is a special category of Marxists who seem to adhere to the small detail rather than to the broad reasoning of *Das Kapital*, which is perhaps most evident in the extensive literature on the labour theory of value. At times such Marxists do not even hesitate to revise the spirit of *Das Kapital*. We have, however, tried to stay far away from such revisions and interpretations. This book was never meant to go down the path of exegesis.

10.2 The linear Marx-Leontief production model

As has been shown, Marx's theory is based on a model of how the capitalist economy works, which allows him to analyse it but also, more importantly, to formulate laws about its dynamics and to embark on risky predictions. The key question, therefore, is: which parts of this model are still standing? And how strong are the model's foundations?

It is abundantly clear that Marx was a "model builder", a *métier* that he learned (among other things) from David Ricardo (1772–1823). Assume that an economy, which produces only grain, has a growing population and therefore each year has to cultivate new land of progressively declining quality. Bearing in mind that the labour force needs a given quantity of grain for subsistence purposes, the question then arises: what will happen to the profits and the rent on the land? This is explored by Ricardo in Chapter 2 of his *On the Principles of Political Economy and Taxation*. Marx proceeds in a very similar manner when he poses the question: "Assume an economy where labour is continuously substituted by machines. What will happen to the profits and the rate of profits?"

In our various discussions, we left some of the basic building blocks of Marx's model undisturbed. One such building block is the juxtaposition of two antagonistic socio-economic classes – the capitalist class and the working class. This is a fundamental assumption. Without classes, there is no exploitation of labour, and without exploitation of labour, there is no Marxian explanation of profits. If there are more than two classes, who will be exploiting whom, exactly? Are only profits, then, linked to exploitation? And what if the notions of the capitalist class and the working class become meaningless and there are no more capitalists or workers, and everyone works harmoniously together? Class struggles and conflicts over wages or the length of the working day will then take on a new meaning and become purely social-psychological phenomena. Furthermore, if there are no workers who receive (in value terms) less than they produce, then there is no threat of underconsumption and no underconsumption crises. If there are no capitalists, the Marxian assumption that capitalists are driven by an inherent urge to accumulate and will attempt to plough back into their companies the maximum level of profits, becomes irrelevant. There is then no conflict between the growth of production capacity, which is desired by the capitalists, and its actual growth, which is constrained by the existing labour force. Also, the thesis that capitalists will introduce new production methods and technologies that replace labour with machines, no longer holds.

Thus, we have simply postulated the existence of a capitalist class and a working class without providing further research or argumentation, since the issue essentially falls outside of the economics domain and is thus beyond our expertise. We ask the reader to follow us when we ask: assume that there is a capitalist class and a working class. What will be . . . ?

Another assumption relates to the consumption required in a society to reproduce the labour force. Although we questioned this occasionally when exploring the concept of "surplus wages", we have nevertheless largely adopted this assumption because of the link between Marx's theory of value and his theory of reproduction. When we compared the physical proportions of the industrial sectors that produce the means of production and consumer goods, respectively, we became convinced that it was appropriate to refer to a defined basket of goods and services which, on average and over extended periods of time, are consumed by the labour force. This, in turn, relates to our use of a linear production model – that is, an input-output type of model that reflects the average conditions for expanded reproduction. In each economy, workers consume all kinds of consumer goods and services which have to be produced but, in the process of "productive consumption", they are also disappearing as a means of accumulating capital and reproducing the economy at large. This basket of goods and services, which in the longer term is consumed by the labour force, should be seen as nothing other than what is generally necessary for a society's consumption.

The linear production model, which was derived from the two-sector representation of the economy *à la* Marx, appeared to be the golden thread running through our analysis of Marx's economic theory. Following some fledgling scholarly papers in the field of input-output analysis, the model was developed in the 1950s and 1960s by, among others, Piero Sraffa, Michio Morishima and András Bródy. This model, the Marx-Leontief model, is both the starting point and the recurring instrument in our analysis. Marx analysed the reproduction of the economic system using his schemes of reproduction, which allowed him to derive the conditions that, on average and in the long run, have to be fulfilled for either simple or expanded reproduction to take place. We have shown that these reproduction schemes can in fact be transformed into mathematical systems of input-output equations, the solutions for which are, on the one hand, the outputs that allow this reproduction and, on the other hand, the dual unit prices/values of these respective outputs that allow "equilibrium" between production and use/consumption. The reproduction schemes from *Das Kapital* can still be used today as an instrument of analysis, particularly in revealing the importance of inter-sectoral proportions and the factors that disturb these proportions. However, the linear model of production, based on Marx's reproduction schemes, provides a much more analytical approach and from a scientific perspective is a more powerful instrument.

The model also allows generalisations in respect of any number of spheres of production, as opposed to Marx's two sectors, the means of production and consumer goods sector, respectively. In this way, it is aligned with the input-output models that were prepared and explored, both mathematically and empirically, by Wassili Leontief (Nobel Prize for Economics, 1973) and others. For a given economy, the model provides as a "solution" those outputs that allow a balanced expansion of the economic system and prices that reflect the labour time required to produce such outputs – the labour values – or that reflect the same rate of profits on the invested capital. However, we need to bear in mind that such balanced expansion is based on average inter-sectoral output flows which apply during a longer period of "normal reproduction". As a concept, this balanced expansion is, in fact, a thought construct which should lay bare its conditions. Therefore, the prices and outputs in a situation of "normal reproduction" should be considered to be "normal prices" and "normal outputs", and not equilibrium prices and equilibrium outputs. Formulating the "normal" inter-sectoral input-output flows as a mathematical system of linear equations allows them to be investigated mathematically.

It goes without saying that in using this linear model of production, a number of assumptions have to be made. Because we are formulating the model mathematically, we are able to get a clear view of these assumptions and their importance. An initial assumption relates to constant returns to scale, which means that when output doubles, all inputs of means of production and labour will also double; no more, no less. This is a restrictive assumption but it offers a firm foundation for further analysis. Moreover, analogous to Marx's reproduction schemes, nothing prevents us from considering these inter-sectoral supplies as average requirements for a specific time period, rather than as fixed and constant quantities.

Another assumption is that all labour is homogeneous or the same. Thus, one hour of labour time in, for example, the steel industry is equal to one hour of labour time in the building/construction industry, which makes them interchangeable. This assumption can also be traced to Marx who indicates that in the process of value creation, abstract labour is relevant, not concrete labour. What is to be done about skilled and unskilled labour? How can one hour of skilled labour be equated to x hours of unskilled labour? We have neglected this issue in earlier chapters of this book. In the scholarly literature on the subject, solutions to this problem are available. However, the degree of substitutability of skilled and unskilled labour by machinery and other means of production seems to be highly variable. An analysis of the linear model with heterogeneous labour should therefore be on the agenda for future research.

Yet another assumption is that no sector produces joint products, such as wheat and chaff, iron and iron slag, etc. If joint products exist, the solutions to the linear equations will show some negative prices and outputs, which evidently is a nonsensical result. It is possible to remedy this mathematically, but it will be at the expense of elegance in the reasoning behind the mathematics. The same problem arises with the introduction of fixed capital in the model. In this case, we have to assume that each item of fixed capital has an economic life of n years, during which time its efficiency remains the same such that it will transfer each year (or production period) 1/n of its value, or, alternatively, that at the end of the production period in question the fixed capital of, say, two years' duration will re-appear as a "joint product" of three years' duration. Both assumptions lead to theoretical complications and we have therefore assumed throughout, for the sake of simplicity, that there is no fixed capital in the economy, or, alternatively, that the economic life of all fixed capital corresponds to the period of "normal reproduction".

A final assumption is that technological change is absent. This evidently makes the linear model a static one, even when depicting a situation of expanded reproduction and balanced growth. Scholarly work has been carried out in the field of input-output models, making the choice of technological innovations by capitalist producers endogenous and showing that in the long run, technology will be introduced which increases labour productivity but keeps capital intensity of production unchanged. This result is at odds with the capital-absorbing characteristic which Marx attributes to technological progress and which leads to a tendency for the rate of profits to fall. This will be explored more fully below.

10.3 Which of the fundamental principles of Marx's economic theory are still intact: value, unproductive labour and the "law of value"?

In applying the linear production model, we have worked through a number of theses relating to Marx's economic theory and attempted an evaluation.

The labour theory of value of *Das Kapital* can thus be interpreted and explained in terms of this linear model. This allowed us to prove mathematically that since labour is considered to be the sole unproduced factor of production, there is a system of prices which, for each product or service, is equal to the quantity of labour directly and indirectly required for their production. This system of prices is unrelated to how the value added is distributed between workers, serfs or slaves, on the one hand, and their masters, on the other hand.[2] According to this theory, the value of each product or service consists of the value of the transferred means of production (the constant capital c), the value of the consumer goods required by the workers (the variable capital v) and the remainder of the value added (the surplus value m).

But what is the relevance of all this? These prices are not the equilibrium prices under conditions of capitalist production, which Marx calls prices of production and which in each sphere of production will lead to a rate of profits that is equal to the average rate. Marx is fully aware of this and states that the average rate of profits is simply the ratio between the total surplus value in the economy and the value of the used constant and variable capital. It appears, however, that this is not correct: if everything that is produced in an economy is totalled in terms of labour values and prices of production, and all profits and surplus value are included in the mix, then the numbers do not add up. This is known as the so-called transformation problem.

In the wake of Piero Sraffa's *Production of Commodities by Means of Commodities* (1960), we know that such equalities between the aggregates in labour values and prices of production hold for a kind of "average sector" – in effect, a "sub-system" of the economy – which Sraffa denotes as a standard system. Without dwelling on the details involved, we can show that the output quantities in such a standard system are the same as those in the economy at large when there is a situation of maximum balanced growth – i.e. when all spheres of production in the economy are expanding at the same rate, in the absence of capitalist consumption. Hence, for an economy following such a growth path, the sum of labour values and prices of production adds up. The average rate of profits will be equal to the ratio between the total surplus value and the total constant and variable capital. But it also appears that in a standard system this is irrelevant, as the maximum rate of growth is then equal to the average rate of profits. If we consider an economy on its maximum balanced growth path, the labour theory of value becomes redundant for explaining the rate of profits!

We advanced two reasons why the labour theory of value remains relevant. The first reason is that a positive rate of exploitation is a necessary condition for a positive, average rate of profits. The matrix algebra of input-output models teaches that, according to the Hawkins-Simon condition, positive outputs and prices are possible only if all economic sectors are producing a surplus over and above what is being used in production. More important, then, is the so-called "Fundamental Marxian Theorem" of Nubuo Okishio and Michio Morishima, which shows that a positive rate of profits is obtained only if the value added in the economy is greater than the value of the labour power.

There is a second, though more debatable, reason why labour values are relevant. If, in the various industrial sectors, workers are not only reclaiming equal pay for equal work (which will determine the average wage rate) but also, stemming from feelings of social injustice, are waging a class struggle over wages in the light of profits, equilibrium prices will tend towards labour values. The fact that workers, assuming they are performing "homogeneous labour", are seeking the same wages/profits ratio has occasionally been suggested by Marx. Evidently, the resultant equilibrium prices will not allow the same rate of profits among the various sectors, such that the capitalists will move their capital to the sectors where the rate of profits is the highest. It follows that the sets of labour values and prices of production are the prices that would be attained if either a class struggle against above-average exploitation, or capital mobility, is carried through. Thus, if we further assume that workers aspire to their wage bill being in the same proportion to the profit level (in other words, they are aiming for the same proportional functional distribution of the value added between labour and capital in all industrial sectors), the logic of the linear production model dictates that prices will move between labour values and prices of production.

Another critical issue in Marxian economics relates to what Marx calls productive and unproductive labour. The linear model explains why there are positive outputs and labour values. The outputs are use values, i.e. goods and services have value in their use. We have proven Marx's thesis that it follows from the dual solutions of the model that activities that do not lead to use value in outputs are not creating labour value, and are therefore unproductive. Although abstract reasoning tells us that labour and means of production that do not give rise to the production of use value are not being used productively, such reasoning creates problems when we consider the *faux frais* of production, which Marx considers to be unproductive. Commerce involves making goods available, while publicity involves creating new use value, with many services of the financial sector being use value. It seems preferable to investigate whether an activity is yielding a surplus product rather than to apply Marx's debatable criterion. If this is the case, the value of the surplus product will be part of the total surplus value and the activity is productive. A related problem stemming from Marx's concept is that the outputs of the unproductive sectors have no labour value, in spite of their having a price of production. As a result, the total output of the economy will be different when measured in labour values and in prices of production, even when it is assumed that the economy is on its maximum balanced growth path. The old transformation problem then reappears with a vengeance. Marx's categorisation of what should be considered productive or unproductive is untenable. Of course, many activities that Marx indicates as being unproductive are wasteful. Here the concept of "potential economic surplus", which Paul Baran and Paul Sweezy launched in the 1960s, is relevant as it captures such wasteful activities, irrespective of whether they are productive or unproductive.

Our thesis that the *faux frais* of production (or cost-increasing inputs) are productive if they give rise to a surplus product which can be used in the next production period, flows from mathematical evidence of an economy on its maximum balanced growth path – i.e. an economy without capitalist consumption and with all surplus value accumulated. We also investigated the relationship between the average rate of profits and the rate of accumulation in an economy, when total surplus value is fully or not fully accumulated. In the former situation, the answer is straightforward as the rate of profits and the rate of accumulation will be identical.[3] However, if the capitalist class is consuming part of the output produced, the same rate of profits as in the former case will be combined with a lower rate of accumulation, which will be even lower as the capitalists consume proportionately more of the profits/surplus value. This leads to the notion of how the so-called Cambridge School models the relationship between the rate of profits and the rate of growth of the economy, particularly since the release of Joan Robinson's *The Accumulation of Capital* (1956), which in turn builds on the work of Michał Kalecki. Robinson emphasises that the capitalists' accumulation will generate the capitalists' profits, or, as Marx would say, will contribute to the realisation of the surplus value. Kalecki famously stated: "Capitalists get what they spend. Workers spend what they get."

This leads us to a discussion on the realisation of the surplus value, which goes to the core of Rosa Luxemburg's reasoning in her *The Accumulation of Capital* (1913). The concept was later reformulated in post-Keynesian neo-Marxist economic theory, which is associated with notable scholars such as Michał Kalecki, Joan Robinson, Josef Steindl, Paul Baran and others. We have called these theories post-Keynesian and neo-Marxist as they put effective demand and the realisation of surplus value at the centre of the debate, such that the rate of profits evolves, in the long run, with effective demand and with the degree of realisation of the surplus value. If there is insufficient effective demand, the total surplus value produced will not be realised and the rate of profits will fall.

We should emphasise, however, that Marx's theory of value or the underlying analysis that uses the linear production model following Sraffa, Morishima and others, bears little resemblance to the dynamics of capitalism – the so-called laws of motion of capitalism – which Marx aims to describe. The post-Keynesian neo-Marxists neglect the Marxist theory of value, which they mostly consider to be "redundant", "useless" or even "metaphysical". The "dynamisation" of the linear model and therefore the linking of its "static" analyses under the assumption that the capitalist economy under consideration is in a state of "normal" reproduction (based on average proportions between the aggregate sectors) with that showing endogenously modelled development, is still in its infancy.[4] This evidently leaves a schizophrenic impression: the theory of value and the theory of growth and development are still methodologically detached from each other.

Do we really need the Marxist theory of value? Yes, if we want to achieve a real synthesis of Marx's views with present-day post-Keynesian economic theory. We do not believe that the post-Keynesian price theory is adequate, either logically or from an historical perspective, for likening Marx's theory of value to the historical relics of economic thought. Only Marx's theory of value provides logical and historical insight into the nature of capitalist exploitation and the role of the institutional "initial condition" that there exists a class owning the means of production and providing micro-foundations of exploitation and class struggle at the shop floor level. However, onto this theory should be grafted post-Keynesian insights into how price formation takes place in monopoly capitalism, if we want to achieve a sufficient degree of theoretical generality.[5] Monopolistic exploitation in circulation thus supersedes exploitation in the sphere of production.

10.4 What about the dynamics in Marx's economic theory: technological innovation, the rate of profits and exploitation in the long run, economic cycles . . . ?

In his analysis of the laws of motion, Marx is interested in assuming sufficient efficient demand. In fact, Marx assumes that, in the long term, the surplus value produced is also realised. How the rate of profits evolves in the long run is primarily determined by the introduction of technological innovations in the capitalist production process. According to Marx, the major feature of this introduction process is that labour is replaced by capital, which increases labour productivity and generates super-profits for the introducing capitalist. In fact, this capitalist hunger for super-profits keeps the process going. Furthermore, due to the introduction of capital-using innovations in production, workers become unemployed. The army of the unemployed expands such that wages fall.

To the extent that the innovations seep through to the other capitalist producers, the super-profits of those who innovated first will disappear. However, due to the increased productivity of labour, the prices of production will be lower than before, thus illustrating the working of what Marx calls the "law of value". How the rate of profits will react depends on the degree to which the innovations replace labour with capital and induce an increase in the amount of capital per unit of output. We have shown that a given increase in the productivity of labour, which keeps the use of the means of production per unit of output constant, leads to a higher percentage change in the rate of surplus value and therefore in the rate of profits. It remains to be seen by how much wages will rise as a result of the increasing productivity of labour – an issue that we did not investigate.

We looked into Marx's argument surrounding the effects of technological innovation in terms of what he calls the technical composition and value composition of capital. These effects are often very complicated and cannot be solved analytically, but they nevertheless lead to relevant conclusions. It is correct to say that if capitalists, in their hunger for super-profits, introduce labour-saving innovations, these can lead to larger percentage increases in the means of production used per unit of output. However, in introducing such labour-saving and capital-using innovations, it does not follow as a law that the value composition of capital necessarily increases, even if the degree of mechanisation – or, in other words, the technical composition of capital – increases.

Yet it is Marx's thesis that where the rate of surplus value remains constant for the sake of the argument, the average rate of profits will fall in the course of capitalist development due to a tendency of the value composition of capital to increase. This is his "law" of the falling rate of profits. At a particular point in time, the average rate of profits will have fallen so much that the capitalists will refrain from accumulating capital and from investing. As a result, capitalism will break down. This is in truth a thesis, based on a logical argumentation, that allows a "risky prediction", both theoretically and empirically.

First and foremost, we investigated this "prediction", again using the linear production model and simulating it. Our simulations showed that rising productivity of labour also allows an increase in the capital intensity of production without inducing a fall in the average (or general) rate of profits. It all depends on the extent to which this capital intensity grows and the amount by which wages will increase with the rising productivity of labour. We reviewed Okishio's theorem which proves that if wages remain constant, a rational capitalist choice of new technologies will actually lead to a *rise* in the average rate of profits. On the other hand, if – as demonstrated in simulations by David Laibman – wages are rising *pari passu* with labour productivity, some rationally chosen innovations might lead to a fall in the rate of profits (as Marx expected) whereas other, similar innovations would not.

What does the empirical literature tell us about the secular behaviour of the rate of profits? For one thing, the literature presents contradictory results which are mostly due to differences in methodology and statistical manipulation of the data. In the United States, for which the best and longest time series are available, we found a volatile rate of profits during the years 1869 to 2009, with phenomenal peaks in 1880 and 1944 and a rock bottom rate in 1932. Visual inspection highlighted a downward trend in the rate of profits in the USA between 1870 and the end of the 1930s, but an increase to an unprecedented level during the Second World War. In the post-war period, the rate of profits tended to fall once more until the start of the 1980s, only to recover and increase slightly thereafter – not exactly in line with Marx's law of the tendential falling rate of profits, but rather a long-term oscillatory movement! In addition, for a mature economy such as Belgium, our estimates of the "return on equity" during the 1911 to 2000 period shows a striking level of stability during the sub-periods.

The evolution in the United States of the value composition of capital C/V, as a measure of the capital intensity of production, does not provide conclusive evidence in favour of Marx's thesis either. On average, it seemed to increase between 1870 and 1914, after which a declining trend set in until the outbreak of the economic crisis in 1929. After the Second World War, C/V was found to first increase and then subsequently decline until the turn of the twenty-first century. Hence, insofar as the capital intensity of production is a factor determining the changes in the rate of profits, a long-term oscillatory movement is prevalent rather than a secular decline.

As Marx's law of the falling rate of profits is debatable on both theoretical and empirical grounds, we investigated other reasons for economic stagnation present in *Das Kapital*, although they are less well developed. Marx's explanation of the dynamics of capitalist development – and, similarly, that of the neo-Marxists – focuses on the rate of profits, which can change in the long run for reasons other than those immediately related to Marx's law. Evidently, technological innovations and changes in the balance of power in the class struggle are also major factors that determine how the rate of surplus value, i.e. the ratio of profits to wages, will change. In Marx's account, this happens at the factory level due to the introduction of labour-saving technology in the production process, as well as the daily struggle between capitalists and workers on the shop floor over wages, the length of the working day and labour intensity. Thus, once again, Marx's theory of exploitation attempts to explain the origin of capitalist profits. If we want to understand capitalist exploitation, we have to start from the "institutional datum" that the capitalists are the owners of the means of production. The relationship between capitalist exploitation and this "institutional datum" also underlies the analysis of the "analytical Marxists". As Marx points out, the intensity of the class struggle and the balance of power between labour and capital determine how the intensity of exploitation, measured as the rate of surplus value, evolves over time.

At the macroeconomic level, this is reflected in how the share of profits in national income evolves. The neo-Marxists, following Kalecki's theoretical insights, have supplemented Marx by emphasising the impact of the sphere of circulation on the share of profits. Free competition is absent in monopoly capitalism and the capitalists apply "mark-up" pricing. Evidently, the "mark-up" applied is a function of the balance of power within a sector between the producers or, what Kalecki calls, the degree of monopoly. The profits share is also influenced by macroeconomic policy and the changing (maybe even cyclical) political influences on policy decision-making processes.

The 1960s provide evidence that the labour movement was strengthened during a period of continuing high employment, which led to rising real wages. According to some scholars, the "profits squeeze" at that time eroded the rate of profits, which in turn must have led to the economic crisis of the 1970s.

Marx also formulated a theory of the business cycle, which was far ahead of its time, in terms of which the available "reserve army of labour" once more plays a determining role. Although Marx advances, among other factors, the evolution of real wages and the rate of surplus value in a bid to explain the business cycle, these are, in fact, less relevant. The downturn in the business cycle sets in because capital accumulation, fired by the capitalists' hunger for profits, proceeds at a faster pace than that at which surplus value can be created. This creates a situation of over-accumulation of capital, which causes a decline in the general rate of profits. The upswing in the business cycle starts once the excess of capital, relative to the available labour force, has disappeared.

Das Kapital also provides a monetary theory, although it is far from fully developed. This monetary theory is clearly different from the monetary orthodoxy in Marx's time. Marx points out, as does Keynes 60 years later, that money is also kept idle. Here we are confronted by a variant of the demand for money due to the Keynesian speculation motive, although Marx devotes attention neither to the determining factors of this demand, nor to its instability – a vision that was first formulated in Keynes's *General Theory* (Keynes, 1936, p. 201). With Marx, as with Keynes, the rate of interest is a purely monetary phenomenon, but in Keynes's economic model the rate of interest links the monetary sphere to the real sphere of the economy, i.e. where production takes place and income is generated. In the second and third volumes of *Das Kapital*, as well as in his unfinished notes, Marx elaborates on the "money crisis" (in other words, a situation of financial and monetary instability) as the possible detonator of an economic crisis. However, he does not relate this "money crisis" to his theory of the business cycle, thus leaving an essentially undeveloped monetary theory. This calls for the intellectual input of the post-Keynesians on the role of uncertainty and of expectations (see Kregel (1976), and the recent summary of the discussions among post-Keynesians in O'Donnell (2013)). Furthermore, the recent theory of "financialisation" provides interesting clues, including a better understanding of the protracted economic crisis in which we in Europe and the United States currently find ourselves.

Das Kapital, however, contains an underconsumption theory, i.e. an explanation of long-run economic stagnation, which is an alternative to Marx's law of the falling rate of profits. Insufficient effective demand due to limited consumption possibilities relative to what can be produced provides a macroeconomic mechanism, which has been analysed by Thomas Robert Malthus and is highly regarded by Keynes (Malthus, 1820, pp. 314–322).[6] On the basis of this approach, underconsumption and stagnation entered Keynes's thinking.

Marx also advances tendencies of underconsumption to explain incomplete surplus value realisation and emphasises the importance of external markets. Yet it is Rosa Luxemburg who analyses the role of these external markets in detail, while also stressing how a situation of expanded reproduction has an impact on effective demand for consumer goods as well as means of production. It is via this "Marxist route" (in contrast to the "classical route" from Malthus to Keynes) that "effective demand" came to Michał Kalecki and the later neo-Marxists.[7] It will become clear that this long-term capitalist development approach offers an almost inexhaustible source of inspiration. A number of the Marxist and neo-Marxist theories that we have investigated are based on this.

First, we have seen how the role of military spending as an "external market" entered the analysis of economic stagnation of Luxemburg and, later, that of Kalecki, Joan Robinson, Paul Baran and Paul Sweezy. However, Kalecki and his aforementioned peers go further by considering technological innovation in production processes as an "external market" of capitalism. Kalecki shows how technological innovations are generating new investments and are preventing the capitalist economy from entering a "stationary state". In addition, he and the post-Keynesian neo-Marxists argue that the slower pace of introducing technological innovations is a major cause of stagnation in monopoly capitalism. Unfortunately, in Kalecki's model, innovations are still exogenous. They drop, as it were, from heaven.

The "endogenisation" of technological change in economic theory has led to interesting results and insights. Important technological innovations, such as the introduction of the steam engine, the combustion engine or electronics, are generating a surge in investment, which will stimulate economic activity for a long time. It is plausible to state that such a surge in investment will also cause an "echo effect" in the future which, in turn, gives rise to a "long cycle" in economic activity. This, in fact, might be behind the Kondratieff cycle. The interaction of the introduction of labour-saving technology, growing unemployment, the evolution of real wages, profits and effective demand, can also generate a cyclical movement – the Goodwin cycle. Alternatively, it is possible to adopt the neo-Schumpeterian thesis of the "clustering" of technological innovations. We looked into the Marxist "long cycle theory" of Ernest Mandel, who asserts that such cyclical movements are the result of the interaction of labour-saving innovations, changes in the balance of power in class struggles, the resultant changes in the rate of surplus value, and the presence of "surplus capital". Mandel's theory does not lead to a cyclical movement of economic activity per se, and it is far from clear how his theory views the role of exogenous events, such as wars and revolutions.

On the other hand, maybe the pattern of long-run economic growth in capitalism only consists of a sequence of "ups" and "downs", not a cyclical pattern with some fixed frequency. This is possibly due to the required institutional changes in the working of the capitalist system which must launch an "upward phase", such as the introduction of a system of limited liability companies, the emergence of monopolistic enterprises and the "managerial revolution", or the establishment of the Bretton Woods System after the Second World War. The absence of appropriate accompanying institutional changes might well explain why a new "upward phase" of the Kondratieff cycle is failing, and why the last three decades have been characterised by global monetary and financial instability, while periodic economic recoveries have been short and fragile.

To summarise, it can be stated that:

- Marx's theory of value is interesting both as a theory of long-term prices in the capitalist economy and as a logical, historical explanation of how capitalist profits emerge and how class struggles on the shop floor influence the evolution of profits over time. On the other hand, this theory of value about price changes offers us little, if any, understanding of the situation in the real world with inflation and oligopolistic price setting. We nevertheless think that Marx's theory of value, when formulated and further explored using the linear model of production (helped by the fact that the "structuralistic" component – the dual structure of the model – is dominant in this approach), still holds a great deal of promise.
- Marx's unproductive labour concept, at least as it refers to labour expended in the process of production or is otherwise related to a product or service

(distribution, publicity, banking, etc.), is theoretically incorrect. To the extent that such activities waste scarce economic resources, they can be analysed and their importance assessed without recourse to Marx's theory of value.

- Marx's assumption of a relentless capitalist drive to accumulate, which is regularly frustrated by labour scarcity, seems correct. However, the interaction between this drive and labour scarcity, as well as the implications for the dynamics of capitalism – in particular, present-day capitalism – can be better analysed using the post-Keynesian (Robinsonian) neo-Marxist theory of economic growth.
- Marx's theory of the individual capitalist's introduction of labour-saving (or labour-augmenting) technology in the production process, which is motivated by the hunger for super-profits, is also essentially correct. The question, though, is whether we need his theory of value, although it does enable us to understand this type of technological innovation and its consequences. Today, however, other types of technological innovation and technological progress are also relevant, which could be either capital-saving and labour-using or capital-augmenting and labour-augmenting. In addition, there is reason to believe that the labour-saving character of technological change that Marx had in mind is hurting the less-skilled workers of today more than the highly skilled ones.
- Marx's theory of the tendency for the composition of capital to rise, which Marx infers from the foregoing, as well as the resultant tendency for the general rate of profits to fall, is not proven and so is debatable on both theoretical and empirical grounds. There is reason to assume the prevalence of a kind of long-term undulatory movement, for which *Das Kapital* presents no explanation.
- Marx's underconsumption theory, as well as the theory surrounding the realisation of surplus value, are inadequately developed. These issues relate to what we would these days call effective demand, and they can be analysed just as well – if not better – with the instruments of present-day, post-Keynesian macroeconomic theory. It is, however, interesting that Marx stresses the role of the external market and that the neo-Marxists have developed his insights further by looking at macroeconomic policies as the result of combining policy instruments that influence the "external markets" in the broadest sense of the word.
- For many decades, Marx's theory of the business cycle was innovative, but the present-day Keynesian theory is more general. Moreover, the monetary part of Marx's economic model is underdeveloped.

In contrast, the linear production model makes possible an analysis of Marx's theory of value and growth by focusing on their interdependence. This, in turn, allows the general rate of profits and the possible rate of growth of the capitalist economy to be explained by the structure of the inter-sectoral supplies of means of production and consumer goods. Although the introduction of technological progress in such a model – certainly endogenous technological progress – is mathematically very complex[8] and still in its infancy, it represents an interesting challenge for future research. Apart from this, simulations of the linear production model can provide more and new insights into the “laws of motion” of capitalism. Furthermore, the introduction of economic classes and the class struggle in the model remains essential.

The above conclusions make it necessary for us to go beyond our findings to date. The question now arises: does Marx’s economic theory, as it stands, contain sufficient substance to develop further into a still-independent economic doctrine?

10.5 Is Marx’s economics an independent doctrine, a module of the post-Keynesian theory or a starting point for a post-Keynesian neo-Marxist synthesis?

The economic theory of Marx, which we have analysed in this book and assessed in terms of its scientific value, belongs to what Marx and Engels like to call “scientific socialism” (Engels, 1880). All the philosophical, sociological and economic theses flowing from dialectical and historical materialism are, for the sake of convenience, referred to as Marxism, of which Marxist economic theory is a part.

In his time, Marx was unable to link his theory to the evolution of economic ideas. The first volume of *Das Kapital*, which was published in German in Hamburg in 1867, remained largely unnoticed in British academic circles. On the other hand, the German economists of the Historical School opposed the abstract reasoning and the model building of the classical economists, and consequently those of Marx as well (Sperber, 2013, pp. 456–463). Why Marx’s economic thinking is still discussed today is because he related it closely to the political praxis of the young socialist labour movement to which he wanted to provide guidance. In this way, Marx’s economic theory, at least for some time, became the economic science of the movement. Later, reformist social democrats began to pay lip service to Marx’s theses but then shifted their attention. In the twentieth century, the social democratic parties succeeded in dominating economic policies, for which Marx’s economics proved to be unusable. More favoured writings were those of economists such as A.C. Pigou and John Maynard Keynes.

Das Kapital therefore did not fall into the slipstream of evolving Western economic thought. The fact that Marx’s economic theory had been state ideology in the Soviet Union for 70 years simply exacerbated the situation. Although originally a source of inspiration, *Das Kapital* and the other writings of Marx and Engels became major reference works and subsequently canons, but with the fall of communism in the Soviet Union and Eastern Europe, Marx was also toppled from his pedestal. As a result, a page in the history of economic thought seems to have been turned.

Notwithstanding these developments, the twentieth century saw Marx’s economics being taken very seriously in some academic circles in the “West”. Joseph Schumpeter, Wassili Leontief, Michał Kalecki, Joan Robinson and others were all seeking answers to the questions that Marx had asked, using new scientific methods.[9] When Piero Sraffa published his *Production of Commodities by Means* *of Commodities* in 1960, it seemed that Marx and the views of the classical economists would be revived. The discussions that followed were of a highly theoretical nature but did not succeed in penetrating or undermining the "mainstream" of neo-classical economics. No doubt, this was attributable to neo-classical economists not being interested in the possible ideological conclusions that might follow on from "Marx after Sraffa", as well as to the seemingly practical usefulness or the perceived sense of reality of neo-classical economics.

When a new scientific paradigm is created, a first phase is generally followed by a series of publications that expand the new theory or introduce further developments. This was the case with neo-classical economics, although the area covered became ever larger – from the theory of prices and the analysis of rational decisions of consumers and producers, to the theory of economic welfare, international trade and economic growth. Moreover, neo-classical economics engulfed (admittedly an impoverished) part of the macroeconomics legacy of John Maynard Keynes. It should be acknowledged that for students and economics professors alike, the "neo-classical synthesis", which was thus created over more than a century, is an impressive construction which offers a beacon and a safe haven.

Neither Marxist nor post-Keynesian economics has been able to offer this. We have seen how Marxist economic analysis aims to deliver a coherent theory of the laws of motion of capitalism. However, based on our investigations in this book, we found that, by using the linear production model (which implicitly supports Marx's methodology and follows his premises), these laws of motion are of a lesser type than the "laws" upheld by Marx and the Marxists. This conclusion holds true in particular for the alleged consequences of substituting machines for labour, but it does not diminish the value or the logic of Marx's model, which is rooted in classical political economy. Based on the average conditions of normal economic reproduction and translated into matrix algebra, this model is able to explain prices, labour values and long-term, balanced growth.[10] Much work still needs to be done to make the model dynamic and to introduce technological change as an endogenous process; yet important steps have already been taken.

Unfortunately, Marx's model lacks a monetary component, and no provision is made for expectations with respect to monetary developments, sales, investment, profitability and wage formation, or the role these play in economic instability. The role of uncertainty and expectations is mostly dealt with in Keynesian and post-Keynesian theory,[11] which suggests that expectations about the future are essentially subjective and irrational, resulting from un-quantifiable uncertainty (Keynes, 1936, ch. 12).[12] Like Marxist economic theory, post-Keynesian theory differs from neo-classical economics in that it starts from the premise that capitalism as a society is divided into antagonistic social classes. The post-Keynesians also regard economic growth and development as an historical process, driven by expectations and decisions in uncertainty, where events are shaped by economic and political institutions (Arestis, 1992, pp. 88–89).

In the post-Keynesian approach, expectations are subject to sudden and violent changes, which are due to – among other things – speculation and changes in the psychological climate. As Kregel (1976) argued, Keynes assumed in his *General Theory* – for the sake of the argument – constant long-period expectations and that particular expectations could be disappointed but could not affect long-term expectations. Being dissatisfied with this approach, Keynes later stressed that when disappointed expectations affect the state of long-period expectations, such "changing views about the future are capable of influencing the present situation" (Keynes, 1936, p. 293). Kregel (1976, p. 221) states:

> Instead of assuming that the future was known (or that there were sufficient future markets and that all future prices could be taken as known) he [Keynes] maintained the assumption that it was in the nature of a monetary economy that the future could not be known. He chose instead to work out the effects of different states of expectations on employment and income under the provisional assumption that differences between expectations and realisations would not affect general expectations – that is, to work with a model of stationary equilibrium. This is precisely the position and assumption that underlies the use of tranquility in the method of comparative dynamics in the post-Keynesian models, but with a different choice of given and dependent variables.

It is right to ask whether the approach of Keynes and the post-Keynesians can be reconciled with that of Marx who considers a self-reproducing economic system, with long-term prices and proportions being relevant for this reproduction. If these prices and proportions are interpreted as equilibrium prices and proportions, this seems at odds with the view of Keynes and the post-Keynesians. However, if such prices and proportions are considered to reflect the long-run conditions of expanded reproduction which are continuously interrupted by all kind of events, then the contradiction between the two views is much less pronounced. The post-Keynesians[13] reject the concept of "economic equilibrium", whereas in Marx's model certain "equilibrium relations" have to be respected as a necessary condition of economic development in the long run. There is no contradiction in as much as both theoretical systems indicate that in the long run the capitalist economic system easily gets out of "equilibrium", if it ever reaches it, making steady expansion impossible.

As to price formation under oligopolistic conditions, which is mostly the case today, we found that profits in the post-Keynesian theory (as with Marx's theory) are not a residue and that the core of the discussion is, rather, how general such monopolistic and oligopolistic price formation is and what its macroeconomic implications are. In addition, wages in post-Keynesian theory are determined by the balance of forces between capital and labour. The fundamental difference with Marx relates to whether wage negotiations focus on nominal wages or real wages, i.e. what Marx calls the value of labour power. The post-Keynesians rightly stress the role of uncertainty and the expectations about future developments that the negotiators are nurturing. In terms of this view, Marx's theory of the value of labour power only holds when the workers equate the nominal wage evolution with that of the real purchasing power (what economists call "money illusion").

On the other hand, it was argued in Chapter 2 that the workers' spending of their wages creates a *physical* consumption pattern, which influences the conditions of simple or expanded reproduction and therefore also the size and proportional composition of the outputs and the attainable rate of economic growth. Depending on what the capitalists are consuming proportionately, this rate of growth will determine the general rate of profits of the economy.

What about other institutions? Marx's historical materialism indicates how existing capitalist institutions – which Marx calls the "relations of production" – are increasingly in contradiction to the evolving "forces of production". No doubt, multinational enterprises and top managers, but also the "military–industrial complex", belong to the set of institutions that determine the functioning of present-day capitalism. Although Thorstein Veblen (1857–1929), father of institutional economics, criticised Marx and the Marxists for their determinism,[14] what is relevant here is the importance attached by both to the evolutionary processes that capitalism is going through.

These are among the reasons why we believe Marx should be linked to present-day economic thinking, particularly post-Keynesian economic theory. There was a time that Paul A. Samuelson (Nobel Prize in Economics, 1970) called Marx a "minor post-Ricardian" (Samuelson, 1962).[15] Given the foregoing discussion, we do not want to give the impression that Marx should also be viewed as a "minor post-Keynesian", nor that he should become one. Nevertheless, Marx's economic theory needs completion and correction, using a number of post-Keynesian insights. We mentioned its integration with monetary theory and the role of uncertainty and expectations, as well as the relevance thereof for long-term macroeconomic dynamics, the full integration of the theory of wage formation, and the integration of technological change and innovation as an endogenous factor in the Marxist economic model. Only in this way, with the insights that Marx's economics provides, can justice be done and the outdated or theoretically questionable views finally be left behind.

Marx is particularly interested in the laws of motion of capitalism, i.e. the long-run developments. We have seen how expanded reproduction is central to his model. We have also seen how the "founding fathers" of what can be called "post-Keynesian neo-Marxism" – Michał Kalecki and Joan Robinson – built their theories about capitalist, long-run development on the methodology and insights of Marx and Rosa Luxemburg. However, an economic model like that of Marx, which is unable to explain short-term developments, is clearly incomplete . . . or incorrect.

Will Marxist political economy thus become a module of post-Keynesian economic theory? We consider this to be highly unlikely. First of all, many post-Keynesians will not be particularly pleased with such a module. For one thing, to be linked to (not to mention, be associated with) Marx is, deplorably, still casting a long, unscientific shadow. Many post-Keynesians have no ideological ties whatsoever with Marx, and some of those who have, might be willing to hide such ties for strategic reasons.[16] Others – not least of which is Paul Davidson – apply a very narrow and limiting definition of what should, and should not, be considered post-Keynesian (Davidson, 2005). Joan Robinson, Geoffrey C. Harcourt and other scholars working in the “Cambridge tradition” see this differently and postulate without diffidence the strong affinity between post-Keynesian macroeconomics (to be precise: non-monetary macroeconomics) and the economic theory developed by Marx. Our plea, however, is not for Marx to be dissolved into post-Keynesianism but rather for a present-day, post-Keynesian neo-Marxist synthesis to be established.[17]

Neo-Marxism, as a school of contemporary economic thought, has reformulated or revised Marx’s economic theory in the light of economic and social developments that have taken place since the publication of *Das Kapital* (monopolisation of the economy, macroeconomic stabilisation and stimulation policies, the concept of the “Third World”, etc.) and developments in non-Marxist economic thinking (Keynes, Leontief, institutionalism, etc.). At the beginning of the 1980s, roughly three strands of neo-Marxist economics could be distinguished: (1) the post-Keynesian, as evidenced in the work of Kalecki, Robinson, Sweezy and Baran, which starts from the “economic surplus” concept and largely sets out to analyse the “laws of motion” of capitalism today; (2) the mathematical, as evidenced in the work of Sraffa and Morishima, which mainly sets out to revise and correct Marx’s theory of value; and (3) the “tiermondist”, as evidenced in the work of Baran, Arghiri Emmanuel, André Gunder Frank, Robinson and others, which analyses the phenomena of international relations, underdevelopment, economic dependence and imperialism. There is a great need for these three strands to be integrated in a coherent way, together with post-Keynesian macroeconomic theory.

Such a synthesis is, in fact, the intersection of Marxist and post-Keynesian economic theory, as illustrated in Figure 10.1. This reveals the common or integrated premises, insights and theses of Marxist and post-Keynesian economics.

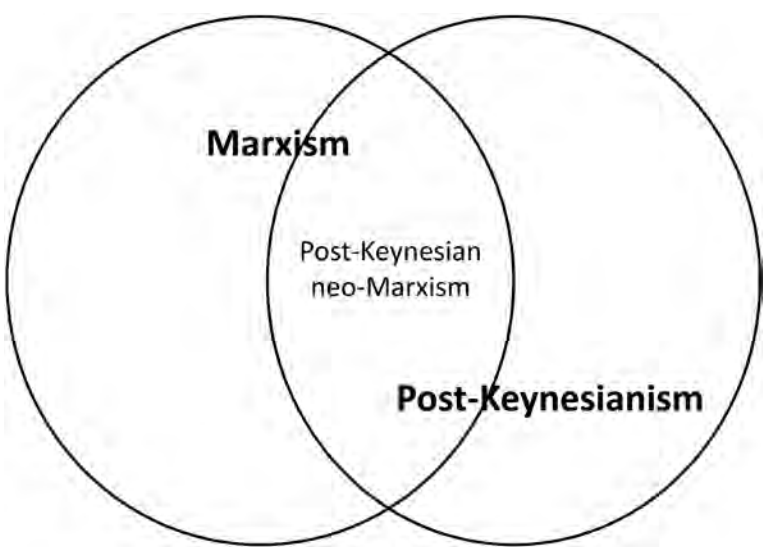

Figure 10.1 The post-Keynesian neo-Marxist synthesis

When Paul Samuelson was once asked whether Keynes was dead, he replied: "Yes, Keynes is dead. And so are Newton and Darwin." It would be nice if we could add Marx to the list of the dead scholars whose theoretical insights have become part of the body of accepted scientific knowledge.

Notes

1 We are consciously referring here only to Volume I of *Das Kapital*, being the only volume that was published by Marx himself. The other volumes were prepared for publication after Marx's death by Friedrich Engels, based on Marx's unfinished manuscripts.

2 Geoffrey Harcourt, a leading post-Keynesian economist whom we view in many respects as being a mentor, rightly pointed out: "Many modern economists find it impossible to accept that there is a distinction between the notion of price as an 'objective' index of reproducibility in the classical tradition and as a 'subjective' index of scarcity in the neo-classical tradition" (Harcourt, 2006, p. 126).

3 This is how we could prove our thesis on the productive character of cost-increasing inputs. When there is no capitalist consumption, the rate of profits and the rate of accumulation are identical. To the extent that cost-increasing inputs can be accumulated, they contribute to the value of the rate of accumulation, and thus also to that of the rate of profits.

4 It should be stressed that renowned scholars working in this field hold conflicting views on the "dynamisation" of a multi-sectoral linear model. Goodwin (1990) and Goodwin and Punzo (1987) explore in a dynamic model the dynamics caused by technological innovation by simulating its impact, while Pasinetti (1993) considers the "dynamisation" to be impossible and introduces technical change in his linear model of a "pure labour economy" with vertically integrated sectors. For more on these views, see Kerr and Scazzieri (2013, pp. 273ff.).

5 The same argument evidently holds for Sraffa's contribution to the classical theory of value in relation to post-Keynesian theory. For a discussion on this issue, see Arena and Blankenburg (2013, pp. 75–79).

6 For a thorough introduction to Malthus's theory of effective demand, see, for example, Eltis (1980). Keynes's appreciation of Malthus can, for example, be found in Keynes (1936, pp. 362–364).

7 As a close collaborator of Keynes in the 1930s, Joan Robinson was influenced from both sides.

8 The introduction of endogenous technological progress in all economic models is very complex. In neo-classical growth models, it was often simplified by adding a "learning-by-doing" equation. Alternatively, Romer (1990) has introduced scientific research as economic activity in a neo-classical model of a one-sector economy, with the rate of technological change being dependent on the interest rate and on the possibility of obtaining monopoly profits. Pasinetti (1993) introduced technological progress in his linear production model of a "pure labour economy" by considering continuously changing sectoral labour input coefficients.

9 Joan Robinson wrote (1942, p. 95): "if there is any hope of progress in economics at all, it must be in using academic methods to solve the problems posed by Marx".

10 Smolinski (1973, p. 1199) wrote: "It would be a difficult task for Marx and, at the early stage of development of mathematical economics at the time, a pioneering venture to reformulate his economic system as a mathematical model using the tools most appropriate for that purpose, such as linear algebra, matrix algebra, and the methods of finite mathematics." Moreover, Smolinski stressed, matrix algebra, although developed in 1868, was known only to a narrow circle of experts.

11 Frederic Lee rightly remarked on our use of "post-Keynesian" (with a hyphen) that it should be distinguished from "post Keynesian" (without a hyphen). "Post-Keynesian" refers to the theories that follow the Cambridge tradition, whereas "post Keynesian" is broader and also encompasses institutional economics and radical economics (Lee, 2009, p. 82). We are using the term "post-Keynesian" in the sense of Lee, but there are evidently also important links between Marx and "post Keynesian" theory, which we do not discuss.

12 The point here is that according to Keynes, there is no way to say something with any degree of probability about some future economic developments, such as, for example, the interest rate 20 years from now or the degree to which an invention will become obsolete.

13 For a good review of the post-Keynesian views about uncertainty and expectations, see Barkley Rosser Jr (2001). From this review it will be clear that there is only limited unanimity among the post-Keynesians about the issue of how expectations should be dealt with. On uncertainty in post-Keynesian economic theory, see also O'Donnell (2013).

14 Veblen criticised the Marxist thesis that the working class, confronted by the growing contradiction between the evolving forces of production and the existing relations of production, would turn against capitalism on rational grounds. See Veblen (1907, pp. 304–306).

15 Samuelson later deplored this statement which was meant to be a joke and which he considered a blunder. It remains to be seen how serious this regret was, taking into account his decades-long crusade against Marx (see Samuelson, 1983, pp. 263–264).

16 For instance, Davidson (2011) mentions Marx on two pages. In Arestis (1992) no mention of Marx can be found, in stark contrast to Harcourt (2006).

17 Lavoie (1992, pp. 5–6) considers Marxist and post-Keynesian economic theory to be part of the "post-classical paradigm", together with a number of non-orthodox theories.

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