# The Contribution of the Minimum Wage to US Wage Inequality over Three Decades: A Reassessment<sup>†</sup>

By DAVID H. AUTOR, ALAN MANNING, AND CHRISTOPHER L. SMITH\*

We reassess the effect of minimum wages on US earnings inequality using additional decades of data and an IV strategy that addresses potential biases in prior work. We find that the minimum wage reduces inequality in the lower tail of the wage distribution, though by substantially less than previous estimates, suggesting that rising lower tail inequality after 1980 primarily reflects underlying wage structure changes rather than an unmasking of latent inequality. These wage effects extend to percentiles where the minimum is nominally nonbinding, implying spillovers. We are unable to reject that these spillovers are due to reporting artifacts, however. (JEL J22, J31, J38, K31)

The rapid expansion of earnings inequality throughout the US wage distribution during the 1980s catalyzed a rich and voluminous literature seeking to trace this rise to fundamental forces of labor supply, labor demand, and labor market institutions. A broad conclusion of the ensuing literature is that while no single factor was solely responsible for rising inequality, the largest contributors included: (i) a slowdown in the supply of new college graduates coupled with steadily rising demand for skills; (ii) falling union penetration, abetted by the sharp contraction of US manufacturing employment early in the decade; and (iii) a 30 log point erosion in the real value of the federal minimum wage between 1979 and 1988 (see overviews in Katz and Murphy 1992; Katz and Autor 1999; Card and DiNardo 2002; Autor, Katz, and Kearney 2008; Goldin and Katz 2008; Lemieux 2008; Acemoglu and Autor 2011).

An early and influential paper in this literature, Lee (1999), reached a markedly different conclusion. Exploiting cross-state variation in the gap between state median wages and the applicable federal or state minimum wage (the "effective minimum"), Lee estimated the share of the observed rise in wage inequality from

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<sup>\*</sup>Autor: Department of Economics, Massachusetts Institute of Technology, 50 Memorial Drive, E52-371, Cambridge, MA 02142, and National Bureau of Economic Research (NBER) (e-mail: dautor@mit.edu); Manning: Centre for Economic Performance and Department of Economics, London School of Economics, Houghton Street, London WC2A 2AE, United Kingdom (e-mail: a.manning@lse.ac.uk); Smith: Federal Reserve Board of Governors, 20th & C Street, NW, Washington DC 20551 (e-mail: christopher.l.smith@frb.gov). We thank Daron Acemoglu, Joshua Angrist, Lawrence Katz, David Lee, Thomas Lemieux, Christina Patterson, Emmanuel Saez, Gary Solon, Steve Pischke, and many seminar participants for valuable suggestions. We also thank David Lee and Arindrajit Dube for providing data on minimum wage laws by state. Any opinions and conclusions expressed herein are those of the authors and do not indicate concurrence with other members of the research staff of the Federal Reserve or the Board of Governors.

1979 through 1988 that was due to the falling minimum rather than changes in underlying ("latent") wage inequality. Lee concluded that *more than* the entire rise of the 50/10 earnings differential between 1979 and 1988 was due to the falling federal minimum wage; had the minimum been constant throughout this period, observed wage inequality would have fallen rather than risen.<sup>1</sup> Lee's work built on the seminal analysis of DiNardo, Fortin, and Lemieux (1996, DFL hereafter), who highlighted the compressing effect of the minimum wage on the US wage distribution prior to the 1980s. Distinct from Lee, however, DFL (1996) concluded that the eroding minimum explained at most 40 to 65 percent of the rise in 50/10 earnings inequality between 1979 and 1988, leaving considerable room for other fundamental factors, most importantly supply and demand.<sup>2</sup>

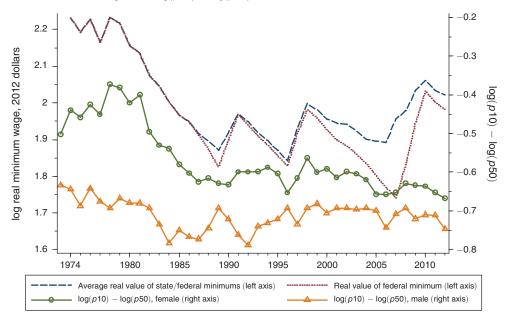
Surprisingly, there has been little research on the impact of the minimum wage on wage inequality since DFL (1996) and Lee (1999), even though the data they use is now over 20 years old. One possible reason is that while lower tail wage inequality rose dramatically in the 1980s, it has not exhibited much of a trend since then (see Figure 1, panel A). But this does not make the last 20 years irrelevant; these extra years encompass 3 increases in the federal minimum wage and a much larger number of instances where state minimum wages exceeded the federal minimum wage. This additional variation will prove crucial in identifying the impact of minimum wages on wage inequality.

In this paper, we reassess the evidence on the minimum wage's impact on US wage inequality with three specific objectives in mind. A first is to quantify how the numerous changes in state and federal minimum wages enacted in the two decades since DFL (1996) and Lee's (1999) data window closed have shaped the evolution of inequality. A second is to understand *why* the minimum wage appears to compress 50/10 inequality despite the fact that the minimum generally binds well below the tenth percentile. A third is to resolve what we see as a fundamental open question in the literature that was raised by Lee (1999). This question is *not* whether the falling minimum wage contributed to rising inequality in the 1980s but whether underlying inequality was in fact rising *at all* absent the "unmasking" effect of the falling minimum. Lee (1999) answered this question in the negative. And despite the incompatibility of this conclusion with the rest of the literature, it has not drawn reanalysis.

We believe that the debate can now be cleanly resolved by combining a longer time window with a methodology that resolves first-order biases in existing literature. We begin by showing why OLS estimates of the impact of the "effective minimum" on wage inequality are likely to be biased by measurement errors and transitory shocks that simultaneously affect both the dependent and independent variables. Following the approach introduced by Durbin (1954), we purge these biases by instrumenting the effective minimum wage with the legislated minimum

<sup>&</sup>lt;sup>1</sup>Using cross-region rather than cross-state variation in the "bindingness" of minimum wages, Teulings (2000 and 2003) reaches similar conclusions. Lemieux (2006) highlights the contribution of the minimum wage to the evolution of residual inequality. Mishel, Bernstein, and Allegretto (2007, chapter 3) also offer an assessment of the minimum wage's effect on wage inequality.

<sup>&</sup>lt;sup>2</sup>See Tables III and V of DFL (1996).



Panel A. Minimum wages and log(p10) - log(p50)

Panel B. Minimum wages and log(p90) - log(p50)

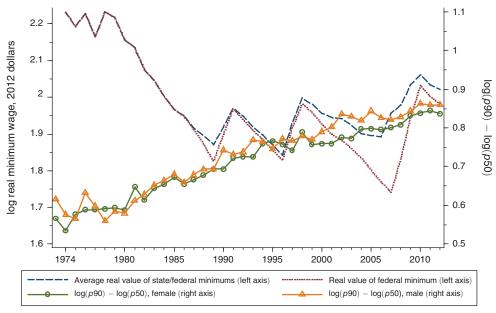


FIGURE 1. TRENDS IN STATE AND FEDERAL MINIMUM WAGES AND LOWER- AND UPPER-TAIL INEQUALITY *Notes:* Data are annual averages. Minimum wages are in 2012 dollars.

(and its square), an idea pursued by Card, Katz, and Krueger (1993) when studying the impact of the minimum wage on employment (rather than inequality).

Our instrumental variables analysis finds that the impact of the minimum wage on inequality is economically consequential but substantially smaller than that reported by Lee (1999). The substantive difference comes from the estimation methodology. Additional years of data and state-level legislative variation in the minimum wage allow us to test (and reject) some of the identifying assumptions made by Lee (1999). In most specifications, we conclude that the decline in the real value of the minimum wage explains 30 to 40 percent of the rise in lower tail wage inequality in the 1980s. Holding the real minimum wage at its lowest (least binding) level throughout the 1980s, we estimate that female 50/10 inequality would have risen by 11-15 log points, male inequality by approximately 1 log point, and pooled gender inequality by 7–8 log points. In other words, there was a substantial increase in underlying wage inequality in the 1980s.

In revisiting Lee's estimates, we document that our instrumental variables strategy-which relies on variation in statutory minimum wages across states and over time-does not perform well when limited to data only from the 1980s period. This is because between 1979 and 1985, only one state aside from Alaska adopted a minimum wage in excess of the federal minimum; the ten additional state adoptions that occurred through 1989 all took place between 1986 and 1989 (Table 1). This provides insufficient variation to pin down a meaningful first-stage relationship between the legislated minimum wage and the effective minimum wage. By extending the estimation window to 1991 (as was also done by Lee 1999), we exploit the substantial federal minimum wage increase that took place between 1990 and 1991 to tighten these estimates; extending the sample further to 2012 lends additional precision. We show that it would have been infeasible using data prior to 1991 to successfully estimate the effect of the minimum wage on the wage distribution. It is only with subsequent data on comovements in state wage distributions and the minimum wage that meaningful estimates can be obtained. Thus, the causal effect estimate that Lee sought to identify was only barely estimable within the confines of his sample (though not with the methods used).

Our finding of a modest but meaningful effect of the minimum wage on 10/50 inequality leaves open a second puzzle: why did the minimum wage have any effect at all? Between 1979 and 2012, there is no year in which more than 10 percent of male hours or aggregate hours were paid at or below the federal or applicable state minimum wage (See Figure 2 and Tables 1A and 1B, columns 4 and 8), and only 5 years in which more than 10 percent of female hours were at or below the minimum wage. Thus, any impact of the minimum wage on 50/10 inequality among males or the pooled gender distribution *must* have arisen from spillovers, whereby the minimum wage must have raised the wages of workers earning above the minimum.<sup>3</sup>

<sup>&</sup>lt;sup>3</sup> If there are disemployment effects, the minimum wage will have spillovers on the observed wage distribution even if no individual wage changes (see Lee 1999, for a discussion of this). The size of these spillovers will be related to the size of the disemployment effect. Although the employment impact of the minimum wage remains a contentious issue (see, for example, Card, Katz, and Krueger 1993; Card and Krueger 2000; Neumark and Wascher 2000; and more recently, see Allegretto, Dube, and Reich 2011 and Neumark, Salas, and Wascher 2014), most estimates are very small. For example, the recent Congressional Budget Office (2014) report on the likely consequences

			A. Females	ales		
	States with > federal minimum (1)	Minimum binding percentile (2)	Maximum binding percentile (3)	Share of hours at or below minimum (4)	Average $\log(p10) - \log(p50)$ (5)	
1979	1	5.0	28.0	0.13	-0.38	
1980	1	6.0	24.0	0.13	-0.40	
1981	1	5.0	24.0	0.13	-0.41	
1982	1	5.0	21.5	0.11	-0.48	
1983	1	3.5	17.5	0.10	-0.51	
1984	1	2.5	15.5	0.09	-0.54	
1985	2	2.0	14.5	0.08	-0.56	
1986	5	2.0	16.0	0.07	-0.59	
1987	6	2.0	14.0	0.06	-0.60	
1988	10	2.0	12.5	0.06	-0.60	
1989	12	1.0	12.5	0.05	-0.61	
1990	11	1.5	14.0	0.05	-0.58	
1991	4	1.5	18.5	0.07	-0.58	
1992	7	2.0	14.0	0.07	-0.58	
1993	7	2.5	11.0	0.06	-0.59	
1994	8	2.5	11.0	0.06	-0.61	
1995	9	2.0	9.5	0.05	-0.61	
1996	11	1.5	12.5	0.05	-0.61	
1997	10	2.5	14.5	0.06	-0.60	
1998	7	2.5	11.5	0.06	-0.58	
1999	10	2.5	11.0	0.05	-0.58	
2000	10	2.0	9.5	0.05	-0.59	
2001	10	2.0	9.0	0.05	-0.59	
2002	11	1.5	9.0	0.04	-0.60	
2003	11	1.5	9.0	0.04	-0.61	
2004	12	1.5	7.5	0.04	-0.63	
2005	15	1.5	8.5	0.04	-0.64	
2006	19	1.5	9.5	0.04	-0.64	
2007	30	1.5	10.0	0.05	-0.63	
2008	31	2.0	13.0	0.06	-0.64	
2009	26	2.5	10.5	0.06	-0.64	
2010	15	3.5	9.5	0.06	-0.64	
2011	19	3.0	10.5	0.06	-0.65	
2012	19	3.0	9.5	0.06	-0.66	

TABLE 1A—SUMMARY STATISTICS FOR BINDINGNESS OF STATE AND FEDERAL MINIMUM WAGES

Note: See text at bottom of Table 1B.

Such spillovers are a potentially important and little understood effect of minimum wage laws, and we seek to understand why they arise.

Distinct from prior literature, we explore a novel interpretation of these spillovers: measurement error. In particular, we assess whether the spillovers found in our samples, based on the Current Population Survey, may result from measurement

of a 25 percent rise in the federal minimum wage from \$7.25 to \$9.00 used a conventional labor demand approach but concluded job losses would represent less than 0.1 percent of employment. This would cause only a trivial spillover effect. In addition, we have explored how minimum wage related disemployment may affect our findings by limiting our sample to 25–64 year olds; because the studies that find disemployment effects generally find them concentrated among younger workers, focusing on older workers may limit the bias from disemployment. When we limit our sample in this way, we find that the effect of the minimum wage on lower tail inequality is somewhat smaller than for the full sample, consistent with a smaller fraction of the older sample earning at or below the minimum. However, using our preferred specification, the contribution of changes in the minimum wage to changes in inequality is qualitatively similar regardless of the sample.

2012

1.5

7.0

0.04

		B. N	Iales		C. Males and females, pooled			
	Minimum binding percentile (1)	Maximum binding percentile (2)	Share of hours at or below minimum (3)	Average log(10) - log(50) (4)	Minimum binding percentile (5)	Maximum binding percentile (6)	Share of hours at or below minimum (7)	Average $log(p10)$ - log(p50) (8)
1979	2.0	10.5	0.05	-0.64	3.5	17.0	0.08	-0.58
1980	2.5	10.0	0.06	-0.65	4.0	15.5	0.09	-0.59
1981	1.5	9.0	0.06	-0.68	2.5	14.5	0.09	-0.60
1982	2.0	8.0	0.05	-0.71	3.5	12.5	0.07	-0.63
1983	2.0	8.0	0.05	-0.73	3.0	11.5	0.07	-0.65
1984	1.5	7.5	0.04	-0.73	2.0	10.5	0.06	-0.67
1985	1.0	6.5	0.04	-0.74	1.5	9.5	0.06	-0.69
1986	1.0	6.5	0.03	-0.74	1.5	10.0	0.05	-0.70
1987	1.0	6.0	0.03	-0.73	1.5	9.0	0.04	-0.70
1988	1.0	6.0	0.03	-0.72	1.5	8.0	0.04	-0.69
1989	1.0	5.0	0.03	-0.72	1.0	7.0	0.04	-0.68
1990	0.5	6.0	0.03	-0.72	0.5	9.0	0.04	-0.67
1991	0.5	9.0	0.04	-0.71	1.0	12.5	0.05	-0.67
1992	1.0	6.5	0.04	-0.72	1.5	9.5	0.05	-0.67
1993	1.0	5.0	0.03	-0.73	1.5	7.5	0.04	-0.68
1994	1.0	4.5	0.03	-0.71	2.0	7.5	0.04	-0.69
1995	0.5	4.5	0.03	-0.71	1.5	6.0	0.04	-0.68
1996	1.0	7.0	0.03	-0.71	1.5	9.0	0.04	-0.67
1997	1.0	7.5	0.04	-0.69	1.5	10.0	0.05	-0.66
1998	1.0	7.0	0.04	-0.69	2.0	8.0	0.05	-0.65
1999	1.0	5.5	0.03	-0.69	2.0	7.0	0.04	-0.65
2000	1.0	6.0	0.03	-0.68	1.5	7.5	0.04	-0.65
2001	0.5	5.5	0.03	-0.68	1.5	7.0	0.04	-0.66
2002	1.0	6.0	0.03	-0.69	1.5	7.5	0.03	-0.65
2003	0.5	5.0	0.03	-0.69	1.5	6.5	0.03	-0.66
2004	1.0	5.0	0.03	-0.70	1.5	6.0	0.03	-0.68
2005	1.0	5.0	0.02	-0.71	1.5	6.5	0.03	-0.68
2006	0.5	6.0	0.02	-0.70	1.0	7.5	0.03	-0.68
2007	0.5	6.0	0.03	-0.70	1.5	7.5	0.04	-0.68
2008	1.0	6.5	0.04	-0.71	1.0	8.5	0.04	-0.69
2009	1.0	6.0	0.04	-0.74	2.0	8.0	0.05	-0.71
2010	2.0	6.5	0.04	-0.73	3.0	7.5	0.05	-0.70
2011	1.5	8.0	0.04	-0.72	2.5	9.0	0.05	-0.69
		= 0	0.04	0 = 1	• •	0.0	0.05	0 54

TABLE 1B—SUMMARY STATISTICS FOR BINDINGNESS OF STATE AND FEDERAL MINIMUM WAGES

Notes: Column 1 in Table 1A displays the number of states with a minimum that exceeds the federal minimum for at least six months of the year. Columns 2 and 3 of Table 1A, and columns 1, 2, 5, and 6 of Table 1B display estimates of the lowest and highest percentile at which the minimum wage binds across states (DC is excluded). The binding percentile is estimated as the highest percentile in the annual distribution of wages at which the minimum wage binds (rounded to the nearest half of a percentile), where the annual distribution includes only those months for which the minimum wage was equal to its modal value for the year. Column 4 of Table 1A and columns 3 and 7 of Table 1B display the share of hours worked for wages at or below the minimum wage. Column 5 of Table 1A and columns 4 and 8 of Table 1B display the weighted average value of the  $\log(p10) - \log(p50)$  for the male or female wage distributions across states.

-0.74

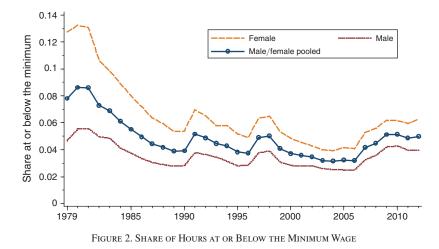
2.0

8.0

0.05

-0.71

artifacts. This can occur if a fraction of minimum wage workers report their wages inaccurately, leading to a hump in the wage distribution centered on the minimum wage rather than (or in addition to) a spike at the minimum. After bounding the potential magnitude of these measurement errors, we are unable to reject the hypothesis that the apparent spillover from the minimum wage to higher (noncovered) percentiles is spurious. That is, while the spillovers are present in the data, they may not be present in the distribution of wages actually paid. These results do not rule



*Notes:* The figure plots estimates of the share of hours worked for reported wages equal to or less than the applicable state or federal minimum wage, corresponding with data from columns 4 and 8 of Tables 1A and 1B.

out the possibility of true spillovers. But they underscore that spillovers estimated with conventional household survey data sources must be treated with caution since they cannot necessarily be distinguished from measurement artifacts with available precision.

The paper proceeds as follows. Section I discusses data and sources of identification. Section II presents the measurement framework and estimates a set of causal effects estimates models that, like Lee (1999), explicitly account for the bite of the minimum wage in estimating its effect on the wage distribution. We compare parameterized OLS and 2SLS models and document the pitfalls that arise in the OLS estimation. Section III uses point estimates from the main regression models to calculate counterfactual changes in wage inequality, holding the real minimum wage constant. Section IV analyzes the extent to which apparent spillovers may be due to measurement error. The final section concludes.

### I. Changes in the Federal Minimum Wage and Variation in State Minimum Wages

In July of 2007, the real value of the US federal minimum wage fell to its lowest point in over three decades, reflecting a nearly continuous decline from a 1979 high point, including two decade-long spans in which the minimum wage remained fixed in nominal terms—1981 through 1990, and 1997 through 2007. Perhaps responding to federal inaction, numerous states have over the past two decades legislated state minimum wages that exceed the federal level. At the end of the 1980s, 12 states' minimum wages exceeded the federal level; by 2008, this number had reached 31 (subsequently reduced to 15 by the 2009 federal minimum wage increase).<sup>4</sup>

<sup>&</sup>lt;sup>4</sup>Table 1 assigns each state the minimum wage that was in effect for the largest number of months in a calendar year. Because the 2009 federal minimum wage increase took effect in late July, it is not coded as exceeding most state minimums until 2010.

Consequently, the real value of the minimum wage applicable to the average worker in 2007 was not much lower than in 1997, and was significantly higher than if states had not enacted their own minimum wages. Moreover, the post-2007 federal increases brought the minimum wage faced by the average worker up to a real level not seen since the mid-1980s. An online Appendix table illustrates the extent of state minimum wage variation between 1979 and 2012.

These differences in legislated minimum wages across states and over time are one of two sources of variation that we use to identify the impact of the minimum wage on the wage distribution. The second source of variation we use, following Lee (1999), is variation in the "bindingness" of the minimum wage, stemming from the observation that a given legislated minimum wage should have a larger effect on the shape of the wage distribution in a state with a lower wage level. Table 1 provides examples. In each year, there is significant variation in the percentile of the state wage distribution where the state or federal minimum wage "binds." For instance, in 1979 the minimum wage bound at the twelfth percentile of the female wage distribution for the median state, but it bound at the fifth percentile in Alaska and the twenty-eighth percentile in Mississippi. This variation in the bite or bindingness of the minimum wage was due mainly to cross-state differences in wage levels in 1979, since only Alaska had a state minimum wage that exceeded the federal minimum. In later years, particularly during the 2000s, this variation was also due to differences in the value of state minimum wages.

# A. Sample and Variable Construction

Our analysis uses the percentiles of states' annual wage distributions as the primary outcomes of interest. We form these samples by pooling all individual responses from the Current Population Survey Merged Outgoing Rotation Group (CPS MORG) for each year. We use the reported hourly wage for those who report being paid by the hour. Otherwise we calculate the hourly wage as weekly earnings divided by hours worked in the prior week. We limit the sample to individuals age 18 through 64, and we multiply top-coded values by 1.5. We exclude self-employed individuals and those with wages imputed by the BLS. To reduce the influence of outliers, we Winsorize the top two percentiles of the wage distribution in each state, year, and sex grouping (male, female, or pooled) by assigning the ninety-seventh percentile value to the ninety-eighth and ninety-ninth percentiles. Using these individual wage data, we calculate all percentiles of state wage distributions by sex for 1979–2012, weighting individual observations by their CPS sampling weight multiplied by their weekly hours worked.<sup>5</sup> For more details on our data construction, see the data Appendix.

Our primary analysis is performed at the state-year level, but minimum wages often change part way through the year. We address this issue by assigning the value of the minimum wage that was in effect for the longest time throughout the calendar

<sup>&</sup>lt;sup>5</sup>Following the approach introduced by DFL (1996), now used widely in the wage inequality literature, we define percentiles based on the distribution of paid hours, thus giving equal weight to each paid hour worked. Our estimates are essentially unchanged if we weight by workers rather by worker hours.

year in a state and year. For those states and years in which more than one minimum wage was in effect for six months in the year, the maximum of the two is used. We have alternatively assigned the maximum of the minimum wage within a year as the applicable minimum wage. This leaves our conclusions unchanged.

### **II.** Reduced Form Estimation of Minimum Wage Effects on the Wage Distribution

# A. General Specification and OLS Estimates

The general model we estimate for the evolution of inequality at any point in the wage distribution (the difference between the log wage at the pth percentile and the log of the median) for state s in year t is of the form:

(1) 
$$w_{st}(p) - w_{st}(50) = \beta_1(p) [w_{st}^m - w_{st}(50)] + \beta_2(p) [w_{st}^m - w_{st}(50)]^2 + \sigma_{s0}(p) + \sigma_{s1}(p) \times time_t + \gamma_t^{\sigma}(p) + \varepsilon_{st}^{\sigma}(p).$$

In this equation,  $w_{st}(p)$  represents the log real wage at percentile p in state s at time t; time-invariant state effects are represented by  $\sigma_{s0}(p)$ ; state-specific trends are represented by  $\sigma_{s1}(p)$ ; time effects represented by  $\gamma_t^{\sigma}(p)$ ; and transitory effects represented by  $\varepsilon_{st}^{\sigma}(p)$ , which we assume to be independent of the state and year effects and trends. Although our state effects and trends are likely to control for much of the economic fluctuations at state level, we also experimented with including the state-level unemployment rate as a control variable. This has virtually no impact on the estimated coefficients in equation (1) for any of our samples.

In equation (1),  $w_{st}^m$  is the log minimum wage for that state-year. We follow Lee (1999) in both defining the bindingness of the minimum wage to be the log difference between the minimum wage and the median (Lee refers to this as the effective minimum) and in modeling the impact of the minimum wage to be quadratic. The quadratic term is important to capture the idea that a change in the minimum wage is likely to have more impact on the wage distribution where it is more binding.<sup>6</sup> By differentiating (1) we have that the predicted impact of a change in the minimum wage on a percentile is given by  $\beta_1(p) + 2\beta_2(p) [w_{st}^m - w_{st}(50)]$ . Inspection of this expression shows how our specification captures the idea that the minimum wage will have a larger effect when it is high relative to the median.

Our preferred strategy for estimating (1) is to include state fixed effects and trends and to instrument the minimum wage.<sup>7</sup> But we start by presenting OLS estimates

<sup>&</sup>lt;sup>6</sup> In this formulation, a more binding minimum wage is a minimum wage that is closer to the median, resulting in a higher (less negative) effective minimum wage. Since the log wage distribution has greater mass toward its center than at its tail, a 1 log point rise in the minimum wage affects a larger fraction of wages when the minimum lies at the fortieth percentile of the distribution than when it lies at the first percentile.

<sup>&</sup>lt;sup>7</sup>Our primary specification does not control for other state-level controls. When we include state-year unemployment rates to proxy for heterogeneous shocks to a state's labor market, however, the coefficients on the minimum wage variables are essentially unchanged.

	OLS (1)	OLS (2)	2SLS (3)	2SLS (4)	OLS Lee Spec. (5)
Panel A. Females					
p(5)	$0.44^{***}$	$0.54^{***}$	0.32***	$0.39^{***}$	0.63***
	(0.03)	(0.05)	(0.04)	(0.05)	(0.04)
p(10)	0.27***	0.46***	0.22***	0.17***	0.52***
	(0.03)	(0.03)	(0.05)	(0.03)	(0.03)
p(20)	0.12***	0.29***	0.10**	0.07**	0.29***
	(0.03)	(0.03)	(0.05)	(0.03)	(0.03)
<i>p</i> (30)	0.07*** (0.01)	0.23*** (0.02)	0.02 (0.02)	0.04 (0.03)	0.15*** (0.02)
p(40)	0.04** (0.02)	0.17*** (0.02)	0.00 (0.03)	0.03 (0.03)	0.06*** (0.01)
<i>p</i> (75)	0.09*** (0.02)	0.23*** (0.03)	-0.03 (0.02)	0.00 (0.03)	$-0.05^{**}$ (0.02)
<i>p</i> (90)	0.15*** (0.03)	0.34*** (0.03)	-0.02 (0.04)	0.04 (0.04)	-0.04 (0.04)
Var. of log wage	0.07	0.04	-0.02	-0.09	-0.20***
	(0.04)	(0.05)	(0.08)	(0.07)	(0.03)
Panel B. Males					
<i>p</i> (5)	0.25***	0.43***	$0.19^{***}$	$0.16^{***}$	$0.55^{***}$
	(0.03)	(0.03)	(0.02)	(0.04)	(0.04)
p(10)	0.12***	0.34***	0.05	$0.05^{*}$	0.38***
	(0.04)	(0.03)	(0.04)	(0.03)	(0.04)
<i>p</i> (20)	0.06**	0.23***	0.02	0.02	0.21***
	(0.03)	(0.02)	(0.02)	(0.03)	(0.03)
<i>p</i> (30)	0.04 (0.02)	0.19*** (0.02)	0.02 (0.02)	0.00 (0.03)	0.09*** (0.02)
<i>p</i> (40)	0.06***	0.15***	0.05*	0.02	0.03***
	(0.01)	(0.02)	(0.02)	(0.04)	(0.01)
<i>p</i> (75)	0.14***	0.23***	0.00	0.02	0.09**
	(0.02)	(0.02)	(0.02)	(0.02)	(0.04)
<i>p</i> (90)	0.16***	0.30***	0.03	0.04	0.14**
	(0.03)	(0.03)	(0.03)	(0.04)	(0.07)
Var. of log wage	0.03	0.01	-0.07	-0.06	$-0.12^{**}$
	(0.03)	(0.05)	(0.05)	(0.07)	(0.05)
Levels/first-differenced	Levels	FD	Levels	FD	Levels
Year FE	Yes	Yes	Yes	Yes	Yes
State FE	Yes	Yes	Yes	Yes	No
State trends	Yes	No	Yes	No	No

TABLE 2A—OLS AND 2SLS RELATIONSHIP BETWEEN $LOG(p) - LOG(p50)$ and
LOG(min. wage) - LOG(p50), FOR SELECT PERCENTILES OF
GIVEN WAGE DISTRIBUTION, 1979–2012

Notes: See text at bottom of Table 2B.

\*\*\*Significant at the 1 percent level.

\*\* Significant at the 5 percent level.

\*Significant at the 10 percent level.

of (1).<sup>8</sup> Column 1 of Tables 2A and 2B reports estimates of this specification. We report the marginal effects of the effective minimum for selected percentiles when

<sup>8</sup>Strictly speaking our OLS estimates are weighted least squares and our IV estimates weighted two-stage least squares.

	OLS (1)	OLS (2)	2SLS (3)	2SLS (4)	OLS Lee Spec. (5)
Panel C. Males and female	s pooled				
<i>p</i> (5)	0.35*** (0.03)	$0.45^{***}$ (0.04)	0.29*** (0.03)	0.29*** (0.06)	0.62*** (0.03)
<i>p</i> (10)	0.17*** (0.02)	0.35*** (0.03)	0.16*** (0.02)	0.17*** (0.04)	0.44*** (0.03)
<i>p</i> (20)	0.08*** (0.02)	0.23*** (0.03)	0.07*** (0.02)	0.04 (0.03)	0.25*** (0.03)
<i>p</i> (30)	0.05*** (0.02)	0.19*** (0.02)	0.02 (0.02)	0.00 (0.02)	0.14*** (0.02)
p(40)	$0.04^{***}$ (0.01)	$0.16^{***}$ (0.02)	$0.02^{**}$ (0.01)	$0.02 \\ (0.03)$	$0.06^{***}$ (0.01)
<i>p</i> (75)	$0.11^{***}$ (0.01)	$0.22^{***}$ (0.02)	$ \begin{array}{c} 0.00 \\ (0.02) \end{array} $	$\begin{array}{c} 0.01 \\ (0.02) \end{array}$	0.01 (0.02)
<i>p</i> (90)	0.15*** (0.03)	0.28*** (0.03)	0.01 (0.04)	0.02 (0.03)	0.04 (0.05)
Var. of log wage	0.04 (0.02)	$0.02 \\ (0.03)$	$-0.05 \\ (0.05)$	-0.07 (0.05)	$-0.18^{***}$ (0.04)
Levels/first-differenced	Levels	FD	Levels	FD	Levels
Year FE	Yes	Yes	Yes	Yes	Yes
State FE State trends	Yes Yes	Yes No	Yes Yes	Yes No	No No

Table 2B—OLS and 2SLS Relationship between log(p) - log(p50) and log(min. wage) - log(p50), for Select Percentiles of Pooled Wage Distribution, 1979–2012

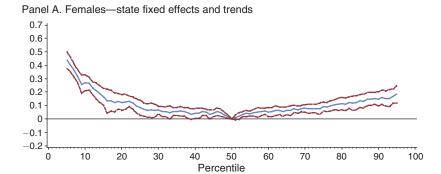
Notes: N = 1,700 for levels estimation, N = 1,650 for first-differenced estimation. Sample period is 1979–2012. For all but the last row, the dependent variable is  $\log(p) - \log(p50)$ , where p is the indicated percentile. For the last row, the dependent variable is the variance of log wage. Estimates are the marginal effects of  $\log(\min, wage) - \log(p50)$ , evaluated at its hours-weighted average across states and years. The last row are estimates of the marginal effects of  $\log(\min, wage) - \log(p50)$ , evaluated at its hours-weighted average across states and years. Standard errors clustered at the state level are in parentheses. Regressions are weighted by the sum of individuals' reported weekly hours worked multiplied by CPS sampling weights. For 2SLS specifications, the effective minimum and its square are instrumented by the log of the minimum, the square of the log minimum, and the log minimum interacted with the average real log median for the state over the sample. For the first-differenced specification, the instruments are first-differenced equivalents.

\*\*\* Significant at the 1 percent level.

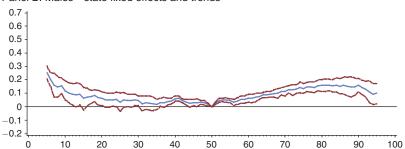
\*\* Significant at the 5 percent level.

\* Significant at the 10 percent level.

estimated at the weighted average of the effective minimum over all states and all years between 1979 and 2012. In the final row we also report an estimate of the effect on the variance, though the upper tail will heavily influence this estimate. Figure 3 provides a graphical representation of these estimated marginal effects for all percentiles. In all three samples (males, females, pooled), there is a significant estimated effect of the minimum wage on the lower tail, but, rather worryingly, there is also a large positive relationship between the effective minimum wage and *upper tail* inequality. This suggests there is some bias in these estimates. This problem also occurs when we estimate the model with first-differences in column 2.



Panel B. Males-state fixed effects and trends



Percentile

Panel C. Males and females-state fixed effects and trends

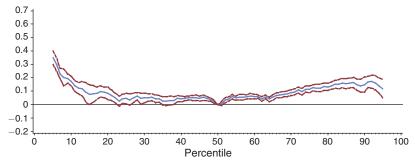


Figure 3. OLS Estimates of the Relationship between log(p) - log(p50) and log(min) - log(p50) and Its Square, 1979–2012

*Notes:* Estimates are the marginal effects of  $\log(\min, wage) - \log(p50)$ , evaluated at the hours-weighted average of  $\log(\min, wage) - \log(p50)$  across states and years. Observations are state-year observations. Ninety-five percent confidence interval is represented by the dashed lines. Estimates correspond with column 1 of Tables 2A and 2B.

In discussing the possible causes of bias in estimates, it is helpful to consider the following model for the median log wage for state *s* in year *t*:

(2) 
$$w_{st}(50) = \mu_{s0} + \mu_{s1} \times time_t + \gamma_t^{\mu} + \varepsilon_{st}^{\mu}$$

Here, the median wage for the state is a function of a state effect,  $\mu_{s0}$ ; a state trend,  $\mu_{s1}$ ; a common year effect,  $\gamma_t^{\mu}$ ; and a transitory effect,  $\varepsilon_{st}^{\mu}$ . With this setup, OLS estimation of (1) will be biased if  $\operatorname{cov}(\varepsilon_{st}^{\mu}, \varepsilon_{st}^{\sigma}(p))$  is nonzero because the median is used in the construction of the effective minimum; that is, transitory fluctuations

in state wage medians are correlated with the gap between the state wage median and other wage percentiles. Is this bias likely to be present in practice? One would naturally expect that transitory shocks to the median do not translate one-for-one to other percentiles. If, plausibly, the effects dissipate as one moves further from the median, this would generate bias due to the nonzero correlation between shocks to the median wage and measured inequality throughout the distribution. This implies that we would expect  $cov(\varepsilon_{st}^{\mu}, \varepsilon_{st}^{\sigma}(p)) < 0$  and that this covariance would attenuate as one considers percentiles further from the median.

How does this covariance affect estimates of equation (1)? This depends on the covariance of the effective minimum wage terms with the errors in the equation. The natural assumption is that  $cov(w_{st}^m - w_{st}(50), w_{st}(50)) < 0$ , that is, even after allowing for the fact that high-wage states may have a state minimum higher than the federal minimum, the minimum wage is less binding in high-wage states. Combining this with the assumption that  $cov(\varepsilon_{st}^\mu, \varepsilon_{st}^\sigma(p)) < 0$  leads to the prediction that OLS estimation of (1) leads to upward bias in the estimate of the impact of minimum wages on inequality in both the lower and upper tail.

We will address this problem by applying instrumental variables to purge biases caused by measurement error and other transitory shocks, following the approach introduced by Durbin (1954). We instrument the observed effective minimum and its square using an instrument set that consists of: (i) the log of the real statutory minimum wage, (ii) the square of the log of the real minimum wage, and (iii) the interaction between the log minimum wage and average log median real wage for the state over the sample period. In this IV specification, identification in (1) for the linear term in the effective minimum wage comes entirely from the variation in the statutory minimum wage, and identification for the quadratic term comes from the inclusion of the square of the log statutory minimum wage and the interaction term.<sup>9</sup> As there are always time effects included in our estimation, all the identifying variation in the statutory minimum comes from the state-specific minimum wages, which we assume to be exogenous to state wage levels of inequality.<sup>10</sup> Our second instrument is the square of the predicted value for the effective minimum from the regression outlined above, and relies on the same identifying assumptions (exogeneity of the statutory minimum wage).

Column 3 of Tables 2A and 2B report the estimates when we instrument the effective minimum in the way we have described. The first-stages for these regressions are reported in Appendix Table A1. For all samples, the three instruments are

<sup>10</sup>We follow almost all of the existing literature and assume the state level minimum wages are exogenous to other factors affecting the state-level wage distribution once we have controlled for state fixed effects and trends. A priori, any bias is unclear, e.g., rising inequality might generate a demand for higher minimum wages as might economic conditions favorable to minimum wage workers. The long lags in the political process surrounding rises in the minimum wages makes it unlikely that there is much response to contemporaneous economic conditions.

<sup>&</sup>lt;sup>9</sup>To see why the interaction is important to include, expand the square of the effective minimum wage,  $\log(\min) - \log(p50)$ , which yields three terms, one of which is the interaction of  $\log(\min)$  and  $\log(p50)$ . We have also tried replacing the square and interaction terms with the square of the predicted value for the effective minimum, where the predicted value is derived from a regression of the effective minimum on the log statutory minimum, state and time fixed effects, and state trends (similar to an approach suggested by Wooldridge 2002; section 9.5.2). 2SLS results using this alternative instrument are virtually identical to the strategy outlined in the main text. In general, using the statutory minimum as an instrument is similar in spirit to the approach taken by Card, Katz, and Krueger (1993) in their analysis of the employment effects of the minimum wage.

jointly highly significant and pass standard diagnostic tests for weak instruments (e.g., Stock, Wright, and Yogo 2002). Compared to column 1, the estimated impacts of the minimum wage in the lower tail are reduced, especially above the tenth percentile. This is consistent with what we have argued is the most plausible direction of bias in the OLS estimate in column 1. And, for all three samples, the estimated effect in the upper tail is now small and insignificantly different from zero, again consistent with the IV strategy reducing bias in the predicted direction.<sup>11</sup>

For robustness, we also estimate these models in first differences. Column 4 shows the results from first-differenced regressions that include state and year fixed effects, instrumenting the endogenous differenced variables using differenced analogues to the instruments described above.<sup>12</sup> Figure 4A shows the results for all percentiles from the level IV specifications; Figure 4B shows the results from the first-differenced IV specifications. Qualitatively, the first-differenced regressions are quite similar to the levels regressions, although they imply slightly larger effects of the minimum wage at the bottom of the wage distribution.

Our 2SLS estimates find that the minimum wage affects lower tail inequality up through the twenty-fifth percentile for women, up through the tenth percentile for men, and up through approximately the fifteenth percentile for the pooled wage distribution. A 10 log point increase in the effective minimum wage reduces 50/10 inequality by approximately 2 log points for women, by no more than 0.5 log points for men, and by roughly 1.5 log points for the pooled distribution. These estimates are less than half as large as those found by the baseline OLS specification, and are considerably smaller than those reported by Lee (1999). What accounts for this qualitative difference in findings? The dissimilarity could stem either from differences in specification and estimation or from the additional years of data available for our analysis. We consider both factors in turn, and show that the first—differences in specification and estimation—is fundamental.

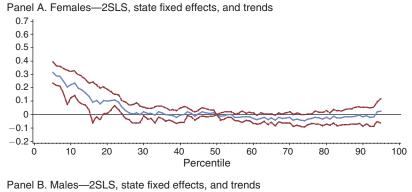
# B. Reconciling with Literature: Methods or Time Period?

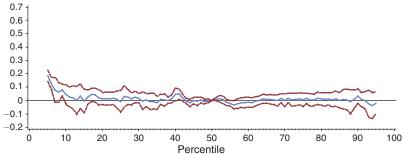
Lee (1999) estimates equation (1) by OLS and his preferred specification excludes the state fixed effects and trends that we have included.<sup>13</sup> Column 5 of Tables 2A and 2B, and Figure 5, shows what happens when we estimate this model on our longer sample period. Similar to Lee, we find large and statistically significant effects of the minimum wage on the lower percentiles of the wage distribution that extend throughout *all* percentiles below the median for the male, female, and pooled wage distributions, and are much larger than the effects in our preferred specifications. Also note that, with the exception of the male estimates, the upper tail "effects" are small and insignificantly different from zero, which might be considered a necessary

<sup>&</sup>lt;sup>11</sup>These findings are essentially unchanged if we use higher order state time trends.

<sup>&</sup>lt;sup>12</sup> The instruments for the first-differenced analogue are  $\Delta w_{st}^m$  and  $\Delta (w_{st}^m - \tilde{w}(50)_{st})^2$ , where  $\Delta w_{st}^m$  represents the annual change in the log of the legislated minimum wage, and  $\Delta (w_{st}^m - \tilde{w}(50)_{st})^2$  represents the change in the square of the predicted value for the effective minimum wage.

<sup>&</sup>lt;sup>13</sup>We include time effects in all of our estimation, as does Lee (1999). We estimate the model separately for each p (from 1 to 99), and impose no restrictions on the coefficients or error structure across equations.







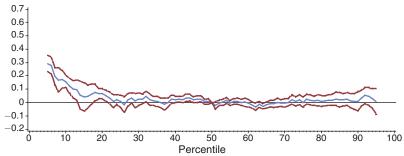
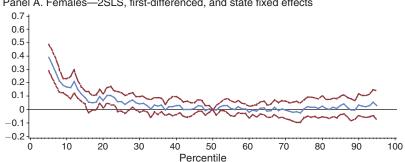


Figure 4A. 2SLS Estimates of the Relationship between  $\log(p) - \log(p50)$  and  $\log(\min) - \log(p50)$  and Its Square, 1979–2012

*Notes:* Estimates are the marginal effects of  $\log(\min, wage) - \log(p50)$ , evaluated at the hours-weighted average of  $\log(\min, wage) - \log(p50)$  across states and years. Observations are state-year observations. Ninety-five percent confidence interval is represented by the dashed lines. Estimates correspond with column 3 of Tables 2A and 2B.

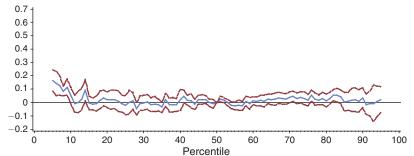
condition for the results to be credible estimates of the impact of the minimum wage on wage inequality at any point in the distribution.

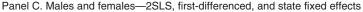
These estimates are likely to suffer from serious biases, however. If state fixed effects and trends are omitted from the specification of (1), estimates of minimum wage effects on wage inequality will be biased if  $(\sigma_{s0}(p), \sigma_{s1}(p))$  is correlated with  $(\mu_{s0}, \mu_{s1})$ , that is, state log median wage levels and latent state log wage inequality are correlated. Lee (1999) is very clear that his specification relies on the assumption of a zero correlation between the level of median wages and inequality. This assumption can be tested if one has a measure of inequality that is unlikely to be



Panel A. Females-2SLS, first-differenced, and state fixed effects







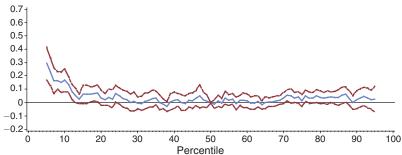


Figure 4B. 2SLS Estimates of the Relationship between log(p) - log(p50) and log(min) - log(p50)IN FIRST-DIFFERENCES, 1979–2012

*Notes:* Estimates are the marginal effects of log(min. wage)  $-\log(p50)$ , evaluated at the hours-weighted average of  $\log(\min, wage) - \log(p50)$  across states and years. Observations are state-year observations. Ninety-five percent confidence interval is represented by the dashed lines. Estimates correspond with column 4 of Tables 2A and 2B.

affected by the level of the minimum wage. For this purpose we use 60/40 inequality, that is, the difference in the log of the sixtieth and fortieth percentiles. Given that the minimum wage never binds very far above the tenth percentile of the wage distribution over our sample period, we feel comfortable assuming that the minimum wage has no impact on percentiles 40 through 60. Under this maintained hypothesis, 60/40 inequality serves as a valid proxy for the underlying inequality of a state's wage distribution.

To assess whether either the level or trend of state latent inequality is correlated with average state wage levels or their trends, we estimate state-level regressions

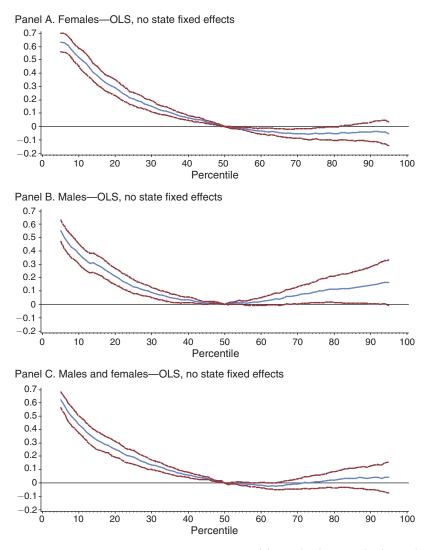


Figure 5. 2SLS Estimates of the Relationship between log(p) - log(p50) and log(min) - log(p50) and Its Square Using Lee (1999) Specification, 1979–2012

*Notes:* Estimates are the marginal effects of log(min. wage)  $-\log(p50)$ , evaluated at the hours-weighted average of log(min. wage)  $-\log(p50)$  across states and years. Observations are state-year observations. Ninety-five percent confidence interval is represented by the dashed lines. Estimates correspond with column 5 of Tables 2A and 2B.

of average 60/40 inequality and estimated trends in 60/40 inequality on average median wages and trends in median wages. Figures 6A and 6B depict scatter plots of these regressions, with regression results reported in Appendix Table A1. Figure 6A depicts the cross-state relationship between the average log(p60)-log(p40) and the average log(p50) for each of our three samples. Figure 6B depicts the cross-state relationship between the trends in the two measures. In all cases but the male trends plot (panel B of Figure 6B), there is a strong, positive visual relationship between the two—and, even for the male trend scatter, there is, in fact, a statistically significant positive relationship between the trends in the log(p60)-log(p40) and log(p50).

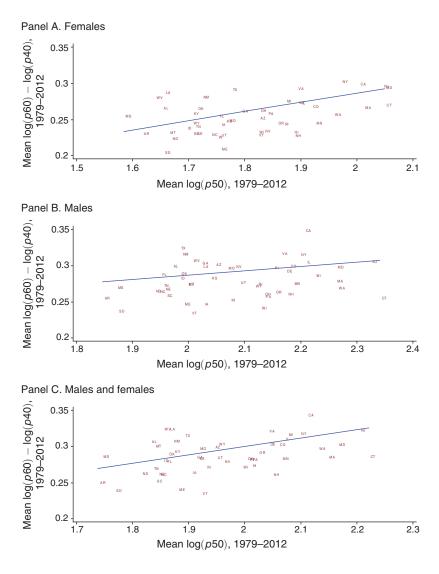
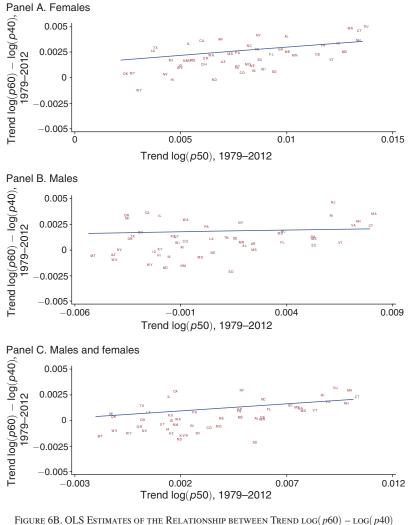


Figure 6A. OLS Estimates of the Relationship between Mean log(p60) - log(p40)and Mean log(p50), 1979–2012

*Notes:* Estimates correspond with regressions from Appendix Table A2. The figures show the cross-state relationship between the average  $\log(p60) - \log(p40)$  and  $\log(p50)$  between 1979 and 2012. Alaska, which tends to be an outlier, is dropped for visual clarity, though this does not materially affect the slope of the line (Appendix Table A2 includes Alaska).

The finding of a positive correlation between underlying inequality and the state median implies there is likely to be omitted variable bias from the exclusion of state fixed effects and trends—specifically, an *upward* bias to the estimated minimum wage effect in the lower tail and, simultaneously, a *downward* bias in the upper tail. To see why, note that higher wage states have lower (more negative) effective minimum wages (defined as the log gap between the legislated minimum and the state median), and the results from Table 2 imply that these states also have higher levels of latent



AND TREND LOG(p50), 1979–2012

*Notes:* Estimates correspond with regressions from Appendix Table A2. The figures show the cross-state relationship between the trend in  $\log(p60) - \log(p40)$  and the trend in  $\log(p50)$  between 1979 and 2012. Alaska, which tends to be an outlier, is dropped for visual clarity, though this does not materially affect the slope of the line (Appendix Table A2 includes Alaska).

inequality; thus they will have a more negative value of the left-hand side variable in our main estimating equation (1) for percentiles below the median, and a more positive value for percentiles above the median. Since the state median enters the right-hand side expression for the effective minimum wage with a negative sign, estimates of the relationship between the effective minimum and wage inequality will be *upward-biased* in the lower tail and *downward-biased* in the upper tail.

Combined with our discussion above on potential biases stemming from the correlation between the transitory error components on both sides of equation (1),

		2SLS cour	terfactuals	OLS counterfactuals Levels, No FE		
	Observed	Levels FE	First diffs			
	change (1)	1979–2012 (2)	1979–2012 (3)	1979–2012 (4)	1979–1991 (5)	
Panel A. 1979–1989						
Females	24.6	11.3*** (3.7)	15.1*** (1.8)	2.9 (2.0)	4.3*** (1.3)	
Males	2.5	1.2(1.4)	1.4 (0.9)	$-6.5^{***}$ (1.4)	$-5.3^{***}$ (1.6)	
Pooled	11.8	6.7*** (1.8)	$8.1^{***}$ (0.8)	-1.2 (1.3)	0.0 (1.2)	
Panel B. 1979–2012						
Females	28.5	14.8*** (3.7)	$18.6^{***}$ (1.8)	$6.4^{***}$ (1.9)	7.4*** (1.3)	
Males	7.9	7.0*** (1.0)	7.1*** (0.7)	3.1*** (1.0)	3.6*** (0.9)	
Pooled	11.4	6.9*** (1.5)	7.9*** (0.7)	1.1 (1.2)	1.8** (0.8)	

TABLE 3—Actual and Counterfactual Changes in log(p50/10) between Selected Years: Changes in log Points ( $100 \times \log$  Change)

*Notes:* Estimates represent changes in actual and counterfactual  $\log(p50) - \log(p10)$  between 1979 and 1989, and 1979 and 2012, measured in log points ( $100 \times \log$  change). Counterfactual wage changes in panel A represent counterfactual changes in the 50/10 had the effective minimum wage in 1979 equaled the effective minimum wage in 1989 for each state. Counterfactual wage changes in panel B represent changes had the effective minima in 1979 and 2012 equaled the effective minimum in 1989. The 2SLS counterfactuals (using point estimates from the 1979–2012 period) are formed using coefficients from estimations reported in columns 3 and 4 of Tables 2A and 2B. The OLS counterfactual estimates (using point estimates from the 1979-2012 period) are formed using coefficients from estimations reported in column 5 of Table 2. Counterfactuals using point estimates from the 1979–1991 period are formed using coefficients from analogous regressions for the shorter sample period. Marginal effects are bootstrapped as described in the text; the standard deviation associated with the estimates is reported in parentheses. \*\*\* Significant at the 1 percent level.

\*\* Significant at the 5 percent level.

\* Significant at the 10 percent level.

which leads to an *upward* bias on the coefficient on the effective minimum wage in both lower and upper tails, we infer that these two sources of bias reinforce each other in the lower tail, likely leading to an overestimate of the impact of the minimum wage on lower tail inequality. Simultaneously, they have countervailing effects on the upper tail. Thus, our finding in the fifth column of Table 2 of a relatively weak relationship between the effective minimum wage and upper tail inequality (for the female and pooled samples) may arise because these two countervailing sources of bias largely offset one another for upper tail estimates. But since these biases are reinforcing in the lower tail of the distribution, the absence of an upper tail correlation is not sufficient evidence for the absence of lower tail bias, implying that Lee's (1999) preferred specification may suffer from upward bias.

The original work assessing the impact of the minimum wage on rising US wage inequality-including DFL (1996), Lee (1999), and Teulings (2000, 2003)-used data from 1979 through the late 1980s or early 1990s. Our primary estimates exploit an additional 21 years of data. Does this longer sample frame make a substantive difference? Figure 7 answers this question by plotting estimates of marginal

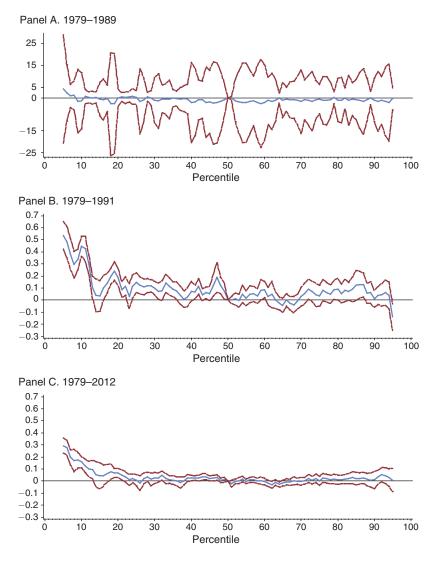


Figure 7. 2SLS Estimates of the Relationship between log(p) - log(p50) and log(min) - log(p50)over Various Time Periods (males and females)

*Notes:* Estimates are the marginal effects of  $\log(\min, wage) - \log(p50)$ , evaluated at the hours-weighted average of  $\log(\min, wage) - \log(p50)$  across states and years. Observations are state-year observations. The 95 percent confidence interval is represented by the dashed lines. Estimates correspond with column 3 of Tables 2A and 2B.

effects of the effect of minimum wage on percentiles of the pooled male and female wage distribution (as per column 3 of Table 2) for each of three time periods: 1979–1989, when there was little state-level variation in the minimum wage; 1979–1991, incorporating an additional two years in which numerous states raised their minimum wage; and 1979–2012. Panel A of Figure 7 reveals that our IV strategy—which relies on variation in statutory minimum wages across states and over time—does *not* perform well when limited to data only from the 1980s period: the

point estimates are enormous relative to both OLS estimates and 2SLS estimates; and the confidence bands are extremely large (note that the scale in the figure runs from -25 to 25, more than an order of magnitude larger than even the largest point estimates in Table 2). This lack of statistical significance is not surprising in light of the small number of policy changes in this period: between 1979 and 1985, only one state aside from Alaska adopted a minimum wage in excess of the federal minimum; the ten additional adoptions through 1989 all occurred between 1986 and 1989 (Table 1). Consequently, when calculating counterfactuals below, we apply marginal effects estimates obtained using additional years of data.

By extending the estimation window to 1991 in panel B of Figure 7 (as was also done by Lee 1999), we exploit the substantial federal minimum wage increase that took place between 1990 and 1991. This federal increase generated numerous cross-state contrasts since nine states had by 1989 raised their minimums above the 1989 federal level and below the 1991 federal level (and an additional three raised their minimum to \$4.25, which would be the level of the 1991 federal minimum wage). As panel B underscores, including these two additional years of data dramatically reduces the standard errors around our estimates, though the estimated marginal effects on a particular percentile are still quite noisy. Adding data for the full sample through 2012 (panel C of Figure 7) reduces the standard errors further and helps smooth out estimated marginal effects across percentiles.

Comparing across the three panels of Figure 7 reveals that it would have been infeasible using data prior to 1991 to successfully estimate the effect of the minimum wage on the wage distribution. It is only with subsequent data on comovements in state wage distributions and the minimum wage that more accurate estimates can be obtained. For this reason, our primary counterfactual estimates of changes in inequality rely on coefficient estimates from the full sample. We also discuss below the robustness of our substantive findings to the use of shorter sample windows (1979–1989 and 1979–1991).

### **III.** Counterfactual Estimates of Changes in Inequality

How much of the expansion in lower tail wage inequality since 1979 can be explained by the declining minimum wage? Following Lee (1999), we present reduced form counterfactual estimates of the change in *latent* wage inequality absent the decline in the minimum wage—that is, the change in wage inequality that would have been observed had the minimum wage been held at a constant real benchmark. These reduced form counterfactual estimates do not distinguish between mechanical and spillover effects of the minimum wage, a topic that we analyze next. We consider counterfactual changes over two periods: 1979–1989 (which captures the large widening of lower-tail income inequality over the 1980s) and 1979–2012.

To estimate changes in latent wage inequality, Lee (1999) proposes the following simple procedure. For each observation in the dataset, calculate its rank in its respective state-year wage distribution. Then, adjust each wage by the quantity:

(3) 
$$\Delta w_{st}(p) = \hat{\beta}_1(p) \big( \tilde{m}_{s,\tau_0} - \tilde{m}_{s,\tau_1} \big) + \hat{\beta}_2(p) \big( \tilde{m}_{s,\tau_0}^2 - \tilde{m}_{s,\tau_1}^2 \big),$$

where  $\tilde{m}_{s,\tau 1}$  is the observed end-of period effective minimum in state *s* in some year  $\tau 1$ ,  $\tilde{m}_{s,\tau 0}$  is the corresponding beginning-of-period effective minimum in  $\tau 0$ , and  $\hat{\beta}_1(p)$ ,  $\hat{\beta}_2(p)$  are point estimates from the OLS and 2SLS estimates in Table 2 (columns 1, 4, or 5).<sup>14</sup> We pool these adjusted wage observations to form a counterfactual national wage distribution, and we compare changes in inequality in the simulated distribution to those in the observed distribution.<sup>15</sup> We compute standard errors by bootstrapping the estimates within the state-year panel.<sup>16</sup>

The first column of the upper panel of Table 3 shows that between 1979 and 1989, the female 50/10 log wage ratio increased by nearly 25 log points. Applying the coefficient estimates on the effective minimum and its square obtained using the 2SLS model fit to the female wage data for 1979 through 2012 (column 2 of panel A), we calculate that had the minimum wage been constant at its real 1989 level throughout this period, female 50/10 inequality would counterfactually have risen by 11.3 log points. Using the first differences specification (column 3), we estimate a counterfactual rise of 15.1 log points. Thus, the minimum wage can explain between 40 and 55 percent of the observed rise in equality, with the complement due to a rise in underlying inequality. These are nontrivial effects, of course, and they confirm, in accordance with the visual evidence in Figure 1, that the falling minimum wage contributed meaningfully to rising female lower-tail inequality during the 1980s and early 1990s.

The OLS estimates preferred by Lee (1999) find a substantially larger role for the minimum wage, however. Using the OLS model fit to the female wage data for 1979 through 2012 (column 4 of panel A), we calculate that female 50/10 inequality would counterfactually have risen by only 2.9 log points. Applying the coefficient estimates for only the 1979–1991 period (column 5), female 50/10 inequality would have risen by 4.3 log points. Thus, consistent with Lee (1999), the OLS estimate implies that the decline in the real minimum wage can account for the bulk (all but 3 to 4 of 25 log points) of the expansion of lower tail female wage inequality in this period.

The second and third rows of Table 3 calculate the effect of the minimum wage on male and pooled gender inequality. Here, the discrepancy between IV and OLS-based counterfactuals is even more pronounced. 2SLS models indicate that the minimum wage makes a modest contribution to the rise in male wage inequality and explains only about 30 to 40 percent of the rise in pooled gender inequality. By contrast, OLS estimates imply that the minimum wage *more than* fully explains both

<sup>&</sup>lt;sup>14</sup>So, for example, taking  $\tau_0 = 1979$  and  $\tau_1 = 1989$ , and subtracting  $\Delta w_{st}^p$  from each observed wage in 1979 would adjust the 1979 distribution to its counterfactual under the realized effective minima in 1989.

<sup>&</sup>lt;sup>15</sup>We use states' observed median wages when calculating  $\tilde{m}$  rather than the national median deflated by the price index as was done by Lee (1999). This choice has no substantive effect on the results but appears most consistent with the identifying assumptions.

<sup>&</sup>lt;sup>16</sup>Our bootstrap takes states as the sampling unit, and thus we start by drawing 50 states with replacement. We next estimate the models in Tables 2A and 2B for the selected states using the percentile estimates and sample weights from the full dataset and, finally, apply the coefficients to the full CPS individual-level sample to calculate the counterfactual in equation (3). Table 3 reports the mean and standard deviation of 1,000 replications of this counterfactual exercise.

the rise in male 50/10 inequality and the rise in pooled 50/10 inequality between 1979 and 1989.

Despite their substantial discrepancy with the OLS models, the 2SLS estimates appear highly plausible. Figure 2 shows that the minimum wage was nominally *nonbinding* for males throughout the sample period, with fewer than 6 percent of all male wages falling at or below the relevant minimum wage in any given year. For the pooled gender distribution, the minimum wage had somewhat more bite, with a bit more than 8 percent of all hours paid at or below the minimum in the first few years of the sample. But this is modest relative to its position in the female distribution, where 9 to 13 percent of wages were at or below the relevant minimum in the first 5 years of the sample. Consistent with these facts, 2SLS estimates indicate that the falling minimum wage generated a sizable increase in female wage inequality, a modest increase in pooled gender inequality, and a minimal increase in male wage inequality.

Panel B of Table 3 calculates counterfactual (minimum wage constant) changes in inequality over the full sample interval of 1979–2012. In all cases, the contribution of the minimum wage to rising inequality is smaller when estimated using 2SLS in place of OLS models, and its impacts are substantial for females, modest for the pooled distribution, and negligible for males.

Figure 8 and the top panel of Figure 9 provide a visual comparison of observed and counterfactual changes in male, female, and pooled-gender wage inequality during the critical period of 1979 through 1989, during which time the minimum wage remained nominally fixed, while lower tail inequality rose rapidly for all groups. As per Lee (1999), the OLS counterfactuals depicted in these plots suggest that the minimum wage explains essentially all (or more than all) of the rise in 50/10 inequality in the female, male, and pooled-gender distributions during this period. The 2SLS counterfactuals place this contribution at a far more modest level. The counterfactual series for males, for example, is indistinguishable from the observed series, implying that the minimum wage made almost no contribution to the rise in male inequality in this period. We see a similarly pronounced discrepancy between OLS and 2SLS models in the lower panel of Figure 9, which plots observed and counterfactual wage changes in the pooled gender distribution for the full sample period of 1979 through 2012 (again holding the minimum wage at its 1988 value).<sup>17</sup>

Consistent with earlier literature, our estimates confirm that the falling minimum wage contributed to the growth of lower tail inequality growth during the 1980s. But while prior work, most notably Lee (1999), finds that the falling minimum fully accounts for this growth, this result appears strongly upward biased by violation of the identifying assumptions on which it rests. Purging this bias, we find that the minimum wage can explain at most half—and generally less than half—of the growth of lower tail inequality during the 1980s. Over the full three decades between 1979 and 2012, at least 60 percent of the growth of pooled 50/10 inequality, 50 percent of

<sup>&</sup>lt;sup>17</sup>We have repeated these counterfactuals using coefficient estimates from years 1979 through 1991 (using the additional cross-state identification offered by the increases in the federal minimum wage over this period) rather than the full 1979–2012 sample period. The counterfactual estimates from this exercise are somewhat smaller but largely consistent with the full sample, both during the critical period of 1979 through 1989 and during other intervals.

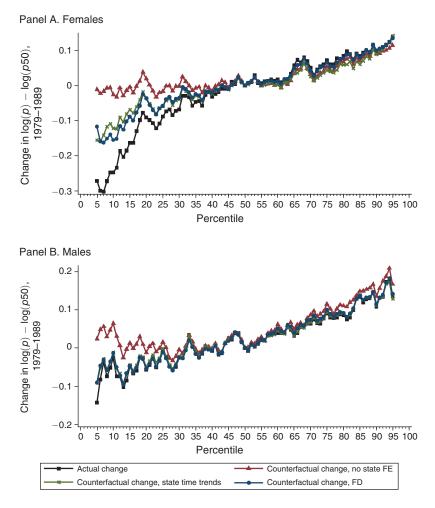


Figure 8. Actual and Counterfactual Change in log(p) - log(p50) Distribution by Sex

*Notes:* Plots represent the actual and counterfactual changes in the fifth through ninety-fifth percentiles of the male wage distribution. Counterfactual changes are calculated by adjusting the 1979 wage distributions by the value of states' effective minima in 1989 using coefficients from OLS regressions without state fixed effects (column 5 of Table 2) and 2SLS regressions with state fixed effects and time trends or first-differenced 2SLS regressions with state fixed effects (columns 3 and 4 of Table 2).

female 50/10 inequality, and 90 percent of male 50/10 inequality is due to changes in the underlying wage structure.

### IV. The Limits of Inference: Distinguishing Spillovers from Measurement Error

Federal and state minimum wages were nominally nonbinding at the tenth percentile of the wage distribution throughout most of the sample (Figure 2); in fact, there is only one 3-year interval (1979 to 1983), when more than 10 percent of hours paid were at or below the minimum wage (Table 1)—and this was only the case for females. Yet our main estimates imply that the minimum wage modestly

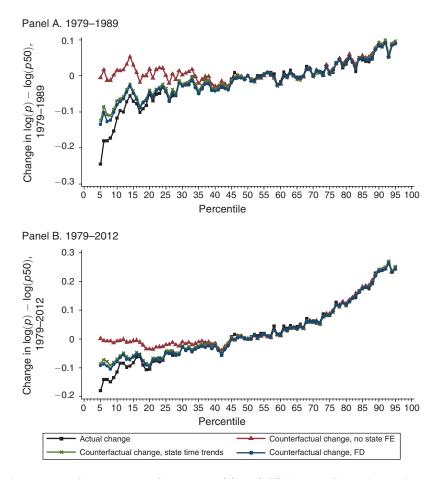


FIGURE 9. ACTUAL AND COUNTERFACTUAL CHANGE IN log(p)-log(p50) Male and Female Pooled Distribution

*Notes:* Plots represent the actual and counterfactual changes in the fifth through ninety-fifth percentiles of the male and female pooled wage distribution. Counterfactual changes in panel A are calculated by adjusting the 1979 wage distributions by the value of states' effective minima in 1989 using coefficients from OLS regressions (column 5 of Table 2) and 2SLS regressions (columns 3 and 4 of Table 2). Counterfactual changes in panel B are calculated by adjusting both the 1979 and 2012 wage distributions by the value of states' effective minima in 1989 using coefficients from OLS regressions (column 5 of Table 2) and 2SLS regressions (column 5 of Table 2).

compressed both male and pooled-gender 50/10 wage inequality during the 1980s. This implies that the minimum wage had spillover effects on percentiles above where it binds.

While these spillovers might arise from several economic forces, such as tournament wage structures or positional income concerns, a mundane but nonetheless plausible alternative explanation is measurement error. To see why, consider a case where the minimum wage is set at the fifth percentile of the latent wage distribution and has no spillover effects. However, due to misreporting, the spike in the wage distribution at the true minimum wage is surrounded by a measurement error cloud that extends from the first through the ninth percentiles. If the legislated minimum wage were to rise to the ninth percentile and measurement error were to remain constant, the rise in the minimum wage would compress the *measured* wage distribution up to the thirteenth percentile, thus reducing the measured 50/10 wage gap. This apparent spillover would be a feature of the data, but it would not be a feature of the true wage distribution.<sup>18</sup>

In this final section of the paper, we quantify the possible bias wrought by these measurement spillovers. Specifically, we ask whether we can reject the null hypothesis that the minimum wage *only* affects the earnings of those earning at or below the minimum—in which case, the apparent spillovers would be consistent with measurement error.<sup>19</sup> Since this analysis relies in part on some strong assumption, it should be thought of as an illustrative exercise designed to give some idea of magnitudes rather than a dispositive test.

# A. General Setup

We use a simple measurement error model to test the hypothesis.<sup>20</sup> Denote by  $p^*$  a percentile of the latent wage distribution (i.e., the percentile absent measurement error and without a minimum wage), and write the latent wage associated with it as  $w^*(p^*)$ . Assuming that there are only direct effects of the minimum wage (i.e., no true spillovers and no disemployment effects), then the true wage at percentile  $p^*$  will be given by

(4) 
$$w(p^*) \equiv \max[w^m, w^*(p)],$$

where  $w^*(p)$  is the true latent log wage percentile and  $w^m$  the log of the minimum wage.

Now, allow for the possibility of measurement error, so that for a worker at true wage percentile  $p^*$ , we observe:

(5) 
$$w_i = w(p^*) + \varepsilon_i,$$

where  $\varepsilon_i$  is an error term with density function  $g(\varepsilon)$ , which we assume to be independent of the true wage. We will make use of the following result proved in section B of the Appendix:

**Result 1:** Under the null hypothesis of no actual spillovers, no disemployment and measurement error independent of the true wage, the elasticity of wages at an

<sup>&</sup>lt;sup>18</sup>This argument holds in reverse for a decline in the minimum wage.

<sup>&</sup>lt;sup>19</sup>Note that we are not testing whether an apparent spillover for a particular percentile, for a particular state/year, is attributable to measurement error—we are testing whether, on average, *all* of the observed spillovers could be attributable to measurement error.

<sup>&</sup>lt;sup>20</sup>In the following discussion, it will be useful to distinguish between three distinct wage distributions: (i) the latent wage distribution, which is the wage distribution in the absence of a minimum wage and measurement error; (ii) the true wage distribution, which is the wage distribution in the absence of measurement error but allowing for minimum wage effects; and (iii) the observed wage distribution, which is the wage distribution allowing for measurement error and a minimum wage (i.e., what is measured from CPS data).

observed percentile with respect to the minimum wage is equal to the fraction of people at that observed percentile whose true wage is equal to the minimum.

The intuition for this result is straightforward: if the minimum wage rises by 10 percent, and 10 percent of workers at a given percentile are paid the minimum wage, and only these have their wage affected, the observed wage at that percentile will rise by 1 percent.

This result has a simple corollary (proved in section C of the Appendix) that we also use in the estimation below:

**Result 2:** Under the null hypothesis of no true spillovers, the elasticity of the *overall* mean log wage with respect to the minimum wage is equal to the fraction of the wage distribution that is truly paid the minimum wage—that is, the size of the true spike.

This result follows from the fact that all individuals who are truly paid the minimum wage must appear *somewhere* in the observed wage distribution. And of course, changes at any point in the distribution also change the mean. Thus, if the true spike at the minimum wage comprises 10 percent of the mass of the true wage distribution, a 10 percent rise in the minimum will increase the *true and observed* mean wage by 1 percent. Note that no distributional assumptions about measurement error are needed for either Result 1 or Result 2 other than the assumption that the measurement error distribution is independent of wage levels.

The practical value of Result 2 is that we can readily estimate the effect of changes in the minimum wage on the mean using the methods developed above. In practice, we estimate a version of equation (1), using as the dependent variable the average log real wage. On the right-hand side, we include the effective minimum wage and its square as endogenous regressors (and instrument for them using the same instruments as in the earlier analysis), state and year fixed effects, state time trends, and the log of the median (to control for shocks to the wage level of the state that are unrelated to the minimum wage, assuming that any spillovers do not extend through the median). We plot these estimates in the three panels of Figure 10, corresponding to females, males, and the pooled wage distribution. The dashed line in each panel represents the marginal effect of the minimum on the mean by year, taking the weighted average across all states for each year. Under the null hypothesis of no true spillovers, this estimate of the effect the minimum on the mean is an estimate of the size of the true spike. Under the alternative hypothesis that true spillovers are present, the marginal effect on the mean will exceed the size of the true spike. To distinguish these alternatives requires a second, independent estimate of the size of the true spike.

## **B.** Estimating Measurement Error

We develop a second estimate of the magnitude of the true spike by directly estimating a model of measurement error and using this estimate to infer the size of the spike absent this error. We exploit the fact that under the assumption of full

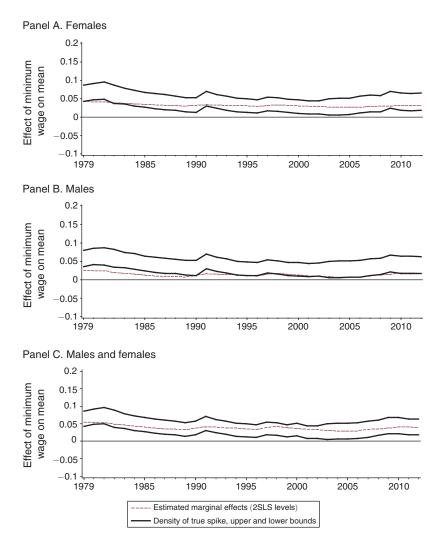


FIGURE 10. COMPARISON OF ESTIMATED EFFECTS OF THE MINIMUM WAGE ON THE MEAN AND DENSITY AT THE TRUE SPIKE

*Notes:* Mean effects represent the average marginal effects of the minimum wage (weighted across states), estimated from 2SLS regressions of  $\log(\text{mean}) - \log(p50)$  on the effective minimum and its square, year and state fixed effects, state time trends, and the log median, where the effective minimum and its square are instrumented as in the earlier analysis. The bounds for the density of the true spike are estimated from a maximum likelihood procedure described in section D of the Appendix.

# compliance with the minimum wage, all observations found below the minimum wage must be observations with measurement error.<sup>21</sup> Of course, wage observations

<sup>21</sup> There are surely some individuals who report sub-minimum wage wages and *actually* receive sub-minimum wages. The largest (but not the only) group is probably tipped workers, who in many states can legally receive a subminimum hourly wage as long as tips push their total hourly income above the minimum. For instance, in 2009, about 55 percent of those who reported their primary occupation as waiter or waitress reported an hourly wage less than the applicable minimum wage for their state, and about 17 percent of all observed subminimum wages were from waitress and waitresses. If we treat the wages of these individuals as measurement error, we will clearly

below the minimum can only provide information on individuals with *negative* measurement error, since minimum wage earners with positive measurement error must have an observed wage above the minimum. Thus, a key identifying assumption is that the measurement error is symmetric, that is  $g(\varepsilon) = g(-\varepsilon)$ .

In what follows, we use maximum likelihood to estimate the distribution of wages below the minimum and the fraction of workers at and above the minimum (for the sample of non-tipped workers as described in footnote 21). We assume that the "true" wage distribution only has a mass point at the minimum wage so that  $w^*(p^*)$ has a continuous derivative. We also assume that the measurement error distribution only has a mass point at zero so that there is a nonzero probability of observing the "true" wage. (Without this assumption, we would be unable to rationalize the existence of a spike in the observed wage distribution at the minimum wage.) Denote the probability that the wage is correctly reported as  $\gamma$ . For those who report an error-ridden wage, we will use, in a slight departure from previous notation,  $g(\varepsilon)$  to denote the distribution of the error.

With these assumptions, the size of the spike in the *observed* wage distribution at the minimum wage, which we denote by  $\tilde{p}$ , is equal to the true spike times the probability that the wage is correctly reported:

(6) 
$$\tilde{p} = \gamma \hat{p}$$

Hence, using an estimate of  $\gamma$ , we can estimate the magnitude of the true spike as  $\hat{p} \approx \tilde{p} / \tilde{\gamma}$ .<sup>22</sup>

# C. Finding: Spillovers Cannot Be Distinguished from Measurement Error

We use the following two-step procedure to estimate  $\gamma$ . Under the assumption that the latent log wage distribution is normal with mean  $\mu$  and variance  $\sigma_w^2$  and that the measurement error distribution is normal with mean zero and variance  $\sigma_{\varepsilon}^2$ , we use observations from the top part of the wage distribution—which we assume are unaffected by changes to the minimum wage—to estimate the median and variance of the observed latent wage distribution, allowing for variation across state and time.<sup>23</sup> Equipped with these estimates, we use the observed fraction of workers who are paid below the minimum for each state and year to estimate ( $\sigma_{\varepsilon}^2$ ,  $\gamma$ ) by maximum likelihood. We assume that ( $\sigma_{\varepsilon}^2$ ,  $\gamma$ ) vary over time but not across states. Exact details of our procedure can be found in section D of the Appendix. As previously noted, we perform this analysis on a sample that excludes individuals from lower-paying occupations that tend to earn tips or commission.

over-state the extent of misreporting. We circumvent this problem by conducting the measurement/spillover analysis on a sample that *excludes* employees in low-paying occupations that commonly receive tips or commission. These are: food service jobs, barbers and hairdressers, retail salespersons, and telemarketers.

<sup>&</sup>lt;sup>22</sup> The assumption on the absence of mass points in the true wage distribution and the error distribution mean that the group of workers who are not paid the minimum but, by chance, have an error that makes them appear to be paid the minimum is of measure zero and so can be ignored.

 $<sup>^{23}</sup>$ This procedure does not account for the type of measurement error induced by heaping of observations around whole numbers (e.g., \$5.50), so the estimates that follow should be treated as suggestive.

Estimates of  $\gamma$  for males, females, and the pooled sample (not shown) generally find that the probability of correct reporting is around 80 percent, and mostly varies from between 70 to 90 percent over time (though is estimated to be around 65 to 70 percent in the early 1980s for females and the pooled distribution). We combine this estimate with the observed spike to get an estimate of the "true" spike in each period, though this will be an estimate of the size of the true spike only for the estimation sample of workers in non-tipped occupations.

This leaves us in need of an estimate of the "true" spike for the tipped occupations. Given the complexity of the state laws surrounding the minimum wage for tipped employees, we do not attempt to model these subminimum wages. Rather we simply note that the spike for tipped employees must be between zero and one, and we use this observation to bound the "true" spike for the entire workforce. Because the fraction of workers in tipped occupations is small, these bounds are relatively tight.

Figure 10 compares these bounds with the earlier estimates of the "true" spike based on the elasticity of the mean with respect to the minimum in each year. Under the null hypothesis that the minimum wage has no true spillovers, the effect on the mean should equal the size of the "true" spike. And indeed, the estimated mean effect lies within the bounds of the estimated "true" spike in almost all years. We are accordingly unable to reject the hypothesis that the apparent effect of the minimum wage on percentiles above the minimum is a measurement error spillover rather than a true spillover.

This analysis rests on some strong assumptions and so should not be regarded as definitive. But if we tentatively accept this null, it has the important implication that changes in the minimum wage may only affect those who are paid the minimum and the apparent effects further up the wage distribution are the consequences of measurement error. A conclusive answer would require better wage data, ideally administrative payroll data.<sup>24</sup>

# V. Conclusion

This paper offers a reassessment of the impact of the minimum wage on the wage distribution by using a longer panel than was available to previous studies, incorporating many additional years of data and including significantly more variation in state minimum wages, and using an econometric approach that purges confounding correlations between state wage levels and wage variances that we find bias earlier estimates. Under our preferred model specification and estimation sample, we estimate that between 1979 and 1989, the decline in the real value of the minimum wage is responsible for 30 to 55 percent of the growth of lower tail inequality in the female, male, and pooled wage distributions (as measured by the differential between the log of the fiftieth and tenth percentiles). Similarly, calculations indicate that during the full sample period of 1979–2012, the declining minimum wage made

<sup>&</sup>lt;sup>24</sup> In Dube, Guiliano, and Leonard's (2015) study of the impact of wage increases on employment and quit behavior at a large retail firm, the authors note that this firm implemented sizable wage spillovers as a matter of corporate policy—with minimum wage increases automatically leading to raises among workers earning as much as 15 percent above the new minimum.

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a meaningful contribution to female inequality, a modest contribution to pooled gender inequality, and a negligible contribution to male lower tail inequality. In net, these estimates indicate a substantially smaller role for the US minimum in the rise of inequality than suggested by earlier work, which had attributed 85 percent to 110 percent of this rise to the falling minimum.

Despite these modest total effects, we estimate that the effect of the minimum wage extends further up the wage distribution than would be predicted if the minimum wage had a purely mechanical effect on wages (i.e., raising the wage of all who earned below it). One interpretation of these significant spillovers is that they represent a true wage effect for workers initially earning above the minimum. An alternative explanation is that wages for low-wage workers are mismeasured or misreported. If a significant share of minimum wage earners report wages in excess of the minimum wage, and this measurement error persists in response to changes in the minimum, then we would observe changes in percentiles above where the minimum wage directly binds in response to changes in the minimum wage. Our investigation of this hypothesis in Section IV is unable to reject the null hypothesis that all of the apparent effect of the minimum wage on percentiles above the minimum is the consequence of measurement error. Accepting this null, the implied effect of the minimum wage on the *actual* wage distribution is even smaller than the effect of the minimum wage on the *measured* wage distribution.

In net, our analysis suggests that there was a significant expansion in latent lower tail inequality over the 1980s, mirroring the expansion of inequality in the upper tail. While the minimum wage was certainly a contributing factor to widening lower tail inequality—particularly for females—it was not the primary one.

### Appendix

### A. Data Appendix

As described in Section I, our primary data comes from individual responses from the Current Population Survey Merged Outgoing Rotation Group (CPS MORG) for each year. For each year, we pool the monthly observations. We use CPS variables (e.g., weekly and hourly wages) as cleaned by the Unicon Research Corporation. Specifically, the hourly wage variable is ERNHR, the weekly wage variable is WKUSERN, ERNWKC, or ERNWK (depending on the year), and the weekly hours variable is HOURS. The respondent weight variable that we use is ERNWGT. As mentioned in the text, in our calculations we weight by a respondent's earnings weight (ERNWGT) multiplied by hours worked (HOURS), although our findings are roughly unchanged if we use ERNWGT instead.

The primary outcome we construct from CPS data is a respondent's wage. For those who report being paid by the hour we take their hourly wage to be their reported hourly wage; otherwise we calculate the hourly wage as weekly earnings divided by hours worked in the prior week. We multiply top-coded values by 1.5. When computing percentiles within a state, we Winsorize the top two percentiles of the wage distribution in each state, year, and sex grouping (male, female, or pooled) by assigning

	A. Females		B. Males		C. Males and females		
	LHS: log(min) - log(p50) (1)	LHS: square of $log(min) - log(p50)$ (2)	LHS: log(min) - log(p50) (3)	LHS: square of $\log(\min) - \log(p50)$ (4)	$\begin{array}{r} \text{LHS:}\\ \log(\min) - \\ \log(p50) \\ (5) \end{array}$	LHS: square of $log(min) - log(p50)$ (6)	
log(min)	0.30	0.61	1.79***	$-2.31^{***}$	$1.15^{***}$	-0.92	
	(0.41)	(0.58)	(0.44)	(0.80)	(0.40)	(0.61)	
Square of log(min)	$-0.34^{*}$	1.32***	$-0.82^{***}$	2.59***	$-0.61^{***}$	2.01***	
	(0.18)	(0.27)	(0.24)	(0.45)	(0.20)	(0.32)	
$\log(\min) \times avg. of state median$	0.79***	-2.76***	0.49*	-2.56***	0.61***	$-2.69^{***}$	
	(0.14)	(0.15)	(0.26)	(0.46)	(0.21)	(0.29)	
<i>F</i> -statistics <i>p</i> -value	251***	362***	370***	394***	587***	684***	
	0.00	0.00	0.00	0.00	0.00	0.00	

TABLE A1—FIRST-STAGE ESTIMATES FOR SPECIFICATIONS THAT INCLUDE	
YEAR FIXED EFFECTS AND STATE TIME TRENDS	

*Notes:* N = 1,700. Sample period is 1979–2012. For columns 1, 3, and 5, the dependent variable is the "effective minimum," that is,  $\log(\min) - \log(p50)$ . For columns 2, 4, and 6, the dependent variable is the square of the effective minimum. The RHS variables included are the three instruments: the log of the minimum wage, the square of the log min, and the interaction between the log of the min multiplied by the average median for the state over the sample. Also included in the regression are year fixed effects, and state-specific trends. The *F*-statistic for testing whether the three instruments are jointly significant, and associated *p*-value, are presented at the bottom. Standard errors clustered at the state level are in parentheses.

\*\*\* Significant at the 1 percent level.

\*\* Significant at the 5 percent level.

\* Significant at the 10 percent level.

	A. Females		B. Males		C. Males and female	
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A. Dependent variable: n	nean $\log(p60)$	$-\log(p40), 1$	1979–2012			
Mean $\log(p50)$ , 1979–2012	0.12***	e a m	0.06		0.11***	
	(0.03)		(0.05)		(0.04)	
Panel B. Dependent variable: t	rend $\log(p60)$	$-\log(p40), 1$	979–2012			
Trend log( <i>p</i> 50), 1979–2012	,	0.16**		0.03		0.14**
		(0.07)		(0.08)		(0.06)

Table A2—OLS Relationship between Mean and Trends in  $\log(p60) - \log(p40)$  and  $\log(p50)$ 

*Notes:* N = 50 (one observation per state). Observations are weighted by the average hours worked per state. Robust standard errors are in parentheses. The dependent variable in the top panel is the mean  $\log(p60) - \log(p40)$  for the state, over the 1979–2012 period. The dependent variable in the bottom panel is the linear trend in the  $\log(p60) - \log(p40)$  for the state, over the 1979–2012 period. Regressions correspond to plots in Figures 6A and 6B.

\*\*\* Significant at the 1 percent level.

\*\* Significant at the 5 percent level.

\* Significant at the 10 percent level.

the ninety-seventh percentile value to the ninety-eighth and ninety-ninth percentiles. Our sample includes individuals age 18–64, and we exclude self-employed individuals as well as respondents with wage data imputed by the BLS.

# B. Proof of Result 1

The density of wages among workers whose true percentile  $p^*$  is given by  $g(w - w(p^*))$ . The density of observed wages is simply the average of  $g(\cdot)$  across true percentiles:

(B1) 
$$f(w) = \int_0^1 g(w - w(p^*)) dp^*$$

And the cumulative density function for observed wages is given by

(B2) 
$$F(w) = \int_{-\infty}^{w} \int_{0}^{1} g(w - w(p^{*})) dp^{*} dx.$$

This can be inverted to give an implicit equation for the wage at observed percentile p, w(p):

(B3) 
$$p = \int_{-\infty}^{w(p)} \int_{0}^{1} g(w - w(p^{*})) dp^{*} dx.$$

By differentiating this expression with respect to the minimum wage, we obtain the following key result:

(B4) 
$$\left[\int_0^1 g\left[w(p) - w(p^*)\right]dp^*\right]\frac{\partial w(p)}{\partial w^m} + \int_{-\infty}^{w(p)} \int_0^1 \frac{\partial g\left[x - w(p^*)\right]}{\partial w^m}dp^*dx = 0.$$

Now we have that

(B5) 
$$\frac{\partial g[x - w(p^*)]}{\partial w^m} = -g[x - w(p^*)]\frac{\partial w(p^*)}{\partial w^m}$$

Which, from (B1), is

(B6) 
$$\frac{\partial g[x - w(p^*)]}{\partial w^m} = \frac{-g[x - w^m]}{0} \quad if \quad p^* \le \hat{p}(w^m)}{0} \quad if \quad p^* > \hat{p}(w^m)}.$$

Substituting (B1) and (B6) into (B4) and re-arranging, we have that

(B7) 
$$\frac{\partial w(p)}{\partial w^m} = \frac{\hat{p}g[w(p) - w^m]}{f[w(p)]}$$

The numerator is the fraction of workers who are really paid the minimum wage but are observed with wage w(p) because they have measurement error equal to  $[w(p) - w^m]$ . Hence, the numerator divided by the denominator is the fraction of workers observed at wage w(p) who are really paid the minimum wage.

## C. Proof of Result 2

One implication of (B7) is the following. Suppose we are interested in the effect of minimum wages on the mean log wage,  $\bar{w}(p)$ . We have that

(C1) 
$$\frac{\partial \overline{w}}{\partial w^m} = \int_0^1 \frac{\partial w(p)}{\partial w^m} dp = \int_0^1 \frac{\hat{p}g[w(p) - w^m]}{f[w(p)]} dp.$$

Change the variable of integration to w(p). We will have

(C2) 
$$dw = w'(p)dp = \frac{1}{f[w(p)]}dp$$

Hence, (C1) becomes

(C3) 
$$\frac{\partial \bar{w}}{\partial w^m} = \hat{p} \int_{-\infty}^{\infty} g[w - w^m] dw = \hat{p}$$

That is, the elasticity of average log wages with respect to the log minimum is just the size of the true spike.

# D. Estimation Procedure for the Measurement Error Model

We first derive the proportion of workers reporting subminimum wages, which we denote by Z. Assuming full compliance with the minimum wage statute, all of these subminimum wages will represent negative measurement error. We therefore have

(D1) 
$$Z = (1 - \gamma) \times \left[ 0.5\hat{p} + \int_{\hat{p}}^{1} G(w^{m} - w^{*}(p^{*})) dp^{*} \right].$$

The symmetry assumption implies that half of those at the true spike who report wages with error will report wages below the minimum, and this is reflected as the first term in the bracketed expression  $(0.5\hat{p})$ . In addition, for workers paid above the minimum, some subset will report with sufficiently negative error that their reported wage will fall below the minimum, thus also contributing to the mass below the statutory minimum. This contributor to Z is captured by the second term in the bracketed expression.

Our assumption is that the true latent log wage is normally distributed according to

(D2) 
$$w^* \sim N(\mu, \sigma_w^2).$$

To keep notation to a minimum we suppress variation across states and time, though this is incorporated into the estimation. The true wage is given by

(D3) 
$$w = \max(w^m, w^*).$$

And the observed wage is given by

(D4) 
$$v = w + D\varepsilon$$
,

where *D* is a binary variable taking the value 0 if the true wage is observed and 1 if it is not. We assume that

(D5) 
$$Pr(D = 1) = 1 - \gamma.$$

We assume that  $\varepsilon$  is normally distributed according to

(D6) 
$$\varepsilon \sim N\left(0, \frac{1-\rho^2}{\rho^2}\sigma_w^2\right).$$

We choose to parameterize the variance of the error process as proportional to the variance of the true latent wage distribution as this will be convenient later. We later show that  $\rho$  is the correlation coefficient between the true latent wage and the observed latent wage when misreported—a lower value of  $\rho$  implies more measurement error so leads to a lower correlation between the true and observed wage. We assume that  $(w^*, D, \varepsilon)$  are all mutually independent.

Our estimation procedure uses maximum likelihood to estimate the parameters of the measurement error model. There are three types of entries in the likelihood function:

- · those with an observed wage equal to the minimum wage
- those with an observed wage above the minimum wage
- those with an observed wage below the minimum wage

Let us consider the contribution to the likelihood function for these three groups in turn.

*Those Observed to Be Paid the Minimum Wage.*—With the assumptions made above, the "true" size of the spike is given by

(D7) 
$$\hat{p} = \Phi\left(\frac{w^m - \mu}{\sigma_w}\right).$$

And the size of the observed "spike" is given by

(D8) 
$$\tilde{p} = \gamma \Phi\left(\frac{w^m - \mu}{\sigma_w}\right).$$

This is the contribution to the likelihood function for those paid the minimum wage.

Those Observed to Be Paid Below the Minimum Wage.—Now let us consider the contribution to the likelihood function for those who report being paid below the minimum wage. We need to work out the density function of actual observed wages w, where  $w < w^m$ . None of those who report their correct wages (i.e., have D = 0) will report a subminimum wage, so we need only consider those who misreport their wage (i.e., those with D = 1). Some of these will have a true wage equal to the minimum and some will have a true wage above the minimum. Those who are truly paid the minimum will have measurement error equal to  $(w - w^m)$ , so, using (D6) and (D7), the contribution to the likelihood function will be

(D9) 
$$(1-\gamma) \frac{\rho}{\sigma_w \sqrt{1-\rho^2}} \phi\left(\frac{\rho(w-w^m)}{\sigma_w \sqrt{1-\rho^2}}\right) \Phi\left(\frac{w^m-\mu}{\sigma_w}\right).$$

Now, consider those whose true wage is above the minimum but have a measurement error that pushes their observed wage below the minimum. For this group, their observed wage is below the minimum and their latent wage is above the minimum. The fraction of those who misreport who are in this category is, with some abuse of the concept of probability,

(D10) 
$$\Pr(v = w, w^* > w^m).$$

Define

(D11) 
$$v^* = w^* + \varepsilon,$$

which is what the observed wage would be if there was no minimum wage and they misreport i.e., D = 1.

From (D2) and (D4),

(D12) 
$$\begin{pmatrix} v^* \\ w^* \end{pmatrix} \sim N \left[ \begin{pmatrix} \mu \\ \mu \end{pmatrix}, \begin{pmatrix} \sigma_w^2 + \sigma_\varepsilon^2 & \sigma_w^2 \\ \sigma_w^2 & \sigma_w^2 \end{pmatrix} \right] \\ \begin{pmatrix} v^* \\ w^* \end{pmatrix} \sim N \left[ \begin{pmatrix} \mu \\ \mu \end{pmatrix}, \sigma_w^2 \begin{pmatrix} 1/\rho^2 & 1 \\ 1 & 1 \end{pmatrix} \right] .$$

This implies the following:

(D13) 
$$\binom{v^*}{w^* - \rho^2 v^*} \sim N \left[ \binom{\mu}{\mu(1-\rho^2)}, \sigma_w^2 \binom{1/\rho^2}{1}, \frac{1}{1-\rho^2} \right],$$

which is an orthogonalization that will be convenient.

Now for those paid above the minimum but whose wage is misreported, the true wage is  $w^*$  and the observed wage is  $v^*$ . So,

(D14) 
$$\Pr(v = w, w^* > w^m) = \Pr(v^* = w, w^* > w^m)$$
  
 $= \Pr(v^* = w, w^* - \rho^2 v^* > w^m - \rho^2 w)$   
 $= \Pr(v^* = w)\Pr(w^* - \rho^2 v^* > w^m - \rho^2 w)$   
 $= \frac{\rho}{\sigma_w} \phi \left(\frac{\rho(w - \mu)}{\sigma_w}\right) \left[1 - \Phi \left(\frac{\left[(w^m - \mu) - \rho^2(w - \mu)\right]}{\sigma_w \sqrt{1 - \rho^2}}\right)\right],$ 

where the third line uses the independence of (D13).

Putting together (D9) and (D14), the fraction of the population observed to be paid at a wage w below the minimum is given by

(D15) 
$$L = (1 - \gamma) \cdot \left[ \left[ \frac{\rho}{\sigma_w \sqrt{1 - \rho^2}} \phi \left( \frac{\rho(w - w^m)}{\sigma_w \sqrt{1 - \rho^2}} \right) \right] \Phi \left( \frac{w^m - \mu}{\sigma_w} \right) + \frac{\rho}{\sigma_w} \phi \left( \frac{\rho(w - \mu)}{\sigma_w} \right) \left[ 1 - \Phi \left( \frac{\left[ (w^m - \mu) - \rho^2(w - \mu) \right]}{\sigma_w \sqrt{1 - \rho^2}} \right) \right] \right].$$

*Those Observed to Be Paid above the Minimum Wage.*—Now let us consider the fraction observed above the minimum wage. These workers might be one of three types:

- Those really paid the minimum wage who misreport a wage above the minimum.
- Those really paid above the minimum wage who do not misreport.
- Those really paid above the minimum wage, who do misreport, but do not report a subminimum wage.

For those who are truly paid the minimum wage and have a misreported wage, a half will be above, so the fraction of those who report a wage above the minimum is:

(D16) 
$$\frac{1}{2}(1-\gamma)\Phi\left(\frac{w^m-\mu}{\sigma_w}\right).$$

Those who do not misreport and truly have a wage above the minimum will be

(D17) 
$$\gamma \left(1 - \Phi\left(\frac{w^m - \mu}{\sigma_w}\right)\right).$$

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Now, consider those whose true wage is above the minimum but who misreport. For this group we know their observed latent wage is above the minimum and that their true latent wage is above the minimum. The fraction who are in this category is

(D18) 
$$\Pr(w^* > w^m, v^* > w^m).$$

Now,

(D19) 
$$\Pr(w^* > w^m, v^* > w^m)$$
  
= 1 -  $\Pr(w^* < w^m) - \Pr(v^* < w^m) + \Pr(w^* < w^m, v^* < w^m)$   
= 1 -  $\Phi\left(\frac{w^m - \mu}{\sigma_w}\right) - \Phi\left(\frac{\rho(w^m - \mu)}{\sigma_w}\right)$   
+  $\Phi\left(\frac{w^m - \mu}{\sigma_w}, \frac{\rho(w^m - \mu)}{\sigma_w}, \rho\right),$ 

where the final term is the cumulative density function of the bivariate normal distribution. Putting together (D16), (D17), and (D19), the fraction of the population observed to be paid above the minimum is given by

$$(D20) \quad (1-\gamma) \cdot \left[\frac{1}{2}\Phi\left(\frac{w^m - \mu}{\sigma_w}\right) + 1 - \Phi\left(\frac{w^m - \mu}{\sigma_w}\right) - \Phi\left(\frac{\rho(w^m - \mu)}{\sigma_w}\right) + \Phi\left(\frac{w^m - \mu}{\sigma_w}, \frac{\rho(w^m - \mu)}{\sigma_w}, \rho\right)\right] + \gamma\left(1 - \Phi\left(\frac{w^m - \mu}{\sigma_w}\right)\right).$$

This is the contribution to the likelihood function for those paid above the minimum.

There are three parameters in this model  $(\sigma_w, \gamma, \rho)$ . These parameters may vary with state or time. In the paper we have already documented how the variance in observed wages varies across state and time, so it is important to allow for this variation. Nevertheless, for ease of computation our estimates assume that  $(\gamma, \rho)$  only vary across time and are constant across states.

We estimate the parameters in two steps. We first use the information on the shape of the wage distribution above the median to obtain an estimate of the median and variance of the latent observed wage distribution for each state/year.<sup>25</sup> This

<sup>&</sup>lt;sup>25</sup> To estimate this, we assume that the latent wage distribution for each state/year is log normal and can be summarized by its median and variance, so that  $w_{st}^*(p) = \mu_{st} + \sigma_{st}F^{-1}(p)$ , where  $\mu_{st}$  is the log median and  $\sigma_{st}$  is the variance. We then assume that the minimum wage has no effect on the shape of the wage distribution above the median, so that upper-tail percentiles are estimates of the latent distribution. To estimate  $\mu_{st}$  and  $\sigma_{st}$ , we pool the fiftieth through seventy-fifth log wage percentiles, regress the log value of the percentile on the inverse CDF of the

assumes that the latent distribution above the median is unaffected by the minimum wage. It also assumes that latent observed wage distribution is normal, which is not consistent with our measurement error model (recall our model assumes that the latent observed wage distribution is a mixture of two distributions, i.e., those who report their wage correctly and those who do not). This does not affect the estimate of the median but does affect the interpretation of the variance. Here we show how to map between this estimate of the variance and the parameters of our measurement error model.

Our measurement error model implies that the log wage at percentile p, w(p) satisfies the following equation:

(D21) 
$$p = \gamma \Phi\left(\frac{w(p) - \mu}{\sigma_w}\right) + (1 - \gamma) \Phi\left(\frac{\rho(w(p) - \mu)}{\sigma_w}\right).$$

Differentiating this, we obtain the following equation for w'(p):

(D22) 
$$1 = \left[\gamma\left(\frac{1}{\sigma_w}\right)\phi\left(\frac{w(p)-\mu}{\sigma_w}\right) + (1-\gamma)\left(\frac{\rho}{\sigma_w}\right)\phi\left(\frac{\rho(w(p)-\mu)}{\sigma_w}\right)\right]w'(p).$$

Our estimated model, which assumes a single normal distribution instead uses the equation

(D23) 
$$1 = \left(\frac{1}{\sigma}\right)\phi\left(\frac{w(p) - \mu}{\sigma}\right)w'(p).$$

And our estimation procedure provides an estimate of  $\sigma$ . Equating the two terms we have the following expression for the relationship between  $\sigma_w$  and  $\sigma$ :

(D24) 
$$\sigma_{w} = \sigma \frac{\left[\gamma \phi \left(\frac{w(p) - \mu}{\sigma_{w}}\right) + \rho(1 - \gamma) \phi \left(\frac{\rho(w(p) - \mu)}{\sigma_{w}}\right)\right]}{\phi \left(\frac{w(p) - \mu}{\sigma}\right)}.$$

If the values of the density functions are similar, then one can approximate this relationship by

(D25) 
$$\sigma_w = \sigma[\gamma + \rho(1 - \gamma)].$$

standard normal distribution, and allowing the intercept ( $\mu_{st}$ ) and coefficient ( $\sigma_{st}$ ) to vary by state and year (and including state-specific time trends in both the intercept and coefficient). Since we assume the wage distribution is unaffected by the minimum wage between the fiftieth and seventy-fifth percentiles, the distribution between the fiftieth and seventy-fifth percentiles, combined with our parametric assumptions, allows us to infer the shape of the wage distribution for lower percentiles. We have experimented with the percentiles used to estimate the latent wage distribution and the results are not very sensitive to the choices made.

This is an approximation, but simulation of the model for the parameters we estimate suggest it is a good approximation. This implies that we can write all elements of the likelihood function as functions of

(D26) 
$$z_{st} = \left(\frac{w_{st}^m - \mu_{st}}{\sigma_{st}}\right).$$

That is,  $z_{st}$  is the standardized deviation of the minimum from the median using the estimate of the observed variance obtained as described above from step 1 of the estimation procedure.

In the second step, we estimate the parameters  $(\rho, \gamma)$  using maximum-likelihood, with the elements of the likelihood function described above.

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